CSCE 221 Cover Page

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Please list all sources in the table below including web pages which you used to solve or implement the current homework. If you fail to cite sources you can get a lower number of points or even zero, read more Aggie Honor System Office http://aggiehonor.tamu.edu/

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I certify that I have listed all the sources that I used to develop the solutions/codes to the submitted work.

"On my honor as an Aggie, I have neither given nor received any unauthorized help on this academic work."

Your Name Rong Xu Date 2020/10/10

Homework 2

due October 16 at 11:59 pm to eCampus

1. (20 points) Given two sorted lists, L1 and L2, write an efficient C++ code to compute L1 $\bigcap$ L2 using only the basic STL list operations.

   (a) Provide evidence of testing: submit your code

```
#include <iostream>
#include <list>
using namespace std;

list<int> inter_list(list<int> L1,
list<int> L2) {
        if(L1.empty() || L2.empty())
                std::exit(-1);
        L1.merge(L2);
        list<int> return_list;
        for(list<int>::iterator it = L1.begin(); it != L1.end(); it++) {
                if(*it == *--it)
                        return_list.push_back(*it);
                ++it;
        }
        return return_list;
}

int main() {
int A1[]={1,2,3,4,5,6};
int A2[]={2,4,6,8,9,10};
list<int> iL1(A1, A1+6);
list<int> iL2(A2, A2+6);
list<int> iL3 = inter_list(iL1, iL2);
list<int>::iterator it = iL3.begin();
while(it != iL3.end()) {
        cout << *it++ << "␣";
}
system("pause");
return 0;
}
```

   (b) What is the running time of your algorithm?

      i. From the official website, the running time of function std::merge() is O(n).
         A. https://en.cppreference.com/w/cpp/algorithm/merge
         B. n is the number of elements of List 1.
      ii. The big-o of for loop is O(n+k).
         A. n is the number of elements of List 1
         B. k is the number of elements of List 2
         C. In this case, the list L1 was merged by List 2. Thus the List 1 now has (n + k) element.
      iii. Base of the statements above, the running time is O(n) + O(n + k) = O(n + k)

2. (20 points) Write a C++ recursive function that counts the number of nodes in a singly linked list.

(a) Test your function using different singly linked lists. Include your code.

```
#include<iostream> using namespace std;
class Node {
        public:
                int data;
                Node *next;
                Node(int da = 0, Node *p = NULL) {
                        this -> data = da;
                        this -> next = p;
                }
};

class List{
        private:
                Node *head,*tail; public:
        List(){head = tail = NULL;};
        ~List(){delete head; delete tail;};
        void print() {
                Node *p = head;
                while (p != NULL) {
                        cout << p -> data << "␣\a";
                        p = p->next;
                }
                cout << endl;
        };
        void Insert(int da) {
                if (head == NULL) {
                        head = tail = new Node(da);
                        head -> next = NULL;
                        tail -> next = NULL;
                } else {
                        Node *p = new Node(da);
                        tail -> next = p;
                        tail = p;
                        tail -> next = NULL;
                }
        };
        Node* first() {return head;}
};

int count_node(Node* n) {
        if(n == NULL)
                return 0;
        else
                return 1 + count_node(n -> next);
}

int main() {
        List l1;
        l1.Insert(1);l1.Insert(2);l1.Insert(3);l1.Insert(4);
        l1.Insert(5);l1.Insert(6);l1.Insert(7);
        l1.print();
        cout << "the␣number␣of␣nodes:␣" << count_node(l1.first()) << endl;
        system("pause");
        return 0;
}
```

(b) Write a recurrence relation that represents your algorithm.

i. Base case: When n is a null pointer, return 0.

A. T(0) = c1 for some constant c1

ii. Recursive case: When n is not a null pointer.

A. T(n) = c2 + T(n - 1) for some constant c2

(c) Solve the recurrence relation using the iterating or recursive tree method to obtain the running time of the algorithm in Big-O notation.

i. If we knew T(n - 1), we could solve T(n).

ii. $T(n) = T(n-1)+c2 = T(n-2)+c2+c2 = T(n-2)+2c2 = T(n-3)+3c2 = ... = T(n-k)+kc2$

iii. So we have $T(n) = T(n-k) + k * c2$ for all k

iv. If we set k = n, we have $T(n) = T(n-n) + nc2 = T(0) + nc2 = c1 + nc2 = O(n)$

3. (20 points) Write a C++ recursive function that finds the maximum value in an array (or vector) of integers *without* using any loops.

   (a) Test your function using different input arrays. Include the code.

```
#include<iostream>

int find_max(int A[], int n) {
        if(n == 1)
                return A[0];
        return std::max(A[n - 1], find_max(A, n - 1));
}

int main() {
int A[] = {1,3,4,6,7,8,9,45,34,54};
int n = sizeof(A) / sizeof(A[0]);
for (int i = 0; i < 10; i++)
        std::cout << A[i] << "␣";
std::cout << "\nthe␣max␣value␣is:␣" << find_max(A, n) << std::endl;
std::system("pause");
return 0;
}
```

```
~ » /Users/abby/Desktop/2020_fall/CSCE221_511/hw2/Q3 ; exit;
1 3 4 6 7 8 9 45 34 54
the max value is: 54
sh: pause: command not found

[Process completed]
```

   (b) Write a recurrence relation that represents your algorithm.

      i. Base case: When n reach to the first element in the array/vector.

