

Experiment #4

Stress Analysis of beams using strain gauges

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Section PI-X

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1. Objective

The objective of this lab is to determine the distribution of stress and strain in a beam with the use of strain gauges and then to compare them the results obtained by the theory of flexure. The strain measurements will also be used to determine the elastic modulus, Poisson's ratio, and the shear modulus.

2. Introduction

It is important for engineers to understand how materials act under pure bending. Bending is usually accompanied with different stresses like transverse shear but for the simplicity of this lab pure bending and a cross section remaining planar will only be considered. For pure bending to occur, the sample must be loading in a specific manner like in the figure below. In Pure bending one side of the beam is in compression and the other is in tension. Which side is determined by the direction of the bending moment.

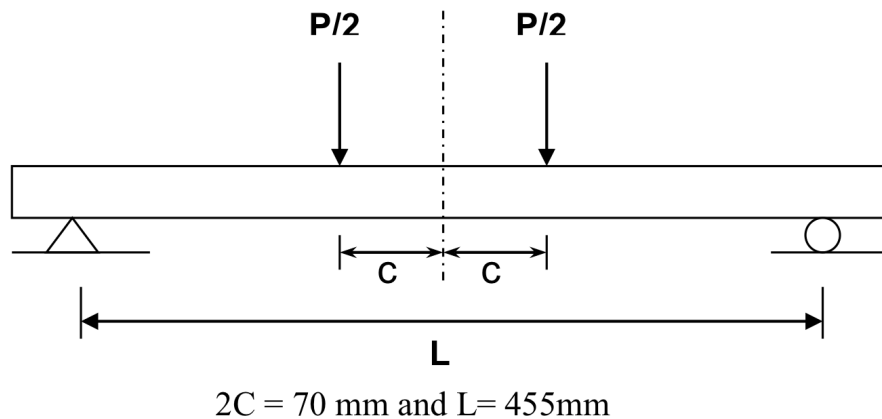


Figure 1: Load arrangement producing pure bending in middle portion of beam

This pure bending leads to a strain that is proportional to the distance of the neutral axis. It is also important to note that lateral strain bears a constant relationship with axial strain during elastic deformation. This relationship is determined by Poisson's ratio. A very important material property that must be considered by engineers when designing structure where dimensional changes due to applied forces must be taken into account.

3. Procedure

Start by measuring the cross section of the steel bar and note that the span length is 455 mm. Record on a sketch the position of each strain gauge. Place and locate the beam symmetrically on the supported with respect to the center line without applying any load. Turn on the strain indicator using the power button on the bottom left. Turn the gauge selector switch to channel 1. Adjust the corresponding potentiometer to balance the gauge until the strain indicator reads zero when there is no force applied. Repeat this procedure for all the other channels. To adjust strain gauge #2, switch to channel #2 and then adjust the potentiometer knob #2. Check individually each strain gauge and recalibrate, if necessary, before beginning the experiment. Be careful not to touch the wires or any of the connections of the experiment. Proceed to slowly load the beam in increments of 1000 N up to 5000 N. Note all the reading of strains for all the six channels at each loading stage and record them. After all the readings are completed slowly unload the beam and record the readings at zero load. This is a verification step where all values should be close to zero.

4. Analysis of the results

$$(1) I = \frac{bh^3}{12}$$

$$(2) \sigma = \frac{My}{I}$$

$$(3) E = \frac{\sigma}{\epsilon}$$

$$(4) \nu = \frac{\frac{\delta \epsilon_T}{\delta p}}{\frac{\delta \epsilon_L}{\delta p}}$$

