Otto-von-Guericke-University Magdeburg
Faculty of Electrical Engineering and Information Technology
Institute for Automation Engineering

Automation Lab

Controller Design/Linear Feedback Control

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by: Md Yasin Arafat

Contents

1 Preparation

In this project, the transfer function obtained from TCL2 (Heater 2 - Sensor 2) is used to determine the parameters of a PI-controller. Afterwards, the controller will be tested through MATLAB/Simulink both for simulation and TCL device.

$$G(s) = \frac{0.3625}{(1 + 15.78s) \cdot (1 + 157.8s)} \tag{1.1}$$

The Bode plots are used in order to analyze the frequency response of the system transfer function. The Bode plot consists of the magnitude and the phase plot. Both are obtained from the transfer function shown in Eq. (1.1) for $G(s = jw)$.

$$G(jw) = \frac{0.3625}{(1 + 15.78jw) \cdot (1 + 157.8jw)} \tag{1.2}$$

Afterwards, the equation can be rewritten for magnitude and phase in the following form:

$$M(w) = \frac{29}{80 \left| -2490w^2 + \frac{8679}{50}wj + 1 \right|} \tag{1.3}$$

$$\phi(w) = \angle \left(\mathrm{Im}\left(\frac{1}{-2490\,w^2 + \frac{8679}{50}wj + 1} \right) j + \mathrm{Re}\left(\frac{1}{-2490\,w^2 + \frac{8679}{50}wj + 1} \right) \right) \tag{1.4}$$

The magnitude plot is then converted into dB scale through the formula $M_{dB}(w) = 20\log(M(w))$. The Bode diagram of the TCL2 transfer function is shown in Fig. 1.1.

1.1 Controller Design in Frequency Domain

In this section, we use gain adjustment and loop shaping to design a PI controller in the frequency domain. The ideal PI controller equation in the time domain is shown below:

$$g(t) = K_p \cdot e(t) + K_i \int e(t)dt \tag{1.5}$$

where $e(t)$ is the difference between the reference signal and the output, and K_i, K_p are the integral and proportional gains. In the Laplace domain the equation is rewritten as below:

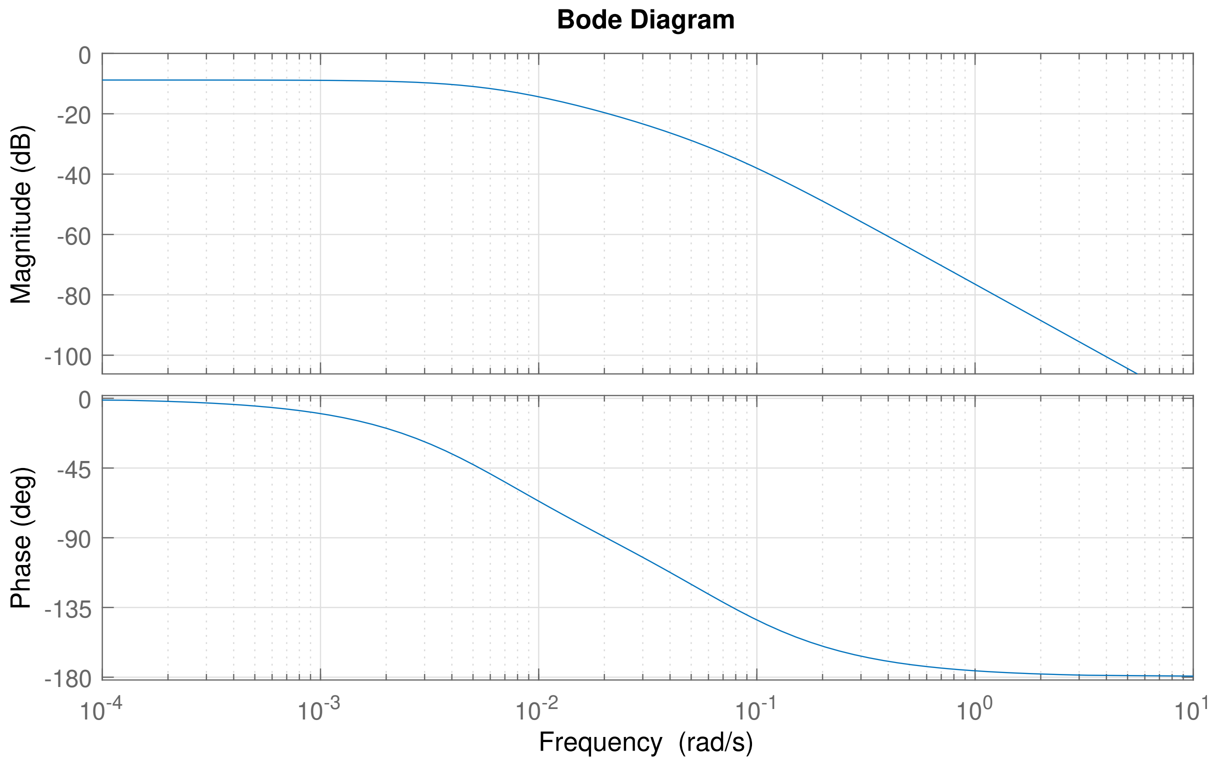

Figure 1.1: Bode diagram of G(s)

$$G_{PI}(s) = K_p + \frac{K_i}{s} \tag{1.6}$$

1.1.1 Gain adjustment

In this section, a PI controller is designed in order to obtain the given values of phase margin $\phi_M = 30°$ and $\phi_M = 70°$.

- Firstly the PI controller equation is re-written in the following form:

$$G_{PI}(s) = \frac{K_i}{s}(1 + s\frac{K_p}{K_i}) \tag{1.7}$$

- $\frac{K_p}{K_i}$ is set to $\frac{K_p}{K_i} = 157.8$ in order to compensate the largest time constant of the process.

- Resulting open-loop transfer function

$$G_0(s) = \frac{K_i}{s}\frac{0.3625}{(1 + 15.78s)} \tag{1.8}$$

- Resulting Bode-diagram of the open-loop for $K_i = 1$ is shown in Fig. 1.2.

- Gain has to be adjusted by -4.36 dB to obtain desired $\phi_M = 30°$ and -23.39 dB to obtain $\phi_M = 70°$.

