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Comparison between Experimental and Simulated Step Response of Temperature Control Kit

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Contents

1 Model identification with aperiodic test signals

1.1 Preparation

In control theory model identification methods serve to find the transfer function and to describe the process mathematically. In Fig. 1.1 are shown the step responses of, (a) PT_1, (b) PT_2 without oscillation, (c) Integrator, (d) PT_2 with oscillation system.

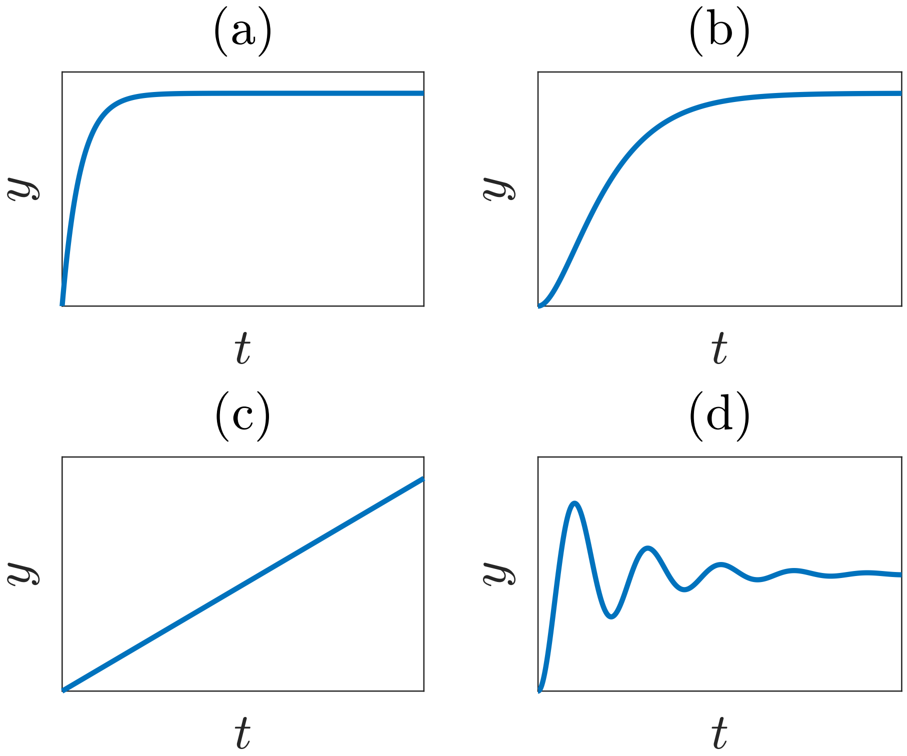

Figure 1.1: Step responses of four basic dynamical systems. [1]

Typical methods for model identification:

- Inflectional tangent method
- Method of Schwarze
- Rangaiah and Krishnaswamy (1994, 1996) [2]

1.1.1 Inflectional tangent method

Requirements for the applicability of the tangent identification method [2]:

- Nearly ideal step input. In some systems, the process equipment such as pumps or control valves cannot be changed instantaneously from one setting to another.
- Low noise level. As the parameters are defined from the point of inflection in systems with high levels of noise finding that single point is difficult.
- Low disturbances. Other process inputs must not change during step input. For example in the TCL 2 case the room temperature must be kept constant during the experiment.

In order to use the inflectional tangent method, we should apply a step input to the system and find the inflection point as shown in Fig. 1.2.

