Otto-von-Guericke-University Magdeburg
Faculty of Electrical Engineering and Information Technology
Institute for Automation Engineering

Automation Lab

Transfer Function Identification using Tangent and Schwarze Methods

submitted: February 6, 2022

by: Md Yasin Arafat

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1 Preparation for model identification

In control theory model identification methods serve to find the transfer function and to describe the process mathematically. In Fig. 1.1 are shown the step responses of, (a) PT_1, (b) PT_2 without oscillation, (c) Integrator, (d) PT_2 with oscillation system.

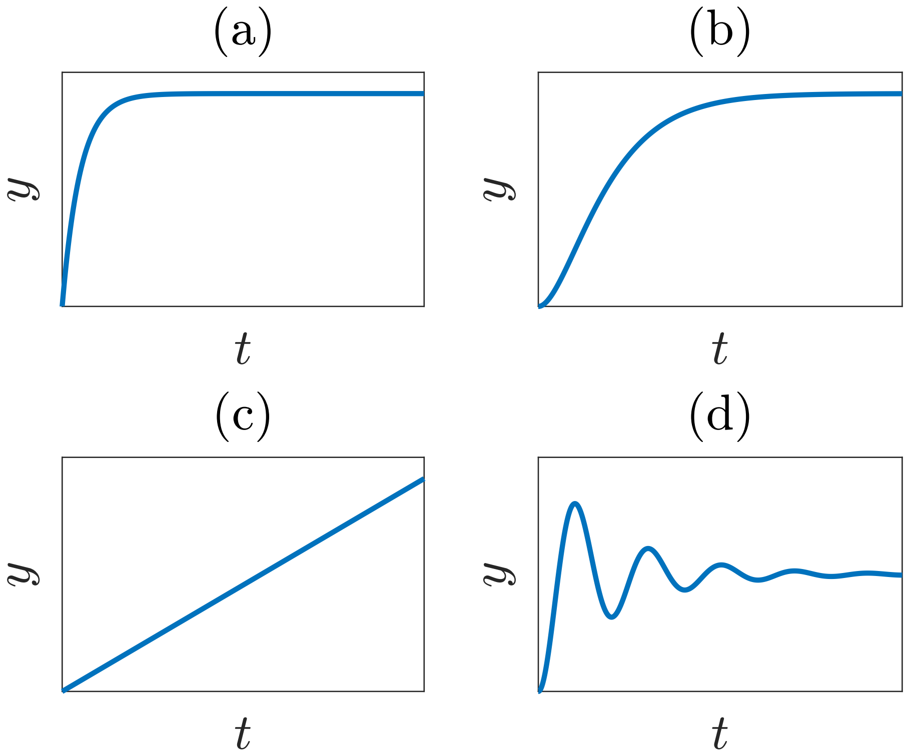

Figure 1.1: Step responses of four basic dynamical systems. [1]

Typical methods for model identification:

- Inflectional tangent method [1]
- Method of Schwarze [1]
- Ziegler-Nichols Closed Loop Method [2]

1.1 Inflectional tangent method

Requirements for the applicability of the tangent identification method [3]:

- Due to the delay action of the system, nearly ideal step input is used.
- In real system low noise level is incorporated, hence finding out single inflection point is difficult.
- Disturbance should be low. Like heater 2 should not be impacted highly. Room temperature plays a very vital role as well, hence it need to be constant.

In order to use the inflectional tangent method, we should apply a step input to the system and find the inflection point as shown in Fig. 1.2.

