راه‌حل سوال ۱:

$NOR \to \overline{a+a} = \bar{a} \to$ not

$NOR \to \overline{\overline{(a+b)} + \overline{(a+b)}} = \overline{(\bar{a}.\bar{b}) + (\bar{a}\bar{b})} = \overline{(\bar{a}.\bar{b})} \cdot \overline{(\bar{a}\bar{b})} = \overline{(\bar{a}.\bar{b})} = a+b$ or

$NOR \to \overline{(\bar{a}+\bar{b})} = a.b$ and

① ثابت کنید گیت NOR یک گیت کامل است. {and, or, not} کامل‌اند

چون توابع را به صورت SOP و POS می‌توان نوشت

$Nor: \overline{a+b} = \bar{a}.\bar{b} \rightsquigarrow$ {and, not} کامل‌اند

② وارون عبارت‌های زیر را به کمک دمورگان به‌دست آورید

$$a)\ f(a,b,c) = \overline{(a\oplus b)(b\oplus c)} \cdot (\overline{\bar{a}+\bar{b}} + \overline{a+\bar{c}}) \xrightarrow{\text{وارون}}$$

$$\text{وارون } f(a,b,c) = \overline{\overline{(a\oplus b)(b\oplus c)} \cdot (\overline{\bar{a}.\bar{b}} + \overline{a+c})} = \overline{\overline{(a\oplus b)(b\oplus c)}} \cdot (a.b + \bar{a}.\bar{c}) =$$

$$\overline{\overline{(a\oplus b)(b\oplus c)}} \cdot (a.b + \bar{a}\bar{c})$$

$$(a.b + \bar{a}\bar{c}) \left[\overline{(a\bar{b}+\bar{a}b).(b\bar{c}+\bar{b}c)} \right] = (a.b+\bar{a}\bar{c}) \left[\overline{(a\bar{b}+\bar{a}b)} + \overline{(b\bar{c}+\bar{b}c)} \right] =$$

$$(a.b+\bar{a}\bar{c}) \left[(\overline{a\bar{b}} . \overline{\bar{a}b}) + (\overline{b\bar{c}} . \overline{\bar{b}c}) \right] = (a.b+\bar{a}\bar{c}) \left[(\bar{a}+b).(a+\bar{b}) + (\bar{b}+c).(b+\bar{c}) \right]$$

$$(a.b+\bar{a}\bar{c}) \left[a\bar{a} + \bar{a}\bar{b} + ab + b\bar{b} + b\bar{b} + \bar{b}\bar{c} + bc + c\bar{c} \right] =$$

$$ab + abc + \bar{a}\bar{b}\bar{c} = ab(1+c) + \bar{a}\bar{b}\bar{c} = ab + \bar{a}\bar{b}\bar{c}$$

$$b)\ f(a,b,c) = (b\oplus\bar{c}) + \bar{a}b\,(\overline{\bar{a}+c}) \quad f^{-1} = \overline{(b\oplus\bar{c})} \cdot \overline{\bar{a}b(\overline{\bar{a}+c})} =$$

$$\overline{(bc+\bar{b}\bar{c})} \cdot (\overline{\bar{a}b} + (\bar{a}+c)) = ((\bar{b}+\bar{c}).(b+c)).(a.b+\bar{a}+c) =$$

۳

a) $f(a,b,c) = \sum m(۱,۴,۵) = m_۱ + m_۴ + m_۵ =$

$\bar{a}.\bar{b}.c + a.\bar{b}.\bar{c} + a\bar{b}c = \bar{b}.(\bar{a}c + a\bar{c} + ac)$

b) $f(a,b,c,d) = \Pi M(۰,۲,۴,۵,۸,۱۱,۱۵)$

$(a+b+c+d).(a+b+\bar{c}+d).(a+\bar{b}+c+d).(a+\bar{b}+c+\bar{d}).(\bar{a}+b+c+d).$

