Date: ۹۷۵۳۲۲۶۵ یاسمین مددی Subject: تمرین سری سوم سیگنال سیستم

۱) برای هر کدام از سیستم ها مشخص کنید که کدام یک از خواص حافظه، علیت، پایداری، خطی و تغییر ناپذیری با زمان را دارند. همچنین برای هر مورد پاسخ ضربه را محاسبه کرده و با داشتن خواص حافظه، علیت، پایداری را بررسی کنید. (n_0 یک عدد ثابت است)

a) $y[n] = x[n+1] - x[n]$

از آنجا که برای $y[n]$ به $x[n+1]$ نیاز داریم و به آینده وابسته است، علیت ندارد
پس غیرعلی است

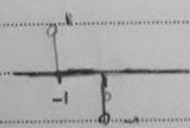
پایداری: $x[n] < M_x$, $x[n+1] < M_x$ $\Big\}$ $x[n+1] - x[n] < M_x - M_x < M_y < +\infty$ پایدار

خطی: $a x_1[n+1] - b x_1[n] + a x_2[n+1] - b x_2[n] =$
$a(x_1[n+1] + x_2[n+1]) - b(x_1[n] + x_2[n])$
$H\{a x_1(n) + b x_2(n)\} = a H\{x_1(n)\} + b H\{x_2(n)\}$ ✓

TI: $y_1[n] = H\{x(n-n_0)\} = x[n-n_0+1] - x[n-n_0]$ $y_1 = y_2$ ✓
$y_2[n] = x[n-n_0+1] - x[n-n_0]$

پاسخ ضربه

$h(n) = \delta(n+1) - \delta(n)$



علی بودن: if $h[n] = 0 \ \forall n < 0$ $n = -1 \rightarrow h[n] \neq 0$ → غیرعلی

پایداری: $\sum_{-\infty}^{\infty} |h[n]| < \infty$ $\sum_{-1}^{0} \delta(n+1) - \delta(n) = 0 < \infty$ محدود

به حافظه نیاز دارد چون برای تعیین $X[n]$ به حافظه نیاز است

Kian ①

b) $y[n] = x[n]\, u(n+5)$

$h(n) = \delta[n]\, u(n+5)$ $\qquad \delta[n] \begin{cases} 1 & n=0 \\ 0 & n\neq 0 \end{cases}$

if $h[n]=0 \ \forall n<0 \rightarrow$ علی $\qquad$ if $n<0 \rightarrow h[n]=0$ علی

$\sum_{-\infty}^{\infty} |h[n]| = \delta[0]\, u(5) = 1 \neq \infty$ محدود پایدار

$TI \longrightarrow \quad y_1 = x[n-n_0]\, u(n+5) \rightarrow n \geq -5$

$y_2 = x[n-n_0]\, u(n-n_0+5) \longrightarrow n \geq n_0 - 5$

TI ✓

$u(n+5) = u(n+n_0-5) \quad n \geq n_0 \leftarrow h[n]=0 \leftarrow n-n_0<0$ ناثابت

چون سیستم LTI بوده و پاسخ ضربه خروجی از تاخیر ضربه است حافظه دارد

$y[n] = (a x_1[n] + b x_2[n])\, u(n+5) = a\, x_1[n]\, u(n+5) + b\, x_2[n]\, u[n+5]$

$= a\, y_1[n] + b\, y_2[n]$ خطی ✓

c) $y[n] = \sum_{k=m}^{n} x[k]$

$h(n) = \sum_{k=m}^{n} \delta[k]$

if $h[n]=0 \ \forall n<0 \rightarrow$ علی $\quad$ if $m>0 \rightarrow h[n]=0$ بسته به مقدار m, n می‌تواند علی یا نباشد

$\sum_{-\infty}^{\infty} |h[n]| = \sum_{m}^{n} \sum_{m}^{n} \delta[k] = (n-m)\cdot 1 \neq \infty$ محدود پایدار

$TI: \quad y_1 = \sum_{k=m}^{n} x[k-t]$, $\quad y_2 = \sum_{k=m}^{n} x[k-t]$ $\quad y_1 = y_2$ $\qquad$ TI ✓

خطی $y[n] = \sum a x_1[k] + b x_2[k] = \sum_{m}^{n} a x_1[k] + \sum_{m}^{n} b x_2[k] = y_1[n] + y_2[n]$ ✓

چون به تمامی مقادیر $x[\,]$ از گذشته نیاز داریم حافظه‌دار است

Date: .... Subject: ....

