Technical Appendix for
Tentative Acceptance Unique Offers Protocol for Automated Negotiation

anonymous

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Abstract

This appendix is provided for two main purposes: (1) We also provide a theoretical analysis of the TAU protocol when using SCS and the proposed WAR strategies for it and show that the later is a Perfect Bayesian Nash Equilibrium of the induced game for discrete outcome spaces with no information about partner strategies or preferences. (2) We provide extended discussion of the proposed approach, its limitations and relationship to existing work in automated negotiation and mechanism design. (3) We elaborate on the results reported in the paper giving more detailed analysis of the improvements introduced by the proposed method and give more details about the scenarios and strategies employed in each experiment.

Contents

Notation and Definitions

Notation

The paper defines all notation used at the point it is introduced to reduce the need to go back to the notation section. In this section, we collect all the notation used in the paper in one section as a reference for the reader and introduce all new notation to be used in the proofs later in this document.

Formal Definitions

Rational Outcome Set $\overline{\Omega}^{\lambda}$: The subset of the outcome space that a rational outcome at least as good as disagreement for everyone: $\left\{\omega : \omega \in \Omega \wedge \succcurlyeq_i \phi \forall i \in \mathcal{N}^{\lambda}\right\}$.

Win-Win deals $\widehat{\Omega}^{\lambda}$: Rational outcomes strictly better than disagreement for at least one agent:
$\left\{\omega : \omega \in \overline{\Omega}^{\lambda} \wedge \exists i \in \mathcal{N}^{\lambda} \rightarrow \omega \succ_i \phi\right\}$.

Pareto Outcome Set $\mathcal{P}$: Rational outcomes that cannot be improved for one agent without making at least one other agent worst off:

$$\left\{\psi : \psi \in \overline{\Omega}^{\lambda} \wedge \left[\neg\exists\left(\omega \in \Omega^{\lambda} \setminus \{\psi\}, b \in \mathcal{N}^{\lambda}\right) : \omega \succ_b \psi \wedge \left(\omega \succcurlyeq_i \psi \forall i \in \mathcal{N}^{\lambda}\right)\right]\right\}.$$

Negotiation Trace We define the *negotiation trace at step*[1] k ($\mathcal{T}^k$) as the ordered tuple of offers sent so far during the negotiation associated with their source and the acceptance/rejection decision for each of them (and their sources): $\mathcal{T}^k \in \mathbb{S}^k(\Omega^+ \times \mathcal{N}^2 \times \{0,1\})$, where $\mathbb{S}^j(X)$ is all subsets of cardinality j of the set X, and $\mathbb{S}(X)$ is the powerset of X. agents can offer

[1] One step corresponds to one offer by one agent (Algorithm 1 in the paper.

ϕ to end the negotiation immediately. The domain of all possible traces is called $\mathbb{T}$ and the domain of all possible traces of length i is called $\mathbb{T}^i$ hereafter.

Bargaining Protocol Formally, a bargaining protocol $\mathcal{P}$ is defined as the tuple $(\zeta, \chi, \gamma, \sigma)$ where $\zeta : \mathbb{T} \times \Re \to \{0, 1\}$ (breaking rule) receives the current trace and number of seconds since the beginning of the negotiation and defines the conditions under which the negotiation is broken (i.e. a predefined number of steps or a predefined number of seconds), $\chi : \mathbb{N} \to \mathcal{N} \times \mathbb{S}(\mathcal{N})$ (activation rule) is a mapping from step number to an offering agent and a set of responding agents, $\gamma : \mathbb{T} \times \mathcal{N} \to \mathbb{S}(\Omega)$ (*offering constraint*) is a mapping from the current negotiation trace and activated agent to a subset of offers available to the next agent to propose (called the *valid offers set* hereafter) and $\sigma : \mathbb{T} \to \Omega^+ \cup \{\triangleright\}$ (*evaluation rule*) is a mapping from the negotiation trace to either $\triangleright$ leading to continuation of the negotiation, ϕ leading to immediate ending of the negotiation with no agreement (assigning each agent its reserved value) or an outcome $\omega_* \in \Omega$ ending the negotiation with agreement ω_* assigning each agent its value for this agreement.

Evaluation Metrics

The paper describes the evaluation metrics used in the evaluation section and theoretical proof. This subsection defines these metrics rigorously. These metrics are based on the proposal by Mohammad Mohammad (2023a) as indicated in the paper.

Evaluating Negotiation Mechanisms (Agent)

The main goal of an agent is to maximize its own **Advantage** (defined as the difference between the value it receives from negotiation and its reserved value) which can be measured for a set of Λ as: $A_a(\pi, \Lambda) \equiv \mathbb{E}_{\lambda\sim\Lambda, i\sim p_\lambda, \Upsilon\sim \mathrm{I}_i^\lambda | i\approx\pi}\left[\hat{u}_i(\mathcal{P}_\Upsilon(\lambda)) - \hat{u}_i(\phi)\right]$.

In some cases, the agent may also be interested in maximizing the expected advantage of its partners, **Partner Welfare** hereafter (e.g. to increase the chance of future trade with them). Partner Welfare can be measured as: $A_p(\pi, \Lambda) \equiv \mathbb{E}_{\lambda\sim\Lambda, i\sim p_\lambda, \Upsilon\sim \mathrm{I}_i^\lambda | i\approx\pi}\left[\mathbb{E}_{j\in\mathcal{N}^\lambda\setminus\{i\}}\hat{u}_j(\mathcal{P}_\Upsilon(\lambda)) - \hat{u}_j(\phi)\right]$.

Moreover, the agent should care about the revelation of its preferences. If this is not a concern, then negotiation is mostly unnecessary as the agents can just exchange their preferences and agree on a predefined bargaining solution point (e.g. Nash or Kalai or Kalai-Smorodinsky Kalai and Smorodinsky (1975) solutions) converting the negotiation into a simple optimization problem[2]. The relative importance of this information revelation varies depending on the application and the way to calculate it depends on the assumptions available to the partners about the protocol and agent strategy. For the *purposes of this paper*, we define the **Privacy** [3] as the expected fraction of outcomes never exchanged during the negotiation: $P(\pi, \Lambda) \equiv \mathbb{E}_{\lambda\sim\Lambda, i\sim p_\lambda, \Upsilon\sim \mathrm{I}_i^\lambda | i\approx\pi}\left[\Omega \setminus \bigcup_{z\in\mathcal{N}^\lambda}\Omega_z\right]$ where Ω_i is the set of offers produced by agent i during of the negotiation.

We can then measure the *quality* of a choice of a protocol $\mathcal{P}$ and a strategy π under $\mathcal{P}$ from the agent's point of view (**Agent Score**) as a linear combination of the aforementioned three measures: $\mathbb{S}_a(\pi, \Lambda) \equiv \beta_a A_a + \beta_p P + (1 - \beta_a - \beta_p)A_p$.

[2]Strictly speaking negotiation may still be needed even if information revelation is not an issue because the agents may not agree about which bargaining solution to use.

[3]Measuring information revelation is an under-researched topic in AN. The proposed measure is obviously inadequate in general because information revelation depends also on the distribution over partner strategies and preferences, the order of offers, etc. For the purposes of this paper, we always compare conditions in which these parameters are almost fixed which justifies the proposed definition as an approximation.

Evaluating Negotiation Mechanisms (Designer)

Agents will choose their strategy given a protocol to maximize the *agent score*. This may not lead to *good* negotiation outcome from the society's (or designer's) point of view because there is no a-priori guarantee that the induced game will not be such that individual maximization of agent scores leads to the *best* result for the whole society (e.g. as in the case of the Prisoner's Dilemma). The designer has no direct control over the choice of strategy by agents but she has control over the set of protocols available to the agents which define the set of available strategies for the agents. In this section we define the most important measures for comparing of a negotiation mechanism $\mathcal{P}_\Upsilon$ (i.e. a protocol $\mathcal{P}$ and SAR Υ) for a predefined set of scenarios Λ from the society's (designer's) point of view.