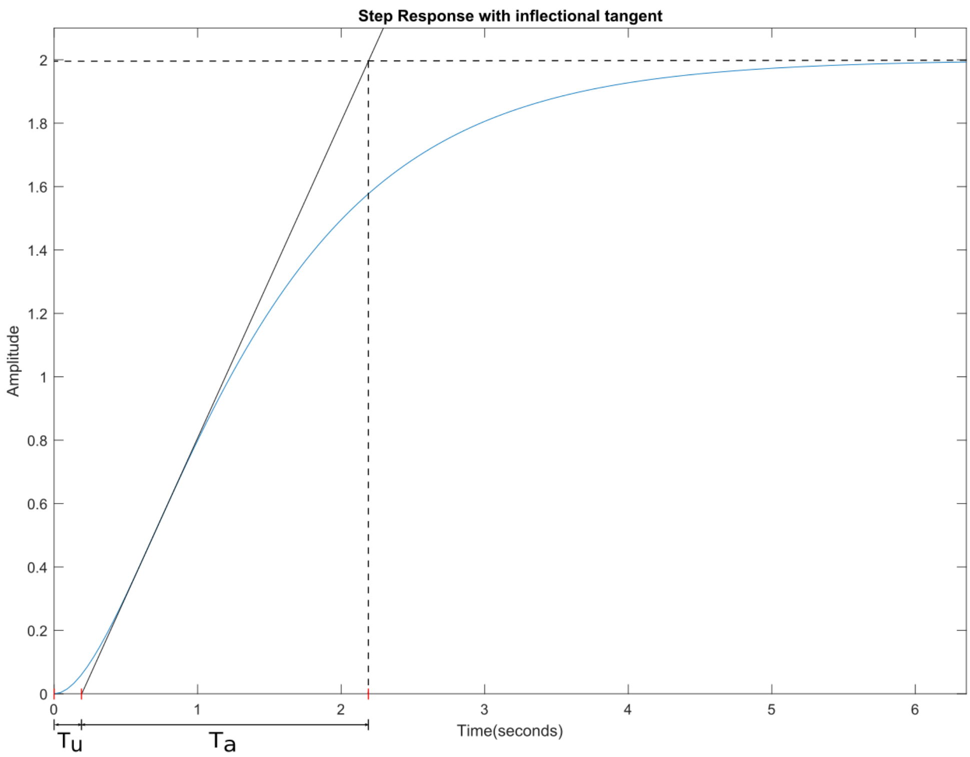

Figure 1.2: Step response of a PT_2-System with inflectional tangent. [1]

The order of the system is defined through Table 1.1.
In case of a second order system the following structure is used [1]:

$$G(s) = \frac{K_S}{(1 + T_1 s) \cdot (1 + T_2 s)} \tag{1.1}$$

where T_1 and T_2 factors are calculated through Table 1.2 coefficients [1].

Table 1.1: System order identification. [1]

n	$\frac{T_a}{T_u}$
2	9.65
3	4.59
4	3.13

Table 1.2: Relation between the factors. [1]

T_2/T_1	T_a/T_1	T_a/T_u
2.0	4.00	10.35
3.0	5.20	11.50
4.0	6.35	12.73
5.0	7.48	13.97
6.0	8.59	15.22
7.0	9.68	16.45
8.0	10.77	17.67
9.0	11.84	18.88

The gain coefficient K_s is calculated from the following equation:

$$K_s = \frac{x_a}{x_e} \tag{1.2}$$

where x_a is the final value of the step response, while x_e is equal to 1 in case of unit step input, in other cases i.e we have a step input from 20% to 60%, x_e would be $x_e = 0.6 - 0.2 = 0.4$.

1.1.2 Method of Schwarze

Requirements for the applicability of the Schwarze identification method [2]:

- Nearly ideal step input. In some systems, the process equipment such as pumps or control valves cannot be changed instantaneously from one setting to another.
- The system has a behavior that can be modeled accurately through Eq. (1.3) model structure.
- Low disturbances. Other process inputs must not change during step input. For example, in the TCL 2 case, the room temperature must be kept constant during the experiment.

In order to use the Schwarze method, a step input must be applied to the system and the time percentage parameters can be obtained as shown Fig. 1.3.