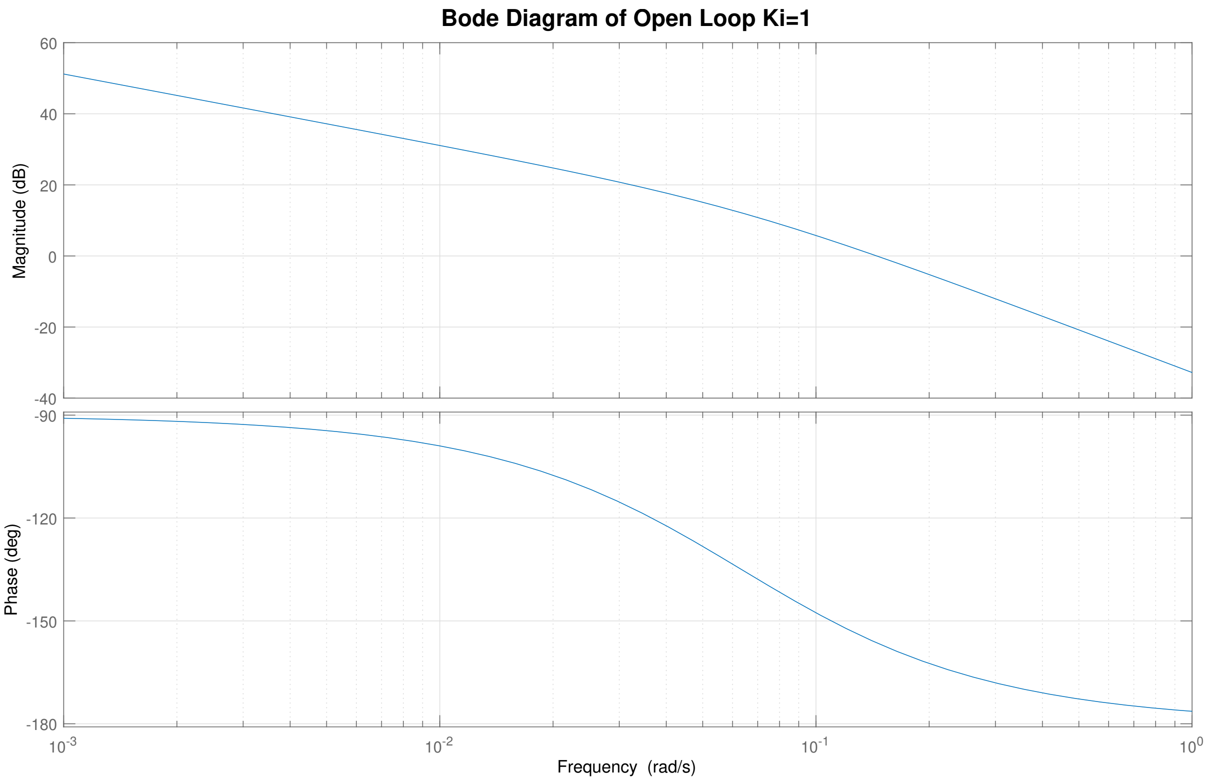

Figure 1.2: Bode diagram of open loop with $K_i = 1$

- Convert from dB to linear scale $K_i = 10^{\frac{-4.36}{20}} = 0.6056$ for $\phi_M = 30°$ and $K_i = 10^{\frac{-23.39}{20}} = 0.0677$ for $\phi_M = 70°$.
- Resulting open-loop transfer function for $\phi_M = 30°$

$$G_0(s) = \frac{0.2195}{s(1 + 15.78s)} \tag{1.9}$$

- Resulting open-loop transfer function for $\phi_M = 70°$

$$G_0(s) = \frac{0.0245}{s(1 + 15.78s)} \tag{1.10}$$

- Finally for $\phi_M = 30°$ we obtain $K_i = 0.6056$ and $\frac{K_p}{K_i} = 157.8, K_p = 95.5637$ and for $\phi_M = 70°$ $K_i = 0.0677$ and $\frac{K_p}{K_i} = 157.8, K_p = 10.6830$. Therefore, the transfer function of the PI controller is given by the following equations:

$$G_{PI30}(s) = 95.5637 + \frac{0.6056}{s} \tag{1.11}$$

$$G_{PI70}(s) = 10.6830 + \frac{0.0677}{s} \tag{1.12}$$

The Bode diagram of the open-loop transfer function is plotted in Fig. 1.3.

