"Learning to program has no more to do with designing interactive software than learning to touch type has to do with writing poetry."

- T. Nelson.

CSE341
Programming Languages

Lecture 11 – December 17, 2015

Prolog

Slides are taken from C. Li & W. He

SWI-Prolog

- http://www.swi-prolog.org/
- Available for:

  Linux, Windows, MacOS

Prolog

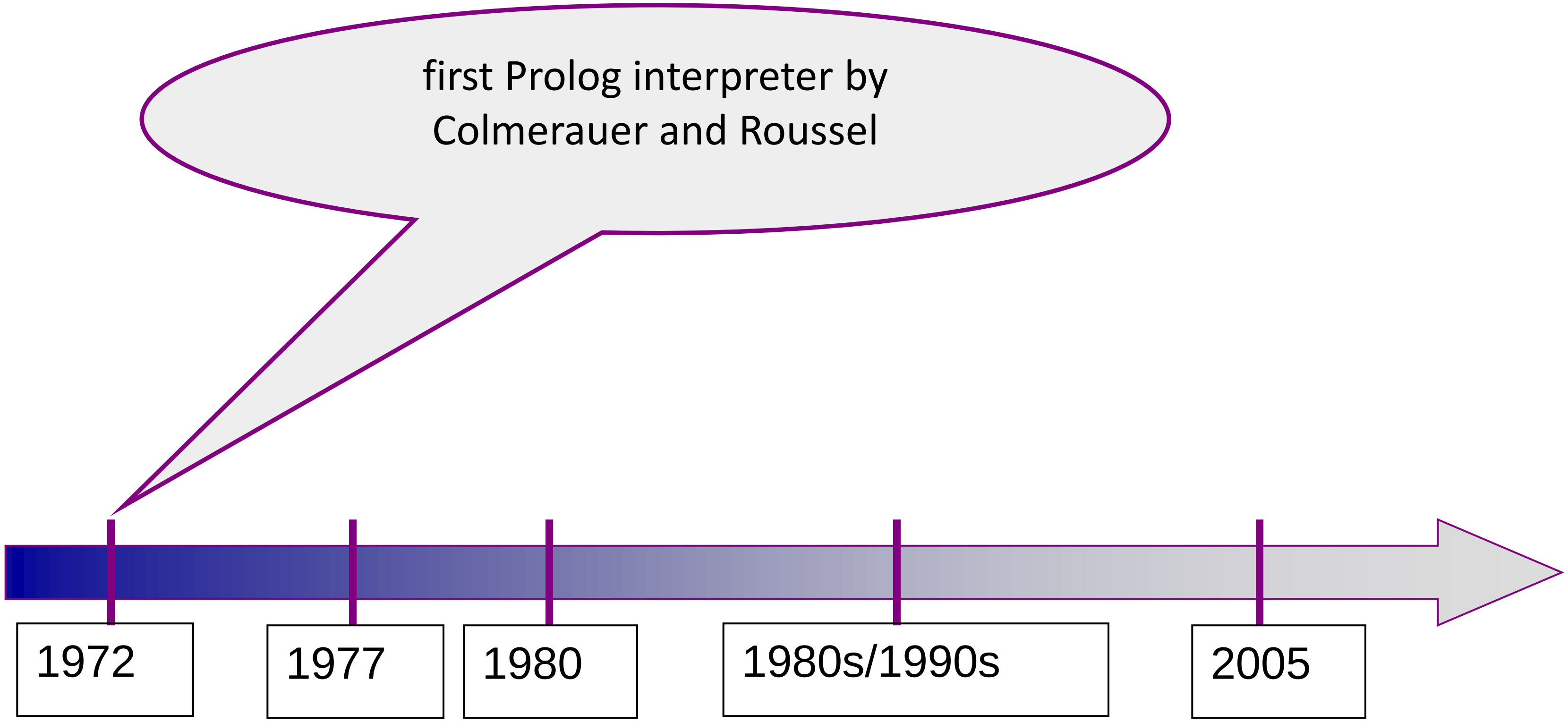
- Prolog:
  “Programming in Logic” (PROgrammation en LOgique)

