①

$$y[n] = x[n] * h[n]$$

a) $x[n] = u[n-1] - 2u[n-4] + u[n-7]$

b) $x[n] = \{u[n] - u[n-12]\} \cdot \sin(\frac{\pi}{4}n)$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$$

$$h[n] = u[n-1] - u[n-14]$$

$$y[n] = \sum_{-\infty}^{\infty} \left(u[k-1] - 2u[k-4] + u[k-7] \right) \cdot \left(u[n-k-1] - u[n-k-14] \right)$$

$$= \sum u[k-1] \cdot u[n-k-1] - 2\sum u[k-4] \cdot u[n-k-1] + \sum u[k-7] \cdot u[n-k-1]$$

$$- \sum u[k-1] \cdot u[n-k-14] + 2\sum u[k-4] \cdot u[n-k-14] - \sum u[k-7] \cdot u[n-k-14]$$

$$= \sum_{1}^{\infty} u[n-k-1] - 2\sum_{4}^{\infty} u[n-k-1] + \sum_{7}^{\infty} u[n-k-1] - \sum_{1}^{\infty} u[n-k-14]$$

$$+ 2\sum_{4}^{\infty} u[n-k-14] - \sum_{7}^{\infty} u[n-k-14]$$

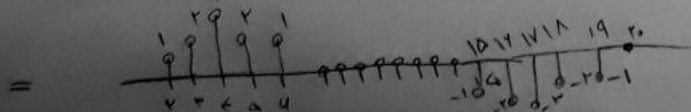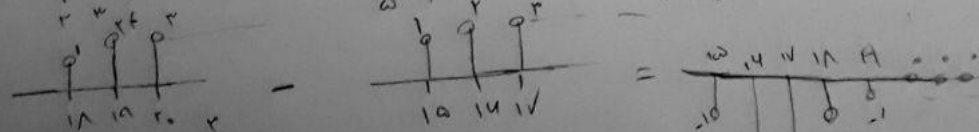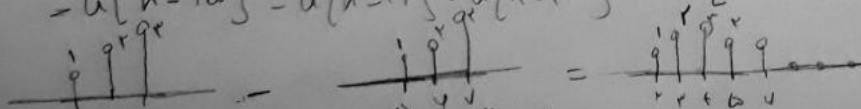$$= \sum_{1}^{7} u[n-k-1] - \sum_{4}^{\infty} u[n-k-1] + \sum_{7}^{\infty} u[n-k-1]$$

$$- \sum_{1}^{7} u[n-k-14] + \sum_{4}^{\infty} u[n-k-14] - \sum_{7}^{\infty} u[n-k-14]$$

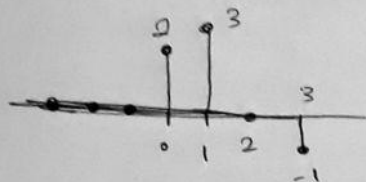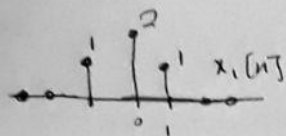$$= \sum_{1}^{7} u[n-k-1] - \sum_{4}^{7} u[n-k-14]$$

$$- \sum_{1}^{7} u[n-k-14] + \sum_{4}^{7} u[n-k-14]$$

$$= u[n-2] + u[n-3] + u[n-4] - u[n-5] - u[n-6] - u[n-7]$$

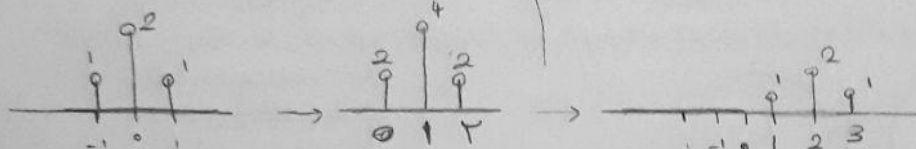$$- u[n-15] - u[n-16] - u[n-17] + u[n-18] + u[n-19] + u[n-20]$$

$x_1[n]$

$y_1[n]$ n

if $|x[n]| < B_x \Rightarrow |y[n]| < B_y$ ← LTI

$\rightarrow \sum_{0}^{\infty} |h[n]| < \infty$

$x_1[n] * h[n] = y_1[n]$
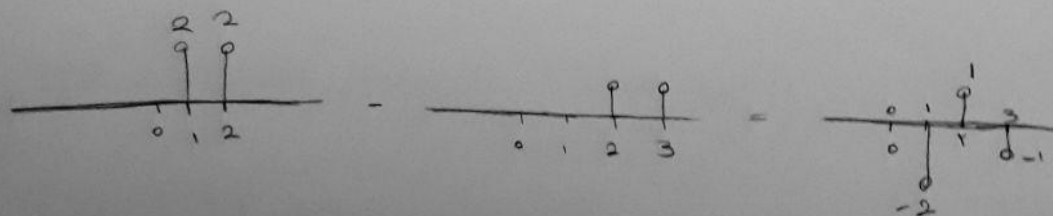
$x_r[n] * h[n] = y_r[n]$

$x_r = 2x_1[n-1] - x[n-2]$

$x_r[n] = 2x_1[n-1] - x[n-2] \xrightarrow{\ h\ } y_r[n] = 2y_1[n-1] - y_1[n-2]$
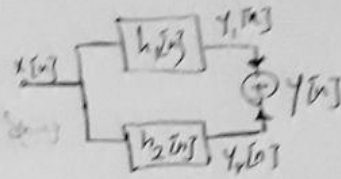
③

$$x[n] = 2\{u[n+1] - u[n-2]\}$$

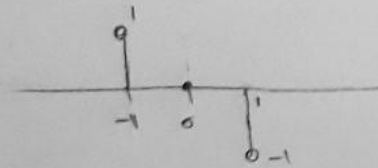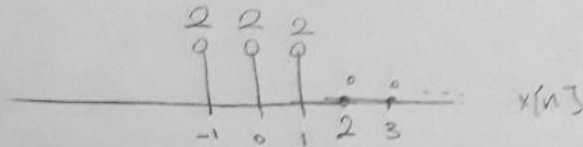$$h_1[n] = \delta[n+1] - \delta[n]$$

$$h_2[n] = \delta[n] - \delta[n-1]$$

$$y_1[n] = x[n] * h_1[n]$$

$$y_r[n] = x[n] * h_r[n] \longrightarrow Y = y_1[n] - y_r[n] = x[n] * [h_1 - h_2]$$

$$x[n]$$

$$y[n]$$

$$y[n] = \sum_{-\infty}^{\infty} 2\, u[n-k+1] - 2u[n-k-2] \cdot \{\delta[n+1] - \delta[n-1]\}$$

$$\sum 2\, u[n-k+1] \cdot \delta[n+1] - \sum 2u[n-k-2] \cdot \delta[n+1] - \sum 2u[n-k+1] \cdot \delta[n-1]$$

$$+ 2\sum u[n-k-2] \cdot \delta[n-1] =$$

$$= 2\, u[n] - 2u[n-3] - 2u[n] + 2u[n-3] = 0$$