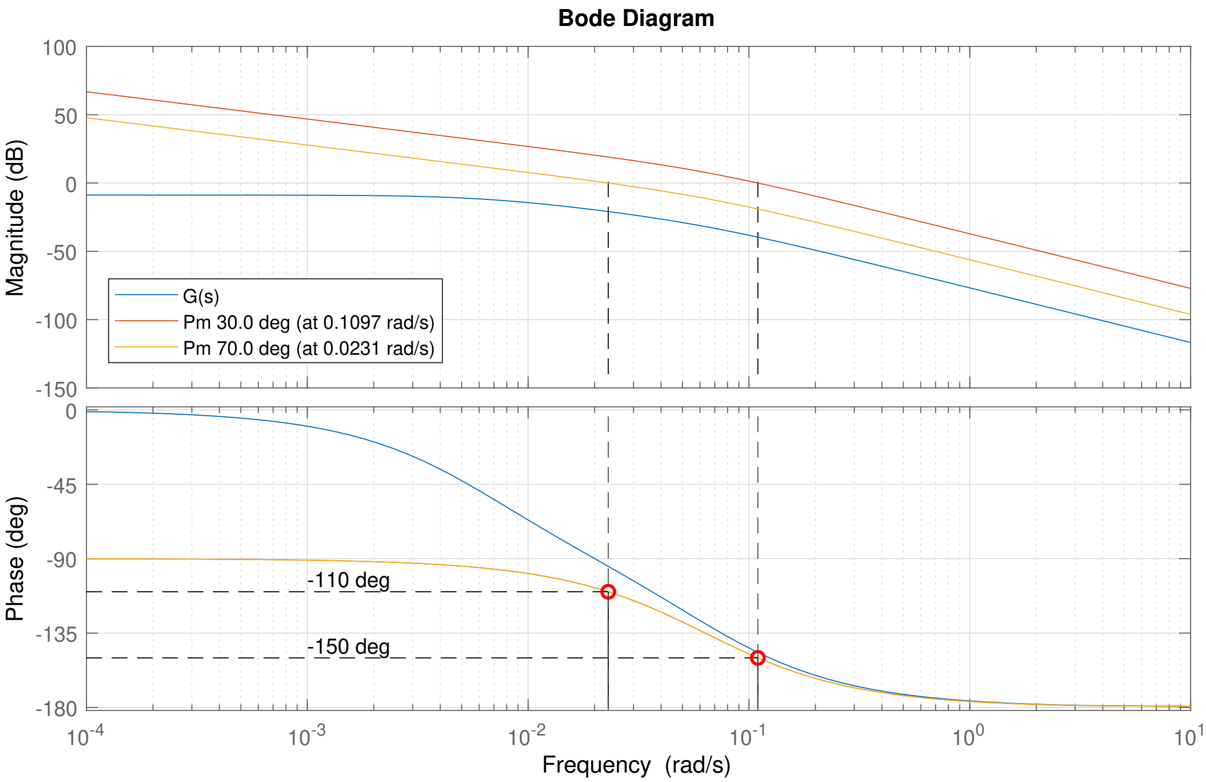

Figure 1.3: Bode diagram of open loop

1.1.2 Increasing the crossover frequency with constant phase margin

In order to increase the crossover frequency, we need to lower the K_i coefficient. A simple way to explain the reason behind this idea is by observing the phase graph of G(s) and the phase graph of a PI controller Fig. 1.4. PI element substantially decreases the phase in the interval from $w = 0$ up to approximately $w = 100 \cdot \frac{K_i}{K_p}$.

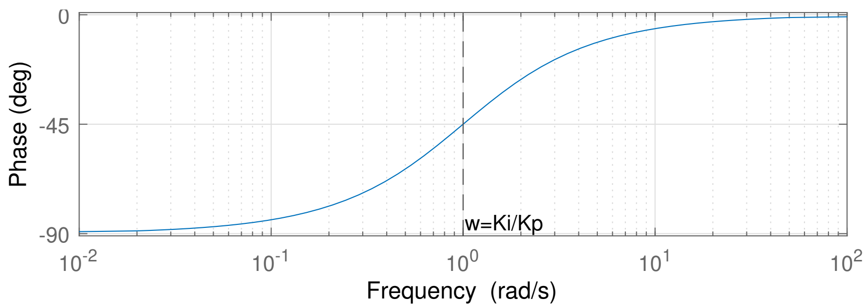

Figure 1.4: PI controller phase graph

In order keep the phase margin at the constant value of 30° and at the same time to increase the frequency of this point (crossover frequency) we need to add phase for high frequencies, but as we are using a PI element adding phase as seen from Fig. 1.4 is not possible. The only thing we can do is shifting the $\frac{K_i}{K_p}$ (shown in Fig. 1.4) at the left of the graph, in such a way that the PI element does not substantially decrease the phase of the open-loop transfer function at the point when $\phi(w) = 30° - 180°$.

In practice, for $\phi_M = 30°$ and $\phi_M = 70°$ we can obtain the PI parameters in the following steps:

- For $\phi_M = 30°$ find w when $\phi(w) = -150°$ from bode plot of G(s), $w = 0.124 rad/s$ and for $\phi_M = 70°$ find w when $\phi(w) = -110°$, $w = 0.0364 rad/s$

- Calculate $\frac{K_i}{K_p}$ factor. For $\phi_M = 30°$,

$$\frac{K_i}{K_p} = \frac{0.124}{100} = 1.24 \cdot 10^{-3} \tag{1.13}$$

  and for $\phi_M = 70°$,

$$\frac{K_i}{K_p} = \frac{0.0364}{100} = 3.64 \cdot 10^{-4} \tag{1.14}$$

- Calculate open-loop transfer function for $\phi_M = 30°$

$$G_0(s) = \frac{K_i}{s} \cdot (1 + s\frac{K_p}{K_i}) \cdot G(s) \tag{1.15}$$

$$G_0(s) = \frac{K_i}{s} \cdot (1 + \frac{s}{1.24 \cdot 10^{-3}}) \cdot \frac{0.3625}{(15.7 \cdot s + 1)(157.8 \cdot s + 1)} \tag{1.16}$$

  and for $\phi_M = 70°$

$$G_0(s) = \frac{K_i}{s} \cdot (1 + \frac{s}{3.64 \cdot 10^{-4}}) \cdot \frac{0.3625}{(15.7 \cdot s + 1)(157.8 \cdot s + 1)} \tag{1.17}$$

- Adjust gain of $G_0(s)$ to obtain the desired ϕ_M. For $\phi_M = 30°$ adjust -16.99dB and for $\phi_M = 70°$ adjust -33.40dB

- Convert from dB to linear scale. For $\phi_M = 30°$, $K_i = 10^{\frac{-16.99}{20}} = 0.1415$ and for $\phi_M = 70°$, $K_i = 10^{\frac{-43.59}{20}} = 0.0066$

- Finally for $\phi_M = 30°$ we have $K_i = 0.1415$, $\frac{K_i}{K_p} = 1.24 \cdot 10^{-3}$, $K_p = 114.12$

$$G_{PI}(s) = 141.12 + \frac{0.1415}{s} \tag{1.18}$$

  and for $\phi_M = 70°$ we have $K_i = 0.0066$, $\frac{K_i}{K_p} = 3.64 \cdot 10^{-4}$, $K_p = 18.17$

$$G_{PI}(s) = 18.17 + \frac{0.0066}{s} \tag{1.19}$$

By decreasing the integrator coefficient K_i, we obtain a faster step response in terms of rise time. However, this comes with the drawback of slower recovery from disturbances.

The highest possible crossover frequency is the one that can be achieved when the integrator element is infinitesimal, thus we have only the P part of the PI controller. A comparison of the speed of the step response based on settling time ($T_{s\%5}$) with a threshold of 5%, peak time (T_p) and rise time from 10%-90% (T_r) is shown in Table 1.1.

Table 1.1: PI comparison

PI coefficients	ϕ_M	w_{ϕ_M}	$T_{s\%5}$	T_p	T_r
$K_i = 0.6056, K_p = 95.5637$	30°	0.1097	90.3	27.6	10.9
$K_i = 0.1415, K_p = 141.12$	30°	0.121	78.6	24.2	10.1
$K_i = 0, K_p = 118.73$	30°	0.1240	80.5	25.1	9.6
$K_i = 0.0677, K_p = 10.6830$	70°	0.0231	86.3	133.7	62.9
$K_i = 0.0066, K_p = 18.17$	70°	0.0358	2912.9	10976	46.6
$K_i = 0, K_p = 18.55$	70°	0.0364	92.5	72.7	34.5

The highest crossover frequency for a fixed given ϕ_M is reached when $K_i = 0$, and as shown in Table 1.1 the rise time is shorter in that case. However due to large steady-state error in the case when only P controller is used with $\phi_M = 70°$ by only using a very small integral value (such as $K_i < 0.05$) causes the system to reach its steady-state value in a very large amount of time.