The system transfer function in this case has the following form [1]:

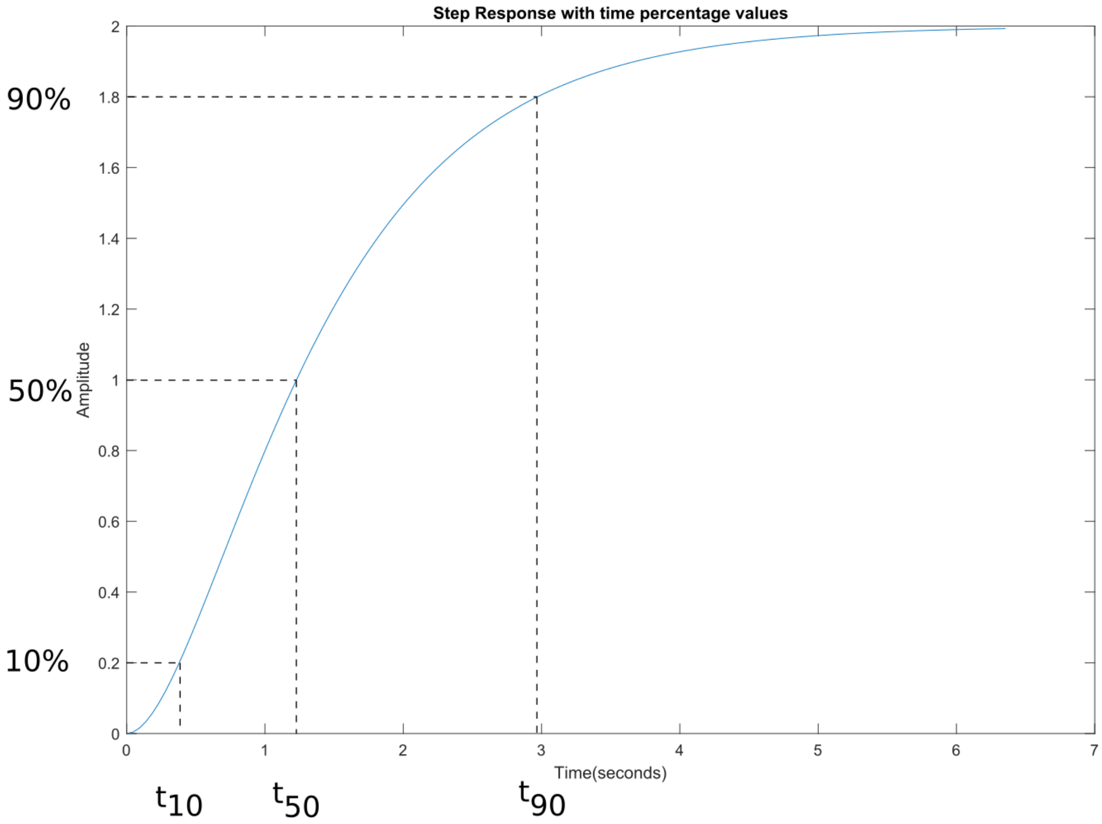

Figure 1.3: Step response of a PT2-System with time percentage values. [1]

$$G(s) = \frac{K_S}{(1 + Ts)^n} \tag{1.3}$$

where the coefficient K_s can be calculated as explained in the inflectional tangent method. In order to calculate the order the Table 1.3 is used.

Table 1.3: System order identification. [1]

n	μ
2	0.137
3	0.207
4	0.261

where μ coefficient can be calculated through the time percentage constants:

$$\mu = \frac{t_{10}}{t_{90}} \tag{1.4}$$

After obtaining the order information, the percentage-based parameters of τ_{10}, τ_{50} and τ_{90} can be taken from Table 1.4.

Afterwards the coefficient T can be calculated through the following equation [1]:

$$T = \frac{1}{3} \cdot \left(\frac{t_{10}}{\tau_{10}} + \frac{t_{50}}{\tau_{50}} + \frac{t_{90}}{\tau_{90}} \right) \tag{1.5}$$

Table 1.4: Percentage-based parameter identification. [1]

n	τ_{10}	τ_{50}	τ_{90}
1	0.105	0.693	2.303
2	0.532	1.678	3.890
3	1.102	2.674	5.322
4	1.745	3.672	6.681

1.2 Practical Part

In this section, the inflectional tangent method is applied in order to obtain the system transfer function.