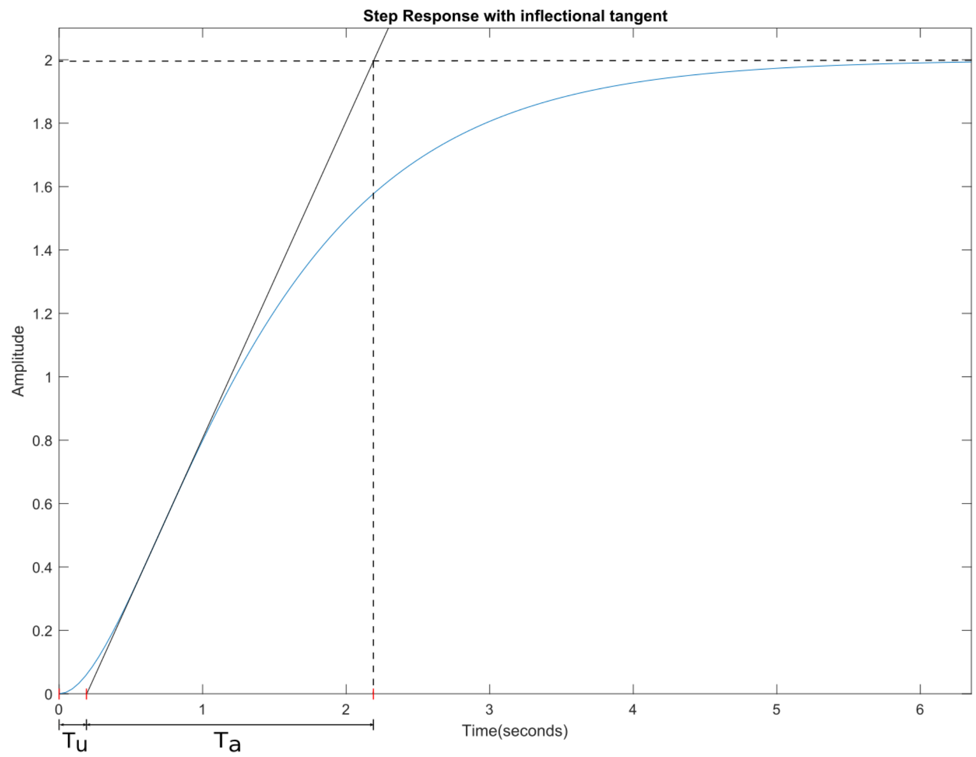

Figure 1.2: Step response of a PT_2-System with inflectional tangent. [1]

The order of the system is defined through Table 1.1.

Table 1.1: System order identification (Tangent method). [1]

n	$\frac{T_a}{T_u}$
2	9.65
3	4.59
4	3.13

In case of a second order system the following structure is used [1]:

$$G(s) = \frac{K_S}{(1 + T_1 s) \cdot (1 + T_2 s)} \tag{1.1}$$

Table 1.2: Relation between the factors. [1]

T_2/T_1	T_a/T_1	T_a/T_u
2.0	4.00	10.35
3.0	5.20	11.50
4.0	6.35	12.73
5.0	7.48	13.97
6.0	8.59	15.22
7.0	9.68	16.45
8.0	10.77	17.67
9.0	11.84	18.88

where T_1 and T_2 factors are calculated through Table 1.2 coefficients [1].

The gain coefficient K_s is calculated from the following equation:

$$K_s = \frac{x_a}{x_e} \tag{1.2}$$

where x_a is the final value of the step response, while x_e is equal to 1 in case of unit step input, in other cases i.e we have a step input from 20% to 60%, x_e would be $x_e = 60 - 20 = 40$.

1.2 Method of Schwarze

Requirements for the applicability of the Schwarze identification method [3]:

- Nearly ideal step input is required.
- The system has a behavior can be expressed using the Eq. (1.3).
- Low disturbances is expected.

In order to use the Schwarze method, a step input must be applied to the system and the time percentage parameters can be obtained as shown Fig. 1.3.