- One (and maybe the only one) successful logic programming languages

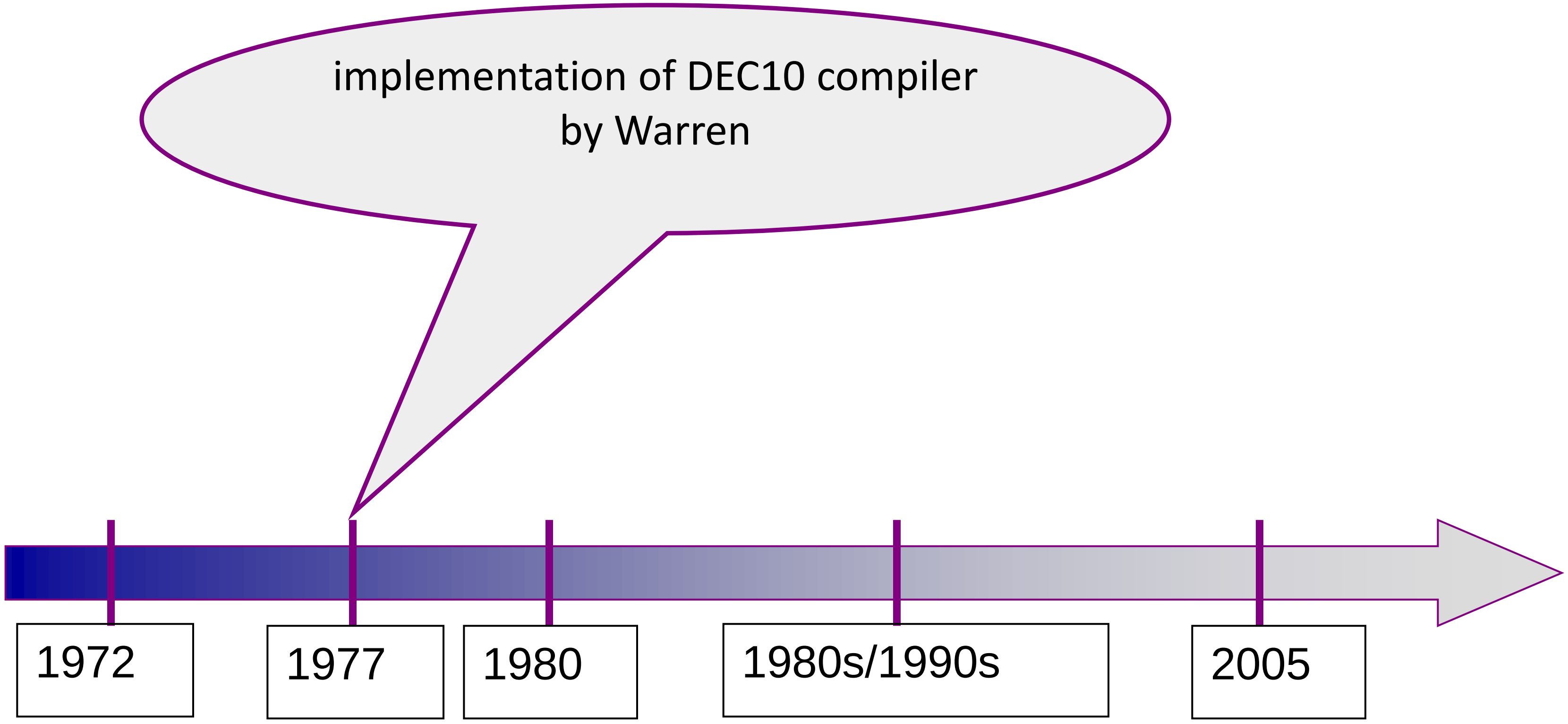
- Useful in AI applications, expert systems, natural language processing, database query languages

- Declarative instead of procedural: “What” instead of “How”