$$(5) \sigma = Ee$$

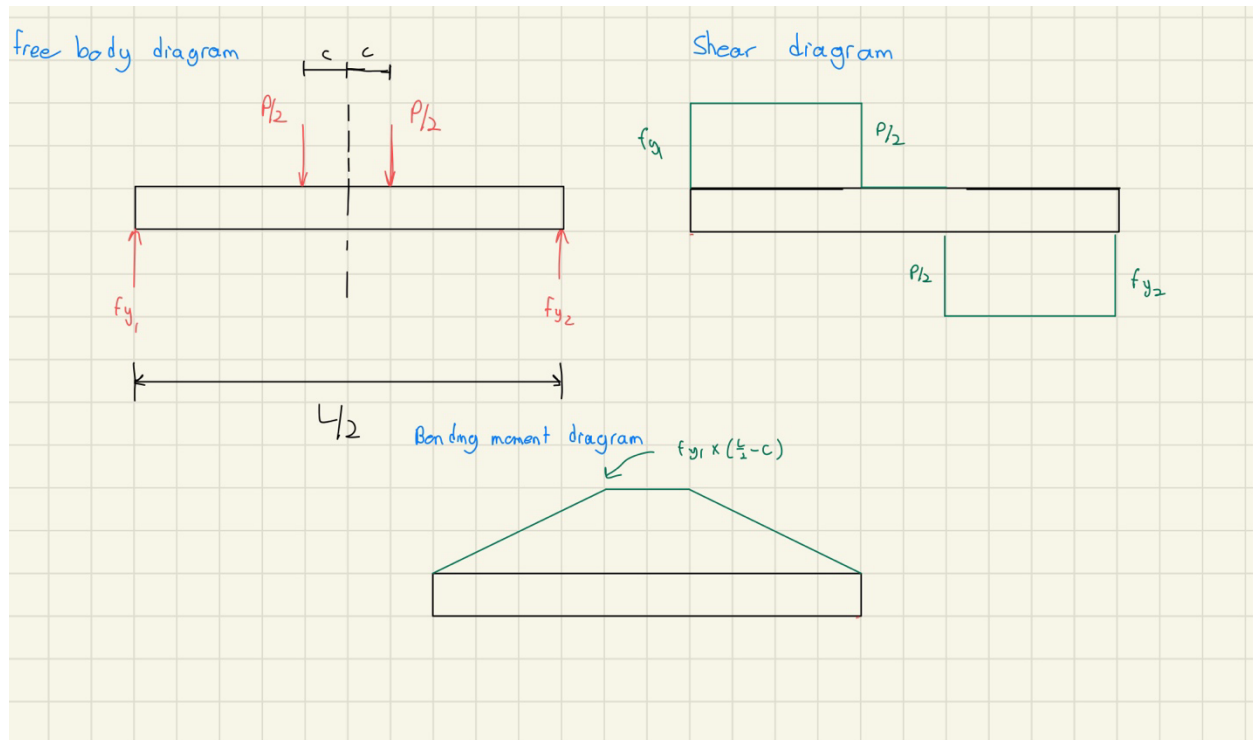
$$(6) G = \frac{E}{2(1+\nu)}$$

Moment of inertia of beam

$$I = \frac{bh^3}{12}$$
$$I = \frac{(19.09)(32.12)^3}{12}$$

$$I = 52717 \text{ mm}^4$$
$$I = 5.2717 \times 10^{-8} \text{ m}^4$$

Free body, shear and bending moment diagrams



Calculated Bending stresses at strain gauge points for each loading stage

Sample calculation for 1000 N applied at point 1

$$M = \left(\frac{L}{2} - c \right) \left(\frac{p}{2} \right)$$

$$M = \left(\frac{455}{2} - 35 \right) \left(\frac{1000}{2} \right)$$

$$M = 96.25 \text{ Nm}$$

$$\sigma = \frac{My}{I}$$

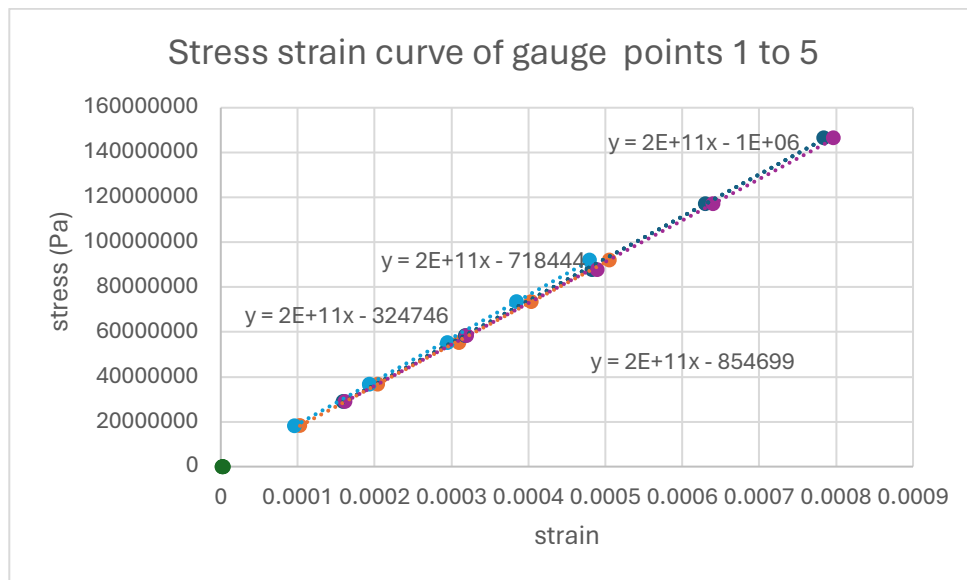
$$\sigma = \frac{96.25 \text{ Nm} (0.01606)}{5.2717 \times 10^{-8} \text{ m}^4}$$