The system transfer function in this case has the following form [1]:

$$G(s) = \frac{K_S}{(1 + Ts)^n} \tag{1.3}$$

where the coefficient K_s can be calculated as explained in the inflectional tangent method. In order to calculate the order the Table 1.3 is used.

where μ coefficient can be calculated through the time percentage constants:

$$\mu = \frac{t_{10}}{t_{90}} \tag{1.4}$$

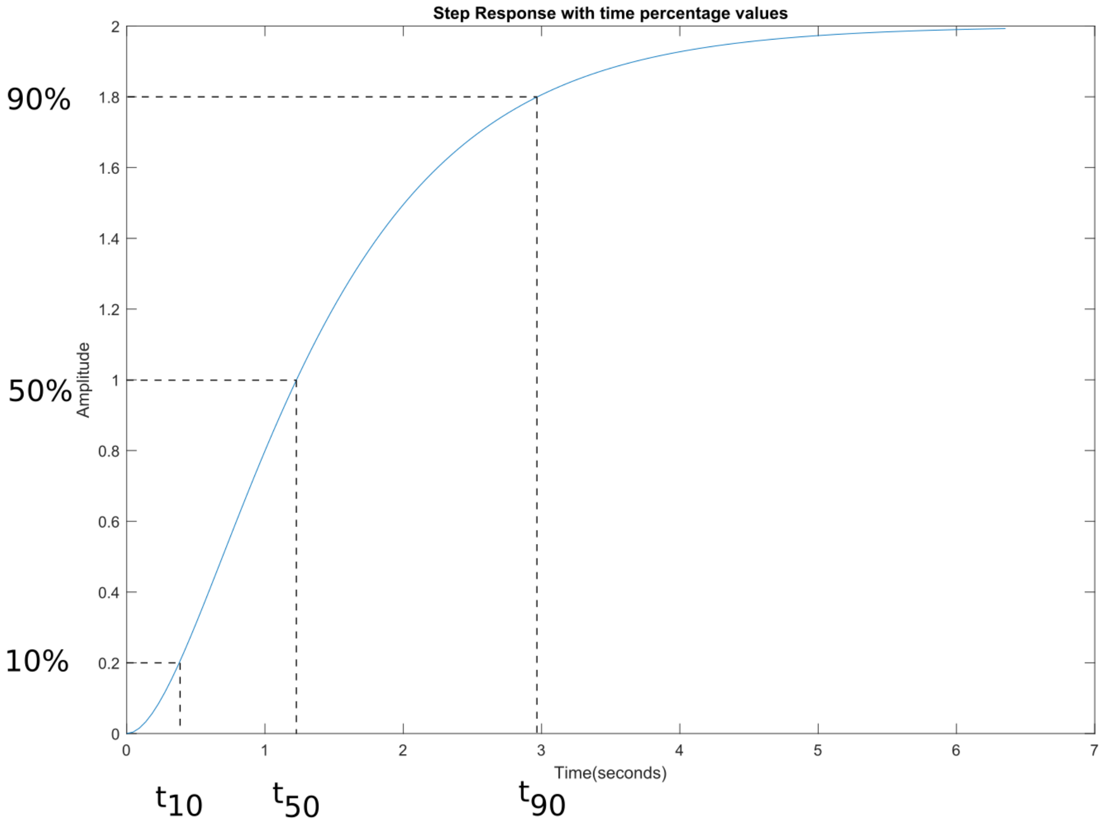

Figure 1.3: Step response of a PT2-System with time percentage values. [1]

Table 1.3: System order identification (Schwarze method). [1]

n	μ
2	0.137
3	0.207
4	0.261

After obtaining the order information, the percentage-based parameters of τ_{10}, τ_{50} and τ_{90} can be taken from Table 1.4.

Table 1.4: Percentage-based parameter identification. [1]

n	τ_{10}	τ_{50}	τ_{90}
1	0.105	0.693	2.303
2	0.532	1.678	3.890
3	1.102	2.674	5.322
4	1.745	3.672	6.681

Afterwards the coefficient T can be calculated through the following equation [1]:

$$T = \frac{1}{3} \cdot \left(\frac{t_{10}}{\tau_{10}} + \frac{t_{50}}{\tau_{50}} + \frac{t_{90}}{\tau_{90}} \right) \tag{1.5}$$

2 Practical part using TCL kit, and Matlab/Simulink

In this section, the inflectional Tangent and Schwarze method are applied in order to obtain the system transfer function.

