CSCE 221 Cover Page
Homework Assignment #3
Due November 23 at 23:59 pm to eCampus

First Name Last Name UIN

User Name E-mail address

Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more on Aggie Honor System Office website: `http://aggiehonor.tamu.edu/`

Type of sources				
People				
Web pages (provide URL)				
Printed material				
Other Sources				

I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.
On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work.

Your Name Date

Homework 3 (100 points)

due November 23 at 11:59 pm to eCampus.

Write clearly and give full explanations to solutions for all the problems. Show all steps of your work.

Reading assignment:

- Balanced Binary Search Trees
- Skip Lists
- Hash Tables
- Heap and Priority Queue
- Graphs

Problems.

1. (10 points) For the following statements about red-black trees, provide a justification for each true statement and a counterexample for each false one.

   (a) A subtree of a red-black tree is itself a red-black tree.

   (b) The sibling of an external node is either external or red.

   (c) There is a unique 2-4 tree associated with a given red-black tree.

   (d) There is a unique red-black tree associated with a given 2-4 tree.

2. (10 points) Modify this skip list after performing the following series of operations: `erase(38)`, `insert(48,x)`, `insert(24,y)`, `erase(42)`. Provided the recorded coin flips for `x` and `y`.

$-\infty$	—	—	—	—	—	$+\infty$
$-\infty$	—	17	—	—	—	$+\infty$
$-\infty$	—	17	—	—	42	$+\infty$
$-\infty$	—	17	—	—	42	$+\infty$
$-\infty$	12	17	—	38	42	$+\infty$
$-\infty$	12	17	20	38	42	$+\infty$

3. (10 points) Draw the 17-entry hash table that results from using the has function: $h(k) = ((3k + 5) \mod 11)$, to hash the keys: 12, 44, 13, 88, 23, 94, 11, 39, 20, 16, 5, assuming collisions are handled by double hashing using the secondary hash function: $h_s(k) = (7 - (k \mod 7))$.

4. (10 points) An airport is developing a computer simulation of air-traffic control that handles events such as landings and takeoffs. Each event has a *time-stamp* that denotes the time when the event occurs. The simulation program needs to efficiently perform the following two fundamental operations:

   - Insert an event with a given time-stamp (that is, add a future event)
   - Extract the event with a smallest time-stamp (that is, determine the next event to process)

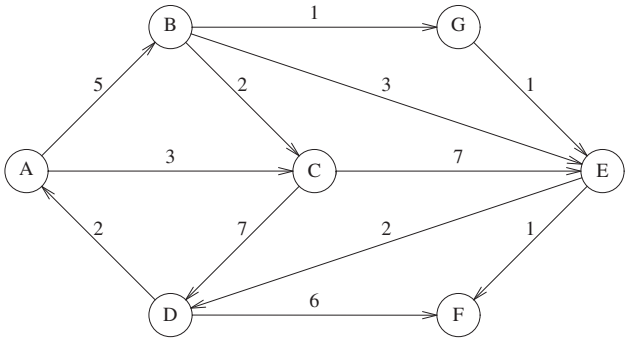
   Which data structure should be used for the above operations? Why? Provide big-O asymptotic complexity for each operation.

5. (15 points) Find the shortest path from D to all other vertices for the graph below.

   (a) Illustrate the minimum priority queue at each iteration Dijkstra's algorithm.

   (b) Draw the Shortest Path Tree.

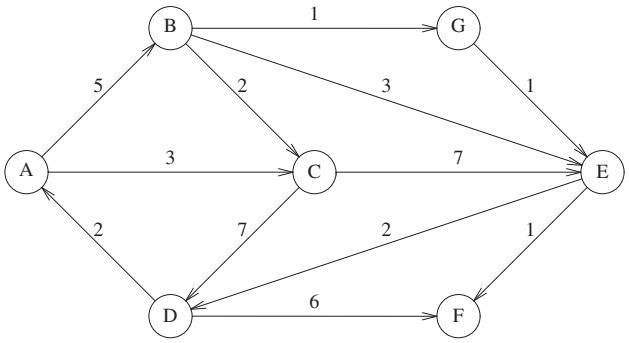
   (c) What is the running time of the Dijkstra's algorithm under the assumption that the graph is implemented based on an adjacency list and the minimum priority queue is implemented based on a binary heap?



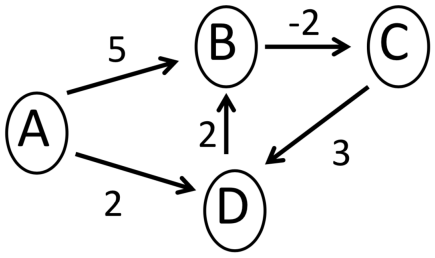
6. (15 points) Find the shortest unweighted path from D to all other vertices for the graph below. You can measure the distance from D by number of edges.

   (a) Which graph algorithm can solve the problem?

   (b) Draw the Shortest Path Tree.



7. (10 points) Apply the Dijkstra's algorithm to find the shortest path from the vertex A to all the vertices in the graph below. Does the algorithm return a correct output? Justify your answer using the Dijkstra's Theorem.



8. (20 points) There are eight small island in a lake, and the state wants to build seven bridges to connect them so that each island can be reached from any other one via one or more bridges. The cost of bridge construction is proportional to its length. The distance between pairs of islands are given in the following table.

   (a) Illustrate the Prim's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-

   (b) Illustrate the Kruskal's algorithm using the graph below. Draw the Minimum Spanning Tree. What is the length of the bridges?

	1	2	3	4	5	6	7	8
1	-	240	210	340	280	200	345	120
2	-	-	265	175	215	180	185	155
3	-	-	-	260	115	350	435	195
4	-	-	-	-	160	330	295	230
5	-	-	-	-	-	360	400	170
6	-	-	-	-	-	-	175	205
7	-	-	-	-	-	-	-	305
8	-	-	-	-	-	-	-	-