The parallel drone scheduling problem with multiple drones and vehicles

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Drones for last-mile delivery

Fig. 4. UPS HorseFly.

Fig. 3. DHL Parcelcopter.

Combined with trucks

- Flying Sidekick Traveling Salesman Problem (FSTSP)
    - drone travels with truck
    - with synchronization
- Parallel Drone Scheduling Traveling Salesman Problem (PDSTSP)
    - drone departs from depot
    - no synchronization

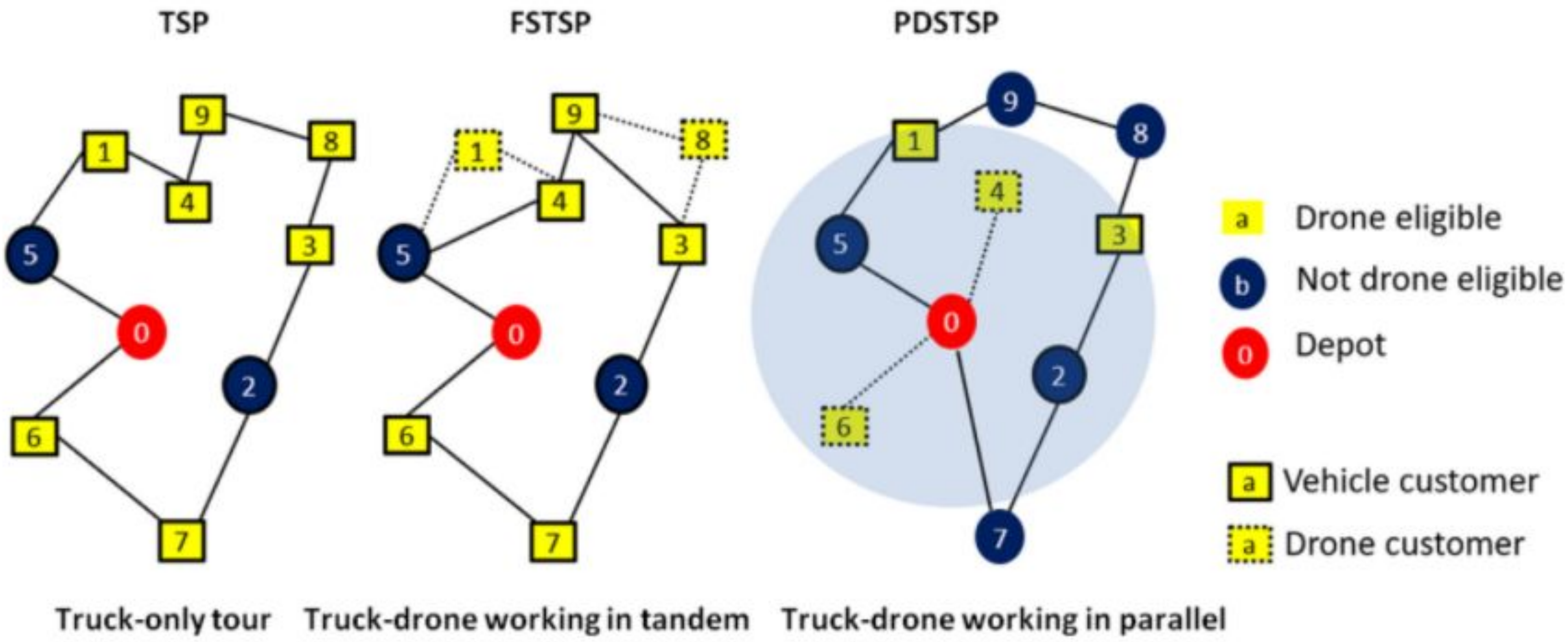


Fig. 5. Illustration of FSTSP and PDSTSP vs truck-only delivery.