1.2.1 Experiment setup

Firstly the ARDUINO device was connected to a computer and an external power supply. Afterwards, the provided SIMULINK model was executed. After a pre-heating phase of 8 min using 20% of the maximal power output regarding Heater 2, a step input was applied to Heater 2 from 20% to 60% of the maximal power output. The obtained data is shown in Fig. 1.4.

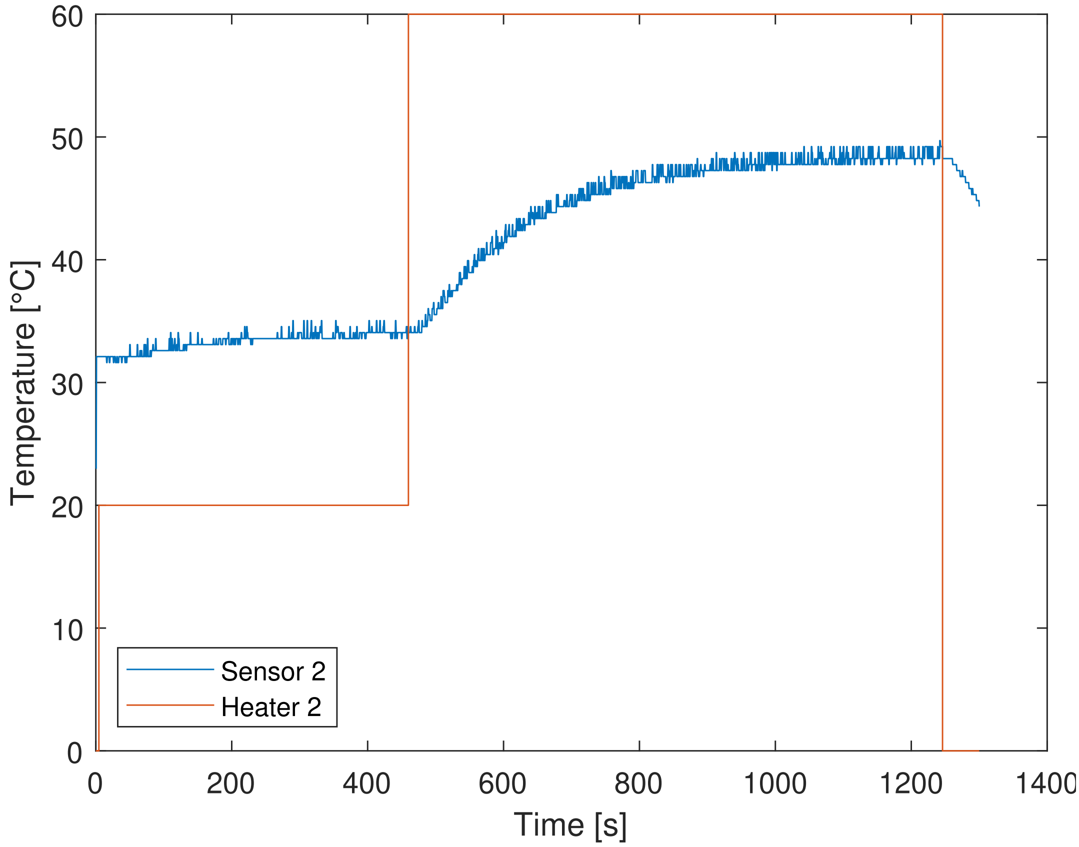

Figure 1.4: Step Response

1.2.2 Calculation of transfer function

As mentioned earlier, the inflectional tangent method was applied in this case. Firstly the signal was windowed in such was that only the relevant information is used for further steps. Later the signal was filtered through a low pass filter in order to remove the random noise. The obtained signal is shown in Fig. 1.5.

