Experiment #5

Deflection of Beams

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Section PI-X

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1. Objective

The objective of this lab is to evaluate the relationship between the load and deflection of a simply supported and cantilever beam. The data obtained is then used to determine the modulus of elasticity of the different samples.

2. Introduction

A beam will always deflect to some extent when subjected to a transverse load. The degree of deflection is dependent upon the magnitude, location of the loads and the span. This knowledge is very important to engineers since many building codes put certain limits upon structural components due to functional considerations.

Equipment
-load machines
-deflection gauges
- vernier caliper
-micrometer
- steel, aluminum and brass bars of rectangular cross section

3. Procedure

Part A (point load on a simply supported beam, hydraulic load machine)

First measure the cross section of each beam at three different locations. Note that the span between the two supports is 455 mm. Position the specimen on the supports and verify that the sample is properly centered on the line of action of the load. Position the two deformations gauges that are mounted to measure the vertical deflection at the center and the quarter points of the beam span. Then proceed to load the beam in increments of 200 N up to 1000 N and record the deflection in the middle of the span and then at the quarters of the beam span. This is done to an accuracy of 0.01mm. Be careful to not exceed the load of 1000 N or the beams will be damaged. Repeat this procedure for the two other beam.

Part B (point load on cantilever beams, STR1 Test Frame)

Note which material is being used. Measure the cross section using the micrometer for precise measurements at here different points. Then place the load hanger at the free end of the beam at the length "L" from the fixed end, and under the display meter. Then proceed to measure the deflection at x=L and x=0.5L for load increments of 100 grams up to the maximum load of 500 grams. Be sure to consider the weight of the load hanger.

3. Analysis of the results

PART A

(1) $\frac{1}{p} = \frac{m(x)}{EI}$

(2) $\frac{1}{p} = \frac{d^2y}{dx^2}$

Expression for elastic curve

$$\frac{1}{p} = \frac{m(x)}{EI} = \frac{d^2y}{dx^2}$$

$$EI\frac{d^2y}{dx^2} = \frac{P}{2}x$$

Integrate with respect to x

$$EI\frac{dy}{dx} = \frac{p}{4}(x^2) + c_1$$

$$x = \frac{L}{2}, \frac{dy}{dx} = 0$$

$$0 = \frac{p}{2}\frac{\left(\frac{l}{2}\right)^2}{2} + c_1$$

$$c_1 = -\frac{PL^2}{16}$$

$$EI\frac{dy}{dx} = \frac{p}{4}(x^2) - \frac{PL^2}{16}$$

Integrate with respect to x again

$$ELy = \frac{Px^3}{12} - \frac{PL^2}{16}x + c_2$$

$$x = 0, y = 0$$

$$EL(0) = \frac{P0^3}{12} - \frac{P0^2}{16}x + c_2$$

$$c_2 = 0$$

$$ELy = \frac{Px^3}{12} - \frac{PL^2}{16}x$$

$$ELy = \frac{P(\frac{L}{2})^3}{12} - \frac{P(\frac{L}{2})^2}{16}x$$

$$y_{max} = \frac{Pl^3}{48EI}$$

Theoretical deflection of the simply supported beam sample calculation

$$y_{max} = \frac{Pl^3}{48EI}$$

$$y_{max} = \frac{(200N)(455mm)^3}{48(200\ 000MPa)(3250mm^4)}$$

$$y_{max} = 0.60382\ mm$$

Tabulated theoretical and experimental values of deflection

Theoretical deflection values (mm)						
load(N)	brass		steel		Aluminum	
	x=L/2	x=L/4	x=L/2	x=L/4	x=L/2	x=L/4
200	1.15440129	0.79365089	0.60382292	0.41512826	1.65006683	1.13442094
400	2.30880259	1.58730178	1.20764583	0.83025651	3.30013366	2.26884189
600	3.46320388	2.38095267	1.81146875	1.24538477	4.95020049	3.40326283
800	4.61760517	3.17460356	2.41529167	1.66051302	6.60026731	4.53768378
1000	5.77200647	3.96825445	3.01911458	2.07564128	8.25033414	5.67210472