$(\bar{a}+b+\bar{c}+\bar{d}).(\bar{a}+\bar{b}+\bar{c}+\bar{d})$

c) $f(a,b,c,d) = \sum m(۰,۲,۵,۸,۹,۱۰,۱۳)$

$(\bar{a}\bar{b}\bar{c}\bar{d}).(\bar{a}\bar{b}c\bar{d})+(\bar{a}b\bar{c}d)+(a\bar{b}\bar{c}\bar{d})+(a\bar{b}\bar{c}d)+(a\bar{b}c\bar{d})+(ab\bar{c}d)$

a) $f(x,y,z) = x\bar{y} + \overline{xz} = x\bar{y} + \bar{x} + \bar{z} \xrightarrow[\text{باعضو همانی ضرب}]{} (z+\bar{z})$ یا $(x+\bar{x})(y+\bar{y})$

$x\bar{y}(z+\bar{z}) + \bar{x}(\bar{z}+z)(y+\bar{y}) + \bar{z}(x+\bar{x})(y+\bar{y}) =$

$x\bar{y}z + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + xy\bar{z} + \bar{x}y\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z}$

$= x\bar{y}z + x\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{x}yz + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + xy\bar{z}$

m(۵) + m(۴) + m(۲) + m(۳) + m(۰) + m(۱) + m(۶)

= Σm(۰,۱,۲,۳,۴,۵,۶) = M(۷)

b) $f(x,y,z,w) = x + xyz + \bar{x}yz + wx$

$x(y+\bar{y})(z+\bar{z})(w+\bar{w}) + xyz(w+\bar{w}) + \bar{x}yz(w+\bar{w}) + wx(y+\bar{y})(z+\bar{z})$

$(xy+x\bar{y})(zw+z\bar{w}+\bar{z}w+\bar{z}\bar{w}) + (wxyz+\bar{w}xyz) + \bar{x}yzw + \bar{x}yz\bar{w} + wx(yz+y\bar{z}+\bar{y}z+\bar{y}\bar{z})$

$= (xyzw + xyz\bar{w} + xy\bar{z}w + xy\bar{z}\bar{w} + x\bar{y}zw + x\bar{y}z\bar{w} + x\bar{y}\bar{z}w + x\bar{y}\bar{z}\bar{w} +$

$xyzw + xyz\bar{w} + \bar{x}yzw + \bar{x}yz\bar{w} + xyzw + xy\bar{z}w + x\bar{y}zw + x\bar{y}\bar{z}w =$

$= xyzw + xyz\bar{w} + xy\bar{z}w + xy\bar{z}\bar{w} + x\bar{y}zw + \bar{x}yzw + x\bar{y}\bar{z}w$

$+ x\bar{y}z\bar{w} + x\bar{y}\bar{z}\bar{w} + \bar{x}yz\bar{w} =$

m(۱۵) + m(۱۴) + m(۱۳) + m(۱۲) + m(۱۱) + m(۷) + m(۹) + m(۱۰) + m(۸) + m(۶)

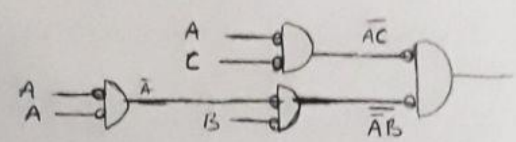
Σm(۶,۷,۸,۹,۱۰,۱۱,۱۲,۱۳,۱۴,۱۵) = ΠM(۰,۱,۲,۳,۴,۵)