This paper

- Parallel Drone Scheduling Travelling Salesman Problem (PDSTSP)
- one depot, multiple trucks and multiple drones
- Mixed Integer Linear Programming Formulation (MILP)
- hybrid metaheuristic

Problem statement

$G = (N \cup \{0\}, A)$

N: customers

N_d: drone-eligible customers

0: depot

K: trucks

M: drones

$t_{i,j}$: travel time for a truck

$\hat{t}_i$: travel and delivery time for a drone

Objective

- Minimize delivery completion time
    - all deliveries completed
    - all trucks and drones at depot

MILP formulation

- decision variables

z_i: customer i visited by vehicle (otherwise drone)
$x_{i,j,k}$: arc (i, j) belongs to vehicle tour k
$w_{i,j}$: arc belongs to a vehicle tour
$y_{i,m}$: customer i assigned to drone m
T: completion time

MILP formulation

- completion time at least as big as greatest truck travel time

$$T \geq \sum_{(i,j)\in A} t_{ij} x_{ijk} \quad (1 \leq k \leq K)$$

MILP formulation

- completion time at least as big as greatest drone travel time

$$T \geq \sum_{i \in N_d} \widehat{t_i} y_{im} \quad (1 \leq m \leq M)$$

MILP formulation

- serve not drone-eligible customers by truck

$$z_i = 1 \quad (i \in N \setminus N_d)$$

MILP formulation

- customer is served by either truck or drone

$$\sum_{1 \le m \le M} y_{im} = 1 - z_i \quad (i \in N_d)$$

MILP formulation

- if customer served by truck, it is part of a truck tour

$$w_{ij} = \sum_{1 \le k \le K} x_{ijk} \quad ((i, j) \in A)$$

$$\sum_{(i,j) \in A} w_{ij} = z_i \quad (i \in N)$$

MILP formulation

- each truck leaves the depot at most once

$$\sum_{(0,j)\in A} x_{0jk} \leq 1 \quad (1 \leq k \leq K)$$

MILP formulation

- flow conservation for truck tours

$$\sum_{(i,j)\in A} x_{ijk} = \sum_{(j,i)\in A} x_{jik} \quad (i \in N,\ 1 \le k \le K)$$

MILP formulation

- Subtour Elimination Constraint

$$\sum_{j \in S} \sum_{l \in N \cup \{0\} \setminus S} w_{jl} \geq z_i \quad (S \subseteq N, S \neq \emptyset, i \in S)$$

MILP formulation

$$z_i \in \{0, 1\} \quad (i \in N)$$

$$x_{ijk} \in \{0, 1\} \quad ((i, j) \in A, \; 1 \leq k \leq K)$$

$$w_{ij} \in \{0, 1\} \quad ((i, j) \in A)$$

$$y_{im} \in \{0, 1\} \quad (i \in N_d, 1 \leq m \leq M)$$

$$T \geq 0$$

Hybrid metaheuristic

- extension of a procedure by Mbiadou Saleu et al. (2018)
    - PDSTSP
    - one truck
    - one depot

Hybrid metaheuristic

- extension of a procedure by Mbiadou Saleu et al. (2018)
    - TSP tour τ visiting all customers
    - decompose τ
        - subsequence τ_{vehicle}
        - subset π_{drones}
        - dynamic programming with labeling

Hybrid metaheuristic

- extension of a procedure by Mbiadou Saleu et al. (2018)
    - TSP tour τ visiting all customers
    - decompose τ
    - re-optimize $\tau_{vehicle}$ with Lin-Kernighan heuristic
    - assign customers from π_{drones} to drones
        - Parallel Machine Scheduling (PMS)
        - greedy heuristic

Hybrid metaheuristic

- extension of a procedure by Mbiadou Saleu et al. (2018)
  - TSP tour τ visiting all customers
  - decompose τ
  - re-optimize $\tau_{vehicle}$ with Lin-Kernighan heuristic
  - assign customers from π_{drones} to drones
  - construct new τ for next iteration

Hybrid metaheuristic

- this paper
    - multiple vehicle tours needed
    - solution S:
        - K customer sequences
        - M customer sets

Hybrid metaheuristic

- algorithm
  - Initialization
  - Decoding
  - Route re-optimization
  - Drone assignment
  - Local search
  - Improve
  - Construct new giant tour

Hybrid metaheuristic

- Initialization
  - giant TSP tour τ
  - nearest-neighbour construction procedure
  - all visitors visited by a single vehicle

Hybrid metaheuristic

- Decoding
    - decompose τ
    - K subsequences
    - one set π_{drones}

Hybrid metaheuristic

- Route re-optimization
    - re-optimize vehicle routes
    - TSP Lin-Kernighan heuristic
    - Helsgaun’s implementation (Helsgaun, 2000)