Rationality R is defined as the fraction of successful negotiations that lead to a *rational* outcome: $\mathbb{E}_{\Lambda|\mathcal{P}_\Upsilon(\lambda)\neq\phi}\left[\overrightarrow{u}(\mathcal{P}_\Upsilon(\lambda)) \in \overline{\Omega}^\lambda\right]$.

Completeness C is is defined as the fraction of negotiations with win-win deals that lead to a win-win outcome: $\mathbb{E}_{\Lambda|\widehat{\Omega}^\lambda\neq\emptyset}\left[\overrightarrow{u}(\mathcal{P}_\Upsilon(\lambda)) \in \widehat{\Omega}^\lambda\right]$.

All the evaluation criteria so far were independent of the evaluation model (single-negotiation or multiple-negotiations). The following criteria — on the other hand — do depend on the specific evaluation model employed. For all of these metrics, we only consider negotiations ending with agreements to avoid double penalizing negotiations leading to disagreement as they are already penalized in the Completeness calculation.

Welfare W is defined as the expected sum of the achieved value for all agents relative to the maximum achievable sum: $W = \mathop{\mathbb{E}}_{\Lambda|\mathcal{P}_\Upsilon(\lambda)\neq\phi}\left[\sum_{i\in\mathcal{N}}\hat{u}_i(\mathcal{P}_\Upsilon(\lambda))/\max_{\omega\in\mathcal{P}}\sum_{i\in\mathcal{N}}\hat{u}_i(\omega)\right]$,

Optimality O is defined as the expected value of one minus the normalized distance of the negotiation outcomes to the rational portion of the Pareto Outcome Set/Pareto frontier for negotiations leading to agreement: $O = \mathbb{E}_{\Lambda|\mathcal{P}_\Upsilon(\lambda)\neq\phi}\left[1 - \min_{\omega\in\mathcal{P}\bigcup\overline{\Omega}} D(\mathcal{P}_\Upsilon(\lambda), \omega)\right]$, where $D(a, b) \equiv \frac{\sqrt{\sum_{i\in\mathcal{N}}(\hat{u}_i(a)-\hat{u}_i(b))^2}}{\max\limits_{\omega\in\Omega\wedge\psi\in\mathcal{P}\bigcup\overline{\Omega}_x}\sqrt{\sum_{i\in\mathcal{N}}(\hat{u}_i(\omega)-\hat{u}_i(\psi))^2}}$.

The reason we only consider the *rational* portion of the Pareto Outcome Set/Frontier is to penalize negotiations ending with Pareto efficient irrational agreements.

Fairness F is defined as the expected value of one minus the normalized distance of the negotiation outcomes to any bargaining solution: $F = \mathbb{E}_{\Lambda|\mathcal{P}_\Upsilon(\lambda)\neq\phi}\left[1 - \min_{\omega\in\Omega^f} D(\mathcal{P}_\Upsilon(\lambda), \phi)\right]$, where Ω^f are the set of bargaining solutions under the single-negotiation and multiple-negotiations model (See the following section for the rationale for this definition and the details of the bargaining solutions set).

We can then measure the *quality* of a choice of a protocol and a strategy from the designer's point of view (**Designer Score**) under a given evaluation model as the product of the five measures above: $\mathbb{S}_d^1(\Lambda, \mathcal{P}_\Upsilon) \equiv RCW_1P_1F_1, \mathbb{S}_d^\infty(\Lambda, \mathcal{P}_\Upsilon) \equiv RCW_\infty P_\infty F_\infty$.

Used in the Paper

$\mathbb{N}$ Integers starting at zero.

$\mathbb{N}^+ \equiv \mathbb{N}$ Natural numbers (positive integers from 1).

$\mathbb{N}^+$ Natural numbers (positive integers starting at 1).

$\{0, 1\}$ The set of booleans (True, False).

$\Re$ The set of real numbers.

$\mathbb{S}(x)$ The power-set of set x (all possible subsets of x).

$\mathbb{S}^i(x)$ Sets of cardinality i belonging to $\mathbb{S}(x)$.

AN Automated Negotiation.

AOP Alternating Offers Protocol.

SAOP Stacked Alternating Offers Protocol.

λ Negotiation scenario.

$\mathcal{P}$ Negotiation Protocol.

$\mathcal{N}^\lambda$ Negotiators for Scenario λ.

Π Strategies associated with negotiators for Scenario λ.

n_A The number of negotiators/agents/preferences in a negotiation.

$\mathcal{D}^\lambda$ Negotiation Domain for scenario λ (Outcome Space + Preferences).

Ω Outcome space (all possible agreements).

Ω_i^v The set of valid outcomes that can be offered by agent i.

n_o The cardinality of Ω (size of the outcome space).

ϕ Represents disagreement as an output of a negotiation, ending negotiation as an offer from an agent, and the empty set otherwise.

Ω^+ Outcome space or disagreement ϕ (all possible negotiation outcomes).

$\Omega_i^\top$ Best set of outcomes for negotiator i (none of these outcomes is better than any other in the set and any of them is better than any outcome not in the set).

$\Omega_i^\perp$ Worst set of outcomes for negotiator i (none of these outcomes is worse than any other in the set and any of them is worse than any outcome not in the set).

$\mathcal{F}^\lambda i$ Preferences of negotiator i in scenario λ.

u_i Utility function of negotiator i (cardinal).

r_i Relative rank of outcomes in Ω^+ for negotiator i (ordinal/cardinal).

$\hat{u}_i$ Value for agent i defined as normalized utility value (cardinal) or normalized relative rank (ordinal) for negotiator i.

$\omega_1 \succcurlyeq_i \omega_2$ Outcome ω_1 is not worse for negotiator i than outcome ω_2 given the preferences $\mathcal{F}^\lambda i$.

$\omega_1 \succ_i \omega_2$ Outcome ω_1 is better for negotiator i than outcome ω_2 given the preferences $\mathcal{F}^\lambda i$.

$\omega_1 \approx_i \omega_2$ Outcome ω_1 is neither better not worse for negotiator i compared with outcome ω_2 given the preferences $\mathcal{F}^\lambda i$.

$\mathcal{I}^\lambda$ Negotiator's Information Sets representing all information available to all negotiators in scenario λ.

$\mathrm{I}^\lambda i(\mathcal{F}^\lambda)$ Information available to negotiator i in scenario λ about preferences of all negotiators $\mathcal{F}^\lambda$ in that scenario.

$\mathbb{D}$ A set of domains.

$\mathcal{D}_i^\lambda$ All the information available to negotiator i in scenario λ.

$\overline{\Omega}_i \subseteq \Omega$**:** The subset of the outcome space Ω satisfying: $\omega \succcurlyeq_i \phi$.

$\overline{\Omega}^\lambda$**:** All possible rational agrements: $\bigcap_{i \in \mathcal{N}^\lambda} \overline{\Omega}_i$.

$\underline{\Omega}_i \subseteq \Omega$**:** The subset of the outcome space Ω satisfying: $\phi \succ_i \omega$.

$\underline{\Omega}^\lambda$**:** All irrational agreementst that should never occur defined as: $\bigcup_{i \in \mathcal{N}^\lambda} \underline{\Omega}_i$.

$\widehat{\Omega}^\lambda$**:** Win-win deals defined as the set of rational outcomes strictly better than disagreement: $\left\{\omega \in \overline{\Omega}^\lambda : \forall i \in \mathcal{N}^\lambda \to \omega \succ_i \phi\right\}$.

$\mathcal{P}$**:** The pareto frontier defined as the set of rational outcomes that cannot be improved for one negotiator without making at least one other negotiator worst off: $\psi \in \mathcal{P} \subseteq \overline{\Omega}^\lambda \leftrightarrow \neg\exists \omega \in \Omega \setminus \{\psi\} : \exists i \in \mathcal{N}^\lambda : \omega \succ_i \psi \wedge \forall x \in \mathcal{N}^\lambda \setminus \{i\}\, \omega \succcurlyeq_i \psi$.

ω_*^λ The outcome of a negotiation using scenario λ.

$\lambda()$ The outcome of a negotiation using scenario λ (same as ω_*^λ).

$\mathcal{T}^k$ The negotiation trace showing all offers and responses (in order) in scenario λ up to round k (i.e. $\mathcal{T}^k = \Omega_i^k, \Omega_j^k, \mathbb{A}_i^k, \mathbb{A}_j^k$).