History of Prolog



History of Prolog



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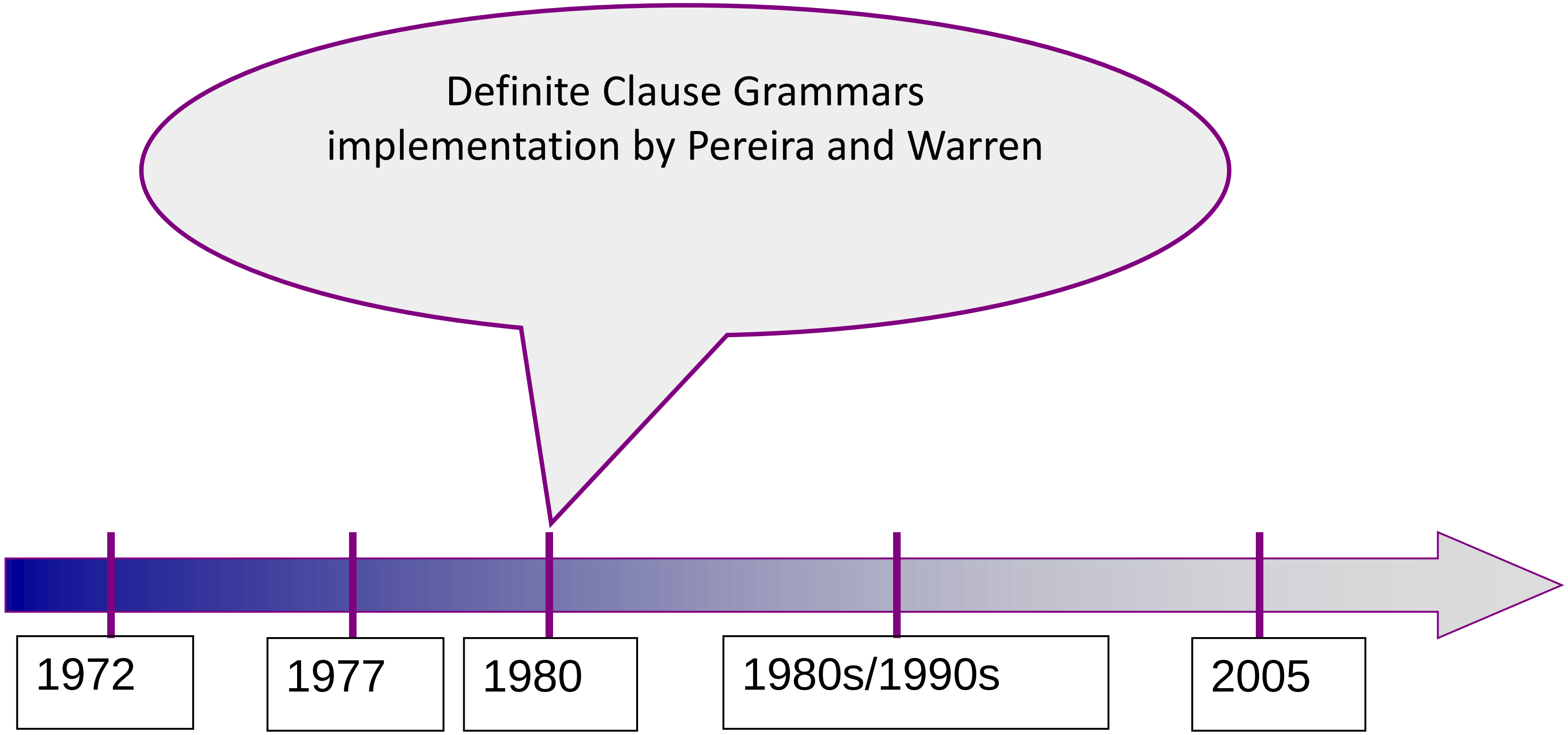

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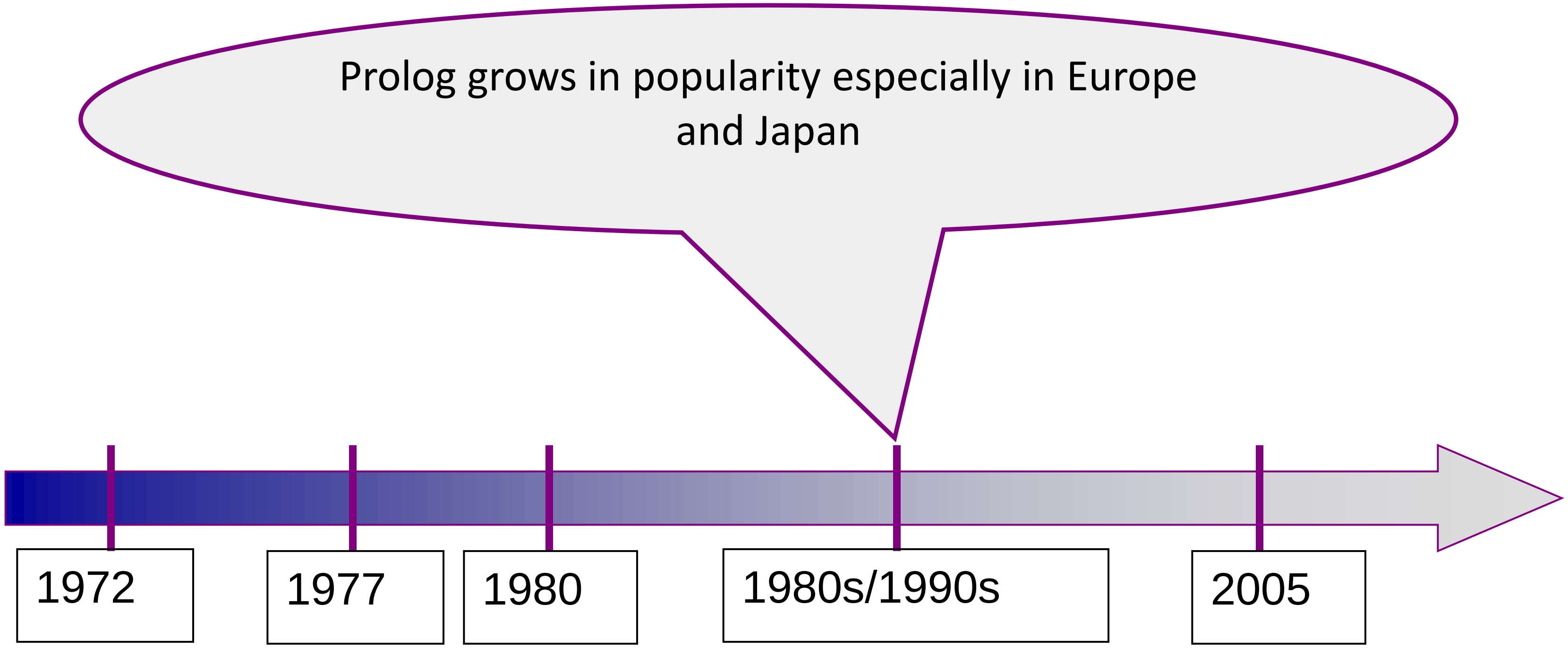

