Experiment 6

Behavior of Columns Under Axial Loads

Yasin Williot (40292208)

Section PI-X

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Lab group members:

Sean Boyle

Alex Bouchard Doyle

1. Objective

The objective of this lab is to determine the experimental determination of buckling loads for columns of various lengths

2. Introduction

Any column subjecting to an increasing load will start deforming and then buckle at a specific load. This load, $P_{cr,}$ depends on many factors. Some of these factors are the column length, the support conditions, the cross-sectional shape of the column and the location of the load in relation to the centroid of the cross section.

3. Procedure

Equipment

- Tension/compression testing machine
- Load measuring devices
- Vernier calipers
- Aluminum tubes and rods: 225,125 and 75 mm long

Start by measuring the outer and inner diameter of each specimen. Then start with the 225 mm specimens and perform a test for each following different support conditions. The three conditions are both ends fixed, one end fixed while the other is pinned and both ends being pinned. Gradually increase the compressive load on every sample with a steady pace and be wary for the onset of bending. Be sure to record the maximum load. Then for the 125 mm and 75 mm length specimens, conduct the test with both ends being pinned.

Sketches of all the samples at failure

Sample 1

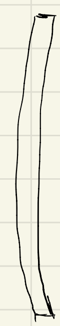
sample 2

sample 3

Sample 4

Sample 5

sample 6



Sample 7

Sample 8

4.Analysis of the results

Formulas

$$(1)A = \pi r^2$$
$$(2)A = \pi(R^2 - r^2)$$
$$(3)\frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$(4)P_{cr} = \frac{\pi^2 EI}{L^2}$$
$$(5)\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$
$$(6)\sigma_{all} = 131\ Mpa\ for\ \lambda < 9.5$$
$$(7)\sigma_{all} = 139 - 0.868\lambda\ Mpa\ for\ 9.5 < \lambda < 66$$
$$(8)\sigma_{all} = \frac{351000}{\lambda^2}\ \ Mpa\ for 66 < \lambda$$
$$(9)r = \sqrt{\frac{I}{a}}$$

Sample calculation

Cross section

$$A = \pi(\frac{6.34}{2})^2$$
$$A = 31.56\, mm^2$$

Radius of gyration

$$r = \sqrt{\frac{I}{a}}$$
$$r = \sqrt{\frac{79.30}{31.56}}$$
$$r = 1.585\ mm$$

Critical load

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$
$$P_{cr} = \frac{\pi^2(70\,000)(79.30)}{(225)^2}$$
$$P_{cr} = 1082.3N$$

Experimental critical stress

$$\sigma = \frac{P_{cr}}{A}$$

$$\sigma = \frac{785}{31.56}$$
$$\sigma = 24.86\ MPa$$

Slenderness ratio

$$\lambda = \frac{KL}{r}$$

K=2 one end fixed, one end free
K=1 both ends pinned
K=$\frac{1}{\sqrt{2}}$ one end fixed, one end pinned
K=$\frac{1}{2}$ both ends fixed

$$\lambda = \frac{KL}{r}$$
$$\lambda = \frac{(225)(1)}{1.585}$$

$$\lambda = 141.955$$

Theoretical critical stress using Euler's formula

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2}$$
$$\sigma_{cr} = \frac{\pi^2 (70\ 000)}{(141.955)^2}$$
$$\sigma_{cr} = 34.24\ MPa$$

