سؤال ۱:

$h[n] = ?$ $\quad s[n] = \checkmark$

$s[n] = u[n] * h[n]$

$s[n] = h[n] * u[n] \rightarrow s[n] = \sum_{k=-\infty}^{\infty} h[k]\, u[n-k]$

$= \sum_{-\infty}^{n} h[k] \longrightarrow h[n] = s[n] - s[n-1]$

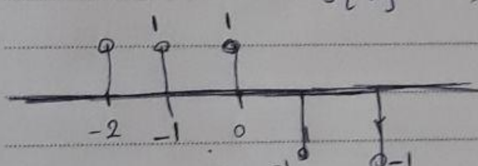
$h[-2] = s[-2] - s[-3] = 1 - 0 = 1$	$h[1] = s[1] - s[0] = 2 - 3 = -1$
$h[-1] = s[-1] - s[-2] = 2 - 1 = 1$	$h[2] = s[2] - s[1] = 1 - 2 = -1$
$h[0] = s[0] - s[-1] = 3 - 2 = 1$	

یا از خواص کانولوشن:

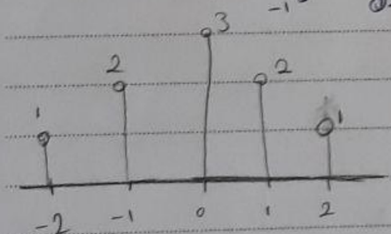
$u[n] \xrightarrow{LTI} s[n]$

$\delta[n] \xrightarrow{LTI} h[n] = ? \rightarrow \delta[n] = u[n] - u[n-1]$ نتیجه ①

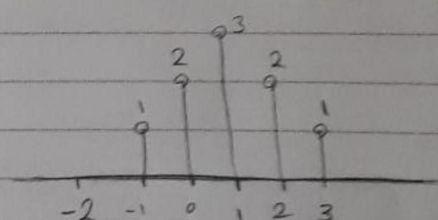
با فرض $s[n]$ داده شده



$\xrightarrow[\text{طبق ①}]{LTI} h[n] = s[n] - s[n-1]$

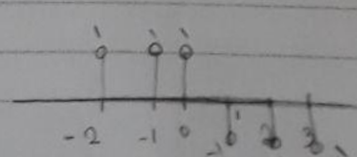


$s[n]$ $-$



$s[n-1]$

$h[n] = s[n] - s[n-1]$



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اردیبهشت

۲۹ 19 May 2019

یک شنبه ۱۴ رمضان ۱۴۴۰

سؤال ② ع

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) \cdot e^{-jk2\pi F_0 t} \, dt \qquad T_0 = 4$$

$$a_k = \frac{1}{4} \int_{-3}^{3} x(t) \, e^{-jk2\pi F_0 t} \, dt =$$

$$= \frac{1}{4} \left(\int_{-2}^{-1} e^{-jk2\pi F_0 t} \, dt + \int_{1}^{2} -e^{-jk2\pi F_0 t} \, dt \right)$$

$$= \frac{1}{4} \left(\frac{1}{-jk2\pi F_0} e^{-jk2\pi F_0 t} \Big|_{-2}^{-1} - \frac{1}{-jk2\pi F_0} e^{-jk2\pi F_0 t} \Big|_{1}^{2} \right)$$

$$= \frac{1}{4} \left(\frac{1}{-jk2\pi F_0} \left(-e^{+4k\pi F_0 j} + e^{4k\pi F_0 j} \right) + \frac{1}{-jk2\pi F_0} \left(e^{-4k\pi F_0 j} - e^{-2\pi k j F_0} \right) \right.$$

$$= \frac{1}{4} \left(\frac{1}{+jk2\pi F_0} \left(e^{4k\pi F_0 j} + e^{-4k\pi F_0 j} \right) - \frac{1}{jk\pi F_0} \left(e^{2k\pi F_0 j} + e^{-2k\pi j F_0} \right) \right)$$

$$a_k = \frac{1}{4jk\pi F_0} \left(\cos(4k\pi F_0) - \cos(4k\pi F_0) \right)$$

$$a_k = \frac{1}{4jk\pi F_0} \left(\cos(4k\pi F_0) - \cos(2k\pi F_0) \right)$$

2π

$2\pi F_0$

a) $x(t) = \sin(10\pi t + \frac{\pi}{4})$

$T = \frac{2\pi}{10\pi} \rightarrow T = \frac{1}{\omega}$ $\quad \omega_0 = \frac{2\pi}{T} = 10\pi$

$\frac{1}{2j} e^{j(10\pi t + \frac{\pi}{4})} - \frac{1}{2j} e^{-j(10\pi t + \frac{\pi}{4})}$

$x(t) = \sum_{-\infty}^{\infty} a_k e^{j\omega_0 k t}$

$= \frac{1}{2j} \times e^{j\frac{\pi}{4}} \cdot e^{10\pi t j} - \frac{1}{2j} e^{-j\frac{\pi}{4}} \cdot e^{-10\pi t j} \rightarrow$

$a_k e^{jk \times 10\pi t} = (\frac{1}{2j} e^{j\frac{\pi}{4}}) \cdot e^{10\pi t j} \rightarrow k=1 \rightarrow a_1 = \frac{1}{2j} e^{j\frac{\pi}{4}}$

$a_k e^{jk10\pi t} = (\frac{-1}{2j} e^{-j\frac{\pi}{4}}) \cdot e^{-10\pi t j} \rightarrow k=-1 \rightarrow a_{-1} = \frac{-1}{2j} e^{-j\frac{\pi}{4}}$

b) $x(t) = 1 + \cos(t)$

$T = \frac{2\pi}{1} = 2\pi$

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$x(t) = 1 + \frac{1}{2} e^{tj} + \frac{1}{2} e^{-tj}$