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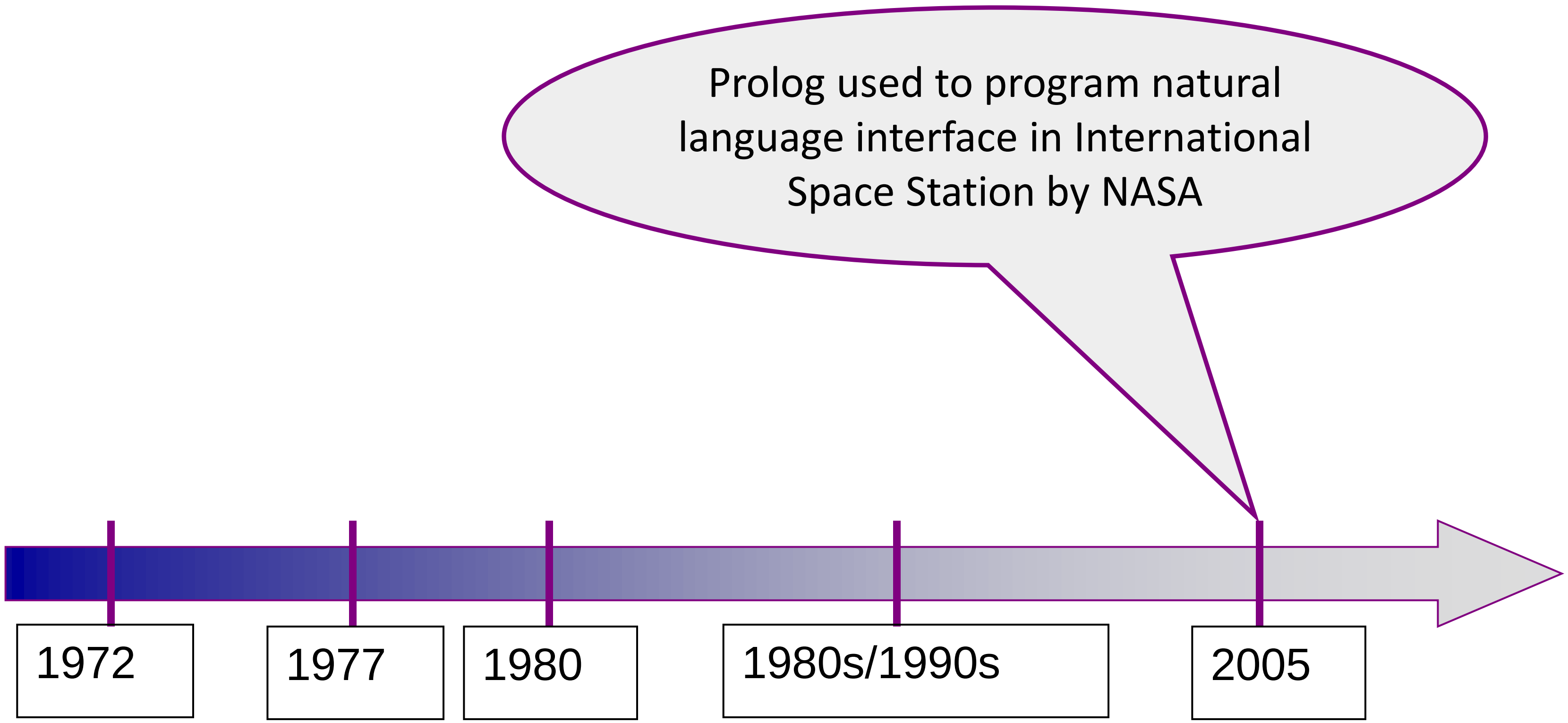


Logic Programming

- Program

  Axioms (facts): true statements

- Input to Program

  query (goal): statement true (theorems) or false?