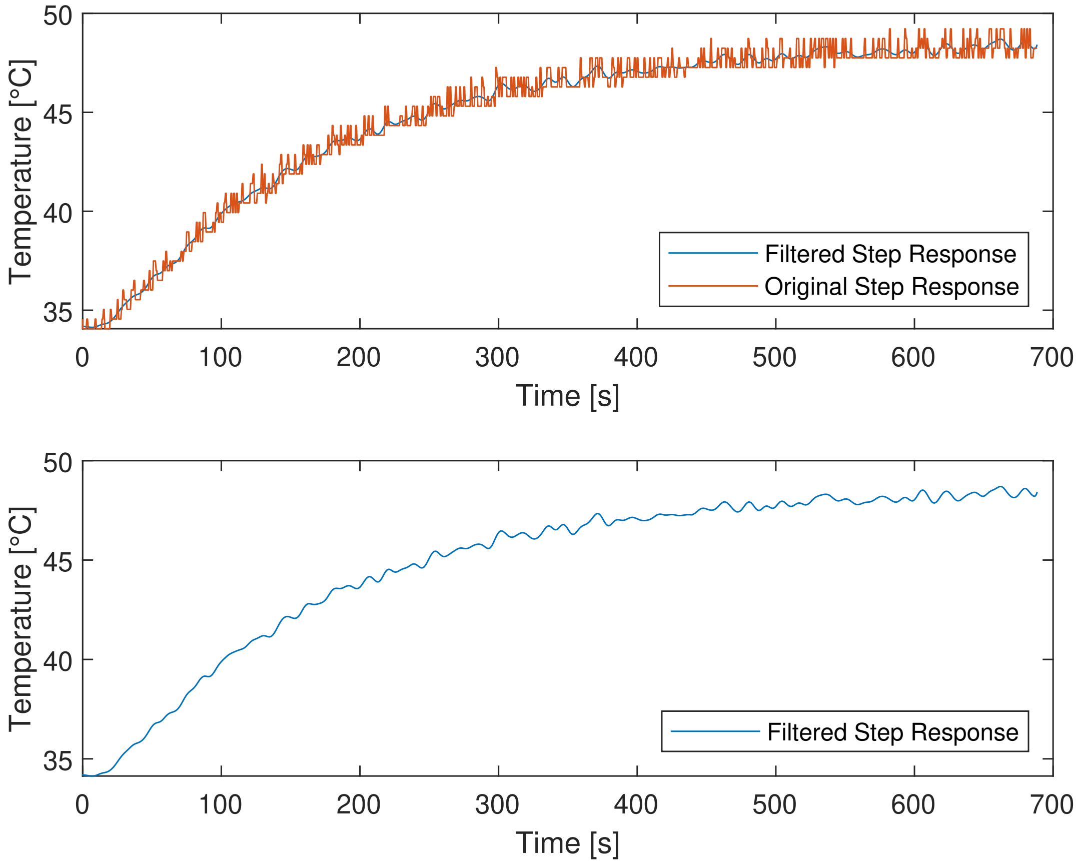

Figure 1.5: Filtered step response

As observed from the filtered signal figure, the signal still contains noise and the calculation of the inflectional point may be inaccurate. In order to find the inflection point another approximation was made, the filtered step response was approximated to a polynomial function. The obtained result is shown in Fig. 1.6.

The idea behind the approximation to a polynomial function is due to the fact that polynomial approximation methods were easier to implement by the author and proved to deliver a good result in this case. However, the approximation to other analytical functions may be beneficial in calculating the inflection point, provided that they do not change the nature of the signal itself but only filter the noise.

As the next step, the inflection point was calculated through the second analytical derivative of the polynomial approximation function. The ratio $\frac{T_a}{T_u}$ as explained in the earlier section 1.1.1 is obtained. In this case $\frac{T_a}{T_u} = 16.48$. In Fig. 1.7 the inflectional tangent is plotted.