1.2 PID Tuning Methods

Firstly the inflectional tangent is drawn and the time constants determined from the tangent are obtained as shown in Fig. 1.5.

For the analyzed step response $T_u = 9.87$ and $T_a = 203.13$. In the next tasks, the following PI controller formula is used:

$$G_{PI}(s) = K_p(1 + \frac{1}{T_i\ s}) \tag{1.20}$$

1.2.1 Ziegler tuning rules

PI controller parameters are obtained from Ziegler tuning rules. K_p and T_i are calculated from the following equations [1]

$$K_p = 0.9\frac{T_a}{K_s T_u} \tag{1.21}$$

$$T_i = \frac{T_u}{0.3} \tag{1.22}$$

For the Eq. (1.1) transfer function $K_p = 0.9\frac{203.13}{0.3625 \cdot 9.87} = 51.0966$ and $T_i = \frac{9.87}{0.3} = 32.9$.

Finally, the PI controller transfer function is obtained:

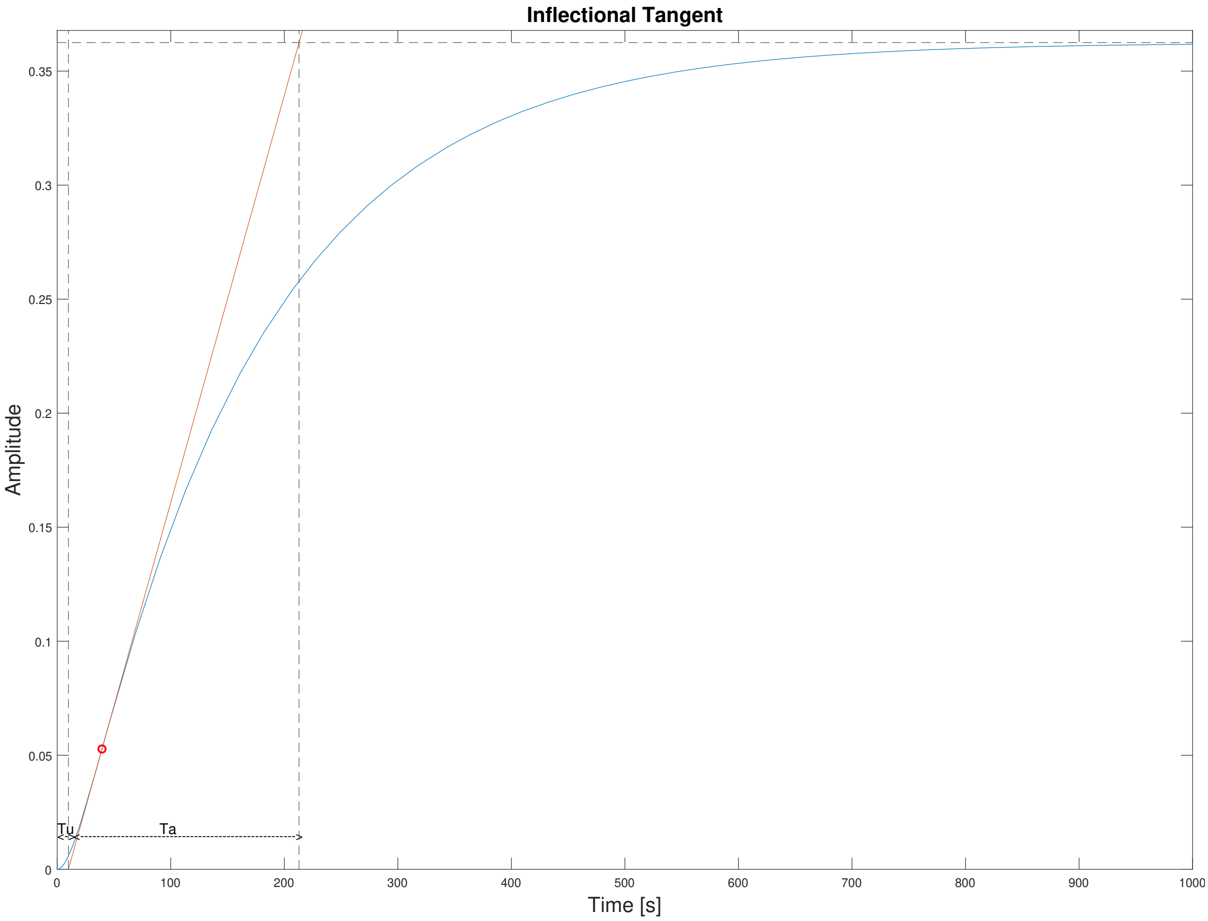

Figure 1.5: Inflectional Tangent

$$G_{PI}(s) = 51.0966(1 + \frac{1}{32.9s}) \tag{1.23}$$

1.2.2 T_Σ rules

PI controller is tuned according to T_Σ rules and K_p and T_i parameters are calculated from the following equations [1]

$$K_p = \frac{1}{K_s} \tag{1.24}$$

$$T_i = 0.7(T_u + T_a) \tag{1.25}$$

Finally we get $K_p = \frac{1}{0.3625} = 2.7586$ and $T_i = 0.7(9.87 + 203.13) = 149.1$
And the following controller equation is obtained:

$$G_{PI}(s) = 2.7586(1 + \frac{1}{149.1 \cdot s}) \tag{1.26}$$

1.2.3 Oppelt tuning rules

PI controller is tuned according to Oppelt method and K_p and T_i parameters are calculated from the following equations [1]

$$K_p = 0.8 \cdot \frac{T_a}{K_s T_u} \tag{1.27}$$

$$T_i = 3T_u \tag{1.28}$$

Finally, we get $K_p = 0.8 \cdot \frac{203.13}{0.3625 \cdot 9.87} = 45.4191$ and $T_i = 3 \cdot 9.87 = 29.61$.
And the following controller equation is obtained:

$$G_{PI}(s) = 45.4191(1 + \frac{1}{29.61 \cdot s}) \tag{1.29}$$

1.3 Simulation

PI controllers determined above are implemented in Matlab/SIMULINK with the SIMULINK diagram shown in Fig. 1.6