2.1 Setup for the experiment

Firstly the ARDUINO device was connected to a computer and an external power supply. Afterwards, the provided SIMULINK model was executed. After a pre-heating phase of 5 min using 20% of the maximal power output regarding Heater 1, a step input was applied to Heater 1 from 20% to 60% of the maximal power output. The obtained data is shown in Fig. 2.1.

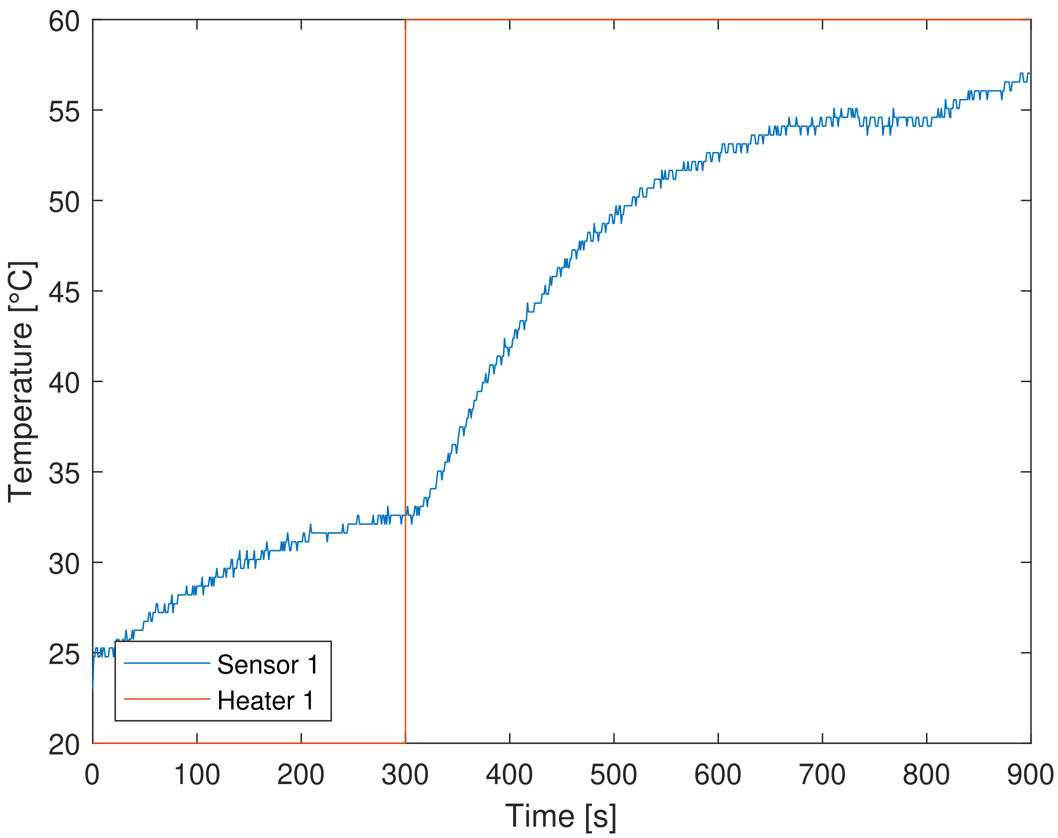

Figure 2.1: Step response

2.1.1 Transfer function using Tangent method

Since, there is a lot of noise, therefore it is difficult to get an inflection point. Therefore a low pass filter was used to reduce the noise. Then we approximated the polynomial, see Fig. 2.2.

