تمرین سری ۷ ام

سوال ۱ :

a) سری فوریه :

$$x(t)=\sum_{-\infty}^{\infty} c_k e^{jk\frac{2\pi}{T}t} = \sum_{-\infty}^{\infty} c_k e^{jk\Omega_0 t}$$

i) $x(t) = (1+\cos(2\pi t)) \times \sin(10\pi t + \frac{\pi}{4})$

$$= \sin(10\pi t + \tfrac{\pi}{4}) + \cos 2\pi t \cdot \sin(10\pi t + \tfrac{\pi}{4}) =$$

$$\left(\frac{e^{(10\pi t+\frac{\pi}{4})j} - e^{-(10\pi t+\frac{\pi}{4})j}}{2j}\right) + \left(\frac{e^{2\pi t j} + e^{-2\pi t j}}{2}\right) \times$$

$$\left(\frac{e^{(10\pi t+\frac{\pi}{4})j} - e^{-(10\pi t+\frac{\pi}{4})j}}{2j}\right) =$$

$$x(t) = \frac{e^{j\frac{\pi}{4}}}{2j} e^{(10\pi t)j} + \frac{-e^{j\frac{\pi}{4}}}{2j} e^{-10\pi t j} + \frac{e^{j\frac{\pi}{4}}}{4j} e^{j(12\pi t)}$$

$$+ \frac{-e^{j\frac{\pi}{4}}}{4j} e^{-j(8\pi t)} + \frac{e^{j\frac{\pi}{4}}}{4j} e^{j(8\pi t)} + \frac{-e^{j\frac{\pi}{4}}}{4j} e^{-j(12\pi t)}$$

$$\boxed{T=1\ ,\ \omega_0 = 2\pi}$$

$$x(t) = a_5 \cdot e^{5\Omega_0 t j} + a_{-5} e^{-5\Omega_0 t j} + a_6 \cdot e^{6\Omega_0 t j} + a_{-6} \cdot e^{-6\Omega_0 t j} +$$

ادامه: $+ a_4 e^{4\Omega_0 t j} + a_{-4} e^{-4\Omega_0 t j}$

سؤال ۱ (ii)

$N = 3 \qquad c_k = \frac{1}{N}\sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n}$

$c_k = \frac{1}{3}\sum_{n=0}^{2} x[n] e^{-jk\omega_0 n}$

چون $N=3$ ← داریم: C_2 و C_1 و C_0

$\cancel{c_k = \frac{1}{3}\left(x[0] + x[1 \right.}$

$c_1 = \frac{1}{3}\sum_{0}^{2} x[n] . e^{\frac{2\pi}{3} j\times 1\times n}$

$c_2 = \frac{1}{3}\sum_{0}^{2} x[n] . e^{\frac{2\pi}{3} j\times 2\times n}$

$\omega_k = \frac{2k\pi}{N}$

$\omega_0 = 0$

$\omega_1 = \frac{2\pi}{3}$

$\omega_2 = \frac{4\pi}{3}$

$c_0 = \frac{1}{3}\sum_{0}^{2} x[n] . 1 = c_0 = \frac{1}{3}\left(1+\frac{1}{2}+\frac{1}{2}\right) = \frac{2}{3}$

$c_1 = \frac{1}{3}\left(1\times 1 + \frac{1}{2}\times e^{\frac{2\pi}{3}j} + \frac{1}{2} e^{\frac{4\pi}{3}j}\right) = \frac{1}{3} + \frac{1}{6} e^{\frac{2\pi}{3}j} + \frac{1}{6} e^{\frac{4\pi}{3}j}$

$c_2 = \frac{1}{3}\left(1\times 1 + \frac{1}{2} e^{\frac{4\pi}{3}j} + \frac{1}{2} e^{\frac{8\pi}{3}j}\right)$

$x[n] = c_0 + c_1 . e^{j\times\frac{2\pi}{3}} + c_2 e^{\frac{4\pi}{3}j}$

ادامه (ii)

$x[n] = \sum_{k=0}^{N-1} c_k . e^{\omega_k n j}$

$x[n] = c_0 + c_1 e^{\frac{2\pi}{3}nj} + c_2 e^{\frac{4\pi}{3}nj}$

$c_0 = \frac{2}{3} \qquad c_1 = \frac{1}{3} + \frac{1}{6} e^{\frac{2\pi}{3}j} + \frac{1}{6} e^{\frac{4\pi}{3}j}$

$c_2 = \frac{1}{3} + \frac{1}{6} e^{\frac{4\pi}{3}j} + \frac{1}{6} e^{\frac{8\pi}{3}j}$

(iii)