- Thus

  Logic programming systems = deductive databases

  datalog

Example

- Axioms:

  0 is a natural number. (Facts)

  For all x, if x is a natural number, then so is the successor of x.

- Query (goal).

  Is 2 natural number? (can be proved by facts)

  Is -1 a natural number? (cannot be proved)

Another Example

- Axioms:

  The factorial of 0 is 1. (Facts)
  If m is the factorial of n - 1, then n * m is the factorial of n.

- Query:

  The factorial of 2 is 3?

First-Order Predicate Calculus

- Logic used in logic programming:

  First-order predicate calculus

  First-order predicate logic

  Predicate logic

  First-order logic

  $\forall x \, (x \neq x+1)$

- Second-order logic

  $\forall S \, \forall \, x \, (x \in S \lor x \notin S)$

First-Order Logic: Review

Slides from Tuomas Sandholm of CMU

First-order Logic

- First-order logic (FOL) models the world in terms of
  - **Objects,** which are things with individual identities
  - **Properties** of objects that distinguish them from other objects
  - **Relations** that hold among sets of objects
  - **Functions,** which are a subset of relations where there is only one “value” for any given “input”
- Examples:
  - Objects: Students, lectures, companies, cars ...
  - Relations: Brother-of, bigger-than, outside, part-of, has-color, occurs-after, owns, visits, precedes, ...
  - Properties: blue, oval, even, large, ...
  - Functions: father-of, best-friend, second-half, one-more-than ...

User Provides

- **Constant symbols,** which represent individuals in the world
  - Mary
  - 3
  - Green
- **Function symbols,** which map individuals to individuals
  - father-of(Mary) = John
  - color-of(Sky) = Blue
- **Predicate symbols,** which map individuals to truth values
  - greater(5,3)
  - green(Grass)
  - color(Grass, Green)

FOL Provides

- **Variable symbols**
  - E.g., x, y, foo
- **Connectives**
  - Same as in PL: not ($\neg$), and ($\wedge$), or ($\vee$), implies ($\rightarrow$), if and only if (biconditional $\leftrightarrow$)
- **Quantifiers**
  - Universal $\forall$**x** or **(Ax)**
  - Existential $\exists$**x** or **(Ex)**

Sentences built from Terms and Atoms

- A **term** (denoting a real-world individual) is a constant symbol, a variable symbol, or an n-place function of n terms.
  - x and $f(x_1, \ldots, x_n)$ are terms, where each x_i is a term.
  - A term with no variables is a **ground term**
- An **atomic sentence** (which has value true or false) is an n-place predicate of n terms
- A **complex sentence** is formed from atomic sentences connected by the logical connectives:
  - $\neg P$, $P \vee Q$, $P \wedge Q$, $P \rightarrow Q$, $P \leftrightarrow Q$ where P and Q are sentences
- A **quantified sentence** adds quantifiers $\forall$ and $\exists$
- A **well-formed formula (wff)** is a sentence containing no "free" variables. That is, all variables are "bound" by universal or existential quantifiers.
  - $(\forall x)P(x,y)$ has x bound as a universally quantified variable, but y is free.

A BNF for FOL

```
S := <Sentence> ;
<Sentence> := <AtomicSentence> |
          <Sentence> <Connective> <Sentence> |
          <Quantifier> <Variable>,... <Sentence> |
          "NOT" <Sentence> |
          "(" <Sentence> ")";
<AtomicSentence> := <Predicate> "(" <Term>, ... ")" |
                    <Term> "=" <Term>;
<Term> := <Function> "(" <Term>, ... ")" |
          <Constant> |
          <Variable>;
<Connective> := "AND" | "OR" | "IMPLIES" | "EQUIVALENT";
<Quantifier> := "EXISTS" | "FORALL" ;
<Constant> := "A" | "X1" | "John" | ... ;
<Variable> := "a" | "x" | "s" | ... ;
<Predicate> := "Before" | "HasColor" | "Raining" | ... ;
<Function> := "Mother" | "LeftLegOf" | ... ;
```

Quantifiers

- **Universal quantification**
  - $(\forall x)P(x)$ means that P holds for **all** values of x in the domain associated with that variable
  - E.g., $(\forall x)$ dolphin(x) $\rightarrow$ mammal(x)
- **Existential quantification**
  - $(\exists\, x)P(x)$ means that P holds for **some** value of x in the domain associated with that variable
  - E.g., $(\exists\, x)$ mammal(x) $\wedge$ lays-eggs(x)
  - Permits one to make a statement about some object without naming it

Translating English to FOL

Every gardener likes the sun.

