Assignment 2

AI1110: Probability and Random Variables
Indian Institute of Techonology Hyderabad

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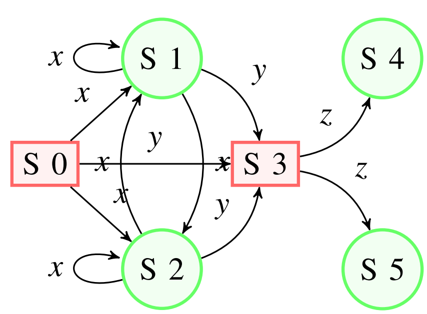
12.13.6.18: Question. Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Answer: 0

Solution: Using Markov Chain Approach,
Let the states **S** be,

1) S 0 : The initial state.
2) S 1 : 3 comes on die throw.
3) S 2 : 6 comes on die throw.
4) S 3 : non-multiple of 3 comes on die throw.
5) S 4 : Heads comes up on coin toss.
6) S 5 : Tails comes up on coin toss.

The Markov Chain for the given states is as follows,



$$x = \frac{1}{6}, y = \frac{4}{6}, z = \frac{1}{2}$$

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The State Transition Matrix is as follows,

	0	1	2	3	4	5
0	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$	0	0
1	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$	0	0
2	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{4}{6}$	0	0
3	0	0	0	0	$\frac{1}{2}$	$\frac{1}{2}$
4	0	0	0	0	1	0
5	0	0	0	0	0	1

We know,

$$p_{ij} = \Pr\left(X = j \mid X = i\right) \tag{1}$$

$$\forall i, j \in S$$

$$\sum_{j=0}^{j=5} p_{ij} = 1 \tag{2}$$

To find,

$$p_{15} = \Pr\left(X = 5 \mid X = 1\right) \tag{3}$$

From (2),

$$\sum_{j=0}^{j=5} p_{1j} = 1 \tag{4}$$

Consider a fair coin,

$$p_{14} = p_{15} \tag{5}$$

Considering a 6 sided unbiased die,

$$p_{11} = p_{12} = \frac{p_{13}}{4} \tag{6}$$

Also,

$$p_{11} = \frac{1}{6} \tag{7}$$

From (6)

$$p_{11} = p_{12} = \frac{p_{13}}{4} = \frac{1}{6} \tag{8}$$

$$\implies p_{13} = \frac{4}{6} \tag{9}$$

From (4),

$$p_{10} + p_{11} + p_{12} + p_{13} + p_{14} + p_{15} = 1 \tag{10}$$

$$\implies 0 + \frac{1}{6} + \frac{1}{6} + \frac{4}{6} + p_{14} + p_{15} = 1 \tag{11}$$

From (5)

$$\implies 2 \times p_{15} = 0 \tag{12}$$

$$\implies p_{15} = 0 \tag{13}$$