         A. T(0) = c1 for some constant c1

      ii. Recursive case: Fund the max element when not reach to the first element..

         A. T(n) = c2 + T(n - 1) for some constant c2

   (c) Solve the recurrence relation and obtain the running time of the algorithm in Big-O notation.

      i. If we knew T(n - 1), we could solve T(n).

      ii. $T(n) = T(n-1)+c2 = T(n-2)+c2+c2 = T(n-2)+2c2 = T(n-3)+3c2 = ... = T(n-k)+kc2$

      iii. So we have $T(n) = T(n-k) + k * c2$ for all k

      iv. If we set k = n, we have $T(n) = T(n-n) + nc2 = T(0) + nc2 = c1 + nc2 = O(n)$

4. (20 points) What is the best, worst and average running time of quick sort algorithm?

   (a) Provide recurrence relations and their solutions.

      i. Best case:

         A. Recurrence Relation: $T(n) = T(n/2) + T(n/2) + O(n)$ and $T(1) = 0$

         B. Solve it by iteration method:
            T(n) =2T(n/2)+O(n)
            =2(2T(n/4)+O(n/2))+O(n/2))+O(n)
            =4T(n/4)+2*O(n)
            =...
            $=2^kT(n/2^k) + K * O(n)$
            $= O(nlog_2n)$

      ii. Worst case:

         A. Recurrence Relation: $T(n) = T(n-1) + T(1) + n = T(n-1) + n$ and $T(1) = 0$

B. Solve it by iteration method:
T(n) = T(n-1)+n
=T(n-2)+(n-1)+n
=T(n-3)+(n-2)+(n-1)+n
=...
=T(n-k+!)+(n-k+2)+...+(n-1)+n
=...
=T(1)+2+3+...+(n-1)+n
=fracn(n+1)2-1
=O(n^2)

iii. Average case:

A. Recureence Relation: $T(n) = T(cn) + T((1-c)n) + n$ and $T(1) = 0$

B. Solve it by interation method: Let hl be the height of left subtree and hr be the hright of right subtree. Notice that hr > hl.

C. solve for hl and hr:
$c^{h_L} = 1/n$
$h_L = -log_c h$
$h_L = log_{1/c} h$

$(1-c)^{h_R} = 1/n$
$h_R = -log_{(1-c)} h$
$h_R = log_{1/(1-c)} h$

D. The big-o is $O(nlog_2 n)$

(b) Provide arrangement of the input and the selection of the pivot point for each case.

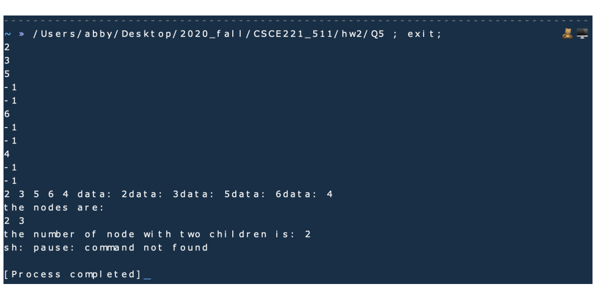
i. For the best case: the input has already been sorted and the pivot is just the middlest element in the list.

ii. For the average case: we usually choose the pivot in the middle or just choose the random index of the pivot.

iii. For the worst case: the input list is reversed and we start to choose the pivot at the beginning of the list until the last one.

5. (20 points) Write a C++ function that counts the total number of nodes with two children in a binary tree (do not count nodes with one or none child). You can use a STL container if you need to use an additional data structure to solve this problem. Use the big-O notation to classify your algorithm. Include your code.

```
#include <iostream>
#include <vector>
using namespace std;

std::vector<int> nodes;
struct BiTNode {
        int data;
        struct BiTNode* lchild;
        struct BiTNode* rchild;
};

void create_tree(BiTNode* &tree) {
        int data;cin >> data;
        if (data != '\n') {
                if (data == -1) {
                        tree = nullptr;
                } else {
                        tree = new BiTNode;
                        tree->data = data;
                        create_tree(tree->lchild);
                        create_tree(tree->rchild);
                }
        }
}

void pre_order_traverse(BiTNode* &tree) {
        if (tree) {
                cout << tree->data << "␣";
                pre_order_traverse(tree->lchild);
                pre_order_traverse(tree->rchild);
        }
}

void count_node(BiTNode* &tree) {
        cout << "data:␣" << tree -> data;
        if(tree -> lchild != nullptr && tree -> rchild != nullptr)
                nodes.push_back(tree -> data);
        if(tree -> lchild == nullptr && tree -> rchild == nullptr)
                return;
        count_node(tree->lchild);
        count_node(tree->rchild);
}

int main() {
    BiTNode* T;
        create_tree(T);
        /*     input here is:     2 3 5 -1 -1 6 -1 -1 4 -1 -1
             2
           /   \
          3     4
         / \
        5   6           */
pre_order_traverse(T); //2, 3, 5, 6, 4
count_node(T);
cout << "\nthe␣nodes␣are:␣" <<endl;
for (int j = 0; j < nodes.size(); j++)
        cout << nodes[j] << "␣";
cout << "\nthe␣number␣of␣node␣with␣two␣children␣is:␣" << nodes.size() << endl;   '
system("pause");␣␣␣␣␣␣␣␣␣
return␣0;
}
```



(a) I think the the big-o of my algorithm is $O(log_2 n)$