a) $F = \bar{A}B + AC$ (2input NAND

$= \overline{(\overline{\bar{A}B} \cdot \overline{AC})}$

a

$\overline{\bar{A} \cdot A}$



b) f(A,B,C) = Σm(٠,٢,٣,٧) = ΠM(1,۴,۵,۶) (3 input or - 2input NAND) → POS

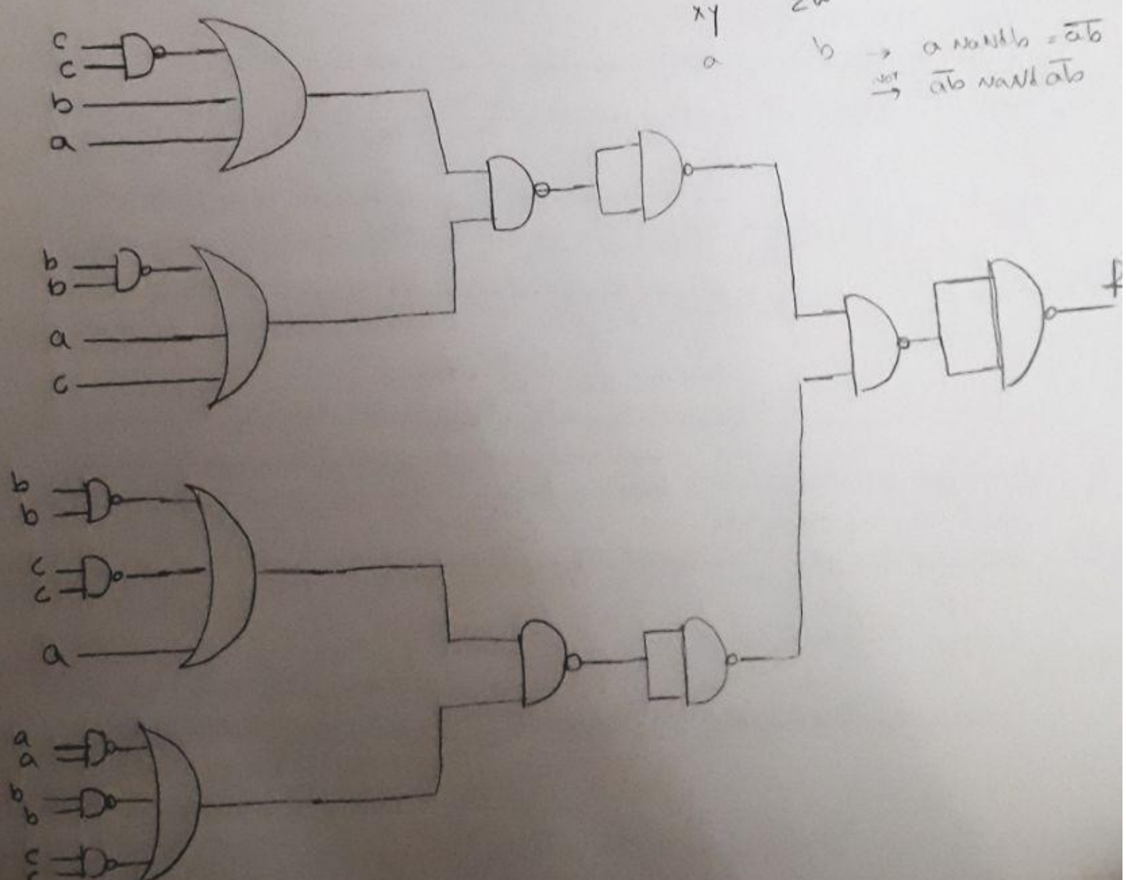
$(a+b+\bar{c}) \cdot (a+\bar{b}+c) \cdot (a+\bar{b}+\bar{c}) \cdot (\bar{a}+\bar{b}+\bar{c})$ → xyzw

$\bar{x}\bar{y}$ $\bar{z}\bar{w}$

xy zw

a b → a NAND b = $\overline{ab}$

$\xrightarrow{not}$ $\overline{ab}$ NAND $\overline{ab}$



(4)

a) $f(x,y,z,w) = x + \bar{w}x + \bar{x}y + wx + xyz + \bar{x}yz$

$x + \bar{x}y + \underbrace{\bar{w}x + wx}_{\text{مجاور}} + \underbrace{xyz + \bar{x}yz}_{\text{مجاور متقارن}} =$

$x + \bar{x}y + x + yz = x + \bar{x}y + yz$

b) $f(x,y,z) = \bar{y}z(\bar{z} + \bar{z}x) + (\bar{x} + \bar{z})(\bar{x}y + \bar{x}z)$

$= \bar{y}z\bar{z} + \bar{y}z\bar{z}x + \bar{x}.\bar{x}y + \bar{z}\bar{x}y + \bar{x}\bar{x}z + \bar{x}y\bar{z} + \bar{z}\bar{x}z$

$= 0 + 0 + \bar{x}y + \bar{x}y\bar{z} + \bar{x}z + \bar{x}y\bar{z} + 0 =$

$\bar{x}y + \bar{x}z + \bar{x}y\bar{z} = \bar{x}y(\underbrace{1+\bar{z}}_{T}) + \bar{x}z = \bar{x}y + \bar{x}z$

c) $f(a,b,c,d) = (a + \bar{c} + \bar{d})(\bar{b} + \bar{c} + d)(a + \bar{b} + \bar{c})$

