Beautiful Linear Programming

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1 Linear Programming

$$\text{minimize } q^T x \tag{1}$$

$$Ax \preceq b \tag{2}$$

To find decision variables that minimize (or maximize) a linear objective function, subject to a set of linear constraints.

The x is the vector of decision variables, q and b are vectors of coefficients, and A is a matrix of coefficients with number of rows and columns equal to the number of linear constraints and decision variable, respectively.

2 Quadratic Programming

$$\text{minimize } \frac{1}{2}x^T Qx + q^T x \tag{3}$$

$$Ax \preceq b \tag{4}$$

To find decision variables that minimize (or maximize) a quadratic objective function, subject to a set of linear constraints.

The x is the vector of decision variable, q and b are vectors of coefficients, A is a matrix of coefficients with number of rows and columns equal to the number of linear constraints and decision variable, respectively, and Q is a diagonal matrix of coefficients with number of rows and columns equal to the number of decision variable.

3 Decision Variables and Objective Functions

Decision variable can be strictly integer or continuous. If an optimization problem consist only strictly integer decision variables, it is called integer programming, If an optimization problem consist of both types of decision variables, it is called mixed-integer programming. If a mixed-integer programming problem has linear objective function it is called mixed-integer linear programming. If a mixed-integer programming problem has quadratic objective function, it is called mixed-integer quadratic programming.

4 Linear Constraints

Each constraint in linear constraints must be expressed in linear expressions terms. It means that each constraint must be expressed as a summation of constants that is multiplied with decision variables raised to the power of 1. The constants itself can be 0 if a decision variable is not used in that constraint. The use of $\preceq$ in constraints means that each row of Ax must be less than the value of b at that row.