۲) کانولوشن سیگنال های زیر را به صورت دستی محاسبه کنید و در کنار استفاده از فرمول ها یک بار هم با استفاده از نمودارهای دو سیگنال را برای موارد b و c
نیز انجام دهید.

a) $x[n] = u[n]$, $h[n] = a^n u[n]$ $(0 < a < 1)$

b) $x[n] = u[n] - u[n-3]$, $h[n] = u[n] - u[n-2]$

c) $x[n] = u[n-5] - 2u[n]$, $h[n] = u[n-3] - u[n+1]$

d) $x[n] = u[n-3] - 2u[n+2]$, $h[n] = u[n-3] - u[n+1]$

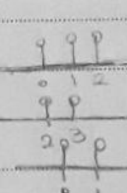
a) $y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h(n-k) = \sum_{k=-\infty}^{k=\infty} u[k] \cdot a^{n-k} u[n-k]$

$a^n \sum u[k] \cdot u[n-k]\, a^{-k}$ $\qquad k \geq 0 \qquad n-k \geq 0 \rightarrow k \leq n$

$= a^n \sum_{k=0}^{k=n} u[k] \cdot u[n-k]\, a^{-k} = a^n \sum_{0}^{n} a^{-k} \rightarrow \frac{(a^{-1})^0 - (a^{-1})^{n+1}}{1 - a^{-1}} \times a^n$

b) $x[n] = u[n] - u[n-3]$, $h[n] = u[n] - u[n-2]$

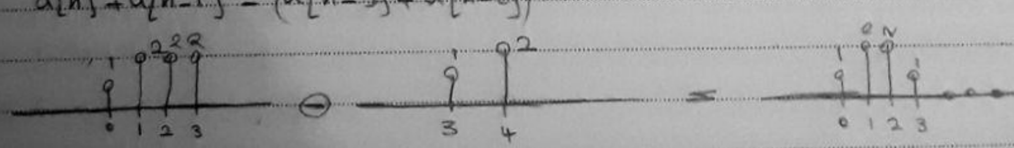
$y[n] = \sum_{-\infty}^{\infty} \left(u[n-k] - u[n-3-k]\right) \cdot \left(u[n] - u[n-2]\right)$



$y[n] = \sum_{0}^{1} u[n-k] - u[n-k-3]$

$u[n] - u[n-3] + u[n-1] - u[n-4]$

$u[n] + u[n-1] - (u[n-3] + u[n-4])$



Date: Subject:

$$\sum_{-\infty}^{\infty} u[n-k] \cdot u[k] - \sum_{-\infty}^{\infty} u[n-k-3] \cdot u[k] - \sum_{-\infty}^{\infty} u[n-k] u[k-2] - \sum_{-\infty}^{\infty} u[k-2] u[n-k-3]$$

$$\sum_{0}^{n}(1) - \sum_{0}^{n-3}(1) - \sum_{2}^{n}(1) + \sum_{2}^{n-3}(1) = n+1-(n-3) - (n-2+1) + n-5+1 =$$

$$1+3-1+2-1-5+1 = 0$$

c) $x[n] = u[n-5] - 2u[n]$

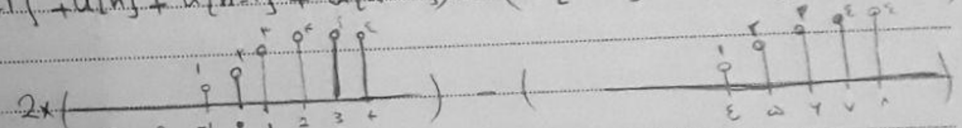
$h[n] = u[n-3] - u[n+1]$

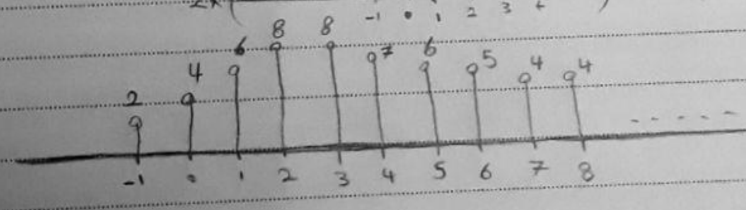
$$y[n] = x[n] * h[n] = \sum_{-\infty}^{\infty} (x[n-k-5] - 2u[n-k])\,(\underbrace{u[k-3] - u[k+1]}_{-[-1,2]})$$

