Beautiful Linear Programming

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1 Linear Programming

$$\text{minimize } q^T x \tag{1}$$

$$Ax \preceq b \tag{2}$$

To find decision variables that minimize (or maximize) a linear objective function, subject to a set of linear constraints.

The x is the vector of decision variables, q and b are vectors of coefficients, and A is a matrix of coefficients with number of rows and columns equal to the number of linear constraints and decision variable, respectively.

2 Quadratic Programming

$$\text{minimize } \frac{1}{2} x^T Q x + q^T x \tag{3}$$

$$Ax \preceq b \tag{4}$$

To find decision variables that minimize (or maximize) a quadratic objective function, subject to a set of linear constraints.

The x is the vector of decision variable, q and b are vectors of coefficients, A is a matrix of coefficients with number of rows and columns equal to the number of linear constraints and decision variable, respectively, and Q is a diagonal matrix of coefficients with number of rows and columns equal to the number of decision variable.

3 Linear Constraints

Each constraint in linear constraints must be expressed in linear expressions terms. It means that each constraint must be expressed as a summation of constants that is multiplied with decision variables raised to the power of 1. The constants itself can be 0 if a decision variable is not used in that constraint. The use of $\preceq$ in constraints means that each row of Ax must be less than the value of b at that row.

4 Types of Decision Variables

Decision variable can be strictly integer or continues. If an optimization problem consist of both types of decision variables, it is called mixed-integer programming. If a mixed-integer programming problem has linear objective function it is called mixed-integer linear programming. If a mixed-integer programming problem has quadratic objective function, it is called mixed-integer quadratic programming.