$\mathbb{T}^\lambda k$ The domain of all possible traces.

f The Filter used in representing a negotiation protocol.

σ The Evaluation Strategy used in representing a negotiation protocol.

$\triangleright$ Continue negotiation (returned by the evaluation policy to signal that the negotiation should go on).

π A negotiation strategy (consists of an offering policy and an acceptance policy).

η_i The offering policy of negotiator i.

ρ_i The Acceptance Policy of negotiator i.

$replace(\pi, \pi')$ Replace strategy π with strategy π'

Ω_x^j An ordered tuple of j most recent **unique** offers proposed by negotiator x so far ordered from earliest to latest.

$\mathbb{A}_x^j$ An ordered tuple of j most **unique** recent offers accepted by negotiator x so far ordered from earliest to latest.

Used only in this Appendix

WAR Wasteful offering Accepting Rational (Proposed).

SCS Slow Concession Strategy.

SCS Slow Concession Strategy (Proposed).

PNP Perfect Negotiation Protocol.

$\Omega_i^v(k)$ The set of valid outcomes that can be offered by agent i at round k.

η_i^k The offer from negotiator i at round k.

a_{ji}^k The response from negotiator i at round k to the offer received from negotiator j.

$\hat{a}_i^k(\omega)$ The outcome ω will be accepted by negotiator i if it was offered at round k.

$\hat{a}_i(\omega)$ The outcome ω will be accepted by negotiator i at some time during the negotiation.

$J_i(\omega)$ The first round at which agent i would make offer ω. We set $J_i(\omega) = \infty$ if i will never make the offer during a negotiation.

$\Omega_i^v(k)$ The set of valid outcomes that can be offered by agent i at round k.

η_i^k The offer from agent i at round k.

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$\hat{a}_i(\omega)$ The outcome ω will be accepted by agent i at some time during the negotiation.

$J_i(\omega)$ The first round at which agent i would make offer ω. We set $J_i(\omega) = \infty$ if i will never make the offer during a negotiation.

T number of offers from the agent with maximum number of offers by the end of the negotiation (i.e. negotiation length).

$i \approx \pi$ agent i uses strategy π.

$\eta_i \approx \eta_\pi$ agent i uses π's offering policy.

$\rho_i \approx \rho_\pi$ agent i uses π's acceptance policy.

Proofs and Theoretical Analysis

Let Λ_D be the set of all scenarios with discrete outcome spaces and let Λ_{DN} but the subset of Λ_D with no information about partner preferences (i.e. $\mathcal{D}^\lambda$ is discrete and $\mathrm{I}_i^\lambda = \{\mathcal{F}_i^\lambda\}$). Note that – crucially – this assumption implies that the agent cannot know which outcomes are rational for its partner. For the rest of this section, we always assume that we are considering Λ_{DN}.

Definition 1. *Let $\succ_l$ be a shared full ordering of the outcome space, SCS's offering policy can be formally written as:*

$$\eta_{SCS}(k) = \omega_i^k = \begin{cases} \sup_{\succ_l} \overline{\Omega}_i^v(k), & \text{if } \underline{\Omega}_i^v(k) = \{\} \wedge \overline{\Omega}_i^v(k) \neq \{\} \\ \omega_i^{k-1}, & \text{otherwise} \end{cases} \tag{1}$$

where $\sup_{\succcurlyeq}(X)$ is the best outcome in the set X according to ordering $\succcurlyeq$, ω_i^{k-1} is the agent's last offer, and $\overline{\Omega}_i^v(k)$is the set of rational outcomes that agent i can offer:

$$\overline{\Omega}_i^v(k) \equiv \{\omega : \omega \in \Omega_i^v(k) \cap \overline{\Omega}_i \wedge \omega \succcurlyeq_i \psi \quad \forall \psi \in \Omega_i^v(k)\}$$

where $\Omega_i^v(k) \equiv \Omega \setminus \Omega_i^{k-1}$, and Ω_i^k is i's offers up to step but not including k.

This strategy offers outcomes in descending order of value for itself repeating when it cannot find any new outcomes to offer. Ties are broken using any predefined lexical ordering on the outcomes. It does not matter what this ordering is. What matters is that it is shared between all agents. Without this shared ordering, exact optimality is lost. It is important that agents not following this lexical ordering rule can never gain any utility because of it which means this rule is incentive compatible. We implicitly assume that $\omega_i^0 = \phi$ to handle the case with no rational outcomes.

Definition 2. *SCS's acceptance policy:*

$$\rho_{SCS}(\omega, k) = \mathbb{1}\left[\omega \succcurlyeq_i \phi \wedge \left(\left|\mathbb{A}_i^{k-1}\right| < 1 \vee \omega \succ_i A_i^{last}\right)\right] \tag{2}$$

Definition 3. *WAR's offering policy is defined as:*

$$\eta_{WAR}(k) = \omega_i^k = \begin{cases} \sim \underline{\Omega}_i^v(k), & \text{if } \underline{\Omega}_i^v(k) \neq \{\} \\ \sup_{\succ_l} \overline{\Omega}_i^v(k), & \text{if } \underline{\Omega}_i^v(k) = \{\} \wedge \overline{\Omega}_i^v(k) \neq \{\} \\ \omega_i^{k-1}, & \text{otherwise} \end{cases} \tag{3}$$

where $\sim X$ is any sample drawn from X, $\underline{\Omega}_i^v(k)$ is the set of irrational outcomes that agent i can offer:

$$\underline{\Omega}_i^v(k) \equiv \{\omega : \omega \in \Omega_i^v(k) \cap \underline{\Omega}_i\}$$

Definition 4. *WAR's acceptance policy is defined as:*

$$\rho_{WAR}(\omega, k) = \mathbb{1}\left[\omega \succcurlyeq_i \phi\right] \tag{4}$$

We will use the following properties of offers directly deducible from Eq. 1 and Eq. 3

- For SCS (but not WAR), a better offer is always offered before a worse offer:
$$\psi \succ_i \omega \wedge \omega_i^k = \omega \implies \psi \in \Omega_i^k \quad \forall \psi, \omega \in \Omega \tag{5}$$
- For SCS and WAR, a better rational offer is always offered before a worse rational offer:
$$\psi \succ_i \omega \wedge \omega_i^k = \omega \implies \psi \in \Omega_i^k \quad \forall \psi, \omega \in \Omega \tag{6}$$
- SCS and WAR never offer ϕ under TAU which means they never choose to explicitly end the negotiation.
$$\neg \exists i \in \mathbb{N} : \omega_i^i = \phi \tag{7}$$
- SCS and WAR never repeat an offer before all offers not worse than it are tried:
$$\omega_i^k = \omega_i^{k-1} \implies \omega \in \Omega_i^{k-1} \quad \forall \omega \in \Omega : \omega \succ_i \omega_i^k \tag{8}$$
- SCS and WAR never repeat an offer before trying all their rational outcomes:
$$\omega_i^k = \omega_i^{k-1} \implies \omega \in \Omega_i^{k-1} \qquad \forall \omega \in \overline{\Omega} \tag{9}$$
Because win-win deals are a subset of rational outcomes:
$$\omega_i^k = \omega_i^{k-1} \implies \omega \in \Omega_i^{k-1} \qquad \forall \omega \in \widehat{\Omega} \tag{10}$$

Considering the acceptance policies of SCS and WAR (Eq. 2, Eq. 4), we can directly deduce the following properties:

- If SCS or WAR are willing to accept an offer, they will accept any better offer anytime not later in the negotiation:
$$\omega \succ_i \psi \wedge \hat{a}_i^i(\psi) \implies \hat{a}_i^j(\omega) \qquad \forall j \leq i \tag{11}$$
The reason that this holds is that SCS will only accept an offer if it is better than everything that was offered before and WAR accepts any rational offer. This means that both will accept any better offer at the same or any earlier round.
- If SCS (but not WAR) rejects an offer, it must have received an offer not worse for itself earlier:
$$\omega_j^k = \omega \wedge \neg a_{ji}^k \implies \exists \psi \in \Omega_j^{k-1} : \psi \succcurlyeq_i \omega \tag{12}$$
- If SCS (but not WAR) actually accepts an offer at step k, it will reject any worse offer coming later:
$$\omega_j^k = \omega \wedge a_{ji}^k \implies \neg \hat{a}_i^j(\omega) \quad \forall j > i \forall \omega \in \Omega \tag{13}$$
- If two outcomes belong to the set of outcomes accepted by SCS (but not WAR), they cannot be as good as each others for it (one of them must be strictly worse than the other):
$$\forall i \in \mathbb{N} \forall \omega, \psi \in \Omega \quad \omega \in \mathbb{A}_i^i \wedge \psi \in \mathbb{A}_i^i \implies \neg \omega \approx_i \psi \tag{14}$$

We can also directly see the following properties of running TAU with SCS and WAR for all negotiators from the definitions above:

- If an outcome is offered by both negotiators and is acceptable to both negotiators by some time step k, the negotiation ends with this outcome as an agreement:
$$\exists \omega \in \Omega : \omega \in \Omega_i^k \wedge \hat{a}_i^j(\omega) \forall j < k \forall i \in \mathcal{N} \implies \omega_* = \omega \tag{15}$$
- For an outcome to become the agreement of a negotiation, it must be offered by all negotiators and be acceptable by each of them before and at the round it received it:
$$\omega_* = \omega \implies \omega \in \Omega_i^{k_i} \wedge \hat{a}_i^j(\omega) \forall j < k_i \forall i \in \mathcal{N} \tag{16}$$

Theoretical Results Summary

All theoretical results presented in the paper can be summarized in Table 1. The following sets of scenarios are used in this table and throughout this document:

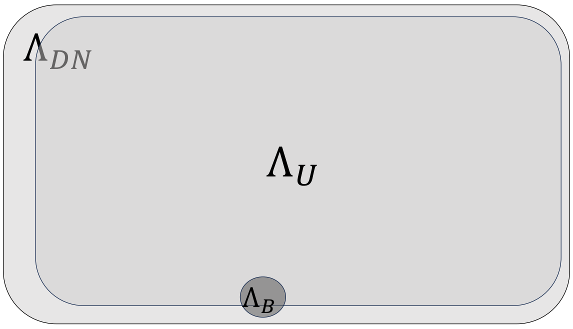


Figure 1: Sets of scenarios used in this work and their relationship. Note that the size of Λ_B is vanishingly small compared to Λ_{DN} while Λ_U is almost equal in size to Λ_{DN}

Λ_{DN} All scenarios with a discrete outcome space and no information about partner preferences.

Λ_U All scenarios in Λ_{DN} with the added requirement that no two outcomes have the same rank (or utility) for any agent.

Λ_B^{RB} Balanced scenarios as defined by de Jonge de Jonge (2022). These are scenarios for which MiCRO is optimal and are characterised with the property that the number of outcomes better than the agreement is almost equal for all agents. Moreover, there are exactly two agents (bilateral negotiations) and all outcomes must be rational for both agents.

Fig. 1 shows these three sets of scenarios and their relationships. Because Λ_U only requires that no two outcomes have *exactly* the same utility value, any scenario Λ_{DN} can be converted to a scenario in Λ_U by adding a vanishingly small value to the utility of some of its outcomes. The constraint on Λ_B – on the other hand – is much stricter as to convert a domain not in Λ_B to one in Λ_B, the number of outcomes above reserved value must be changed which is a discrete change.

TAU Always Terminates for Λ_{DN}

In the paper, Section "Tentative Acceptance Unique Offers Protocol", we claimed that TAU always terminates.

Theorem 1. *TAU always terminates in a finite number of steps T not greater than n_o+1 for any strategy assignment rule on Λ_{DN}.*

$$T \leq n_o + 1 \tag{17}$$

Proof. The size of valid set is the difference between the outcome space size and the number of unique offers for every negotiator:

$$n_v \equiv \min_{i\in\mathcal{N}} |\Omega_i^v| = \min_{i\in\mathcal{N}} n_o - |\Omega_i^\infty| \leq n_o - \max_{i\in\mathcal{N}} |\Omega_i^\infty| \tag{18}$$

Let $n_v(k)$ be the size of the valid set of outcomes from all negotiators as defined above at round k.

For any round k if all negotiators repeat their last offer, the negotiation terminates:

$$\omega_i^k = \omega_i^k \forall i \in \mathcal{N} \Rightarrow terminates(TAU, k).$$

Table 1: Theoretical Result Summary

Set	Strategy	Complete	Optimal	Fair	Equilibrium
Λ_{DN}	TAU-SCS	x	✓	x	x
	TAU-WAR	✓	x	?	✓
	AOP-MiCRO	x	x	x	?
Λ_{DN}^B	TAU-SCS	✓	✓	x	x
	TAU-WAR	✓	x	?	✓
	AOP-MiCRO	x	x	x	?
Λ_{DN}^{RB}	TAU-SCS	✓	✓	✓	✓
	TAU-WAR	✓	x	✓	x
	AOP-MiCRO	x	x	x	?
Λ_U^B	TAU-SCS	✓	✓	x	x
	TAU-WAR	✓	✓	✓	✓
	AOP-MiCRO	x	✓	✓	?
Λ_B^{RB}	TAU-SCS	✓	✓	✓	✓
	TAU-WAR	✓	✓	✓	x
	AOP-MiCRO	✓	✓	✓	✓

? Unknown ^{B} Bilateral ^{R} All Rational
✓x Conjecture (No Proof Provided) ✓ Proven

From this we conclude that if the negotiation is not terminated at round k, there is at least one negotiator not repeating at this round:

$$\neg terminates(TAU, k) \Rightarrow \exists y \in \mathcal{N} : \omega_j^k \neq \omega_j^k$$

$$\therefore \neg \text{terminates}(TAU, k) \Rightarrow \exists y \in \mathcal{N} : \quad |\omega_i^\infty|_k > |\omega_i^\infty|_{k-1} \tag{19}$$

From Eq. 19 and Eq. 18,

$$\therefore \neg terminates(TAU, k) \Rightarrow n_v(k) < n_v(k-1)$$

Which means that if a negotiation did not end at round k, the size of the valid set of outcomes n_v will go down.

$$\therefore \lim_{k\to\infty} n_v(k) = 0$$

Negotiators will eventually run out of unique outcomes to offer and the negotiation will terminate.

$\therefore$ TAU will always terminate. □

Empirical support comes from the fact that in all our experiments TAU terminated.

TAU($\Upsilon_{WAR,SCS}$) is Exactly Rational on Λ_{DN}

Theorem 2. *TAU is Exactly Rational with any strategy assignment rule that always assigns strategies with the AcceptRational or AcceptBetter acceptance policies ($TAU(\Upsilon_{*B,*R})$) on Λ_{DN}.*

$$\neg\exists\lambda \in \Lambda_{DN} \exists i \in \mathcal{N}^\lambda : \lambda_{TAU}(\Upsilon_{*B,*R}) \prec_i \phi \tag{20}$$

Proof. $\because$ From Eq. 2 and Eq. 4, an agent using AcceptBetter or AcceptRational will only accept an offer if it is better than disagreement.

$\because$ From Eq. 16, an agreement can only be reached if all agents accept it.

$\therefore$ An outcome that is worse than disagreement for any negotiator using AcceptBetter (Eq: 2) or AcceptRational (Eq: 4) cannot be the output of a negotiation.

$$\therefore \omega_*^\lambda \neq \phi \implies \omega_* \in \overline{\Omega} \tag{21}$$

□

Lemma 1. *TAU($\Upsilon_{SCS,WAR}$) is exactly rational.*

Proof. This follows directly from Theorem 2 the fact that SCS uses AcceptBetter and WAR uses AcceptRational. □

Empirical support comes from the fact that in all cases in all experiments, TAU never ended with an agreement with a utility lower than the reserved value of any negotiator involved. This can be seen in from the fact that Pareto Optimality was always exactly one implying that all outcomes are on the Pareto-frontier which contains only rational outcomes by definition.