$$y[n] = \sum_{-1}^{2} -u[n-k-5] + 2u[n-k]$$

$$-u[n-4] + 2u[n+1] - u[n-5] + 2u[n] - u[n-4] + 2u[n-1] - u[n-7] + 2u[n-2] =$$

$$2(u[n+1] + u[n] + u[n-1] + u[n-2]) - (u[n-4] + u[n-5] + u[n-6] - u[n-7])$$

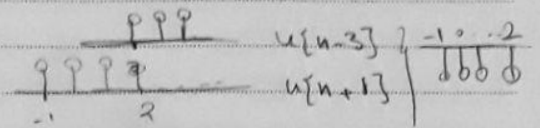




Date: **Subject:**

d) $x[n] = u[n-3] - 2u[n+2]$

$h[n] = u[n-3] - u[n+1]$



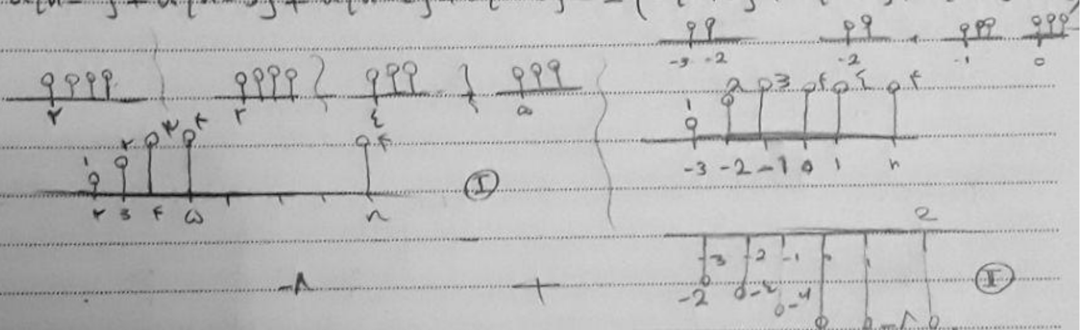
$y[n] = \sum_{-\infty}^{\infty} \left(u[n-k-3] - 2u[n-k+2]\right) \cdot \left(u[n-3] - u[n+1]\right) =$ $[-1, 2]$

$y[n] = (-1) \sum_{k=-1}^{2} u[n-k-3] - 2u[n-k+2]$

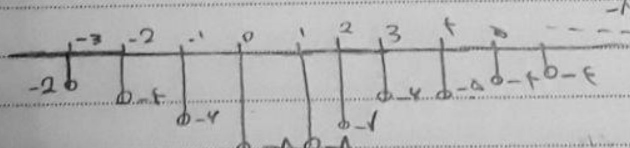
$-\Big[\ u[n-2] - 2u[n+3] + u[n-3] - 2u[n+2] + u[n-4] + 2u[n+1]$

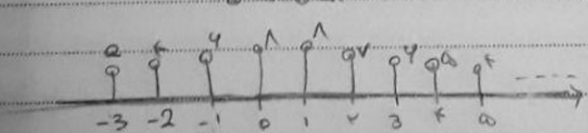
$u[n-5] - 2u[n]\ \Big] =$

$-\Big[u[n-2] + u[n-3] + u[n-4] + u[n-5] - 2\left(u[n+3] + u[n+2] + u[n+1] + u[n]\right)$



$-(\text{I} + \text{II}) \rightarrow$





$\begin{cases} n \geq 5 \rightarrow 4 \\ -3 \leq n \leq 1 \rightarrow 2n+8 \\ 1 \leq n < 5 \rightarrow 9-n \\ n < -3 \quad 0 \end{cases}$

5

Date: Subject:

① سؤال ۲:

$h_1[n] = \left(-\frac{1}{2}\right)^n u[n] \qquad x[n] = u[n]$

$y[n] = \sum_{-\infty}^{\infty} w_1(k) \cdot h_2[n-k]$

$h_2[n] = u[n] + \frac{1}{2} u[n-1]$

$y[n] = \sum_{k=-\infty}^{\infty} \left(u[k] * h_1[k]\right) \cdot h_2[n-k]$

a) $y[n] = w[n] * h_2[n]$

$y[n] = \sum_{-\infty}^{\infty} \left(\sum_{t=-\infty}^{\infty} h_1[t] \cdot u[k-t]\right) \cdot h_2[n-k]$

$w[n] = x[n] * h_1[n]$

$\sum_{-\infty}^{\infty} h_1[t] \cdot u[k-t] = \sum_{-\infty}^{\infty} \left(-\frac{1}{2}\right)^t u[t] \cdot u[k-t]$

b) $y[n] = g[n] * h_1[n]$

$= \sum_{0}^{n} \left(-\frac{1}{2}\right)^t = \frac{\left(-\frac{1}{2}\right)^0 - \left(-\frac{1}{2}\right)^{n+1}}{1+\frac{1}{2}} = \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1}\right)$

$g[n] = x[n] * h_2[n]$

$\sum_{-\infty}^{\infty} \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1-k}\right) \cdot \left(u[k] + \frac{1}{2} u[k-1]\right)$

$= \sum_{k=0}^{\infty} \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1-k}\right) + \sum_{k=1}^{\infty} \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1-k}\right) \times \frac{1}{2} =$

$+ \frac{2}{3} \sum_{1}^{\infty} \left(1 - \left(-\frac{1}{2}\right)^{n-k+1}\right) + \frac{1}{3} \sum_{1}^{\infty} \left(1 - \left(-\frac{1}{2}\right)^{n-k+1}\right) = \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1}\right) + \sum_{n=1}^{\infty} \left(1 - \left(-\frac{1}{2}\right)^{n-k+1}\right)$

$\frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1}\right) + \sum_{1}^{\infty} (1) - \left(-\frac{1}{2}\right)^{n+1} \sum_{1}^{\infty} \left(-\frac{1}{2}\right)^{-k} \qquad \frac{\left(\left(-\frac{1}{2}\right)^{-1}\right)^1 - \left(\left(-\frac{1}{2}\right)^{-1}\right)^{\infty}}{1 - \left(-\frac{1}{2}\right)^{-1}} = \frac{-2-\infty}{1+2}$

$\frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1}\right) + \sum_{1}^{\infty} (1) + \left(-\frac{1}{2}\right)^{n+1} \times \frac{2}{3} - \infty$

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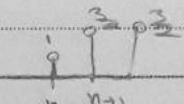
$\sum_{-\infty}^{\infty} \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{k+1}\right) \cdot \left(u[n-k] + \frac{1}{2} u[n-k-1]\right)$



$\left(\sum_{n+1}^{\infty} \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{k+1}\right) \cdot \frac{3}{2}\right) + \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1}\right) =$

$\sum_{n+1}^{\infty} 1 - \sum_{n+1}^{\infty} \left(-\frac{1}{2}\right)^{k+1} + \frac{2}{3}\left(1 - \left(-\frac{1}{2}\right)^{n+1}\right) = \sum_{n+1}^{\infty}(1) - \frac{1}{3}\left(-\frac{1}{2}\right)^{n+1} + \frac{2}{3}$

$\sum_{n+1}^{\infty} 1 \qquad \frac{\left(-\frac{1}{2}\right)^{n+2} - \left(-\frac{1}{2}\right)^{\infty}}{1+\frac{1}{2}} = \frac{2}{3}\left(-\frac{1}{2}\right)^{n+2} , \frac{2}{3}\left(-\frac{1}{2}\right)^{n+1}\left(-\frac{1}{2}\right)$

$\frac{1}{3}\left(-\frac{1}{2}\right)^{n+1} - \frac{2}{3}\left(-\frac{1}{2}\right)^{n+1} = -\frac{1}{3}\left(-\frac{1}{2}\right)^{n+1}$

Date: .... Subject: ....