$N = 5$ $\qquad \omega = \frac{2\pi}{N} = \frac{2\pi}{5}$

$$C_k = \frac{1}{5}\sum_{n=0}^{4} x[n] \cdot e^{-jk\frac{2\pi}{5}n}$$

$\omega_0 = 0 \quad \omega_1 = \frac{2\pi}{5} \quad \omega_2 = \frac{4\pi}{5}$

$\omega_3 = \frac{6\pi}{5} \quad \omega_4 = \frac{8\pi}{5}$

چون $N=5$: C_0 تا C_4 داریم

$$C_0 = \frac{1}{5}\sum_{0}^{4} x[n] \cdot e^{0} = \frac{1}{5}\left(1+\frac{1}{2}+0+0+\frac{1}{2}\right) = \frac{2}{5}$$

$$C_1 = \frac{1}{5}\sum_{0}^{4} x[n] \cdot e^{-j\frac{2\pi}{5}n} = \frac{1}{5}\left(1\times1+\frac{1}{2}\times e^{-\frac{2\pi}{5}j}+0+0+\frac{1}{2}\times e^{-j\frac{8\pi}{5}}\right)$$

$$C_1 = \frac{1}{5}+\frac{1}{10}e^{-\frac{2\pi}{5}j}+\frac{1}{10}e^{-j\frac{8\pi}{5}}$$

$$C_2 = \frac{1}{5}\sum_{0}^{4} x[n] \cdot e^{-j\frac{4\pi}{5}n} = \frac{1}{5}\left(1+\frac{1}{2}\times e^{-j\frac{4\pi}{5}}+\frac{1}{2}e^{-\frac{16\pi}{5}j}\right)$$

$$C_3 = \frac{1}{5}\sum_{0}^{4} x[n] \cdot e^{-j\frac{6\pi}{5}n} = \frac{1}{5}\left(1+\frac{1}{2}e^{-j\frac{6\pi}{5}}+\frac{1}{2}e^{-j\frac{24\pi}{5}}\right)$$

$$C_4 = \frac{1}{5}\sum_{0}^{4} x[n] \cdot e^{-j\frac{8\pi}{5}n} = \frac{1}{5}\left(1+\frac{1}{2}e^{-j\frac{8\pi}{5}}+\frac{1}{2}e^{-j\frac{32\pi}{5}}\right)$$

$$x[n] = \left(C_0 + C_1 \cdot e^{\frac{2\pi}{5}nj} + C_2 e^{\frac{4\pi}{5}nj} + C_3 e^{\frac{6\pi}{5}nj} + C_4 e^{\frac{8\pi}{5}nj}\right)$$

$$x[n] = ۱ + \cos(۲\pi n) + \sin(n\pi) + \cos(\frac{n\pi}{۲})$$

$$x[0] = ۱ + ۱ + 0 + ۱ = 3$$

$$x[۱] = ۱ + ۱ + 0 + 0 = ۲$$

$$x[۲] = ۱ + ۱ + 0 + -۱ = ۱$$

$$x[3] = ۱ + ۱ + 0 + 0 = ۲$$

$$c_۳ = \frac{۱}{۴}(۳ + ۲e^{\frac{۳\pi}{۴}j} + ۱e^{۳\pi j} + ۲e^{\frac{۹\pi}{۴}j})$$

$$c_0 = \frac{1}{N}\sum_{n=0}^{N-1} x[n] \cdot ۱ = \frac{۱}{۴}(۲ + ۲ + ۱ + ۳) = ۲$$

$$c_۱ = \frac{۱}{۴}\sum_{0}^{۳} x[n] e^{-\frac{\pi}{۲}nj} = \frac{۱}{۴}(۳ + ۲e^{-\frac{\pi}{۲}j} + ۱e^{-\pi j} + ۲e^{-\frac{۳}{۲}\pi j})$$

$$c_۲ = \frac{۱}{۴}\sum_{0}^{۳} x[n] e^{-\pi nj} = \frac{۱}{۴}(۳ + ۲e^{-\pi j} + ۱e^{-۲\pi j} + ۲e^{-۳\pi j})$$

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$$x[n] = \sum_{k=0}^{N-1} c_k e^{j\frac{2\pi}{N}kn} =$$

$$x[n] = 2 + \frac{1}{4}\left(3 + 2e^{-\frac{\pi}{2}j} + e^{-\pi j} + 2e^{-\frac{3\pi}{2}j}\right) e^{\frac{\pi}{2}nj} +$$

$$+ \frac{1}{4}\left(3 + 2e^{-\pi j} + e^{-2\pi j} + 2e^{-3\pi j}\right) e^{\pi nj} +$$

$$\frac{1}{4}\left(3 + 2e^{-\frac{3\pi}{2}j} + e^{-3\pi j} + 2e^{-\frac{9\pi}{2}j}\right) \cdot e^{\frac{3\pi}{2}nj}$$