$(a\bar{b} + a\bar{c} + ad + \bar{c}\bar{b} + \bar{c} + \bar{c}d + \bar{d}\bar{b} + \bar{d}\bar{c} + \underbrace{\bar{d}d}_{0})(a + \bar{b} + \bar{c})$

$a\bar{b} + a\bar{b} + a\bar{b}\bar{c} + a\bar{c} + a\bar{b}\bar{c} + a\bar{c} + ad + a\bar{b}d + a\bar{c}d + a\bar{b}\bar{c} + \bar{b}\bar{c} + \bar{b}\bar{c}$

$+ a\bar{c} + \bar{b}\bar{c} + \bar{c} + a\bar{c}d + \bar{b}\bar{c}d + \bar{c}d + a\bar{b}\bar{d} + \bar{b}\bar{d} + \bar{b}\bar{c}\bar{d} + a\bar{d}\bar{c} + \bar{b}\bar{d}\bar{c} + \bar{d}\bar{c}$

$= a\bar{b} + a\bar{c} + ad + \bar{b}\bar{c} + \bar{b}\bar{d} + \bar{c}d + \bar{d}\bar{c} + \bar{c} + a\bar{b}d + a\bar{c}d + a\bar{b}\bar{c}$

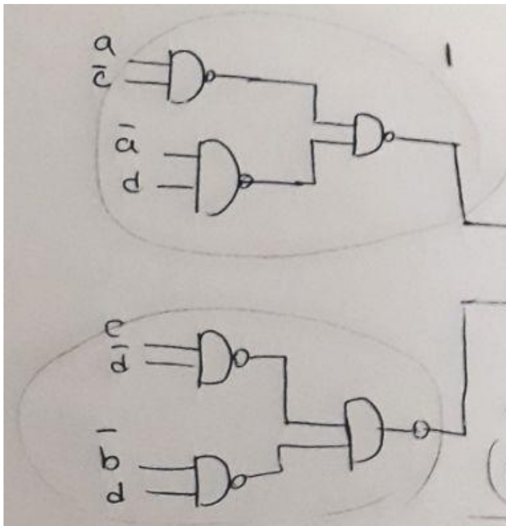
$+ a\bar{d}\bar{c} + \bar{b}\bar{c}d + a\bar{b}\bar{d} + \bar{b}\bar{c}\bar{d} =$

$a\bar{b} + a\bar{c} + ad + \bar{b}\bar{c} + \bar{b}\bar{d} + \bar{c} + \bar{b}\bar{c} + a\bar{b} + a\bar{c} + a\bar{b}\bar{c} =$

$\bar{c} + a\bar{b} + a\bar{c} + ad + \bar{b}\bar{c} + \bar{b}\bar{d} + a\bar{b}\bar{c} =$

$(\underbrace{\bar{c} + a\bar{c}}_{c(1+a)}) + (\underbrace{a\bar{b} + a\bar{b}\bar{c}}_{a\bar{b}(1+\bar{c})}) + ad + \bar{b}\bar{c} + \bar{b}\bar{d} = \bar{c} + a\bar{b} + ad + \bar{b}\bar{c} + \bar{b}\bar{d} = \bar{c} + a\bar{b} + ad + \bar{b}$

(۷)



۱: $\overline{\overline{(a\bar c)}\cdot\overline{(\bar a d)}}$

$\overline{(\bar a+c)\cdot(a+\bar d)}$

$\overline{(\bar a+c)}+\overline{(a+\bar d)}$

① nand ② = f

۲: $\overline{\overline{(c\bar d)}\cdot\overline{(\bar b d)}}$

$\overline{\left(\overline{(\bar a+c)}+\overline{(a+\bar d)}\right)\cdot\left(\overline{(\bar c+d)}+\overline{(b+\bar d)}\right)} =$

$= \overline{\left(\overline{(\bar a+c)}+\overline{(a+\bar d)}\right)} + \overline{\left(\overline{(\bar c+d)}+\overline{(b+\bar d)}\right)}$

۱: $\overline{(\bar a+c)}+\overline{(a+\bar d)}$

۲: $\overline{(\bar c+d)\cdot(b+\bar d)} = \overline{(\bar c+d)}+\overline{(b+\bar d)}$