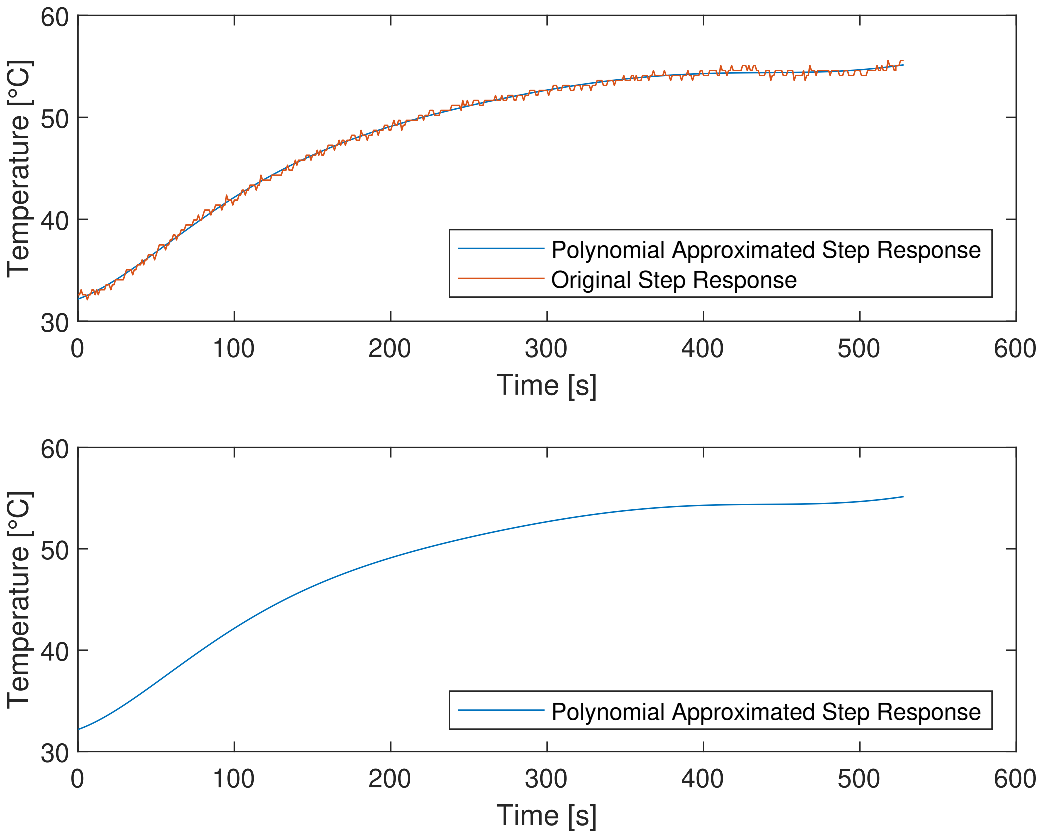

Figure 2.2: Approximated polynomial step response

The inflection point was calculated through the second analytical derivative of the polynomial approximation function. The ratio $\frac{T_a}{T_u}$ 1.1 is obtained. In this case $\frac{T_a}{T_u}$ = 23.44914. In Fig. 2.3 the inflectional tangent is plotted.

Afterwards, the order was calculated through Table 1.1 data. In this case, the calculated value of T_u is sufficiently large to model the process using a second order model. As described in subsection 1.1 the coefficients T_1 and T_2 were calculated through Table 1.2 factors. In this case, with $T_a = 205.1565$ and $T_u = 8.7490$, we get $\frac{T_a}{T_u} = 23.44914$ was approximated to 23.45 value which is found on the table. As the last step, the gain coefficient K_s is calculated through Eq. (1.2), and we get $\frac{X_a}{X_e} = 0.57437$, where $X_a = Max - Min = 55.1473 - 32.1781$ and $X_e = 40 - 20$ The following transfer function is obtained:

