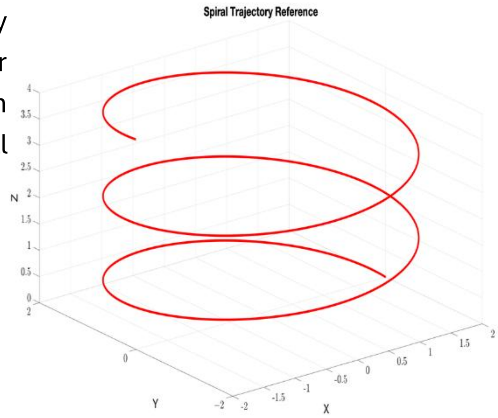
CENG597 Numerical Optimization
Project Assignment

In this project, you will simulate and reconstruct a drone trajectory that follows a spiral path around an object (e.g., a building, car, or tower). You will then recover the true path from a noisy observation using numerical optimization techniques. The project will emphasize:



- Synthetic data generation and visualization
- Cost function design and derivative analysis
- Optimization using Line Search, Trust Region, and Conjugate Gradient methods
- Analysis of convexity and convergence behavior

Submit a ZIP file containing two Jupyter Notebooks: *01_generate_data.ipynb* & *02_optimize_path.ipynb*.
Each notebook must begin with a Markdown cell listing: *Student Name(s)* & *Student ID(s)*:
You will do it alone or as two-students team. Only one member will submit the ZIP file on Aybuzem.

Notebook 1 — Generate Synthetic Drone Trajectory

- Simulate a spiral trajectory around a target object.
  Use parametric equations: $x = r\cos(\theta)$, $y = r\sin(\theta)$, $z = h\theta$
  where $\theta \in [0, 4\pi]$, r and h are constants controlling spiral radius and height.
- Add noise and deformation to obtain a realistic observed trajectory:
- For each trajectory point define a 6-DOF vector: $p_i = [x_i, y_i, z_i, n_{x,i}, n_{y,i}, n_{z,i}]$ where $[n_{x,i}, n_{y,i}, n_{z,i}]$ represents the camera orientation vector pointing toward the object center.
- Visualize the noise-free (unknown) and noisy (observed) trajectories in 3D (use matplotlib's 3D plotting). Show both the ideal (noise-free) and noisy trajectories.
- Save all generated data (positions, orientations, noise levels, etc.) to a compressed NumPy file (.npz).

Notebook 2 — Path Reconstruction via Optimization

- Given a noisy observed trajectory $[\ldots, p_{i-1}, p_i, p_{i+1}, \ldots]$, reconstruct the true (unknown) trajectory.
- Define a cost function that balances data fidelity (reconstructed path must be close to noisy curve) and smoothness (smooth path).
- Balance between data fidelity and smoothness must be controlled with λ which will be a positive coefficient in the smoothness regularization term. The first term (data fidelity) should enforce similarity to observed (noisy) data. While $\lambda > 0$ controls regularization strength, the second term should enforce smoothness along the trajectory.
- Show the gradient and Hessian of the cost function.
- Implement solvers:
  - Line Search Methods (first-order & Newton-based second-order)
  - Trust Region Method
  - (Optional) Use Conjugate Gradient (CG) and/or Preconditioned CG (PCG)
- Record and visualize: (a) Cost vs. Iteration plots for each solver, (b) Reconstructed trajectory compared to noisy and ground-truth trajectories
- Discuss convergence: (a) How fast and stable each method converges, (b) How noise and regularization affect results
- Theoretical Analysis (Markdown Cells)
- Each notebook must include clear explanations with LaTeX-formatted equations:
  - Derivation of the cost function and its gradient/Hessian
  - Mathematical background of chosen solvers
  - Convexity analysis (convex or strictly convex, or not convex at all), does it have a unique global minimum? How does λ affect convexity?