سوال ۳:

a) $x(t) = e^{-at} u(t) \quad , a>0$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} dt = \int_{-\infty}^{\infty} e^{-at} u(t) \cdot e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} dt = \int_{0}^{\infty} e^{-(a+j\omega)t} dt$$

$$= \frac{1}{-(a+j\omega)} \cdot e^{-(a+j\omega)t} \Big|_{0}^{\infty} = 0 - \frac{1}{-(a+j\omega)} = \frac{1}{a+j\omega}$$

b) $x(t) = e^{-a|t|} \quad a>0$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|} \cdot e^{-j\omega t} dt =$$

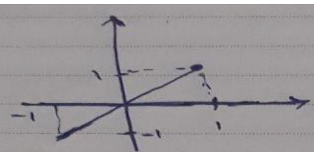
$$\int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t} + \int_{-\infty}^{0} e^{at} \cdot e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-(a+j\omega)t} dt + \int_{-\infty}^{0} e^{(a-j\omega)t} dt$$

$$= \frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \Big|_{0}^{\infty} + \frac{e^{(a-j\omega)t}}{(a-j\omega)} \Big|_{-\infty}^{0} =$$

$$0 - \frac{1}{-(a+j\omega)} + \frac{1}{a-j\omega} - 0 = \frac{1}{a+j\omega} + \frac{1}{a-j\omega}$$

c)



$x(t) = t \quad -1 < t < 1$

$$X(jw) = \int_{-1}^{1} \underbrace{t}_{u} \cdot \underbrace{e^{-jwt}\, dt}_{dv}$$

انتگرال	مشتق
e^{-jwt}	t
$\frac{1}{-jw} e^{-jwt}$	1
	0

$u = t \rightarrow du = dt$

$dv = e^{-jwt} \rightarrow v = \frac{1}{-jw} e^{-jwt}$

$$uv - \int \frac{1}{-jw} e^{-jwt} \cdot dt =$$

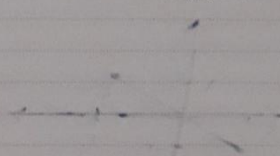
$$\frac{t}{-jw} e^{-jwt} \Big|_{t=-1}^{t=1} - \int_{-1}^{1} \frac{1}{-jw} e^{-jwt} dt =$$

$$\left(\frac{1}{-jw} e^{-jw} - \frac{1}{jw} e^{jw}\right) - \left(\frac{1}{-j^2 w^2} e^{-jwt} \Big|_{-1}^{1}\right) =$$

$$\left(\frac{1}{-jw}\left(e^{-jw} + e^{jw}\right)\right) - \left(\frac{1}{-w^2}\left(e^{-jw} - e^{jw}\right)\right) =$$

$$= \frac{1}{-jw}\left(e^{-jw} + e^{jw}\right) + \frac{1}{w^2}\left(e^{-jw} - e^{jw}\right)$$

d) $x[n] = \delta[n]$



$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} x[n] \cdot e^{-jwn}$$

$$X(e^{jw}) = \sum_{n=-\infty}^{\infty} \delta[n] \cdot e^{-jwn} = \sum_{0}^{0} e^{-jwn} = 1$$

e) $x[n] = a^n u[n] \quad |a| < 1$

$$X(e^{jw}) = \sum_{-\infty}^{\infty} a^n u[n] \cdot e^{-jwn} = \sum_{0}^{\infty} a^n e^{-jwn} =$$

$$\sum_{0}^{\infty} \left(a e^{-jw}\right)^n = \frac{(ae^{-jw})^0 - (ae^{-jw})^\infty}{1 - ae^{-jw}} =$$

$$\frac{1 - 0}{1 - ae^{-jw}} = \frac{1}{1 - ae^{-jw}}$$

f) $x[n] = a^{|n|} , \quad |a| < 1$

$$X(e^{j\omega}) = \sum_{-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{-\infty}^{\infty} a^{|n|} e^{-j\omega n} =$$

$$\sum_{-\infty}^{-1} a^{-n} e^{-j\omega n} + \sum_{0}^{\infty} a^{n} e^{-j\omega n} =$$

$$\sum_{-\infty}^{-1} (a^{-1} e^{-j\omega})^{-n} + \sum_{0}^{\infty} (a e^{-j\omega})^{n} =$$

$$\frac{1}{1 - a^{-1} e^{-j\omega}} + \frac{1}{1 - a e^{-j\omega}}$$

تیر

ش	ی	د	س	چ	پ	ج
۱	۲	۳	۴	۵	۶	۷
۸	۹	۱۰	۱۱	۱۲	۱۳	۱۴
۱۵	۱۶	۱۷	۱۸	۱۹	۲۰	۲۱
۲۲	۲۳	۲۴	۲۵	۲۶	۲۷	۲۸
۲۹	۳۰	۳۱				