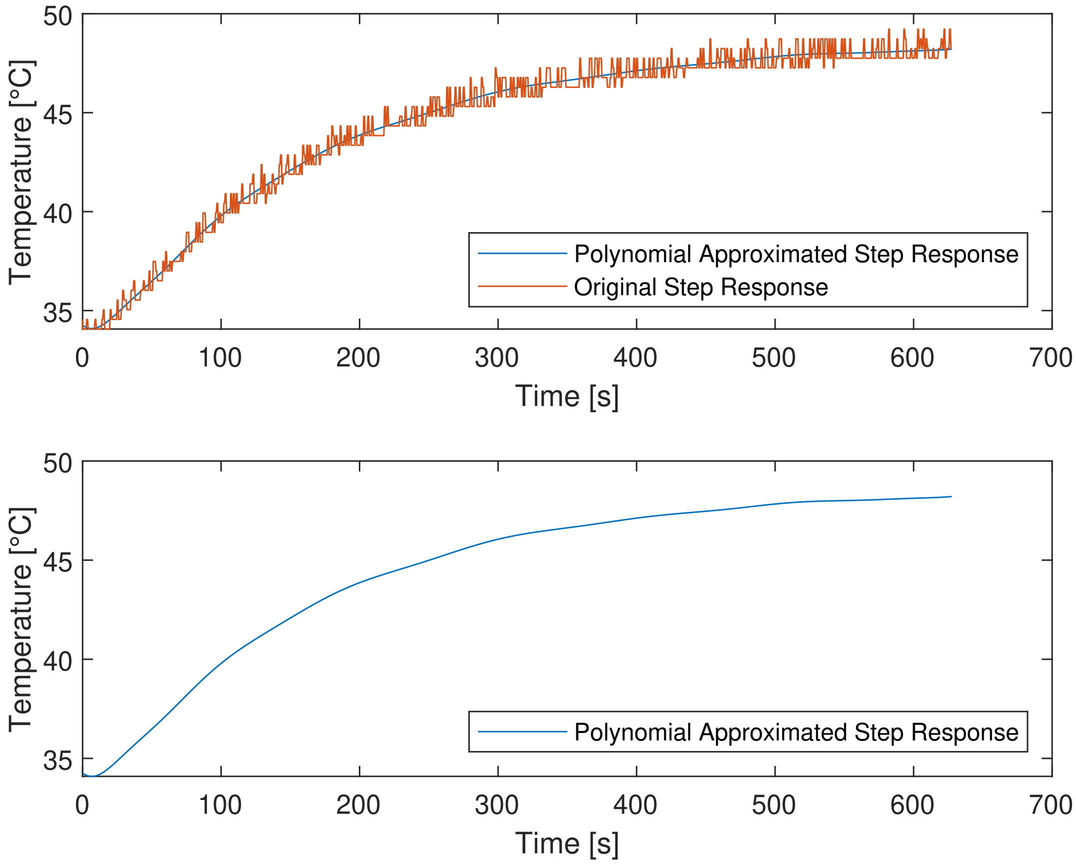

Figure 1.6: Polynomial approximated step response

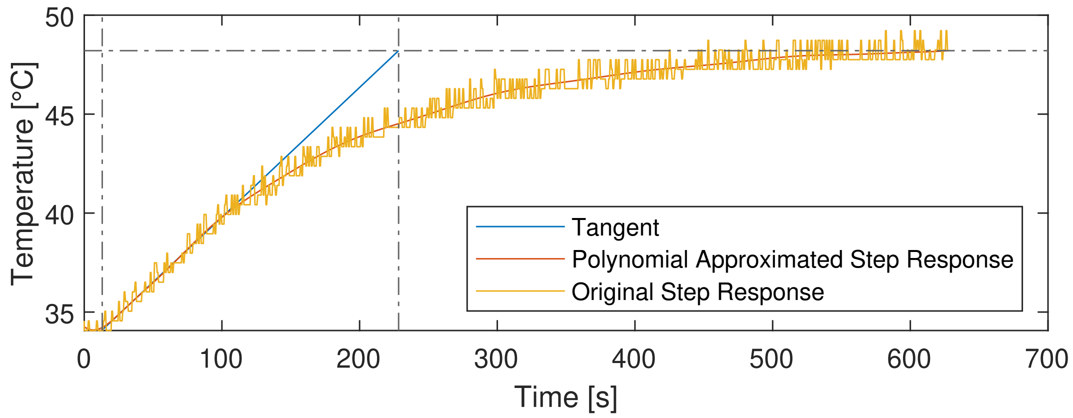

Figure 1.7: Step response with inflectional tangent.