$y[n] = \sum_{-\infty}^{\infty} g[k] \cdot h_1[n-k]$ سوال ۴ الف ③

$g[n] = \sum_{t=-\infty}^{\infty} u[t] \cdot h_2[\cancel{n}-t] = \sum_{-\infty}^{\infty} u[n-t] \cdot h_2[t] = \sum_{-\infty}^{\infty} u[n-t] \cdot u[t] + \frac{1}{2} \sum u[n-t]\, u[t-1]$

$\sum_{0}^{n} (1) + \frac{1}{2} \sum_{1}^{n} (1) = 1 + \sum_{1}^{n} (1) + \frac{1}{2} \sum_{1}^{n} (1) = 1 + \frac{3}{2} \sum_{1}^{n} (1) = 1 + \frac{3}{2} n$

$y[n] = \sum_{-\infty}^{\infty} \left(\frac{-1}{2}\right)^k u[k] \cdot 1 + \frac{3}{2}(n-k) = \sum_{0}^{\infty} \left(\frac{-1}{2}\right)^k + \left(\frac{-1}{2}\right)^k \times \frac{3}{2}(n-k)$

$\sum_{0}^{\infty} \left(\frac{-1}{2}\right)^k + \frac{3}{2} n \sum_{0}^{\infty} \left(\frac{-1}{2}\right)^k - \sum_{0}^{\infty} \left(\frac{-1}{2}\right)^k \cdot (-k)$

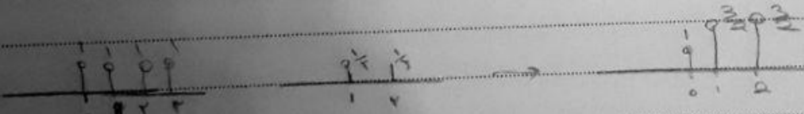
$= \frac{1}{\frac{3}{2}} + \frac{3}{2} n \times \frac{2}{3} - \sum_{0}^{\infty} \left(\frac{-1}{2}\right)^k \cdot (-k) = \frac{2}{3} + n - \sum_{0}^{\infty} \left(\frac{-1}{2}\right)^k (-k)$

$w = \sum_{-\infty}^{\infty} u[n-t] \cdot \left(\frac{-1}{2}\right)^t u[t] = \sum_{0}^{n} \left(\frac{-1}{2}\right)^t = \frac{1-\left(\frac{-1}{2}\right)^{n+1}}{1+\frac{1}{2}} = \frac{2}{3}\left(1-\left(\frac{-1}{2}\right)^{n+1}\right)$

$\sum_{-\infty}^{\infty} \frac{2}{3}\left(1-\left(\frac{-1}{2}\right)^{n-k+1}\right) \cdot \left(u[n] + \frac{1}{2} u[n-1]\right)$

$\sum_{-\infty}^{\infty} \frac{2}{3} u[k] - \sum_{-\infty}^{\infty} \frac{2}{3}\left(\frac{-1}{2}\right)^{n-k+1} \cdot u[k] + \sum_{-\infty}^{\infty} \frac{2}{3} \times \frac{1}{2} u[k-1] - \sum_{-\infty}^{\infty} \left(\frac{-1}{2}\right)^{n-k+1} \cdot \frac{1}{2} u[k-1]$

$= \sum_{0}^{\infty} \frac{2}{3} - \frac{2}{3} \times \left(\frac{-1}{2}\right)^{n+1} \sum_{0}^{\infty} \left(\frac{-1}{2}\right)^{-k} + \frac{1}{3} \sum_{1}^{\infty} (1) -$



$\sum_{1}^{\infty} \frac{2}{3}\left(1-\left(\frac{-1}{2}\right)^{n-k+1}\right) \times \frac{3}{2} + \frac{2}{3} \cdot \left(1-\left(\frac{-1}{2}\right)^{n+1}\right)$

از آنجا که کانولوشن خاصیت

$(x[n] * h_1[n]) * h_2[n] = (x[n] * h_2[n]) * h_1[n]$ ④

جابجایی پذیری دارد می توان ترتیب کانولوشن ها را عوض کرد

Kian

سؤال ۵

a) $y_1 = x[n] * [1,1,1,1,1]$

$y_2 = x[n] * [1,-1,-1,1]$

$y_1 + y_2 = x[n] * [1,1,1,1,1] + x[n] * [1,-1,-1,-1,1] = x[n] * ([1,1,1,1,1] + [1,-1,-1,-1,1])$

(1) $x[n] * [2,0,0,0,2] * [1,1,1] = y$; (1), (2) $\rightarrow h = [2,0,0,0,2] * [1,1,1]$

(2) $y = h * x[n]$

$h[n] = [2,0,0,0,2] * [1,1,1]$