Experimental deflection values (mm)						
load(N)	Brass		Steel		aluminum	
	x=L/2	x=L/4	x=L/2	x=L/4	x=L/2	x=L/4
200	1.41	1.05	0.65	0.49	1.8	1.3
400	3	2.17	1.3	0.99	3.61	2.58
600	4.6	3.24	1.98	1.48	5.38	3.83
800	6.22	4.45	2.67	1.98	7.22	5.12
1000	7.84	5.59	3.37	2.49	9.02	6.39

Sample calculation of E

$$E = \frac{Pl^3}{48YI}$$

$$E = \frac{(200N)(455mm)^3}{48(0.65mm)(3250mm^4)}$$

$$E = 185\ 791\ MPa$$

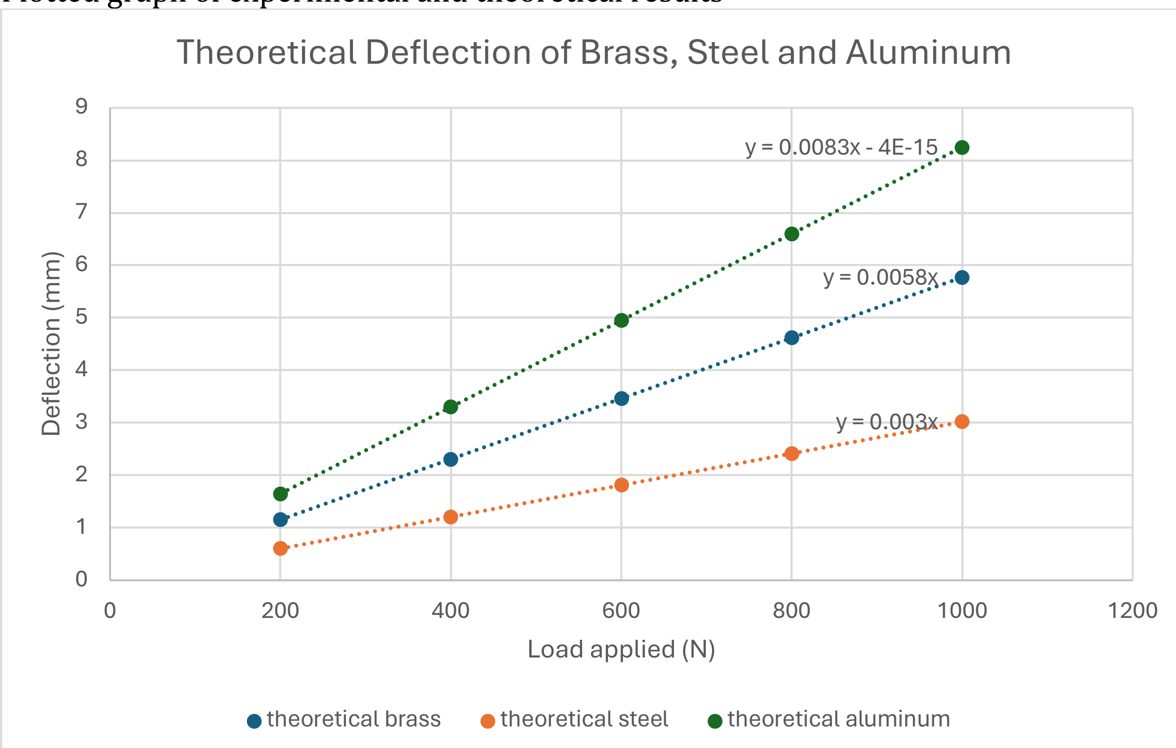
Tabulated experimental values of E

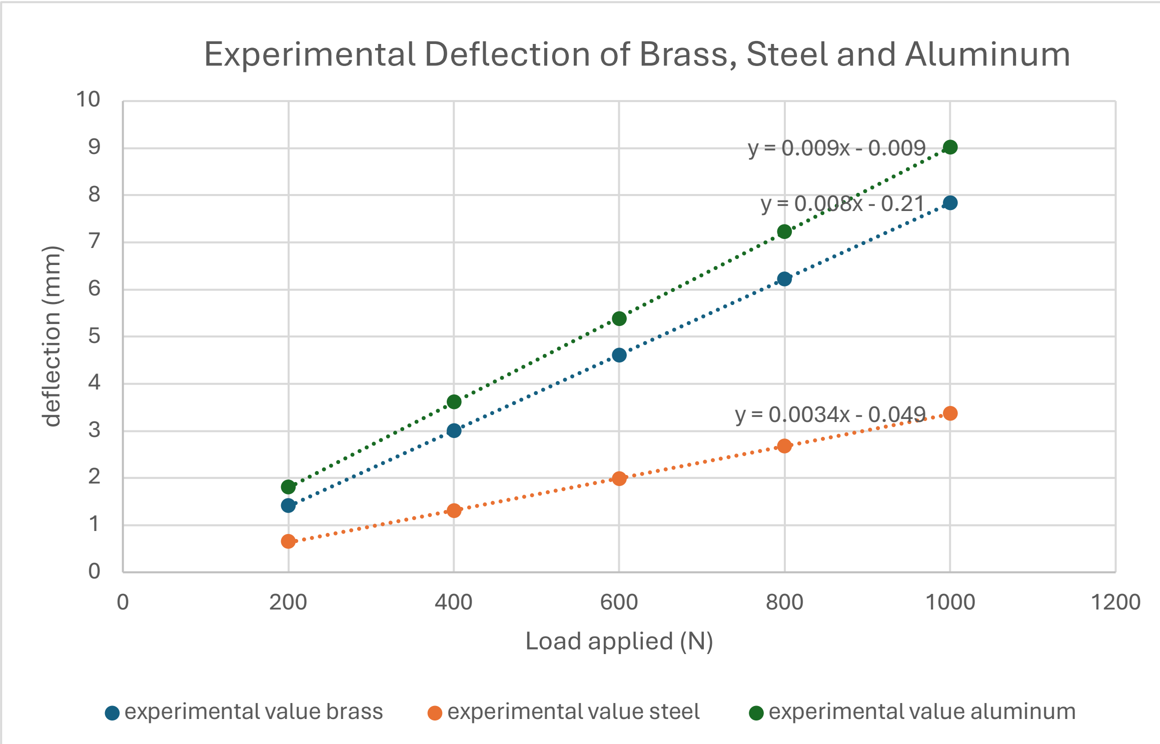
Experimental Modulus of Elasticity						
Load (N)	Brass		Steel		aluminum	
	x=L/2	x=L/4	x=L/2	x=L/4	x=L/2	x=L/4
200	85966.0538	79365.0889	185791.667	170068.048	64169.2656	61084.2047
400	80808.0906	76804.9248	185791.667	168350.189	63991.5114	61557.7257
600	79051.3929	77160.5031	182976.641	168918.939	64407.813	62200.6262
800	77949.9266	74906.3761	180920.724	168350.189	63991.5114	62038.6454
1000	77303.6581	74537.8742	179175.94	167336.031	64026.9834	62135.7325

Average E for brass = 78.385 GPa
Average E for steel = 175.768 GPa
Average E for aluminum=62.960 GPa

The published value for brass is 105 GPa and the experimental value is 78.385. 26.615 GPa is quite a significant inaccuracy. The published value for steel is 200 GPa and the experiment value is 175.768 GPa. Again, a difference 0f 24.232 GPa is quite large. The published value for aluminum is 70 GPa and the experimental value is 62.90 GPa. A difference of 7.1 GPa is much more accurate than the other results.