TAU(Υ_{SCS}) is Exactly Optimal on Λ_{DN}

Proposition 1 (Acceptability)**.** *In a bilateral negotiation of Λ_{DN} with TAU(Υ_{SCS}), an agent accepts an outcome iff it is Pareto-efficient or as good as a Pareto-efficient outcome for its partner.*

Proof. The acceptance policy of SCS (AcceptBetter) will only accept an outcome if it is better than everything it received so far (Eq. 11). Therefore, for an agent i running SCS:

$$\hat{a}_i^i(\omega) \implies \omega \succ_i \psi \qquad \forall \psi \in \Omega_j^i$$

Moreover, SCS offers outcomes in descending order. Therefore, for an agent j running SCS:

$$\omega_j^i = \omega \implies \omega \succcurlyeq_j \psi \qquad \forall \psi \in \Omega \setminus \Omega_j^i$$

Agent i will accept ω iff it was offered by j at some step i at which it was acceptable by i.

$$a_{ji}^k \implies \omega_j^k = \omega \wedge \hat{a}_i^j(\omega)$$

$$\therefore a_{ji}^k \implies \omega \succ_i \psi \forall \psi \in \Omega_j^i \wedge \omega \succcurlyeq_j \psi \forall \psi \in \Omega \setminus \Omega_j^i$$

The inequality case gives the Pareto frontier:

$$\psi \in \Omega_j^i \wedge \omega \succ_j \psi \forall \psi \in \Omega \setminus \Omega_j^i \implies \omega \in \mathcal{P}$$

The equality case, gives outcomes that are as good as a Pareto efficient outcome for agent j:

$$\psi \in \Omega_j^i \wedge \exists \psi \in \Omega \setminus \Omega_j^i \omega \approx_j \psi \implies \omega \in \{\alpha \in \Omega : \alpha \approx_j \omega\}$$

But if there is another outcome that is the same for j as ω, the Pereto-frontier must have at least one such outcome:

$$\psi \in \Omega_j^i \wedge \exists \psi \in \Omega \setminus \Omega_j^i \omega \approx_j \psi \implies \exists \alpha \in \mathcal{P} : \alpha \approx_j \omega$$

Taken together:

$$\therefore a_{ji}^k \implies \omega \in \mathcal{P} \vee \exists \alpha \in \mathcal{P} : \alpha \approx_j \omega$$

□

As a simple example illustrating a case where Pareto-inefficient outcome is accepted consider the following outcome space ($\Omega = \{\omega_1, \omega_2, \omega_3\}$) with the following preferences: $\omega_1 \approx_j \omega_2 \succ_j \omega_3 \succ_j \phi$ and $\omega_3 \succ_i \omega_2 \succ_i \omega_1 \succ_i \phi$. Assume that j started by offering ω_1, as this is the first offer for agent i, it will accept it. Nevertheless ω_1 is Pareto-inefficient because ω_2 weakly dominates it. Note that in this case, the other condition is satisfied because $\omega_1 \approx_j \omega_2$. [4]

Proposition 2 (Pareto →Accepted)**.** *In a bilateral negotiation of Λ_{DN} with TAU(Υ_{SCS}), every Pareto-efficient outcome is accepted by at least one agent or has the same value as the agreement for all agents.*

Proof. From Theorem 4 (Completeness), we know that if the negotiation ends in disagreement, there are NO rational outcomes, and no outcomes on the Pareto frontier and the proposition holds trivially. Therefore, we need only concern ourselves with proper agreements in finite time $T \leq n_o$:

$$\omega_* \in \Omega$$

Let's assume that there is another outcome ψ with unidential value for all negotiators on the Perto-frontier that is not accepted by one of the negotiators:

$$\psi \in \mathcal{P} \wedge \neg\psi \approx_z \omega_* \forall z \in \mathcal{N} \wedge \exists i \in \mathcal{N} \neg \exists k \leq T : a_{ji}^k$$

This means it is either never offered by j or was not acceptable by i when it was offered by j:

$$\psi \notin \Omega_j \vee \exists k \leq T : \omega_i^k = \psi \wedge \neg \hat{a}_k^i(\psi)$$

Case 1: ψ not offered $\psi \notin \Omega_j$

Because the negotiation ends with agreement ω_*, all outcomes better for any agent z must have been offered (descending order of offering):

$$\omega_* \in \Omega \implies \alpha \in \Omega_z \qquad \forall \alpha \succ_z \omega_* \forall z \in \mathcal{N}$$

But,

$$\because \psi \in \mathcal{P}$$

$$\therefore \exists z \in \mathcal{N} \psi \succ_z \omega_* \vee \psi \approx_z \omega_* \forall z \in \mathcal{N}$$

The first subcase implies that ψ must have already been offered by z before the agreement ω_*:

$$\exists z \in \mathcal{N} \psi \succ_z \omega_* \implies \exists k < T : \omega_z^k = \psi$$

which contradicts the assumption of Case 1. The second subcase implies that ψ must have the same value for all agents which directly contradicts our assumption about ψ.

$\therefore$ Case 1 leads to a contradiction.

Case 2: ψ was not acceptable $\exists k \leq T : \omega_i^k = \psi \wedge \neg \hat{a}_i^k(\psi)$

For ψ not to be acceptable by agent i at step k, a better outcome (for i) must have been offered and accepted by i before that:

$$\neg \hat{a}_i^k(\psi) \implies \exists \omega \in \Omega \exists z \in \mathcal{N} \exists j < k : \omega_z^j = \omega \wedge a_{zi}^j \wedge \omega \succ_i \psi$$

$$\therefore \neg \hat{a}_i^k(\psi) \implies \omega \succ_i \psi$$

But for ω to be offered by z before ψ, ω must be at least as good as ψ for z:

$$\omega_z^j = \omega \wedge \omega_z^i = \psi \implies \omega \succcurlyeq_i \psi$$

$$\therefore \omega \succcurlyeq_i \psi \wedge \omega \succ_i \psi$$

$$\therefore \psi \notin \mathcal{P}$$

[4] There is nothing special about ω_1 being the first offer here. We could have just added two other outcomes in either side of the ordering and got the same result.

Which contradicts our assumption.

$\therefore$ Case 2 leads to a contradiction.

$\therefore$ Both cases lead to a contradiction and there are no other possibilities.

$\therefore$ ψ cannot exist.

$$\therefore \neg\exists\psi : \psi \in \mathcal{P} \wedge \neg\psi \approx_z \omega_* \forall z \in \mathcal{N} \wedge \exists i \in \mathcal{N} \neg\exists k \leq T : a_{ji}^k$$

□

Theorem 3 (Optimal). *TAU(Υ_{SCS}) is Exactly Optimal with any strategy assignment rule that uses only SCS on Λ_{DN}.*

$$\omega_*^\lambda \in \mathcal{P}^\lambda \cup \phi \qquad \forall \lambda \in \Lambda_{DN} \tag{22}$$

Proof. We will assume that both negotiators are using SCS. We need to consider too cases.

Case 1 (No win-win deals): $\left|\widehat{\Omega}\right| = 0$.

In this case, there are no outcomes that strictly dominate disagreement.

$\because$ TAU(Υ_{SCS}) is Exactly Rational (Lemma 1)

$\therefore$ It will either lead to disagreement or to some agreement $\omega_* \succcurlyeq_i \phi \forall i \in \mathcal{N}$. This is the optimal result.

$$\left|\widehat{\Omega}\right| = 0 \implies optimal(TAU) \tag{23}$$

Case 2 (some win-win deals): $\left|\widehat{\Omega}\right| > 0$.

There are some win-win outcomes which means that the Pareto-frontier is not empty. We now need to show that: $\left|\widehat{\Omega}\right| > 0 \implies \omega_* \in \mathcal{P}$

We will use a proof by contradiction. Assume that $\omega_* \notin \mathcal{P}$ and that agreement was reached at round T:

From Theorem 1, we know that ω_* must be rational: $\omega_* \in \overline{\Omega}$.