$x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$a_k e^{jkt} = \frac{1}{2} e^{tj} \rightarrow k=1 \rightarrow a_1 = \frac{1}{2}$

$a_k e^{jkt} = 1 \rightarrow jkt = 0 \rightarrow k=0 \rightarrow a_0 = 1$

$a_k e^{jkt} = \frac{1}{2} e^{-tj} \rightarrow k=-1 \rightarrow a_{-1} = \frac{1}{2}$

c) $1 + \cos(2t)$

$x(t) = 1 + \frac{1}{2} e^{2tj} + \frac{1}{2} e^{-2tj}$

$T = \frac{2\pi}{2} = \pi$

$\omega_0 = \frac{2\pi}{T} = 2$

$x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t}$

$a_k e^{2jkt} = \frac{1}{2} e^{2tj} \rightarrow k=1 \rightarrow a_1 = \frac{1}{2}$

$a_k e^{2jkt} = \frac{1}{2} e^{-2tj} \rightarrow k=-1 \rightarrow a_{-1} = \frac{1}{2}$

$a_k e^{2jkt} = 1 \rightarrow k=0 \rightarrow a_0 = 1$

d

$\sin(10\pi t + \frac{\pi}{4}) + \cos(2\pi t) \cdot \sin(10\pi t + \frac{\pi}{4}) =$

$\sin(10\pi t + \frac{\pi}{4}) + \frac{1}{2}\sin(12\pi t + \frac{\pi}{4}) + \frac{1}{2}\sin(8\pi t + \frac{\pi}{4})$

$T_1 = \frac{1}{6}$

دوره تناوب = ک.م.م دوره تناوب‌ها

$T_2 = \frac{1}{5}$

$T = \frac{1}{6} \cup \frac{1}{5} \cup \frac{1}{4} = \cancel{\ } \; 1$

$T_3 = \frac{1}{4}$

$\omega_0 = \frac{2\pi}{1} = 2\pi$

$x(t) = \frac{1}{2}\left(\frac{1}{2j}\left(e^{(12\pi t + \frac{\pi}{4})j} - e^{-(12\pi t + \frac{\pi}{4})j}\right)\right) + \frac{1}{2j}\left(e^{10\pi t j} - e^{-10\pi t j}\right)$

$+ \frac{1}{2}\left(\frac{1}{2j}\left(e^{(8\pi t + \frac{\pi}{4})j} - e^{-(8\pi t + \frac{\pi}{4})j}\right)\right) =$

$\frac{1}{4j} e^{\frac{\pi}{4}j} e^{12\pi t j} - \frac{1}{4j} e^{-\frac{\pi}{4}j} e^{-12\pi t j} + \frac{1}{2j} e^{10\pi t j} - \frac{1}{2j} e^{-10\pi t j}$

$+ \frac{1}{4j} e^{\frac{\pi}{4}j} e^{8\pi t j} - \frac{1}{4j} e^{-\frac{\pi}{4}} e^{-8\pi t j} =$

2

$x(t) = \sum_{-\infty}^{\infty} a_k e^{jk\omega_0 t} \qquad \omega_0 = 2\pi$

$\frac{1}{4j} e^{\frac{\pi}{4}j} e^{12\pi t j} = a_k e^{2\pi t k j} \rightarrow k = 6 \rightarrow a_6 = \frac{e^{\frac{\pi}{4}j}}{4j}$

$-\frac{1}{4j} e^{-\frac{\pi}{4}} e^{-12\pi t j} = a_k e^{2\pi t k j} \rightarrow k = -6 \rightarrow a_{-6} = \frac{-1}{4j} e^{-\frac{\pi}{4}j}$

$\frac{1}{2j} e^{10\pi t j} = a_k e^{2\pi t j k} \rightarrow k = 5 \rightarrow a_5 = \frac{1}{2j}$

$-\frac{1}{2j} e^{-10\pi t j} = a_k e^{2\pi t j k} \rightarrow k = -5 \rightarrow a_{-5} = \frac{-1}{2j}$

$\frac{e^{\frac{\pi}{4}}}{4j} e^{8\pi t j} = a_k e^{2\pi t j k} \rightarrow k = 4 \rightarrow a_4 = \frac{e^{\frac{\pi}{4}}}{4j}$

$-\frac{1}{4j} e^{-\frac{\pi}{4}} e^{-8\pi t j} = a_k e^{2\pi t j k} \rightarrow k = -4 \rightarrow a_{-4} = \frac{-e^{-\frac{\pi}{4}}}{4j}$

3

e) $x(t) = \dfrac{1}{\frac{1}{2}\cos(2t) + \frac{j}{2}\sin(2t)} = \dfrac{1}{\frac{1}{2}\left(\frac{1}{2}e^{2tj} + \frac{1}{2}e^{-2tj}\right) + \frac{j}{2}\left(\frac{1}{2j}e^{2tj} - \frac{1}{2j}e^{-2tj}\right)}$

$= \dfrac{1}{\frac{1}{4}\left(e^{2tj} + \cancel{e^{-2tj}} + e^{2tj} - \cancel{e^{-2tj}}\right)} = \dfrac{4}{2e^{2tj}} = 2e^{-2tj}$

$= 2\left(\cos(-2t) + i\sin(-2t)\right) = 2\left(\cos(2t) - i\sin(2t)\right)$

$T = \dfrac{2\pi}{2} = \pi \rightarrow w_0 = \dfrac{2\pi}{T} = \dfrac{2\pi}{\pi} = 2 \rightarrow a_k e^{jk2t} = 2e^{-2tj}$

$\boxed{a_{-1} = 2}$ $\quad -2 = 2k$

$k = -1$