Hybrid metaheuristic

- Drone assignment
    - assign customers to drones
    - Parallel Machine Scheduling (PMS)
    - longest processing time heuristic (Pinedo & Hadavi, 1992)

Hybrid metaheuristic

- Local search
  - improve current solution
- Update best solution
  - smaller completion time
  - same completion time and smaller total travel time

Hybrid metaheuristic

- Construct new giant tour
    - concatenate vehicle tours in random order
    - randomly insert customers assigned to drones
    - optimize result with 2-opt
- continue iterated local search (ILS)
- stop when time limit is reached

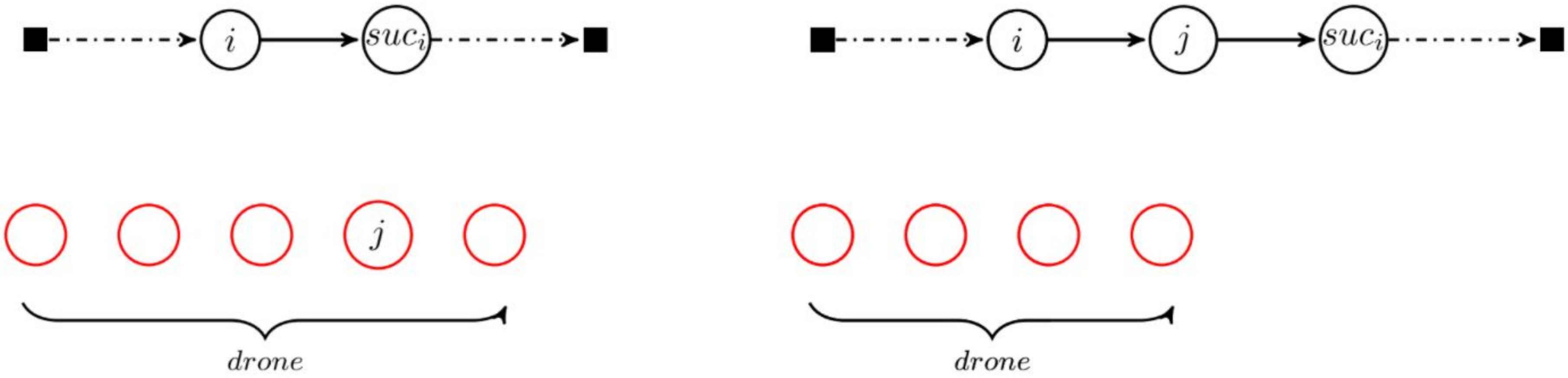
Local search

- improve current solution
- moves:
    - Transfer
    - Exchange move drone - vehicle / vehicle - vehicle
    - Relocate
    - Cross move
- apply moves until local optimum