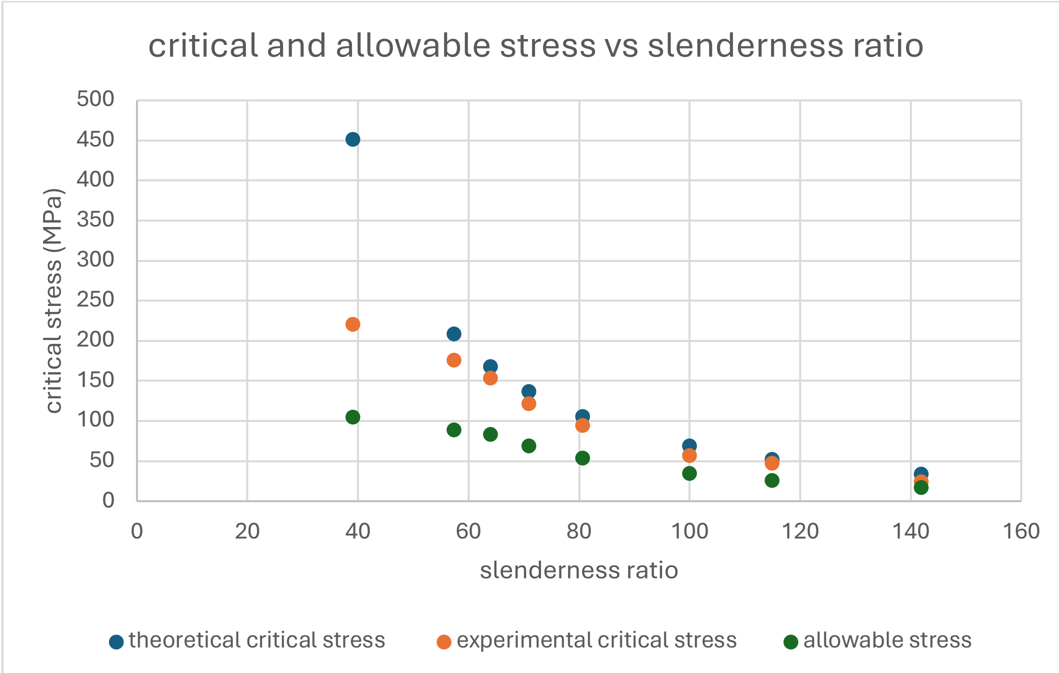
Table of tabulated section properties

column	L(mm)	connection	K	effective L	inner d	outer d
hollow	75	pin-pin	1	75	4.3	6.35
	125	pin-pin	1	125	4.51	6.38
	225	pin-pin	1	225	4.52	6.39
	225	pin-fixed	0.707	159.075	4.6	6.4
	225	fixed-fixed	0.5	112.5	4.56	6.37
solid	225	pin-pin	1	225	6.34	
	225	pin-fixed	0.707	159.075	6.36	
	225	fixed-fixed	0.5	112.5	6.34	

Respective calculations of each sample

A (mm^2)	I(mm^4)	r(mm)	λ ratio	σall (mpa)	theo. Pcr(N)	theo.σcr	exp. Pcr(N)	exp.σcr
17.1472054	63.0293761	1.91723271	39.118882	105.04481	7741.37788	451.465862	3792	221.143907
15.9940838	61.0219281	1.95327449	63.9951017	83.4522517	2698.13506	168.695818	2467	154.244534
16.0234577	61.3523174	1.95675912	114.986049	26.5470833	837.266511	52.2525491	767	47.8673214
15.5508836	60.3763057	1.97040605	80.7320907	53.8535971	1648.39185	105.999883	1471	94.5926955
15.5377675	59.597535	1.95848315	57.442414	89.1399846	3253.27696	209.378662	2737	176.151432
31.5695504	79.3098138	1.585	141.955836	17.4180933	1082.32996	34.2839839	785	24.8657326
31.7690416	80.3153139	1.59	100.04717	35.0669102	2192.76597	69.0221003	1821	57.319954
31.5695504	79.3098138	1.585	70.977918	69.6723733	4329.31983	137.135936	3849	121.92128

Graph of critical and allowable stress vs slenderness ration



5. Discussion

1. It can be observed that for all the samples, the experimental critical stress obtained is significantly lower than the theoretical values obtained with Euler's formula. Some factors that might account for the discrepancies and the lower values are that all the equipment used and the samples are not ASTM approved, that the pins holding the samples are not correctly aligned creating unequal stressed and the tubes were not completely straight.
2. All the critical stresses of the samples are still higher than the allowable stress in aluminum based on their slenderness ratio. This is expected since the allowable stress equations were formulated to account for defect in the materials.
3. The main factors that affect the buckling strength of real columns are length, geometry, elastic modulus of the material used. Some other factors are imperfections in the column, the end conditions of the column, the temperature or environmental factors and dynamics effects like cyclic loading or resonance conditions.
4. The tube provides stronger resistance to a buckling load since it has a greater radius of gyration due to it's smaller area. This leads to a smaller slenderness ratio which creates a smaller denominator in the σ_{cr} formula leading to a larger stress.

5. References

Beer F. and Johnston, R. (1992) Mechanics of Materials, McGraw-Hill.

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