From the definition of pareto-optimality:

$$\therefore \exists\psi \neq \omega_* \exists z \in \mathcal{N} : \psi \succcurlyeq_i \omega_* \forall i \in \mathcal{N} \wedge \psi \succ_z \omega_*$$

$$\because \psi \succ_z \omega_* \tag{24}$$

because SCS always offer outcomes from best to worst (Eq. 5), ψ must have been offered by z:

$$\therefore J_z(\psi) < J_z(\omega_*)$$

but ω_* was also offered by z to become an agreement in the first place (Eq. 16):

$$\omega_* \in \Omega_z \tag{25}$$

$$\therefore J_z(\psi) < J_z(\omega_*) \leq T$$

$$\therefore \psi \in \Omega_z \tag{26}$$

ψ was offered by z earlier. Now consider how would other agents respond to this offer. ω_* must be acceptable by all agents at the end of the negotiation ($\hat{a}_i^T(\omega_*)$):

$$\because \hat{a}_i^T(\omega_*) \wedge \psi \succcurlyeq_i \omega_* \quad \forall i \in \mathcal{N}$$

From Eq. 11:

$$\therefore \hat{a}_i^i(\psi) \quad \forall i < T \quad \forall i \in \mathcal{N} \setminus \{z\}$$

Because ψ is acceptable to all agents and is offered at step $k \equiv J_z(\psi)$ by z, it must have been accepted by everyone:

$$\because \omega_i^k = \psi \wedge \hat{a}_i^k(\psi) \forall i \in \mathcal{N} \setminus \{z\}$$

$$\therefore a_{zi}^k \forall i \in \mathcal{N} \setminus \{z\} \tag{27}$$

Consider any agent $i \neq z$, we know that it accepted ψ and ω:

$$\because \psi \in \mathbb{A}_i \wedge \omega_* \in \mathbb{A}_i \wedge \psi \succcurlyeq_i \omega_* \tag{28}$$

From Eq. 14[5]:

$$\therefore \psi \succ_i \omega_*$$

This is exactly the form of Eq. 24, and by the same steps (applied for all agents) we can arrive at:

$$\psi \in \Omega_i \forall i \in \mathcal{N} \tag{29}$$

and

$$a_{iz}^k \forall i \in \mathcal{N} \forall z \in \mathcal{N} \setminus \{i\} \tag{30}$$

Which means that ψ is offered and accepted by every negotiator and therefore must be the agreement:

$$\psi = \omega_*$$

This is a contradiction as we assumed $\omega_* \neq \psi$.

$$\therefore \left|\widehat{\Omega}^\lambda\right| > 0 \implies \omega_* \in \mathcal{P}$$

Combining the two cases above completes the proof. □

Empirical support comes from the fact that in all experiments, TAU(Υ_{SCS}) found agreements on the Pareto-frontier. This can be seen from the fact that Pareto Optimality was always exactly one which implies that all outcomes are on the Pareto-frontier.

TAU(Υ_{WAR}) is NOT Exactly Optimal on Λ_{DN}

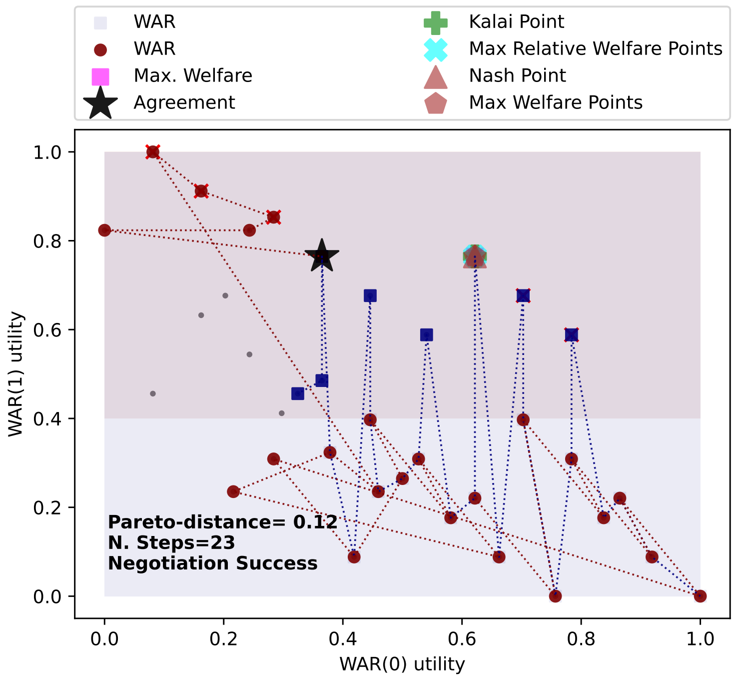


Figure 2: Example from ANAC 2016 domains (Defensive Charms) showing that TAU(Υ_{WAR}) is not exactly optimal.

[5]This is exactly where the proof breaks for WAR as this is not guaranteed in that case.

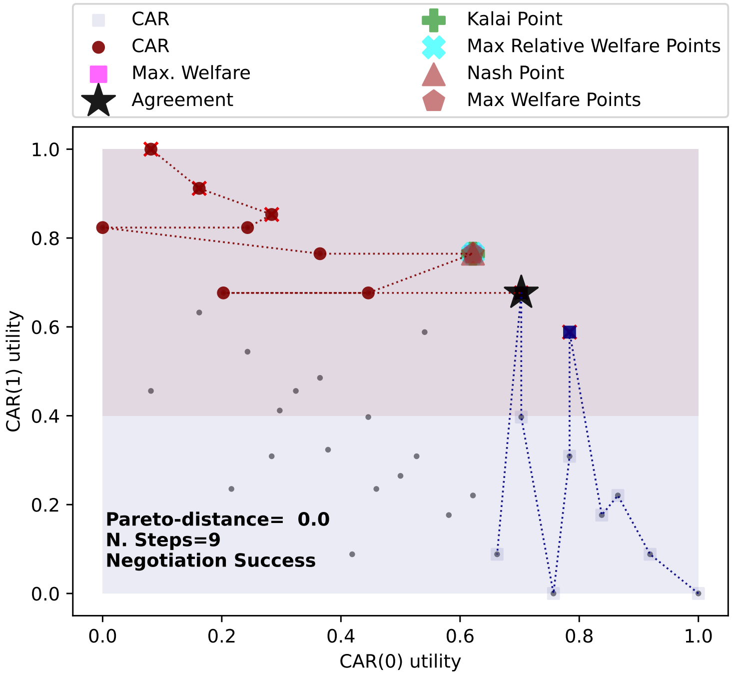


Figure 3: Example from ANAC 2016 domains (Defensive Charms) showing that TAU(Υ_{SCS}) can – in this case – find an agreement on the Perto-frontier. Compare this to WAR (Fig. 3)

Proposition 3 (WAR is not exactly optimal). *TAU(Υ_{WAR}) is NOT exactly optimal.*

Proof. We need only provide a single counter example to prove this proposition. Fig. 2 shows an example from the Defensive Charms domain (ANAC).[6] A smaller example can be constructed with only three outcomes $(\omega_1, \omega_2, \omega_3)$ with the following rankings $\omega_1 \succ_i \omega_2 \approx_i \omega_3 \succ_i \phi$ and $\omega_3 \succ_j \omega_2 \succ_j \omega_1 \succ_j \phi$ for agents i and j. Agent i can offer the outcomes in the order $\langle \omega_1, \omega_2, \omega_3 \rangle$ while Agent j is forced to offer them in the following order $\langle \omega_3, \omega_2, \omega_1 \rangle$. Let's assume that at the second round, agent j offered first. In this case, once agent i offers ω_2, it will become the agreement: $\omega_*^\lambda = \omega_2$ but $\omega_2 \notin \mathcal{P}$ because ω_3 weakly dominates it. □

The only case this can happen in a bilateral negotiation (the only point at which the previous proof failed) is when there are two outcomes ω, ψ such that ($\approx_i (\omega, \psi)$ and $\succ_j (\psi, \omega)$). Moreover, both outcomes are rational for both agents. If j have already offered ω (implying it also offered ψ) by the time i offers ω, and it happened that i offers ω before ψ (which is only possible because $\approx_i (\omega, \psi)$), then ω will be offered by both agents and accepted by both at that point and will become the agreement despite the fact that it is weakly dominated by ψ and cannot lie on the Pareto-frontier. It is important to note that the whenever TAU(Υ_{WAR} reaches a non-optimal agreement, it will be with no loss of utility for the agent that accepted it. The reason SCS is not subject to this problem is that it only accepts outcomes that are strictly better than what it accepted before so in our example above agent i will not accept ω_2 when offered to it because it already accepted ω_3 and $\omega_3 \approx_i \omega_2$.