$\forall x\ gardener(x) \rightarrow likes(x,Sun)$

You can fool some of the people all of the time.

$\exists x\ \forall t\ \ person(x) \wedge time(t) \rightarrow can\text{-}fool(x,t)$

You can fool all of the people some of the time.

$\forall x\ \exists t\ (person(x) \rightarrow time(t) \wedge can\text{-}fool(x,t))$

$\forall x\ (person(x) \rightarrow \exists t\ (time(t) \wedge can\text{-}fool(x,t))$

Equivalent

All purple mushrooms are poisonous.

$\forall x\ (mushroom(x) \wedge purple(x)) \rightarrow poisonous(x)$

No purple mushroom is poisonous.

$\neg\exists x\ purple(x) \wedge mushroom(x) \wedge poisonous(x)$

$\forall x\ \ (mushroom(x) \wedge purple(x)) \rightarrow \neg poisonous(x)$

Equivalent

There are exactly two purple mushrooms.

$\exists x\ \exists y\ mushroom(x) \wedge purple(x) \wedge mushroom(y) \wedge purple(y)\ \wedge \neg(x=y) \wedge \forall z\ (mushroom(z) \wedge purple(z)) \rightarrow ((x=z) \vee (y=z))$

Clinton is not tall.

$\neg tall(Clinton)$

First-Order Predicate Calculus: Example

- `natural(0)`
  `∀ X, natural(X) → natural(successor(x))`

- `∀ X and Y, parent(X,Y) → ancestor(X,Y).`
  `∀ A, B, and C, ancestor(A,B) and ancestor(B,C) → ancestor(A,C).`
  `∀ X and Y, mother(X,Y) → parent(X,Y).`
  `∀ X and Y, father(X,Y) → parent(X,Y).`
  `father(bill,jill).`
  `mother(jill,sam).`
  `father(bob,sam).`

- `factorial(0,1).`
  `∀ N and M, factorial(N-1,M) → factorial(N,N*M).`

First-Order Predicate Calculus: Statements

Symbols in statements:

- *Constants (a.k.a. atoms)*

  numbers (e.g., `0`) or names (e.g., `bill`).

- *Predicates*

  Boolean functions (true/false) . Can have arguments. (e.g. `parent(X,Y)`).

- *Functions*

  non-Boolean functions (`successor(X)`).

- *Variables*

  e.g., `X`.

- *Connectives (operations)*

  `and, or, not`

  `implication (→):a→b (b or not a)`

  `equivalence (↔): a↔b (a→b and b→a)`

First-Order Predicate Calculus: Statements

- *Quantifiers*

  universal quantifier "for all“ ∀

  existential quantifier "there exists" ∃

  bound variable (a variable introduced by a quantifier)

  free variable
- *Punctuation symbols*

  parentheses (for changing associativity and precedence.)

  comma

  period

- Arguments to predicates and functions can only be ***terms***:
  - Contain constants, variables, and functions.
  - Cannot have predicates, qualifiers, or connectives.

Problem Solving

- Program = Data + Algorithms
- Program = Object.Message(Object)
- Program = Functions Functions
- Algorithm = Logic + Control

Programmers: facts/axioms/statements

Logic programming systems: prove goals from axioms

- We specify the logic itself, the system proves.
  - Not totally realized by logic programming languages. Programmers must be aware of how the system proves, in order to write efficient, or even correct programs.
- Prove goals from facts:
  - Resolution and Unification

Proving things

- A **proof** is a sequence of sentences, where each sentence is either a premise or a sentence derived from earlier sentences in the proof by one of the rules of inference.
- The last sentence is the **theorem** (also called goal or query) that we want to prove.
- Example for the “weather problem”

1 Hu	Premise	“It is humid”
2 $Hu \rightarrow Ho$	Premise	“If it is humid, it is hot”
3 Ho	Modus Ponens(1,2)	“It is hot”
4 $(Ho \wedge Hu) \rightarrow R$	Premise	“If it’s hot & humid, it’s raining”
5 $Ho \wedge Hu$	And Introduction(1,3)	“It is hot and humid”
6 R	Modus Ponens(4,5)	“It is raining”

Horn Clause

- First-order logic too complicated for an effective logic programming system.
- Horn Clause: a fragment of first-order logic

```
b ← a1 and a2 and a3 … and an.
```

head

body

no "or" and no quantifier

```
b ←.
```

fact

```
  ←b.
```

query

- Variables in head: universally quantified
  Variables in body only: existentially quantified
- Need "or" in head?   Multiple clauses

