

# Assignment 2

AI1110: Probability and Random Variables  
Indian Institute of Technology Hyderabad

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**12.13.6.18: Question.** Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event ‘the coin shows a tail’, given that ‘at least one die shows a 3’.

**Answer: 0**

**Solution:** Using Markov Chain Approach,  
Let the states **S** be,

- 1) S 1 : 3 comes on die throw.
- 2) S 2 : 6 comes on die throw.
- 3) S 3 : non-multiple of 3 comes on die throw.
- 4) S 4 : Heads comes up on coin toss.
- 5) S 5 : Tails comes up on coin toss.

We know,

$$p_{ij} = \Pr(X = j | X = i) \quad (1)$$

$$\forall i, j \in S$$

$$\sum_{j=1}^{j=5} p_{ij} = 1 \quad (2)$$

To find,

$$p_{15} = \Pr(X = 5 | X = 1) \quad (3)$$

From 2,

$$\sum_{j=1}^{j=5} p_{1j} = 1 \quad (4)$$

Consider a fair coin,

$$p_{14} = p_{15} \quad (5)$$

Considering a 6 sided unbiased die,

$$p_{11} = p_{12} = \frac{p_{13}}{4} \quad (6)$$

Also,

$$p_{11} = \frac{1}{6} \quad (7)$$

From (6)

$$p_{11} = p_{12} = \frac{p_{13}}{4} = \frac{1}{6} \quad (8)$$

$$\Rightarrow p_{13} = \frac{4}{6} \quad (9)$$

From (4),

$$p_{11} + p_{12} + p_{13} + p_{14} + p_{15} = 1 \quad (10)$$

$$\Rightarrow \frac{1}{6} + \frac{1}{6} + \frac{4}{6} + p_{14} + p_{15} = 1 \quad (11)$$

From (5)

$$\Rightarrow 2 \times p_{15} = 0 \quad (12)$$

$$\Rightarrow p_{15} = 0 \quad (13)$$

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