Plotted graph of experimental and theoretical results





Part B

Experimental Cantilever Beam Deflexion (mm)						
Load (gm)	brass		steel		aluminum	
	x=L/2	x=L	x=L/2	x=L	x=L/2	x=L
119.4	0.39	1.1	0.2	0.53	0.46	1.43
218.3	0.66	2.05	0.39	1.1	0.9	2.77
314	0.98	3.1	0.57	1.66	1.33	4.09
422	1.28	4.12	0.75	2.2	1.77	5.46
523.04	1.58	5.14	0.95	2.8	2.21	6.79

Theoretical Cantilever Beam Deflexion (mm)						
Load (N)	brass		steel		aluminum	
	x=L/2	x=L	x=L/2	x=L	x=L/2	x=L
1.171314	0.27155397	0.86897271	0.18727179	0.59926971	0.44261759	1.41637629
2.141523	0.496484353	1.58874993	0.34239054	1.09564973	0.80924137	2.5895724
3.08034	0.714136907	2.2852381	0.49249029	1.57596893	1.16400271	3.72480867
4.13982	0.959763614	3.07124356	0.66188185	2.11802193	1.56436033	5.00595305
5.1310224	1.189561044	3.80659534	0.82035707	2.62514263	1.93891713	6.2045348

Estimated Elastic modulus values					
brass		steel		aluminum	
x=L/2	x=L	x=L/2	x=L	x=L/2	x=L
73110.6843	82947.3946	187271.785	226139.514	67354.8508	69333.1051
78986.1471	81374.99638	175584.893	199209.043	62940.9958	65440.4577
76514.6686	77403.22606	172803.61	189875.774	61263.3005	63749.7817
78730.609	78271.98406	176501.828	192547.448	61867.3577	64178.8853
79053.1074	77761.18889	172706.752	187510.188	61413.6646	63964.2763

Average E for brass=78.415 GPa
Average E for steel =188.015 GPa
Average E for aluminum=64.150 GPa