$$G(s) = \frac{0.57437}{785.8792s^2 + 84.7654s + 1} \tag{2.1}$$

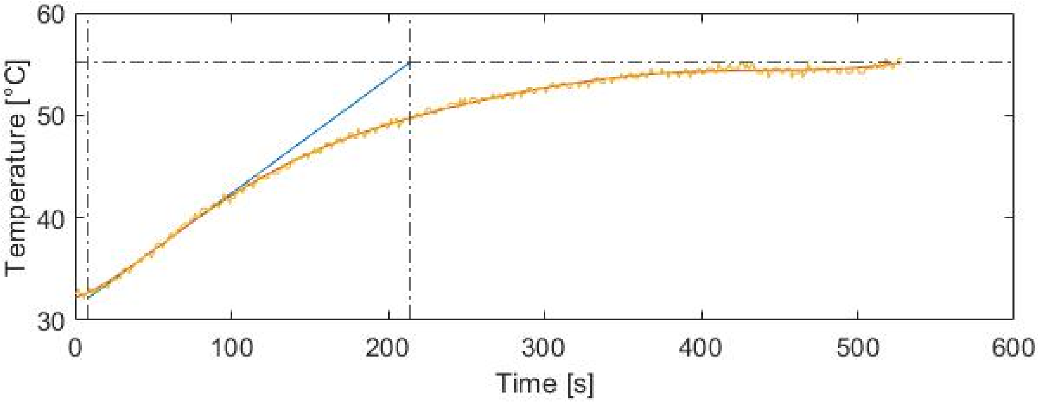

Figure 2.3: Step response with inflection tangent

2.1.2 Transfer function using Schwarze method

From the Fig. 2.1, we have calculated the value of $t_{10} = 17$, $t_{50} = 62$, and $t_{90} = 167$. Using these values and 1.3, we get $\mu = 0.102$ (approximately). Hence the system order is $n = 2$. Using the system gain from Tangent method part, that is $K_s = 0.57423$. We get the transfer function:

$$G(s) = \frac{0.57437}{(1 + 37.2781s)^2} \tag{2.2}$$

2.2 Comparison between the experimental and simulated results

Both transfer functions of Eq. (2.1) and Eq. (2.2) have been put in the TCL-block consecutively. A constant signal was added, due to room temperature. That is equal to $24[°C]$. It is assumed that the environment and pre-heating are constant. The edited Simulink model is shown in Fig. 2.4.