$$\sigma = 29.32 \text{ GPa}$$

Table of bending stresses (Pa) at different gauge points

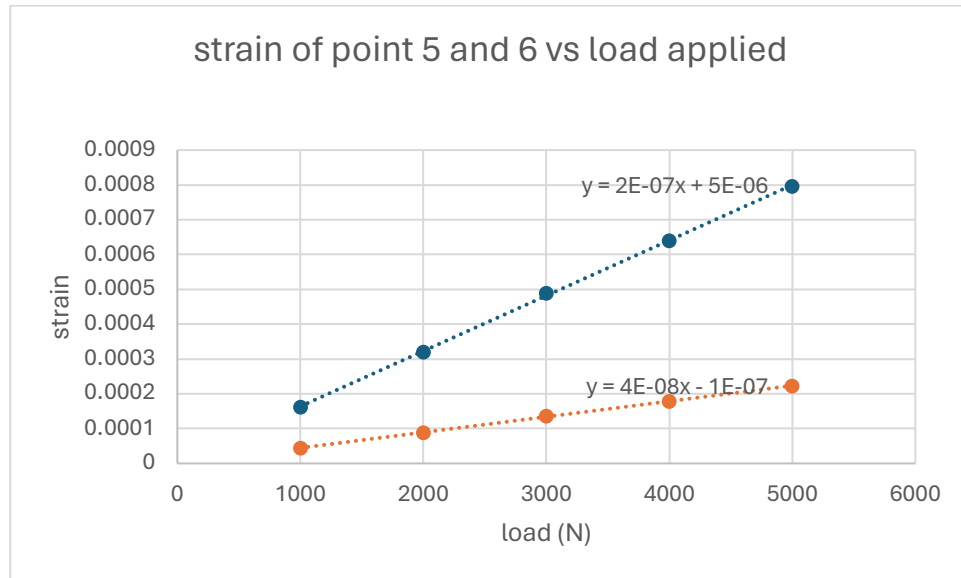
load (N)	moment (Nm)	stress 1 (Pa)	stress 2 (Pa)	stress 3 (Pa)	stress 4 (Pa)	stress 5 (Pa)	stress 6 (Pa)
1000	96.25	29322079.6	18440411.2	0	18440411.2	29322079.6	29322079.6
2000	192.5	58644159.1	36880822.4	0	36880822.4	58644159.1	58644159.1
3000	288.75	87966238.7	55321233.5	0	55321233.5	87966238.7	87966238.7
4000	385	117288318	73761644.7	0	73761644.7	117288318	117288318
5000	481.25	146610398	92202055.9	0	92202055.9	146610398	146610398

Stress-Strain curve for gauge points 1 to 5



According to (1) $\sigma = Ee$ and the stress strain curve of the 5 gauge points the elastic modulus of the steel bar is 2×10^{11} Pa or 200 GPa.