Afterwards, the order was calculated through Table 1.1 data. In this case, the calculated value of T_u is sufficiently large to model the process using a second order model. As described in subsection 1.1.1 the coefficients T_1 and T_2 were calculated through Table 1.2 factors. In this case, $\frac{T_a}{T_u} = 16.48$ was approximated to 16.45 value which is found on the table. As the last step, the gain coefficient K_s is calculated through Eq. (1.2). The following transfer function is obtained:

$$G(s) = \frac{35.2819}{3461.7709s^2 + 177.9058s + 1} \tag{1.6}$$

1.2.3 Comparison between the experimental step response and the simulated step response

The SIMULINK model was edited and the transfer function shown in Eq. (1.6) replaced the TCL-block. Furthermore, at the output of the transfer function block a constant signal was added. This added signal, in this case, corresponds to the room temperature which is equal to $27[°C]$. It is assumed that the environment and pre-heating are constant. The edited Simulink model is shown in Fig. 1.8.

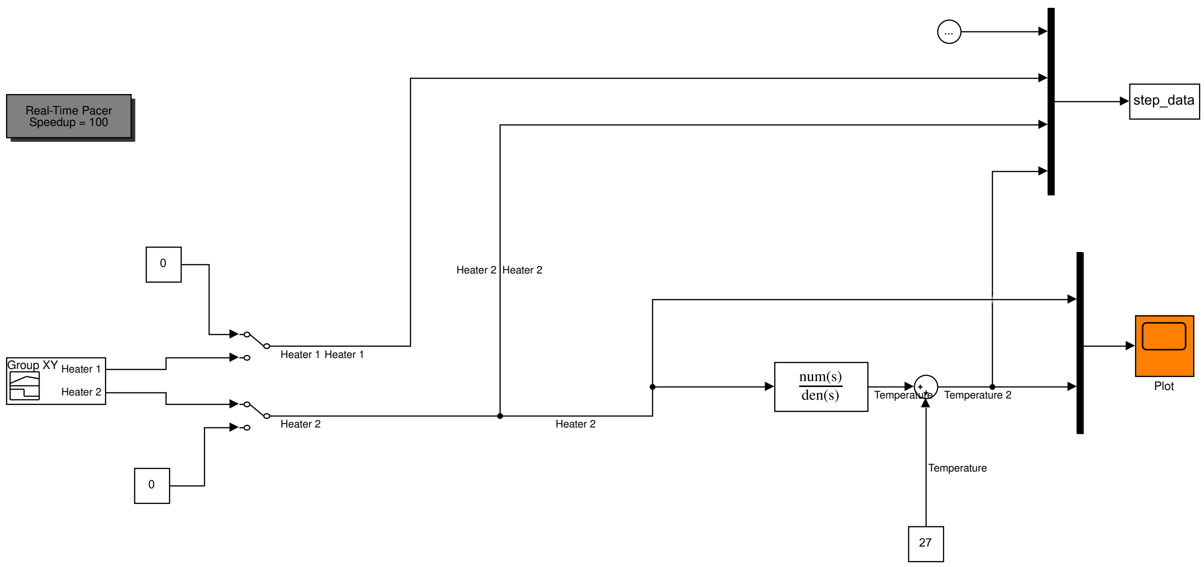

Figure 1.8: Simulink model

As the last step, the data from the simulation is compared against the original signal in Fig. 1.9.