TAU(Υ_{WAR}) is Exactly Optimal on Λ_U

Proposition 4. *TAU(Υ_{WAR}) is exactly optimal for Λ_U.*

[6] SCS still finds a pareto efficient outcome in this case as shown in Fig. 3.

Proof. The proof follows exactly the same as in the proof for Theorem 3 up until and including Eq. 28. Instead of using Eq. 14 to continue the proof, we use the fact that no two outcomes have the same utility value in any scenario that belongs to Λ_{DN} to show that:

$$\therefore \psi \succ_i \omega_*$$

The rest of the proof follows exactly as in Theorem 3. □

TAU(Υ_{SCS}) is Exactly Complete on Λ_{DN}^B

Theorem 4 (Completeness). *TAU(Υ_{SCS}) is Exactly Complete for bilateral negotiations on Λ_{DN}.*

$$\left|\widehat{\Omega}^\lambda\right| \neq 0 \rightarrow \omega_*^\lambda \in \widehat{\Omega}^\lambda \quad \forall \lambda \in \Lambda_{DN}^B \tag{31}$$

Proof. We will assume that both negotiators are using SCS. For TAU(Υ_{SCS}) to be incomplete, we must have at least one win-win deal in a negotiation that ends either with disagreement or an agreement not in the set of win-win deals. Formally:

$$\neg complete(TAU) \implies \left|\widehat{\Omega}\right| > 0 \wedge \omega_* \notin \widehat{\Omega}$$

$\therefore$ we only need to consider negotiations with $\left|\widehat{\Omega}\right| > 0$ and show that in all such negotiations $\omega_* \neq \phi \wedge \omega_* \in \widehat{\Omega}$.

$\because$ The negotiation always terminates (Theorem 1) after a finite number of rounds T.

$\therefore$ Either an agreement is reached or the negotiation ends in disagreement because of repetition.

Case 1 (agreement): $\left|\widehat{\Omega}\right| > 0 \wedge \omega_* \neq \phi$

Let's consider first the case where a negotiation ends with an agreement: $\omega_* \neq \phi$

From 20, we know that the agreement is rational:

$$\omega_* \neq \phi \implies \omega_* \in \overline{\Omega}$$

Because TAU (with SCS) is Exactly Optimal (Theorem 3), we know:

$$\omega_* \neq \phi \implies \omega_* \in \mathcal{P}$$

From the definition of the Pareto-frontier:

$$\left|\widehat{\Omega}\right| > 0 \implies \mathcal{P} \subset \widehat{\Omega}$$

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_* \neq \phi \implies \omega_* \in \widehat{\Omega} \tag{32}$$

Case 2 (disagreement) $\left|\widehat{\Omega}\right| > 0 \wedge \omega_* = \phi$.

Can this happen? From Eq 16, disagreement ($\omega_* = \phi$) can happen only in one of two ways:

1. One negotiator ends the negotiation: $\exists i \in \mathcal{N} : \omega_i^T = \phi$. This cannot happen because SCS never explicitly end a negotiation (See Eq. 7).

2. Both negotiators are repeating offers: $\forall i \in \mathcal{N} : \omega_i^T = \omega_i^{T-1}$. We need to show that this is also cannot happen when there are any win-win deals.

$\therefore$ We only need to show that it is never the case that $\left|\widehat{\Omega}\right| > 0 \wedge \exists T \in \mathbb{N} : \omega_i^T = \omega_i^{T-1} \quad \forall i \in \mathcal{N}$.

We will do a proof by contradiction again. To arrive at round T, all rational outcomes for i must have been offered by the agent (because SCS and WAR never repeat before trying all of their rational outcomes – Eq 9):

$$\omega_i^T = \omega_i^{T-1} \implies \omega \in \Omega_i^{:T-1} \quad \forall \omega \in \overline{\Omega}_i \tag{33}$$

This is true for both negotiators which means:

$$\omega_i^T = \omega_i^{T-1} \forall i \in \mathcal{N} \implies \omega \in \Omega_i^{:T-1} \quad \forall \omega \in \overline{\Omega}_i \forall i \in \mathcal{N} \tag{34}$$

This means that all possible rational outcomes for both negotiators have been offered.

$$\because \widehat{\Omega} \subseteq \overline{\Omega} \subseteq \overline{\Omega}_i \forall i \in \mathcal{N}$$
$$\therefore \omega \in \Omega_i^{:T} \quad \forall \omega \in \widehat{\Omega} \quad \forall i \in \mathcal{N} \tag{35}$$

Now consider negotiator i that rejects win-win deal $\omega \in \widehat{\Omega}$. From Eq. 12, we know that a better or equivalent offer for i must have been proposed earlier by the other agent j:

$$\omega_j^k = \omega \wedge \omega \in \widehat{\Omega} \wedge \neg a_{ji}^k \implies \exists \psi \in \Omega_j^{:k-1} \cap \widehat{\Omega} : \psi \succcurlyeq_i \omega \tag{36}$$

Because ψ was offered first, it is also not worse than ω for agent j:

$$\because J_j(\psi) < J_j(\omega)$$
$$\therefore \psi \succcurlyeq_j \omega \wedge \psi \succcurlyeq_i \omega$$
$$\therefore \psi \succcurlyeq \omega$$
$$\because \omega \in \widehat{\Omega}$$
$$\therefore \psi \in \widehat{\Omega}$$

This means that a negotiator can reject a win-win deal only if it accepted one earlier. Therefore no negotiator rejects all win-win deals.

$$\forall i \in \mathcal{N} \exists \omega_i \in \widehat{\Omega} : \omega_i \in \mathbb{A}_i$$

Let $Q_i \subset \widehat{\Omega}$ be the set of win-win deals rejected by negotiator i and $q_i^\top$ ($q_i^\perp$) be the best (worst) such outcome in Q_i for i. By definition

$$q_i^\top \succcurlyeq_i q_i^\perp \wedge q_j^\top \succcurlyeq_j q_j^\perp \tag{37}$$

Because a rejected offer must have a non-dominated offer accepted before it (Eq. 12), we have:

$$\exists \alpha_i \in \widehat{\Omega} - Q_i : \alpha_i \succcurlyeq_i q_i^\top \tag{38}$$

Because the negotiation did not end in agreement until step T and all win-win deals were offered by both negotiators (Eq. 58), we must have no win-win deal that is accepted by both negotiators:

$$\left|\widehat{\Omega} \setminus (Q_i \cup Q_j)\right| = 0 \tag{39}$$

From Eq. 38, Eq. 37, and E. 39 (and their symmetric counterparts), we get:

$$q_j^\top \succcurlyeq_i \alpha_i \succcurlyeq_i q_i^\top \tag{40}$$

By symmetry

$$q_i^\top \succcurlyeq_j \alpha_j \succcurlyeq_j q_j^\top \tag{41}$$

From Eq. 40 and Eq. 41 we get:

$$q_i^\top \approx_i q_j^\top \wedge q_i^\top \approx_j q_j^\top \tag{42}$$

In summary:

$$\left|\widehat{\Omega} \setminus (Q_i \cup Q_j)\right| = 0 \implies q_i^\top \approx_i q_j^\top \wedge q_i^\top \approx_j q_j^\top \tag{43}$$

Because of the lexical ordering rule in Eq. 1, we know that one of these two outcomes must have been received first by both negotiators. Moreover, by defintion, they both have been rejected. Therefore, there is another outcome that must have been accepted by both negotiators before them which must be a win-win deal and cannot be a member of either Q_i or Q_j:

$$\exists \psi \in \widehat{\Omega} : \psi \notin Q_i \wedge Q_j \tag{44}$$

From Eq. 43 and Eq. 44 we have:

$$\left|\widehat{\Omega} \setminus (Q_i \cup Q_j)\right| = 0 \implies \left|\widehat{\Omega} \setminus (Q_i \cup Q_j)\right| > 0 \tag{45}$$

which is a contradiction disproving the third way of ending the negotiation with no agreement.