Longitudinal and transversal strain vs applied load



(1) According to equation (4) $\nu = \frac{\frac{\delta \epsilon_T}{\delta p}}{\frac{\delta \epsilon_L}{\delta p}}$

$$\nu = \frac{4 \times 10^{-8}}{2 \times 10^{-7}}$$

$$\nu = 0.2$$

Stress at different Gauge points

Sample calculation for stress of gauge point 1 at 1000 N

$$\sigma = E \epsilon$$

$$\sigma = (200 \times 10^9)(.000159)$$

$$\sigma = 318.159 \text{ Mpa}$$

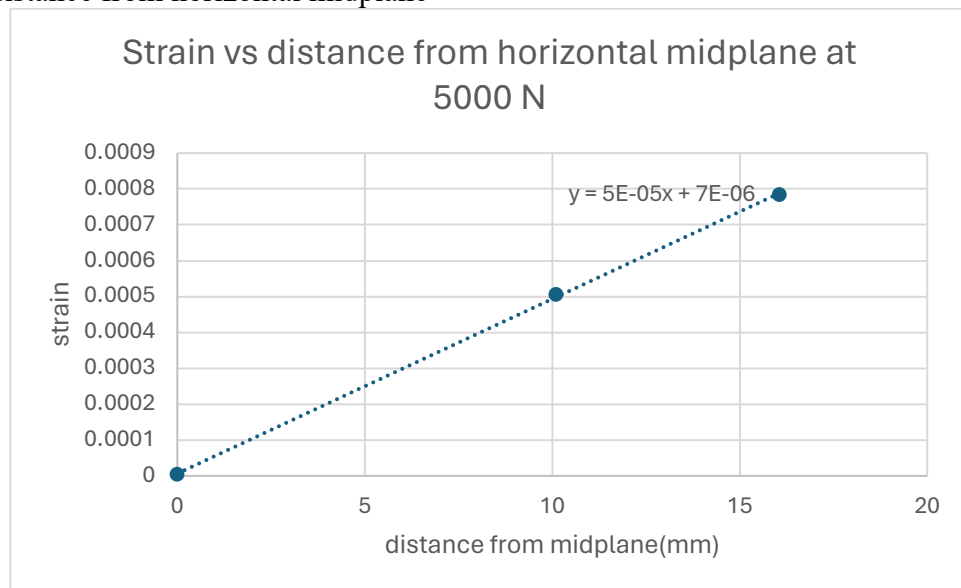
stress 1	stress 2	stress 3	stress 4	stress 5	stress 6
318.159	206.103	4.002	192.096	324.162	88.044
636.318	408.204	4.002	386.193	640.32	178.089
966.483	620.31	6.003	590.295	980.49	272.136
1260.63	808.404	6.003	770.385	1280.64	356.178
1568.784	1010.505	8.004	960.48	1594.797	446.223

Experimental shear modulus calculation

$$G = \frac{E}{2(1 + \nu)}$$
$$G = \frac{(200 \text{ GPa})}{2(1 + 0.2)}$$
$$G = 83.33 \text{ GPa}$$

The published value for the shear modulus of steel is 77 GPa which resembles the experimental value of 83.33 GPa obtained. There is a significant discrepancy present.

Strain vs distance from horizontal midplane



To find the experimental location of the neutral axis...

$$\delta = 5 \times 10^{-5}x + 7 \times 10^{-6}$$
$$0 = 5 \times 10^{-5}x + 7 \times 10^{-6}$$
$$7 \times 10^{-6} = 5 \times 10^{-5}x$$
$$x = -0.14$$

The experimental location of the neutral axis is determined to -0.14 mm. The negative signifies that it is one the bottom half by 0.14 mm. The theoretical position is supposed to be at the exact center of the beam. 0.14 of one millimeter is very close to the theoretical value.