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جمعه

28 June 2019

۲۴ شوال ۱۴۴۰

g) $x[n] = \begin{cases} 1 & |n| \leq N_1 \\ 0 & o.w \end{cases}$

$$\sum_{-\infty}^{\infty} x[n] e^{-j\omega n} = \sum_{-N_1}^{N_1} 1 \times e^{-j\omega n} =$$

$$\sum_{-N_1}^{N_1} (e^{-j\omega n}) = \frac{e^{j\omega N_1} - e^{-j\omega N_1}}{1 - e^{-j\omega}}$$

h) $x[n] = \begin{cases} 1 & n = ۰,۱,۲,۳ \\ 0 & o.w \end{cases}$

$$X(e^{wj}) = \sum_{-\infty}^{\infty} x[n]\, e^{-jwn} = \sum_{0}^{۳} 1 \times e^{-jwn} = e^{-jw} + e^{-۲jw} + e^{-۳jw} + 1$$

سوال ۴ a

$$x[n] = \sum_{k=0}^{N-1} C_k \cdot e^{j\omega kn} \xrightarrow{DFS} C_k = \sum_{n=0}^{N-1} x[n] e^{-j\omega kn}$$

$$y[n] = \sum_{r=-\infty}^{\infty} x[n+rN]$$

$$y[n] = \sum_{r=-\infty}^{\infty} \sum_{k=0}^{N-1} C_k \cdot e^{j\omega k(n+rN)} =$$

$$\sum_{r=-\infty}^{\infty} e^{rN} \underbrace{\sum_{k=0}^{N-1} C_k e^{j\omega kn}}_{x[n]} = \sum_{r=-\infty}^{\infty} e^{rN} \cdot x[n] = y[n]$$

$$y[n] = \sum_{r=-\infty}^{\infty} e^{rN} \cdot x[n] \rightarrow , a_k = \frac{1}{N} \sum_{n=0}^{N-1} y[n] \cdot e^{-\omega jkn}$$

$$\Rightarrow a_k = \frac{1}{N} \sum_{n=0}^{N-1} \left(\sum_{r=-\infty}^{\infty} e^{rN} \cdot x[n] \right) e^{-j\omega kn} =$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} e^{-j\omega kn} \cdot x[n] \cdot \sum_{r=-\infty}^{\infty} e^{rN}$$

$$a_k = \sum_{r=-\infty} C_k$$

سوال ۴ b

$$y[n] = \cdots + x[n-N] + x[n] + x[n+N] + \cdots$$

$$\xrightarrow{DFT} Y(e^{j\omega}) = \cdots + e^{-j\omega N} X(e^{j\omega}) + X(e^{j\omega}) + e^{j\omega N} X(e^{j\omega})$$

$$Y(e^{jw}) = X(e^{jw}) \cdot \sum_{t=-\infty}^{t=\infty} \cdot e^{+jwNt}$$

$$a_K \cdot N = X(e^{jw}) \sum_{t=-\infty}^{t=\infty} e^{jwNt} \rightarrow$$

$$a_K = \frac{1}{N} X(e^{jw}) \sum_{t=-\infty}^{t=\infty} e^{jwNt}$$

سوال ۵:

برای محاسبه ضرایب فوریه با تبدیل فوریه می توان حاصل تبدیل را در عکس دوره تناوب ضرب کرد همانطور که در برنامه انجام شده و حاصل هر دو روش یکسان خواهد بود نمونه خروجی دو تابع سوال ۱و ۵

[(2.272727272727273+0j), (-1.076685229061551-0.3161433078078853j), (0.02076780642374363+0.013346658769925671j), (-0.0862445972542397-0.099531571044986j), (0.013340610646024431+0.02921186259856054j), (-0.0075422271176132-0.05245734184240964j), (-0.007542227117613159+0.052457341842409105j), (0.013340610646024856-0.02921186259856093j), (-0.08624459725423923+0.09953157104498542j), (0.020767806423745056-0.013346658769926033j), (-1.07668522906155+0.3161433078078827j)]

[2.27272727+0.j   -1.07668523-0.31614331j 0.02076781+0.01334666j

-0.0862446 -0.09953157j 0.01334061+0.02921186j -0.00754223-0.05245734j

-0.00754223+0.05245734j 0.01334061-0.02921186j -0.0862446 +0.09953157j

0.02076781-0.01334666j -1.07668523+0.31614331j]