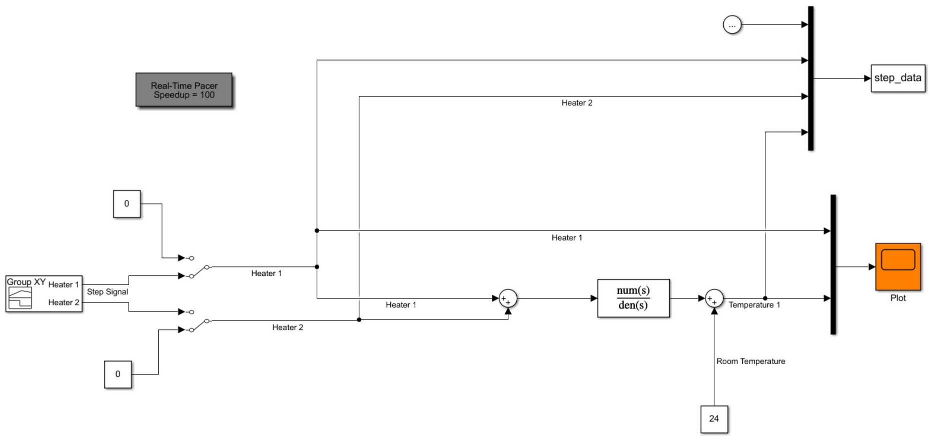

Figure 2.4: Simulink model

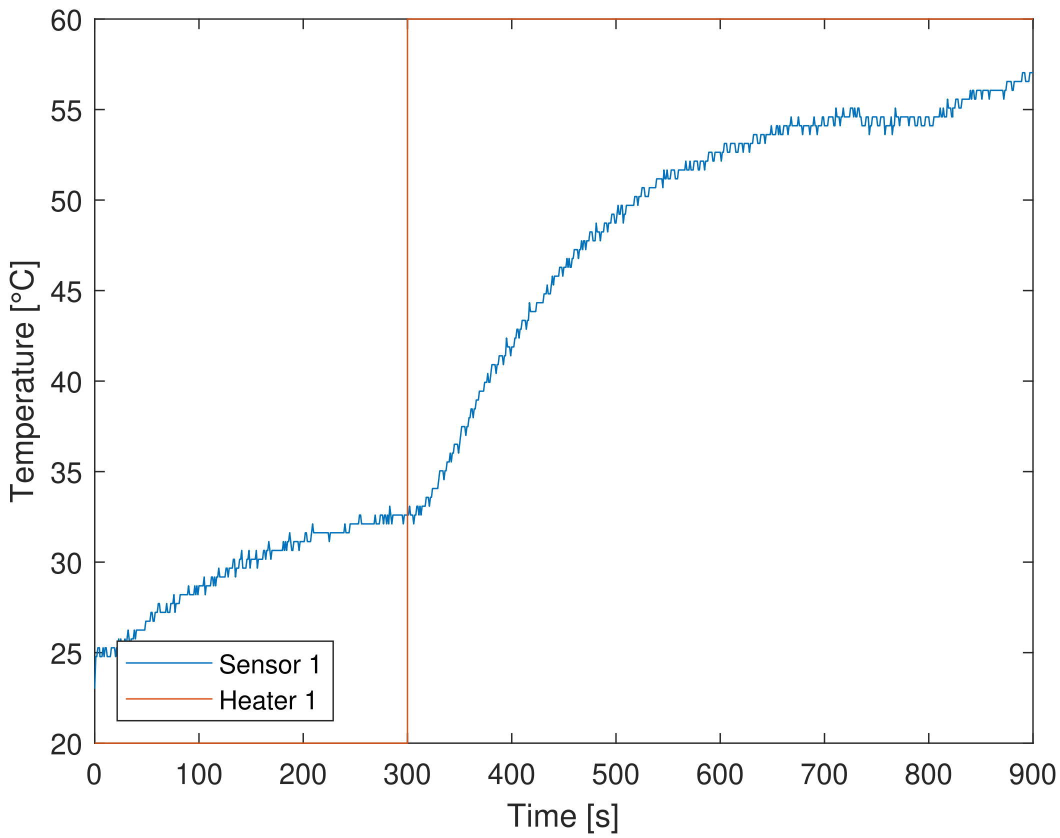

Figure 2.5: Generated step response using TCL kit

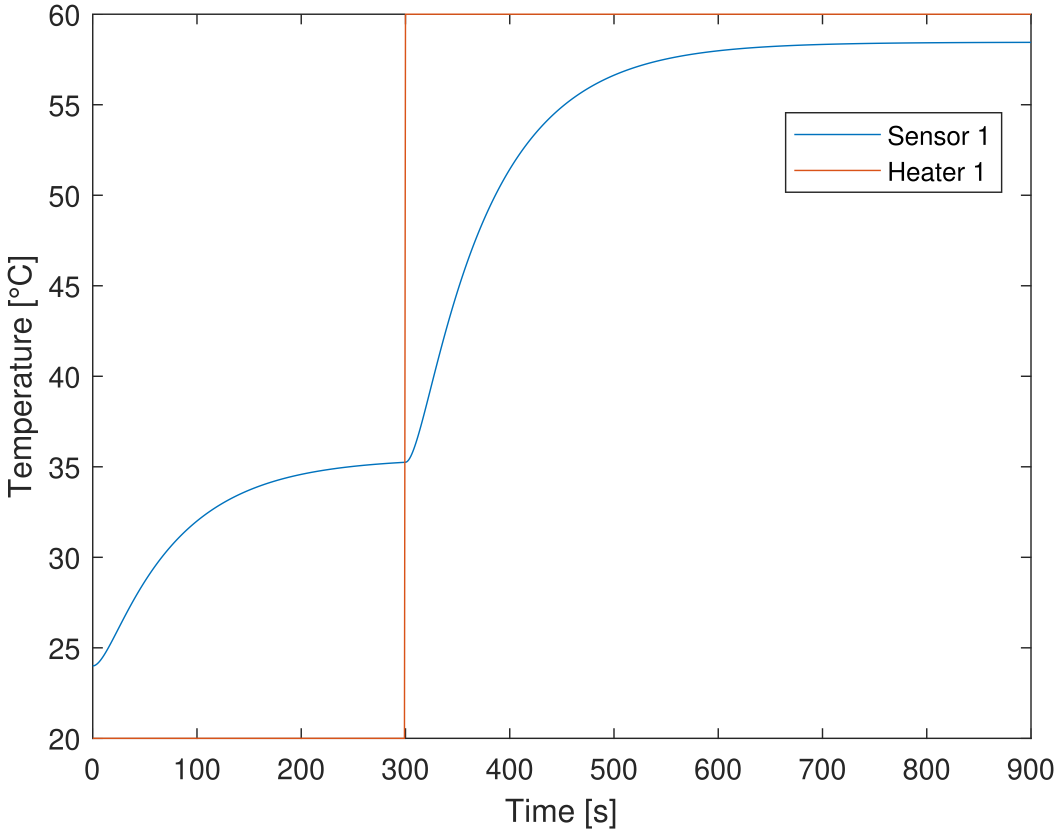

Figure 2.6: Simulated step response using Tangent method

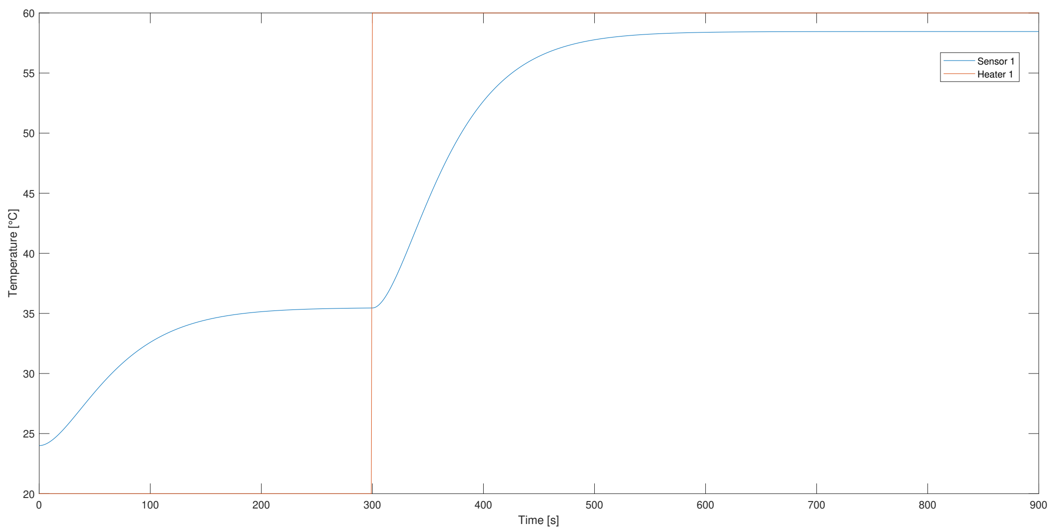

Figure 2.7: Simulated step response using Schwarze method

3 Interpretation of the results

During experiment, unfortunately we opened the window after 700s, hence there is a distortion in temperature profile, see Fig. 2.5. The response of original signal in order to reach steady state is slow. However, the simulated results in Fig. 2.6 and Fig. 2.7 are very close to the original step response. We observe that the original step response responding slower than the simulated outputs as well.The correlations are high between simulated and original outputs.

Bibliography

[1] Handout-Temperature Control Lab.

[2] Process Control Lectures.

[3] Duncan A. Mellichamp Dale E. Seborg, Thomas F. Edgar. *Process Dynamics and Control*. 2 edition, 2003.