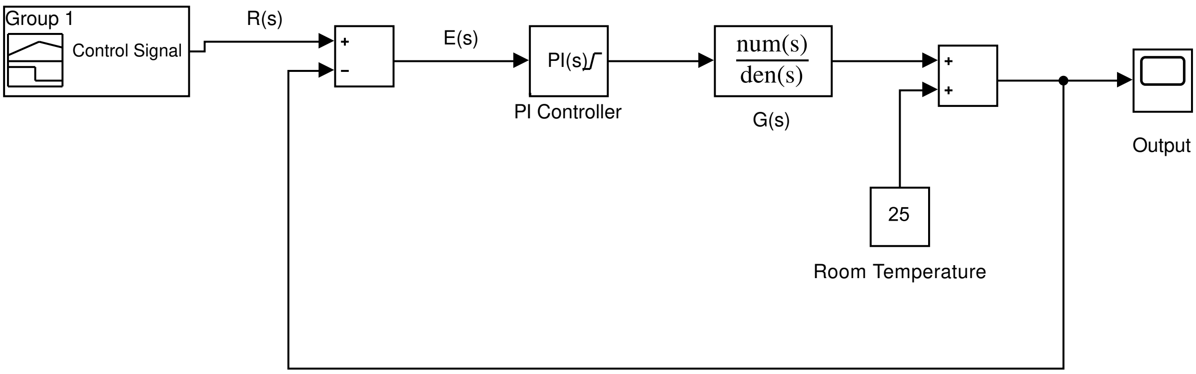

Figure 1.6: Closed loop

The performance of the PI controller is evaluated through 3 indicators: Integral Absolute Error (IAE), Integral Squared Error (ISE), and Integral Time-weighted Absolute Error (ITAE) which are defined as:

$$\begin{aligned} ISE &= \int e(t)^2 dt \\ IAE &= \int |e(t)| dt \\ ITAE &= \int t|e(t)| dt \end{aligned} \tag{1.30}$$

where $e(t)$ signal is obtained from the difference between the control signal and the output of the system. Smaller value of these indicators translates into better performance of the system.

- Integral Absolute Error (IAE), is the integral of the absolute value of the error function, and as such gives an evaluation of the overall error during all the time.
- Integral Squared Error (ISE), this indicator penalizes large error values while tolerates small error values.
- Integral Time-weighted Absolute Error (ITAE), this indicator penalizes errors which happen latter in time while tolerates error values which are at the beginning of the response.

The performance of the PI controllers obtained above is evaluated through these 3 indicators for few different set points, meanwhile, the anti-windup method of clamping was used for all PI controllers, and the over-saturation limits were set to [0% 100%] in SIMULINK.

Table 1.2: Performance comparison with set point 35[°C]

Set point 35[°C]	ISE	IAE	ITAE
PI Ziegler	2761	407.1	$1.28 \cdot 10^4$
PI T_Σ	8461	1494	$1.889 \cdot 10^5$
PI Oppelt	2760	408	$1.293 \cdot 10^4$

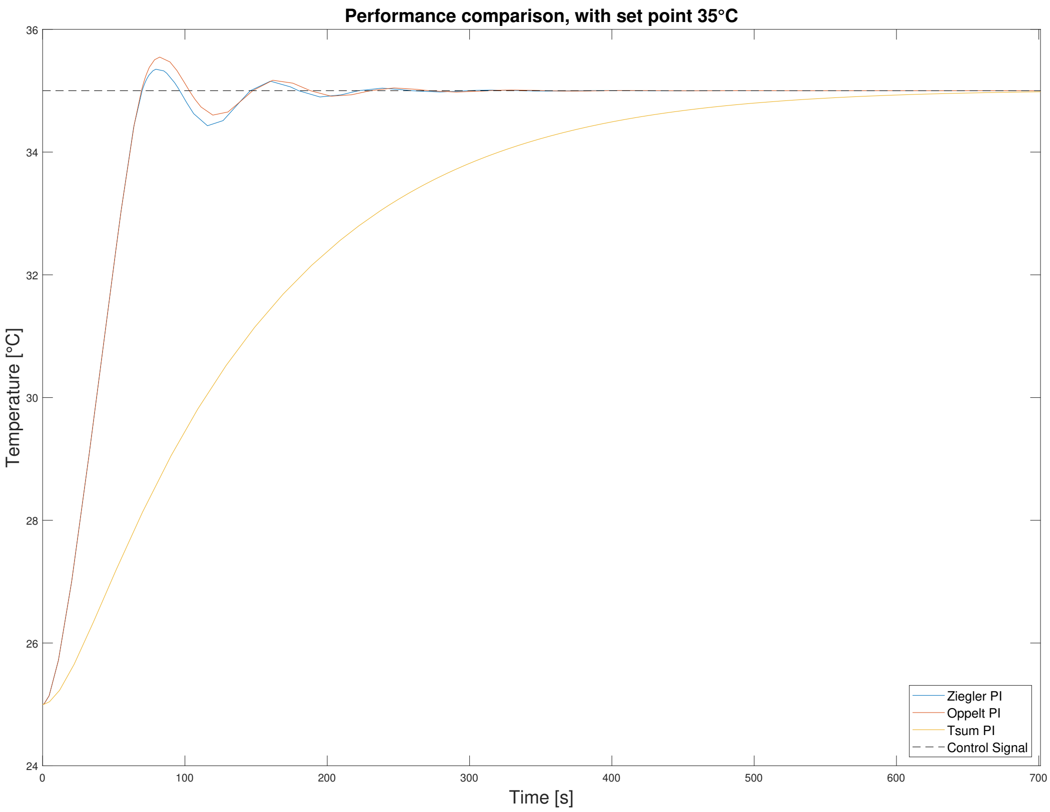

Figure 1.7: Performance comparison with set point 35[°C]

Table 1.3: Performance comparison with set point 45[°C]

Set point 45[°C]	ISE	IAE	ITAE
PI Ziegler	$1.918 \cdot 10^4$	1420	$6.777 \cdot 10^4$
PI T_Σ	$3.384 \cdot 10^4$	2988	$3.778 \cdot 10^5$
PI Oppelt	$1.918 \cdot 10^4$	1418	$6.76 \cdot 10^4$