1.2.4 Interpretation of the results

The obtained transfer function G(s) describes the process behaviour with relatively high accuracy. As shown in Fig. 1.9 the experimental data fit nearly perfectly with the simulation results. It is important to note that during all the approximations the results of the simulated transfer function were compared against the experimental data through normalized correlation. Given through the following equation:

$$norm\_corr = \frac{\sum_{n=0}^{m} x[n]y[n]}{\sqrt{\sum_{n=0}^{m} x[n]^2 \sum_{n=0}^{m} y[n]^2}} \tag{1.7}$$

where $x[n]$ and $y[n]$ are the compared signals, $norm\_corr$ is the obtained value of the normalized correlation.

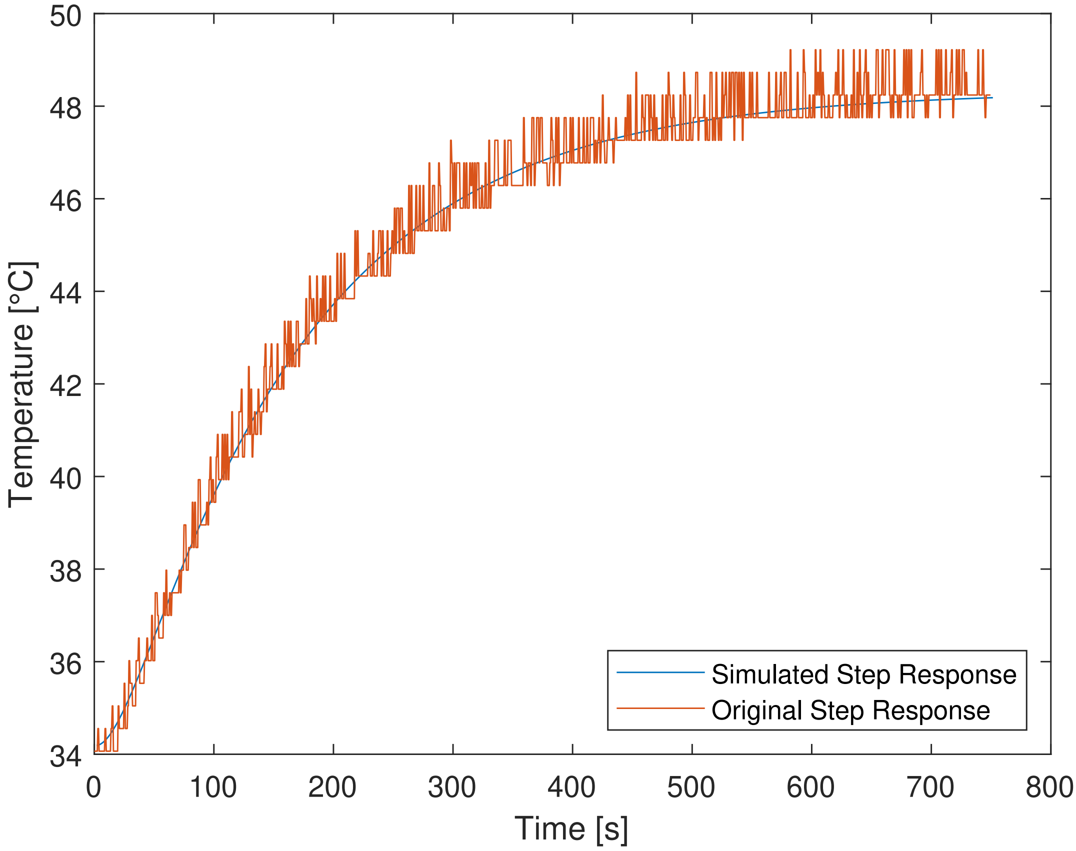

Figure 1.9: Comparison between the experimental step response and the simulated step response

In the final run, the normalized correlation value between the simulated step response and the experimental one was equal to $norm\_corr \approx 0.99995$.

References

[1] Handout-Temperature Control Lab 2.

[2] Duncan A. Mellichamp Dale E. Seborg, Thomas F. Edgar. *Process Dynamics and Control*. 2 edition, 2003.