Horn Clauses: Example

- First-Order Logic:

```
natural(0).
∀X, natural(X) → natural(successor(x)).
```

- Horn Clause:

```
natural(0).
natural(successor(x)) ← natural(X).
```

Horn Clauses: Example

- First-Order Logic:

```
factorial(0,1).
∀ N and ∀ M, factorial(N-1,M) → factorial(N,N*M).
```

- Horn Clause:

```
factorial(0,1).
factorial(N,N*M) ← factorial(N-1,M).
```

Horn Clauses: Example

- Horn Clause:

```
ancestor(X,Y) ← parent(X,Y).
ancestor(A,C) ← ancestor(A,B) and ancestor(B,C).
parent(X,Y) ← mother(X,Y).
parent(X,Y) ← father(X,Y).
father(bill,jill).
mother(jill,sam).
father(bob,sam).
```

Horn Clauses: Example

- First-Order Logic:

```
∀ X, mammal(X) → legs(X,2) or legs(X,4).
```

- Horn Clause:

```
legs(X,4) ← mammal(X) and not legs(X,2).
legs(X,2) ← mammal(X) and not legs(X,4).
```

Prolog syntax

- :- for ←

  , for `and`

```
ancestor(X,Y) :- parent(X,Y).
ancestor(X,Y) :- ancestor(X,Z), ancestor(Z,Y).
parent(X,Y) :- mother(X,Y).
parent(X,Y) :- father(X,Y).
father(bill,jill).
mother(jill,sam).
father(bob,sam).
```

Prolog BNF Grammar

<program> ::= <clause list> <query> | <query>
<clause list> ::= <clause> | <clause list> <clause>
<clause> ::= <predicate> . | <predicate> :- <predicate list>.
<predicate list> ::= <predicate> | <predicate list> , <predicate>
<predicate> ::= <atom> | <atom> (<term list>)
<term list> ::= <term> | <term list> , <term>
<term> ::= <numeral> | <atom> | <variable> | <structure>
<structure> ::= <atom> (<term list>)
<query> ::= ?- <predicate list>.
<atom> ::= <small atom> | ' <string> '
<small atom> ::= <lowercase letter> | <small atom> <character>
<variable> ::= <uppercase letter> | <variable> <character>
<lowercase letter> ::= a | b | c | ... | x | y | z
<uppercase letter> ::= A | B | C | ... | X | Y | Z | _
<numeral> ::= <digit> | <numeral> <digit>
<digit> ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
<character> ::= <lowercase letter> | <uppercase letter> |
<digit> | <special>
<special> ::= + | - | * | / | \ | ^ | ~ | : | . | ? | | # | $ | &
<string> ::= <character> | <string> <character>

Resolution and Unification

Resolution

- Resolution: Using a clause, replace its head in the second clause by its body, if they “match”.

- $a \leftarrow a_1, \ldots, a_n.$

  $b \leftarrow b_1, \ldots, b_i, \ldots, b_m.$

  if b_i matches a;

  $b \leftarrow b_1, \ldots, a_1, \ldots, a_n, \ldots, b_m.$

Resolution: Another view

- Resolution: Combine two clauses, and cancel matching statements on both sides.

- $a \leftarrow a_1, \ldots, a_n.$
  $b \leftarrow b_1, \ldots, b_i, \ldots, b_m.$

  $\cancel{a}, b \leftarrow a_1, \ldots, a_n, b_1, \ldots, \cancel{b}_i, \ldots, b_m.$

Problem solving in logic programming systems

- Program:
  - Statements/Facts (clauses).
- Goals:
  - Headless clauses, with a list of subgoals.

- Problem solving by resolution:
  - Matching subgoals with the heads in the facts, and replacing the subgoals by the corresponding bodies.
  - Cancelling matching statements.
  - Recursively do this, till we eliminate all goals. (Thus original goals proved.)

Example

- Program:

```
mammal(human).
```

- Goal:

```
← mammal(human).
```

- Proving:

```
mammal(human) ← mammal(human).
                  ←.
```

Example

- Program:

```
legs(X,2) ← mammal(X), arms(X,2).
legs(X,4) ← mammal(X), arms(X,0).
mammal(horse).
arms(horse,0).
```

- Goal:

```
 ← legs(horse,4).
```

- Proving: ?

Unification

- Unification: Pattern matching to make statements identical (when there are variables).
- Set variables equal to patterns: instantiated.
- In previous example:
  legs(X,4) and legs(horse,4) are unified.
  (X is instantiated with horse.)