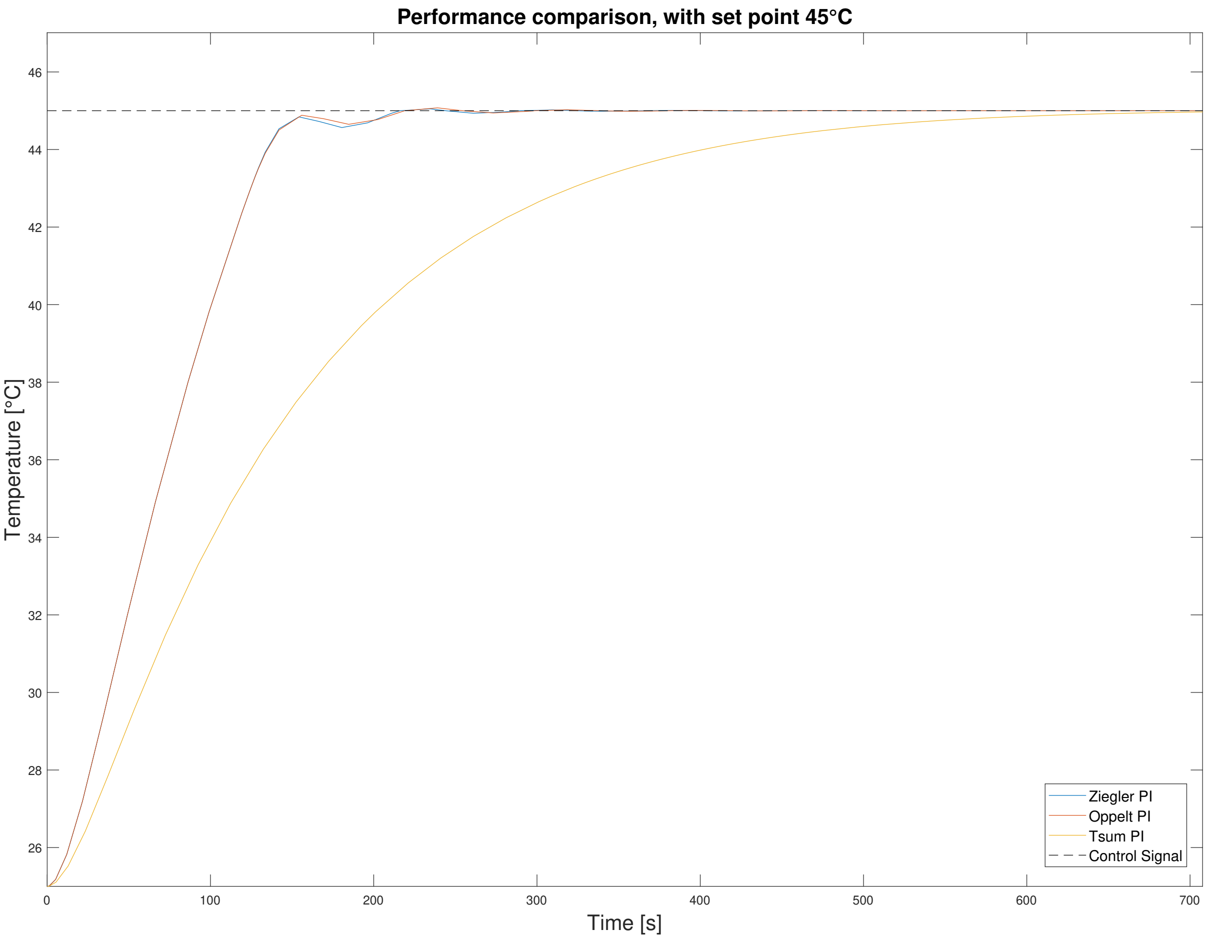

Figure 1.8: Performance comparison with set point 45[°C]

2 Practical Part

2.1 Controller Design

In this section, a PI controller is designed with the experimental data from the TCL. For this purpose, Ziegler, Oppelt and $T_{\sum}$ methods were used to calculate the controller parameters.

Step response data is taken from the TCL device from 3 different runs. In each run, the room temperature was kept a constant value. Afterwards, the step response data was filtered with a low pass filter in order to remove the noise, and by applying the inflectional tangent method the T_u and T_a time constants were obtained as well as the system gain K_s. As expected T_u, T_a and K_s values shown in Table 2.1 are similar to those obtained earlier from the step response of $G(s)$ given on Eq. (1.1). Therefore, the controllers which were designed earlier can be applied to control the TCL device as the transfer function $G(s)$ accurately describes the behaviour of the system.

Table 2.1: Time constants calculated from the TCL step response

N	T_a	T_u	K_s
1	200.81	10.18	0.364
2	202.45	10.54	0.365
3	203.05	9.94	0.361
Average	202.10	10.22	0.363

2.2 Controller Performance

PI controllers designed earlier from Ziegler, Oppelt and $T_{\sum}$ methods were implemented in SIMULINK with the Arduino TCL device. The anti-windup method of clamping was selected with a saturation limit [0% 100%].

2.2.1 Disturbance Rejection

The command value of $45°C$ was given to the system. Heater 1 was used in order to add disturbance to the system. During all the experiments Heater 1 was set to 50% by changing the state on/off every 25 seconds. Each experiment was executed in 1000 seconds. System response is shown in Fig. 2.1. Performance indicators for each run are shown in Table 2.2.

