Planning and search

Logical agents; FOL

Outline

- ♢ First-order logic
- ♢ Syntax
- ♢ Semantics
- ♢ What can we express

First-order logic

Whereas propositional logic assumes world contains **facts**,
first-order logic (like natural language) assumes the world contains

- Objects: people, houses, numbers, theories, Ronald McDonald, colors, baseball games, wars, centuries . . .
- Relations: red, round, bogus, prime, multistoried . . .,
  brother of, bigger than, inside, part of, has color, occurred after, owns, comes between, . . .
- Functions: father of, best friend, third inning of, one more than, end of . . .

Logics in general

Language	Ontological Commitment	Epistemological Commitment
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief
Fuzzy logic	facts + degree of truth	known interval value

Syntax of FOL: Basic elements

Constants	$KingJohn,\ 2,\ UCB, \ldots$
Predicates	$Brother,\ >, \ldots$
Functions	$Sqrt,\ LeftLegOf, \ldots$
Variables	$x,\ y,\ a,\ b, \ldots$
Connectives	$\land \quad \lor \quad \neg \quad \Rightarrow \quad \Leftrightarrow$
Equality	$=$
Quantifiers	$\forall\ \exists$

Atomic sentences

Atomic sentence $= predicate(term_1, \ldots, term_n)$
or $term_1 = term_2$

Term $= function(term_1, \ldots, term_n)$
or $constant$ or $variable$

E.g., $Brother(KingJohn, RichardTheLionheart)$
$> (Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))$

Complex sentences

Complex sentences are made from atomic sentences using connectives

$$\neg S, \quad S_1 \land S_2, \quad S_1 \lor S_2, \quad S_1 \Rightarrow S_2, \quad S_1 \Leftrightarrow S_2$$

E.g. $Sibling(KingJohn, Richard) \Rightarrow Sibling(Richard, KingJohn)$
$>(1,2) \lor \leq(1,2)$
$>(1,2) \land \neg >(1,2)$

Truth in first-order logic

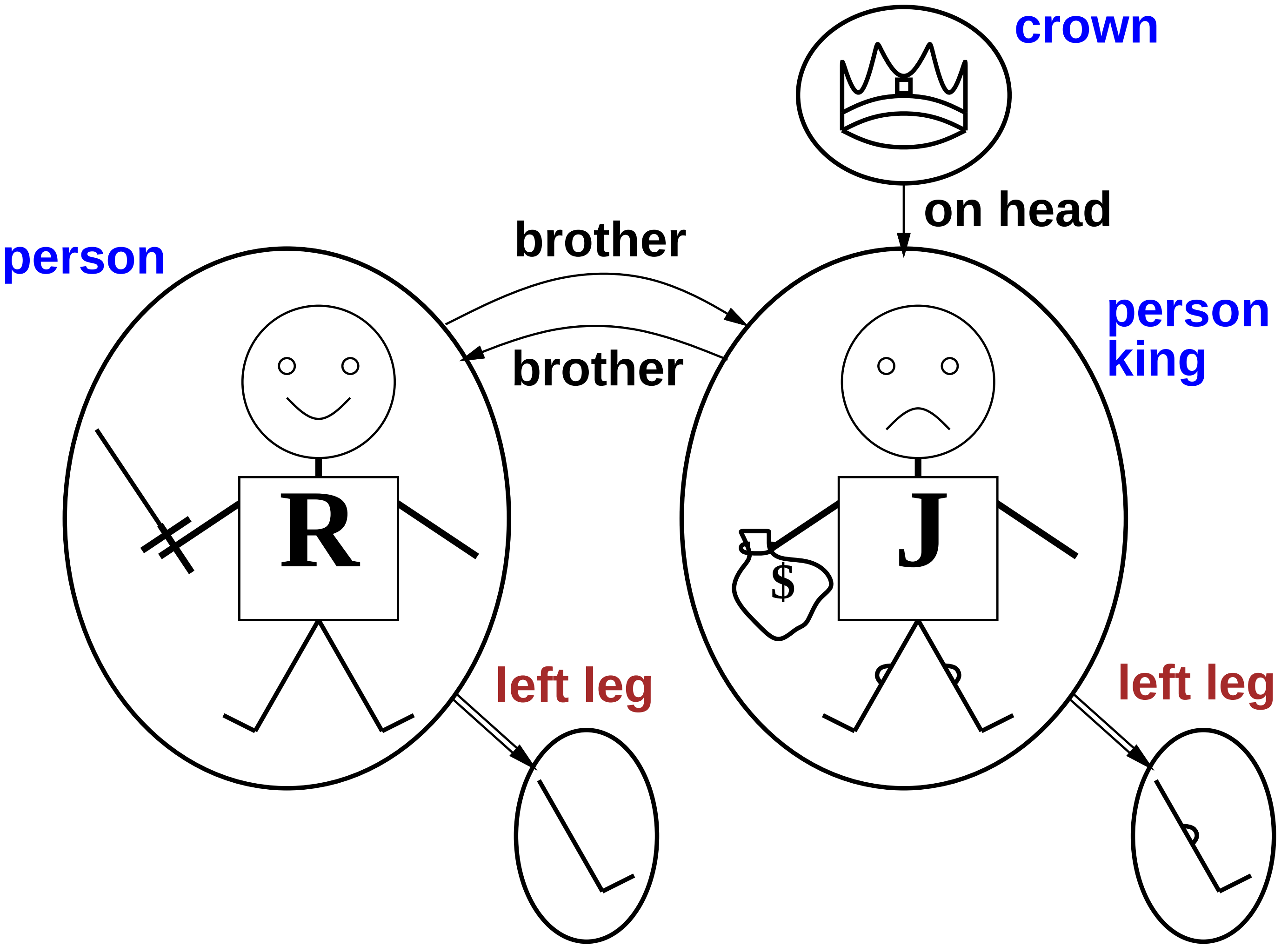
Sentences are true with respect to a model and an interpretation