Sample calculation of theoretical deflection

$$y = \frac{pl^3}{3EI}$$

$$y = \frac{(1.17N)(250mm)^3}{3(50.9mm^4)(200000Mpa)}$$

$$y = 0.5999mm$$

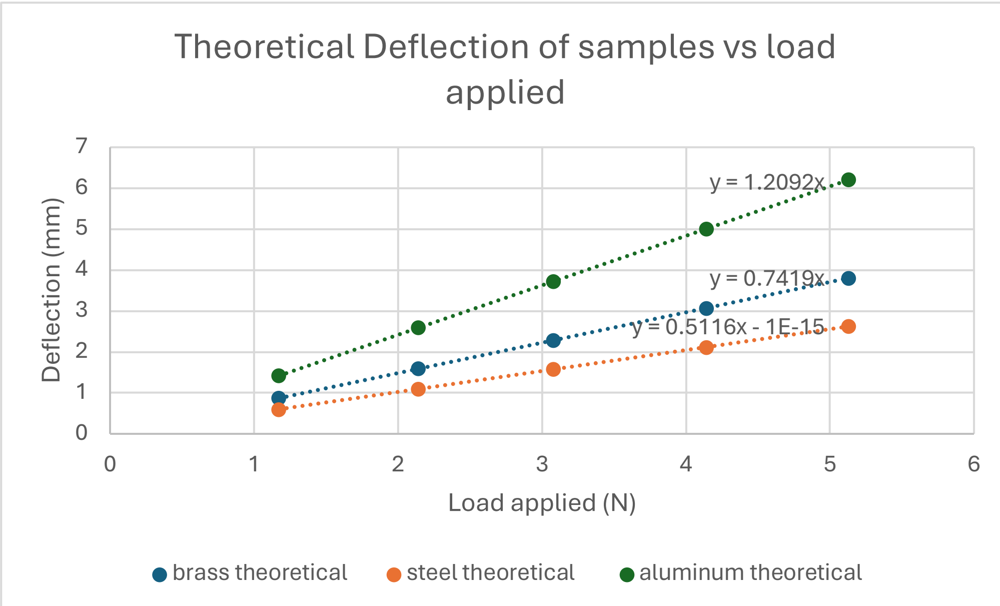
Sample calculation of E

$$E = \frac{pl^3}{3YI}$$

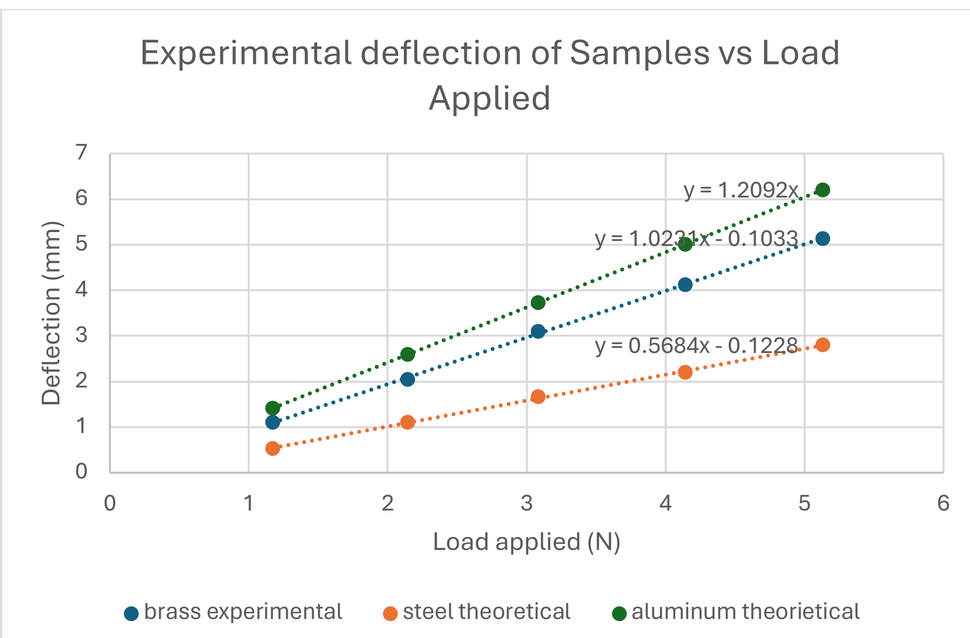
$$E = \frac{(1.17N)(250mm)^3}{3(50.9mm^4)(0.53mm)}$$

$$E = 226\ 139\ MPa$$

Theoretical deflections of cantilever beam



Experimental delfections of cantilever beams



4. Discussion

The accuracy of the theoretical equations is dependent on the measured values in the equations. So the accuracy of these equations is dependent on how accurate the load applied was measure and the accuracy of the caliper or micrometer used to measure the width, height and thickness. Since the load machine is not ASTM approved, these equations should not be regarded as accurate. The experimental deflections in both the simply supported beam and the cantilever beam across all samples and spans were significantly higher than the theoretical values obtain. This indicates that the materials were all weakened. The probable cause for this is that the materials were all slightly plastically deformed due to the numerous trials done on them. Some of these beams were already warped from the start indicating plastic deformation. This also means that these beams are obviously not ASTM approved.

Sample	simply supp.	Cantilever	published
Brass E	78.385 GPa	78.415 GPa	105 GPa
Steel E	175.768 GPa	188.015 GPa	200 GPa
aluminum E	62.960 GPa	64.150 GPa	70 GPa

All the experimental data E values obtained for both loading scenarios show smaller resistance to elastic deformation which the causes for were explained previously. It seems that the cantilever data obtained for E was closer to the published values possibly indicating that the cantilever test was more accurate.

5. References

Beer F. and Johnston, R. (1992) Mechanics of Materials, McGraw-Hill.

Engineeringtoolbox, Editor. “Cantilever Beams - Moments and Deflections.” *Engineering ToolBox*, 7 Nov. 2023, www.engineeringtoolbox.com/cantilever-beams-d_1848.html.

“Beam Deflection Tables.” *MechaniCalc*, mechanicalc.com/reference/beam-deflection-tables. Accessed 3 Apr. 2024.