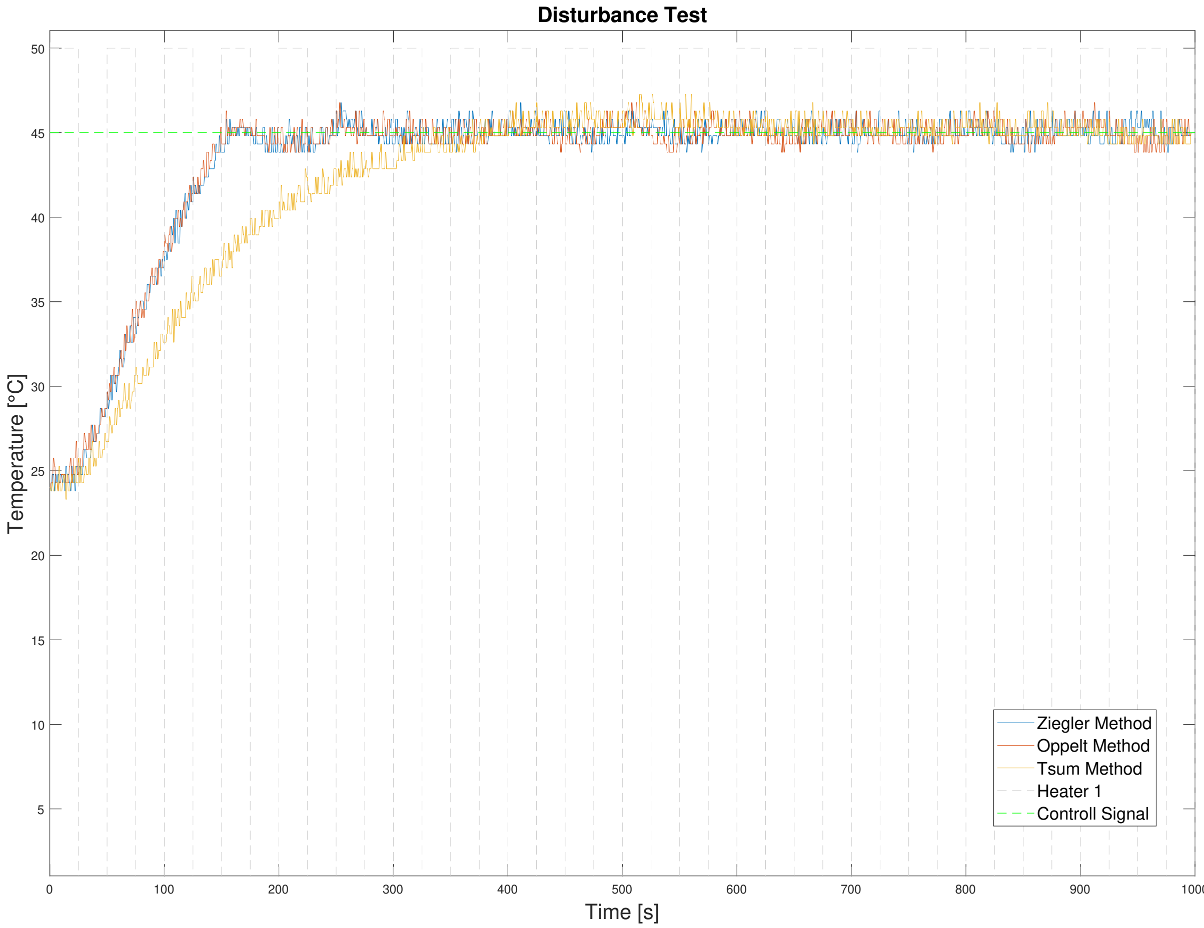

Figure 2.1: Disturbance test with set point 45[$°C$]

Table 2.2: Disturbance rejection comparison

Set point 45[$°C$]	ISE	IAE	ITAE
PI Ziegler	$2.705 \cdot 10^4$	2149	$3.156 \cdot 10^5$
PI T_Σ	$3.973 \cdot 10^4$	3243	$5.095 \cdot 10^5$
PI Oppelt	$2.616 \cdot 10^4$	2119	$3.3 \cdot 10^5$

2.2.2 Set Point Change

The command values of 35$°C$, 45$°C$ and were given to the system. Each experiment was executed in 1000 seconds. System response when set point equals 35$°C$ is shown in Fig. 2.2, this plot was filtered with a low-pass filter with a pass-band frequency of 0.05[Hz]. Performance indicators for each run are shown in Table 2.3.

Table 2.3: Set point change is 35[$°C$]

Set point 35[$°C$]	ISE	IAE	ITAE
PI Ziegler	5631	914.8	$1.563 \cdot 10^5$
PI T_Σ	$1.074 \cdot 10^4$	1834	$3.098 \cdot 10^5$
PI Oppelt	4925	890	$1.573 \cdot 10^5$