Unification: Example

- Euclid's algorithm for greatest common divisor

- Program:

```
gcd(U,0,U).
gcd(U,V,W) ← not zero(V), gcd(V, U mod V, W).
```

- Goals:

```
← gcd(15,10,X).
```

Unification: Example

```
gcd(U,0,U).
gcd(U,V,W) ← not zero(V), gcd(V, U mod V, W).
```

1. gcd(15,10,X) does not match the first clause…
2. gcd(15,10,X) matches the second clause
   1. ← not zero(10), gcd(10, 15 mod 10, X)
   2. ← gcd(10, 5, X)
   3. ← not zero(5), gcd(5, 10 mod 5, X)
   4. ← gcd(5, 0, X)

Things unspecified

- The order to resolve subgoals.
- The order to use clauses to resolve subgoals.

- Possible to implement systems that don’t depend on the order, but too inefficient.

- Thus programmers must know the orders used by the language implementations. (Search Strategies)

Example

- Program:

```
ancestor(X,Y) :- ancestor(X,Z), parent(Z,Y).
ancestor(X,Y) :- parent(X,Y).
parent(X,Y) :- mother(X,Y).
parent(X,Y) :- father(X,Y).
father(bill,jill).
mother(jill,sam).
father(bob,sam).
```

- Goals:

```
← ancestor(bill,sam).
← ancestor(X,bob).
```

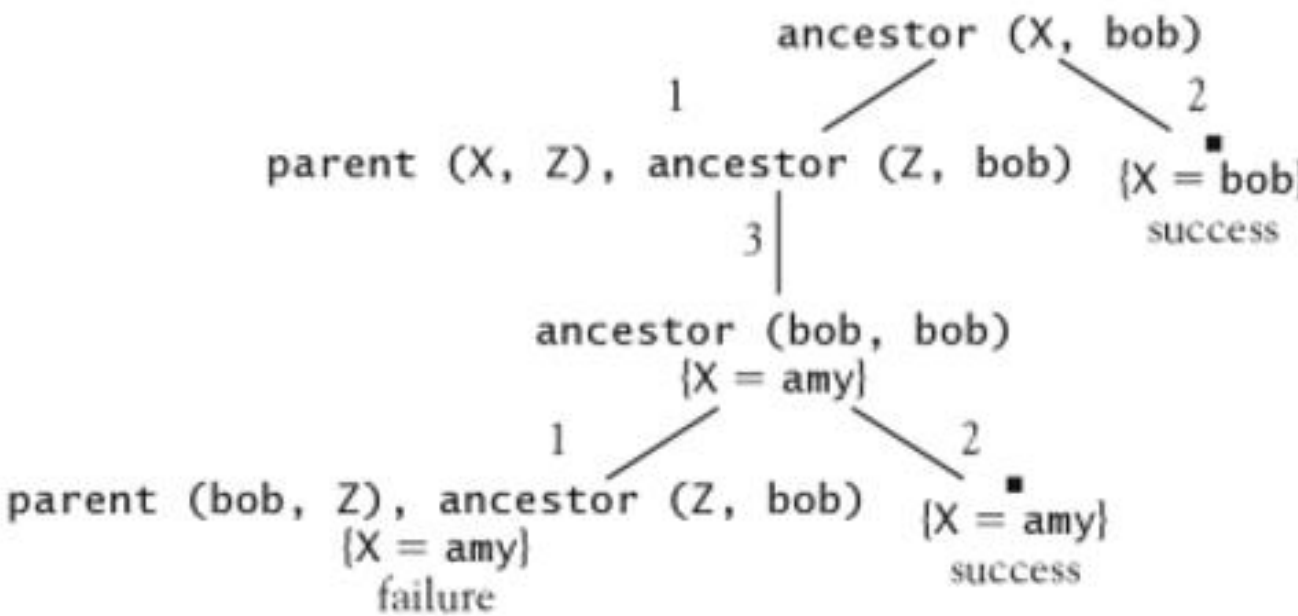
Prolog Search Strategy

- Applies resolution in strictly linear fashion
  - Replacing goals left to right
  - Considering clauses top to bottom order

➔ A depth-first search on a tree of possible choices…

Prolog Search Strategy

```
(1) ancestor(X, Y) :- parent(X, Z), ancestor(Z, Y).
(2) ancestor(X, X).
(3) parent(amy, bob).
```



Prolog Loops and Controls…

```
printpieces(L) :-append(X, Y, L),
                  write(X),
                  write(Y),
                  nl,
                  fail.
```

```
?- printpieces([1, 2]).
```

```
[][1,2]
[1][2]
[1,2] []
no
```

Backtracking…

Prolog Loops and Controls…

```
(1) num(0).
(2) num(X) :- num(Y), X is Y + 1.
```