Model contains ≥ 1 objects (domain elements) and relations among them

Interpretation specifies referents for
constant symbols $\rightarrow$ objects
predicate symbols $\rightarrow$ relations
function symbols $\rightarrow$ functional relations

An atomic sentence $predicate(term_1, \ldots, term_n)$ is true
iff the objects referred to by $term_1, \ldots, term_n$
are in the relation referred to by $predicate$

Models for FOL: Example



Truth example

Consider the interpretation in which
$Richard$ → Richard the Lionheart
$John$ → the evil King John
$Brother$ → the brotherhood relation

Under this interpretation, $Brother(Richard, John)$ is true just in case Richard the Lionheart and the evil King John are in the brotherhood relation in the model

Models for FOL: Lots!

Entailment in propositional logic can be computed by enumerating models

We **can** enumerate the FOL models for a given KB vocabulary:

For each number of domain elements n from 1 to ∞
    For each k-ary predicate P_k in the vocabulary
        For each possible k-ary relation on n objects
            For each constant symbol C in the vocabulary
                For each choice of referent for C from n objects . . .

Computing entailment by enumerating FOL models is not easy!

Universal quantification

$\forall \langle variables \rangle \ \ \langle sentence \rangle$

Everyone at Berkeley is smart:
$\forall x \ \ At(x, Berkeley) \ \Rightarrow \ Smart(x)$

$\forall x \ \ P$ is true in a model m iff P is true with x being **each** possible object in the model

Roughly speaking, equivalent to the conjunction of instantiations of P

$$
\begin{aligned}
& (At(KingJohn, Berkeley) \ \Rightarrow \ Smart(KingJohn)) \\
\land \ & (At(Richard, Berkeley) \ \Rightarrow \ Smart(Richard)) \\
\land \ & (At(Berkeley, Berkeley) \ \Rightarrow \ Smart(Berkeley)) \\
\land \ & \ldots
\end{aligned}
$$

A common mistake to avoid

Typically, $\Rightarrow$ is the main connective with $\forall$

Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall\, x \;\; At(x, Berkeley) \wedge Smart(x)$$

means "Everyone is at Berkeley and everyone is smart"

Existential quantification

$\exists \langle variables \rangle \ \ \langle sentence \rangle$

Someone at Stanford is smart:
$\exists\, x \ \ At(x, Stanford) \land Smart(x)$

$\exists\, x \ \ P$ is true in a model m iff P is true with x being **some** possible object in the model

Roughly speaking, equivalent to the disjunction of instantiations of P

$$
\begin{array}{ll}
 & (At(KingJohn, Stanford) \land Smart(KingJohn)) \\
\lor & (At(Richard, Stanford) \land Smart(Richard)) \\
\lor & (At(Stanford, Stanford) \land Smart(Stanford)) \\
\lor & \ldots
\end{array}
$$

Another common mistake to avoid

Typically, $\wedge$ is the main connective with $\exists$

Common mistake: using $\Rightarrow$ as the main connective with $\exists$:

$$\exists\, x \;\; At(x, Stanford) \Rightarrow Smart(x)$$

is true if there is anyone who is not at Stanford!

Properties of quantifiers

$\forall\, x\ \ \forall\, y$ is the same as $\forall\, y\ \ \forall\, x$ (why??)

$\exists\, x\ \ \exists\, y$ is the same as $\exists\, y\ \ \exists\, x$ (why??)

$\exists\, x\ \ \forall\, y$ is **not** the same as $\forall\, y\ \ \exists\, x$

$\exists\, x\ \ \forall\, y\ \ Loves(x, y)$
"There is a person who loves everyone in the world"

$\forall\, y\ \ \exists\, x\ \ Loves(x, y)$
"Everyone in the world is loved by at least one person"

Quantifier duality: each can be expressed using the other

$\forall\, x\ \ Likes(x, IceCream) \qquad \neg\exists\, x\ \ \neg Likes(x, IceCream)$

$\exists\, x\ \ Likes(x, Broccoli) \qquad \neg\forall\, x\ \ \neg Likes(x, Broccoli)$

Fun with sentences

Brothers are siblings

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$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y).$

"Sibling" is symmetric

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Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

One's mother is one's female parent

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

"Sibling" is symmetric

$\forall\, x, y \;\; Sibling(x, y) \;\Leftrightarrow\; Sibling(y, x)$.

One's mother is one's female parent

$\forall\, x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y))$.

A first cousin is a child of a parent's sibling

Fun with sentences

Brothers are siblings

$\forall\, x, y \;\; Brother(x, y) \;\Rightarrow\; Sibling(x, y)$.

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$\forall\, x, y \;\; Mother(x, y) \;\Leftrightarrow\; (Female(x) \land Parent(x, y))$.

A first cousin is a child of a parent's sibling

$\forall\, x, y \;\; FirstCousin(x, y) \;\Leftrightarrow\; \exists\, p, ps \;\; Parent(p, x) \land Sibling(ps, p) \land Parent(ps, y)$