Transfer move

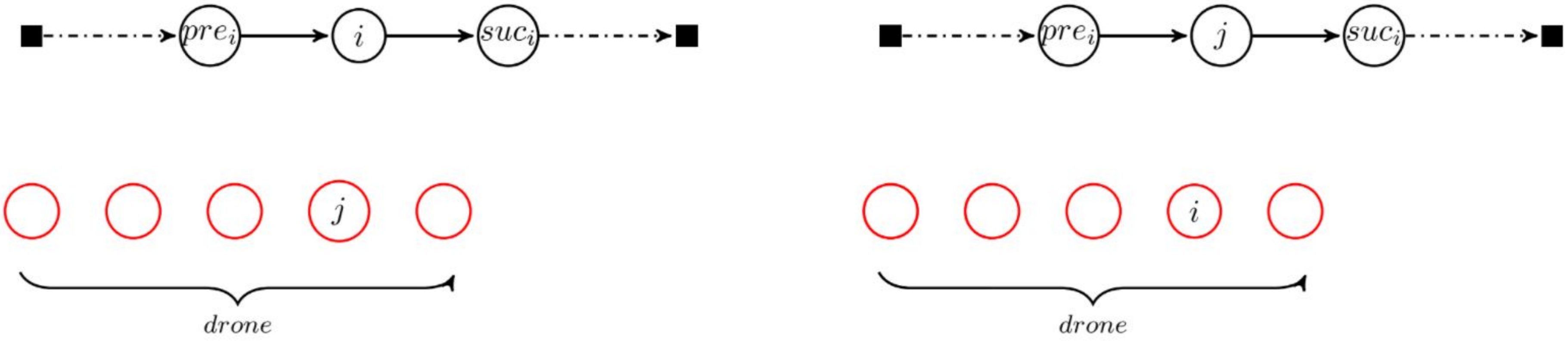
(a) transfer from a vehicle to a drone

Transfer move



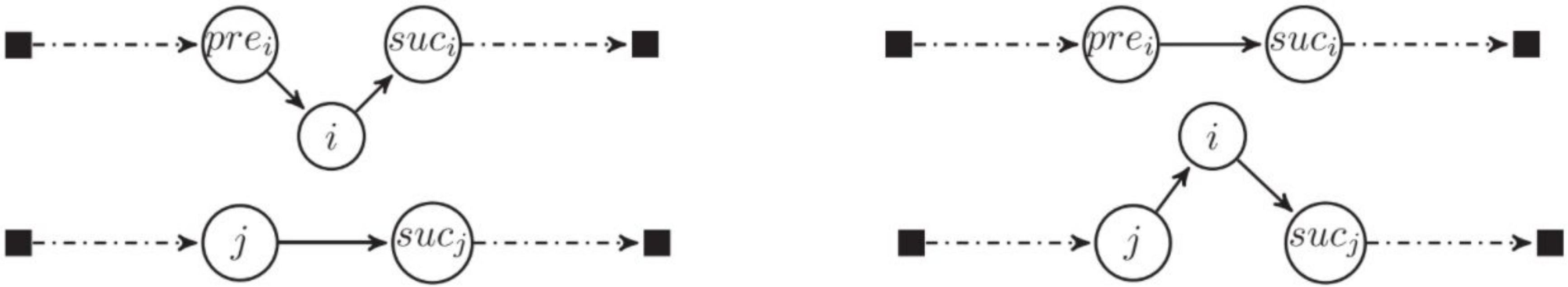
(b) transfer from a drone to a vehicle

Exchange move drone - vehicle



(c) exchange move drone-veh

Relocate move

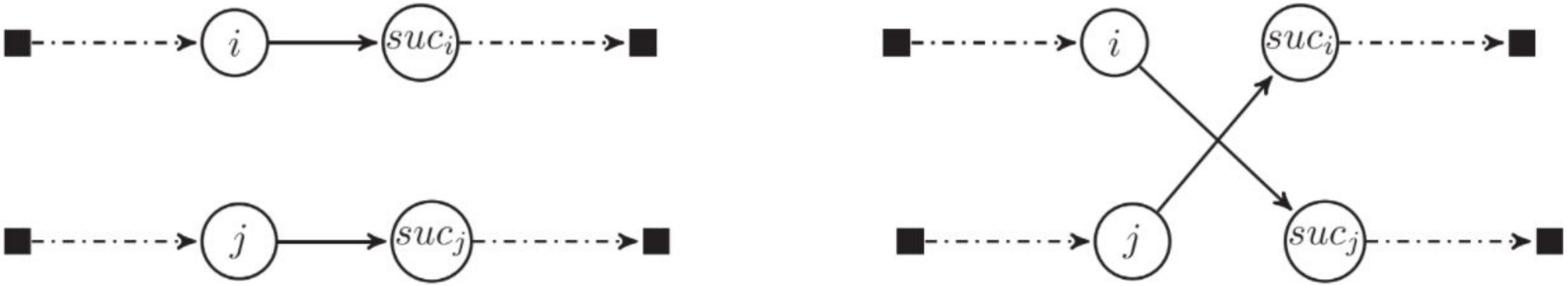


(a) relocate move

Exchange move vehicle - vehicle

(b) exchange move veh-veh

Cross move



(c) cross move

Decoding

- split procedure:
  - extracts K vehicle tours and set of customers served by drones
  - introduce acyclic directed graph
  - solve multi-criteria shortest path problem by dynamic programming (Climaco & Martins, 1982)
  - assign labels to nodes

Decoding

- split procedure
- upper bounds
- lower bounds
- bounding mechanism for pruning labels

Experiments

- CVRPLIB dataset
    - 20 instances
    - depot near barycenter of all customers
    - between 50 and 199 customers