5. Report

Experimental data

stress 1 (MPa)	stress 2 (MPa)	stress 3 (MPa)	stress 4 (MPa)	stress 5 (MPa)	stress 6 (MPa)
31.8159	20.6103	0.4002	19.2096	32.4162	8.8044
63.6318	40.8204	0.4002	38.6193	64.032	17.8089
96.6483	62.031	0.6003	59.0295	98.049	27.2136
126.063	80.8404	0.6003	77.0385	128.064	35.6178
156.8784	101.0505	0.8004	96.048	159.4797	44.6223

Theoretical data

stress 1 (Pa)	stress 2 (Pa)	stress 3 (Pa)	stress 4 (Pa)	stress 5 (Pa)	stress 6 (Pa)
29322079.6	18440411.2	0	18440411.2	29322079.6	29322079.6
58644159.1	36880822.4	0	36880822.4	58644159.1	58644159.1
87966238.7	55321233.5	0	55321233.5	87966238.7	87966238.7
117288318	73761644.7	0	73761644.7	117288318	117288318
146610398	92202055.9	0	92202055.9	146610398	146610398

The stress in points 1 to 3 are in compression and all the points below are in tension except stress 6 which is transversal. It seems both the theoretical and experimental values follow a similar trend and the experimental value does hold some noticeable inaccuracy but still resembles the theoretical values.

6. Discussion

Plane section before bending remains plane after bending means that strain at any point on cross section of the beam is proportional to its distance which is depicted by a linear strain-distance diagram. The diagram obtained in this experiment was in fact linear meaning that it substantiates the usual assumption that a plane section before bending remains plane after bending.

The discrepancies due to the theoretical and experimentally determined values have a high chance of being linked to the fact that the sample nor any of the material was ASTM approved. The beam has been placed under pure bending multiple times which has already warped it (plastic deformation) which is probably the cause for the neutral axis not being in the complete center. The stress values were off by a substantial amount which is probably due to the load applied by the non-ASTM approved pump.

$\sigma_m = \frac{My}{I}$ $\sigma = \frac{E\epsilon}{\rho}$
 $\frac{1}{\rho} = \frac{M}{EI}$ $\sigma_1 = E_1 \epsilon_1 = \frac{E_1 y}{\rho}$
 $\frac{EI}{\rho} = M$ $\sigma_2 = E_2 \epsilon_2 = \frac{E_2 y}{\rho}$

$E_{Al} = 70 \text{ GPa}$
 $E_{St} = 200 \text{ GPa}$

① $\frac{E_2}{E_1} = \frac{200}{70} = 2.86$
 $n = 2.86$
 $30(2.86) = 85.8$
 24.477

② $\bar{y} = \frac{\sum Ay}{\sum A} = \frac{(50 \text{ mm})(60) + (85.8)(40)}{(30)(20) + (85.8)(40)}$
 $\bar{y} = \frac{9600}{4020} = 24.477 \text{ mm}$

③ Moment of inertia

$I_c = \frac{bk^3}{12} + D^2a$
 $= 4.56 \times 10^{-7} + 7.78 \times 10^{-8}$
 $= 5.33 \times 10^{-7} \text{ m}^4$

$I_c = \frac{bk^3}{12} + n^2a$
 $= 2 \times 10^{-8} + 3.9 \times 10^{-7}$
 $= 4.1 \times 10^{-7} \text{ m}^4$
 $I = 5.438 \times 10^{-7} \text{ m}^4$

④ $\sigma = \frac{My}{I}$
 $\sigma = \frac{(2000)(0.024477)}{5.438 \times 10^{-7}}$
 $\sigma = 76.27 \text{ MPa at the top of loose section}$
 $\sigma = 51.869 \text{ MPa}$

7. References

Beer F. and Johnston, R. (1992) Mechanics of Materials, McGraw-Hill.

“Mechanics of Materials: Bending – Normal Stress ” Mechanics of Slender Structures: Boston University.” *Mechanics of Slender Structures RSS*, www.bu.edu/moss/mechanics-of-materials-bending-normal-stress/#:~:text=Bending%20results%20from%20a%20couple,to%20as%20the%20neutral%20axis. Accessed 30 Mar. 2024.

“Neutral Axis.” *Wikipedia*, Wikimedia Foundation, 10 Aug. 2023, en.wikipedia.org/wiki/Neutral_axis.