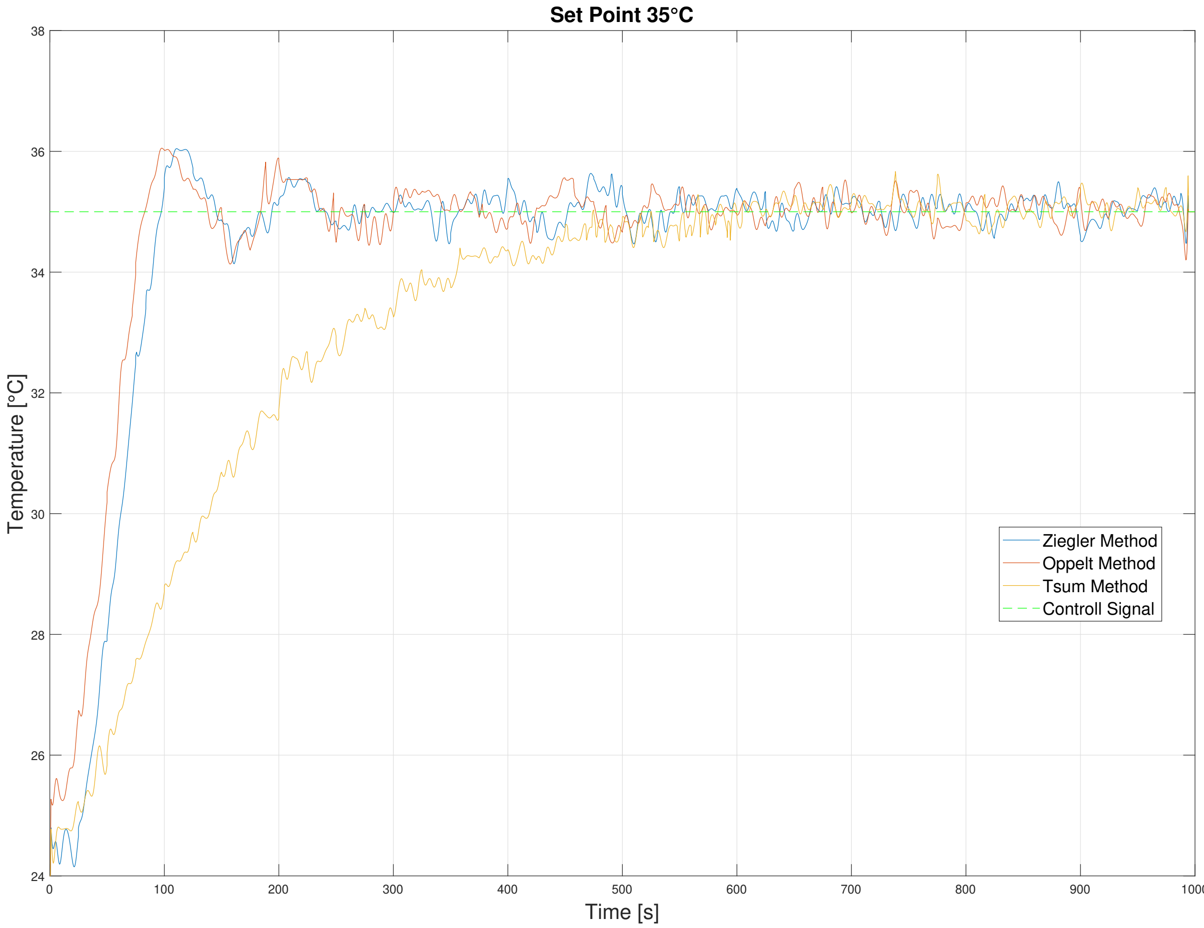

Figure 2.2: Set point 35[°C]

System response when set point equals 45°C is shown in Fig. 2.3, this plot was filtered with a low-pass filter with a pass-band frequency of 0.05[Hz]. Performance indicators for each run are shown in Table 2.4.

Table 2.4: Set point change is 45[°C]

Set point 45[°C]	ISE	IAE	ITAE
PI Ziegler	$2.532 \cdot 10^4$	2120	$3.141 \cdot 10^5$
PI T_Σ	$4.029 \cdot 10^4$	3529	$5.903 \cdot 10^5$
PI Oppelt	$2.588 \cdot 10^4$	2148	$3.139 \cdot 10^5$

2.3 Conclusions

The experimental data obtained in cases when the command signal was set to 35°C and 45°C is similar to the simulation results obtained in chapter 1.3.

In disturbance rejection experiments, the performance indicators of Ziegler and Oppelt controllers were better compared to all performance indicators of T_Σ controller. During all the experiments and simulations, the PI controller output was measured and for both

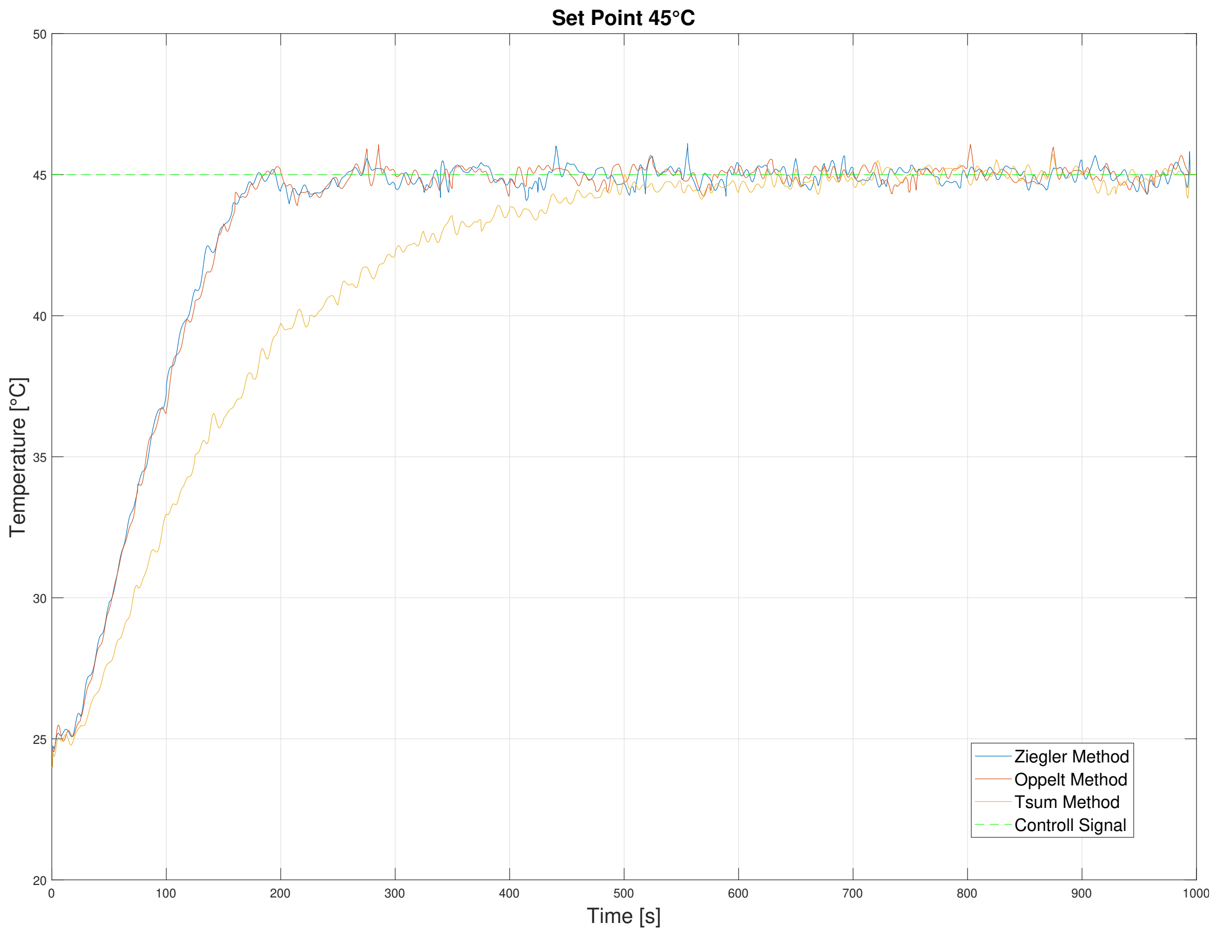

Figure 2.3: Set point $45[°C]$

Ziegler and Oppelt method it showed similar output values, meanwhile for T_Σ controller the output of PI was almost always slower and the saturation limits [0% 100%] were rarely reached.

From the experiments with set point $35°C$ and $45°C$, the performance of Ziegler and Oppelt controllers were almost identical to each other and the performance indicators had similar values. These controllers had better performance indicators and faster rise time compared to T_Σ controller.

References

[1] Handout-Temperature Control Lab 3.