Results

- MILP
  - branch-and-cut algorithm
  - up to 100 customers
  - time limit of 3 hours
  - no optimal solutions found

Results

- Hybrid metaheuristics (HM)
- different variants:
  - HMb
  - MS
  - HM(LL), HMb(LL), MS(LL)
  - HM(UB), HMb(UB), MS(UB)
- time limit of 1000 seconds

Hybrid metaheuristics (HM) variants

- HMB
    - modified reconstruction of the giant tour
    - parameter X
    - concatenate first X vehicle tours of best solution in random order
    - best insertion for remaining customers

Hybrid metaheuristics (HM) variants

- HMB
  - update X:
  - current solution better than best solution:
    - set X = K
  - otherwise:
    - set X = max (0, X - 1)

Hybrid metaheuristics (HM) variants

- MS
  - generate giant tour with randomized nearest-neighbour heuristic
  - randomly choose one of three nearest neighbours
  - standard multi-start (instead of ILS)

Hybrid metaheuristics (HM) variants

- HM(LL), HMb(LL), MS(LL)
    - labels in the decoding step are limited
    - less efficient but faster

Hybrid metaheuristics (HM) variants

- HM(UB), HMb(UB), MS(UB)
    - decoding step limited to computation of upper bound
    - no computation of lower bound and labels
    - less efficient but faster

Table 5
Solution values.

Instance	LB	Method									
		HM	HMb	MS	HM(LL)	HMb(LL)	MS(LL)	HM(UB)	HMb(UB)	MS(UB)	B&C
CMT1 (50,3,2)	145.86	168	168	188	**166**	168	196	174	174	204	188
CMT2 (75,5,5)	101.54	**130.23**	133.60	148	132	133.41	152	140	140	152	3630.86
CMT3 (100,4,4)	160.86	**184**	186	208	187.04	186.17	204	195.42	197.24	216	4537.11
CMT4 (150,6,6)	115.62	**160.38**	150	184	162	162	180	166	164	192	
CMT5 (199,9,8)	72.65	**138**	139.29	152	140	**138**	154	142.04	140	152	
E-n51-k5 (50,3,2)	145.86	**168**	**168**	182	**168**	**168**	180	168.86	174	196	188
E-n76-k8 (75,4,4)	126.95	**154**	**154**	168	156	156	182	161.86	174	196	2975.51
E-n101-k8 (100,4,4)	160.86	186	**184**	208	188	190.17	224	196	196	216	4537.11
M-n151-k12 (150,6,6)	116.23	**154**	158.96	186	164	162	182	168	169.88	182	
M-n200-k16 (199,8,8)	80.69	**144**	148	162	148	146	156	152	152	168	
P-n51-k10 (50,5,5)	81.34	**111.07**	114	118	112.69	114	122	118	118	133.25	230
P-n55-k7 (54,4,3)	101.46	128	128	138	128	**126**	142	130	132	148	308
P-n60-k10 (59,5,5)	84.30	**114**	116	124	114.86	116	124	122	120	124	246
P-n65-k10 (64,5,5)	94.62	**126**	**126**	138	128	**126**	142	134	131.36	154	580
P-n70-k10 (69,5,5)	99.23	129.29	**128**	138	136	132	146	138	136.56	158	3166.25
P-n76-k5 (75,3,2)	181.34	202	**200**	214	202	202	243.44	210	210	258	280
P-n101-k4 (100,2,2)	321.74	342.69	**342**	396	346	348	388	353.26	354	422	4725.47
X-n110-k13 (109,7,6)	1189.78	**1864**	1898	2080	1898	1898	2044	1926	1960	1970	
X-n115-k10 (114,5,5)	1676.05	**2258**	2300	2658	2262	2274	2504	2316	2332	2862	
X-n139-k10 (138,5,5)	1582.46	2928.64	2740	3144	2534	**2492**	2696	2594	2550	3022	

Conclusion

- hybrid metaheuristics
    - first paper with multiple trucks (PDSMTSP)
    - no comparison with literature possible
- branch-and-cut
    - no competitive results

Future work

- branch-and-price
- better compromises between decoding quality and number of labels in decoding step
- Constraint programming framework
- more realistics models (time windows, drone recharching)
- investigate different ratios for drone and truck fleet sizes