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_i^T = \omega_i^{T-1} \forall i \in \mathcal{N} \implies \omega_* \neq \phi$$

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_* = \phi \implies \omega_* \neq \phi \tag{46}$$

which is a contradiction for Case 2. From Eq. 46 and Eq. 32, we get:

$$\therefore \left|\widehat{\Omega}\right| > 0 \implies \omega_* \in \widehat{\Omega} \tag{47}$$

$\therefore$ TAU is Exactly Complete on Λ_{DN}^B for any strategy assignment rule that employ only SCS.

□

This is supported empirically in the paper by the fact that in all experiments, the agreement rate (completeness) of TAU with SCS when negotiating against itself was always 100%.

TAU(Υ_{WAR}) is Exactly Complete on Λ_{DN}^B

Theorem 5 (Completeness)**.** *TAU(Υ_{WAR}) is Exactly Complete for bilateral negotiations on Λ_{DN}.*

$$\left|\widehat{\Omega}^\lambda\right| \neq 0 \rightarrow \omega_*^\lambda \in \widehat{\Omega}^\lambda \quad \forall \lambda \in \Lambda_{DN}^B \tag{48}$$

Proof. We will assume that both negotiators are using WAR. For TAU(Υ_{WAR}) to be incomplete, we must have at least one win-win deal in a negotiation that ends either with disagreement or an agreement not in the set of win-win deals. Formally:

$$\neg complete(TAU) \implies \left|\widehat{\Omega}\right| > 0 \wedge \omega_* \notin \widehat{\Omega}$$

$\therefore$ we only need to consider negotiations with $\left|\widehat{\Omega}\right| > 0$ and show that in all such negotiations $\omega_* \neq \phi \wedge \omega_* \in \widehat{\Omega}$.

$\because$ The negotiation always terminates (Theorem 1) after a finite number of rounds T.

$\therefore$ Either an agreement is reached or the negotiation ends in disagreement because of repetition.

Case 1 (agreement): $\left|\widehat{\Omega}\right| > 0 \wedge \omega_* \neq \phi$

Let's consider first the case where a negotiation ends with an agreement: $\omega_* \neq \phi$

From 20, we know that the agreement is rational:

$$\omega_* \neq \phi \implies \omega_* \in \overline{\Omega}$$

From the agreement assumption and Eq. 16, the agreement was offered and was acceptable by all negotiators.

$$\omega_* \neq \phi \implies \omega_* \in \Omega_i^T \wedge \hat{a}_i^j(\omega_*^\lambda) \forall j < T \forall i \in \mathcal{N} \tag{49}$$

Now consider a rational outcome that is not a win-win deal: $\psi \in \overline{\Omega} \setminus \widehat{\Omega}$

$$\because \psi \in \overline{\Omega} \setminus \widehat{\Omega}$$
$$\therefore \omega \succ_i \psi \qquad \forall i \in \mathcal{N} \forall \omega \in \widehat{\Omega} \tag{50}$$

From Eq. 10, every outcome in $\widehat{\Omega}$ was offered before ψ by all negotiators.

From Eq. 11, assuming that ψ is acceptable at time T implies that every outcome in $\widehat{\Omega}$ will be acceptable if offered anytime before T. From Eq. 5, we then have:

$$\omega_* = \psi \implies \hat{a}_i^T(\psi) \longrightarrow \hat{a}_i^k(\omega)\forall k < T \tag{51}$$

From Eq 50 and Eq 51 we have:

$$\therefore \omega_* = \psi \implies \exists i_i < T : \omega_i^{i_i} = \omega \wedge \hat{a}_j^{i_i}(\omega)\forall i, y \in \mathcal{N}\forall\omega \in \widehat{\Omega}$$

From Eq. 5, and Eq 50, we can conclude that all win-win deals were offered during the negotiation by all negotiators:

$$\omega_* \notin \widehat{\Omega} \implies \omega_* \in \Omega_i^T \quad \forall\omega \in \widehat{\Omega} \quad \forall i \in \mathcal{N} \tag{52}$$

From Eq. 11, and Eq 50, we can conclude that all win-win deals were acceptable during the negotiation by all negotiators:

$$\omega_* \notin \widehat{\Omega} \implies \hat{a}_i^j(\omega) \quad \forall j < T\forall\omega \in \widehat{\Omega}\forall i \in \mathcal{N} \tag{53}$$

Substituting Eq. 52, Eq. 53 into Eq. 15, we get:

$$\begin{aligned} \omega_* \notin \widehat{\Omega} &\implies \hat{a}_i^j(\omega) \wedge \omega_* \in \Omega_i^T \quad \forall j < T\forall\omega \in \widehat{\Omega}\forall i \in \mathcal{N} \\ &\implies \omega_* \in \widehat{\Omega} \end{aligned} \tag{54}$$

We have proven a contradiction.

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_* \neq \phi \implies \omega_* \in \widehat{\Omega} \tag{55}$$

Case 2 (disagreement) $\left|\widehat{\Omega}\right| > 0 \wedge \omega_* = \phi$**.**

Can this happen? From Eq 16, disagreement ($\omega_* = \phi$) can happen only in one of two ways:

1. One negotiator ends the negotiation: $\exists i \in \mathcal{N} : \omega_i^T = \phi$. This cannot happen because WAR never explicitly end a negotiation (See Eq. 7).
2. Both negotiators are repeating offers: $\forall i \in \mathcal{N} : \omega_i^T = \omega_i^{T-1}$. We need to show that this is also cannot happen when there are any win-win deals.

$\therefore$ We only need to show that it is never the case that $\left|\widehat{\Omega}\right| > 0 \wedge \exists T \in \mathbb{N} : \omega_i^T = \omega_i^{T-1} \quad \forall i \in \mathcal{N}$.

We will do a proof by contradiction again. To arrive at round T, all rational outcomes for i must have been offered by the agent (because WAR never repeats before trying all of its rational outcomes – Eq 9):

$$\omega_i^T = \omega_i^{T-1} \implies \omega \in \Omega_i^{:T-1} \quad \forall\omega \in \overline{\Omega}_i \tag{56}$$

This is true for both negotiators which means:

$$\omega_i^T = \omega_i^{T-1}\forall i \in \mathcal{N} \implies \omega \in \Omega_i^{:T-1} \quad \forall\omega \in \overline{\Omega}_i\forall i \in \mathcal{N} \tag{57}$$

This means that all possible rational outcomes for both negotiators have been offered.

$$\begin{gathered} \because \widehat{\Omega} \subseteq \overline{\Omega} \subseteq \overline{\Omega}_i\forall i \in \mathcal{N} \\ \therefore \omega \in \Omega_i^{:T} \quad \forall\omega \in \widehat{\Omega} \quad \forall i \in \mathcal{N} \end{gathered} \tag{58}$$

WAR never rejects a win-win deal $\omega \in \widehat{\Omega}$ because it accepts all rational offers. From Eq. 58, we now know that all win-win deals have been offered and accepted by all agents. This means that at least one of them must be an agreement.

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_i^T = \omega_i^{T-1}\forall i \in \mathcal{N} \implies \omega_* \neq \phi$$

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_* = \phi \implies \omega_* \neq \phi \tag{59}$$

which is a contradiction for Case 2. From Eq. 59 and Eq. 55, we get:

$$\therefore \left|\widehat{\Omega}\right| > 0 \implies \omega_* \in \widehat{\Omega} \tag{60}$$

$\therefore$ TAU is Exactly Complete on Λ_{DN}^B for any strategy assignment rule that employ only WAR.

□

This is supported empirically in the paper by the fact that in all experiments, the agreement rate (completeness) of TAU with WAR when negotiating against itself was always 100%.

Lemma 2 ($\Upsilon_{SCS,WAR}$ **Not Complete).** *TAU($\Upsilon_{SCS,WAR}$) (i.e. WAR negotiating against SCS) is not exactly complete on Λ_{DN}.*

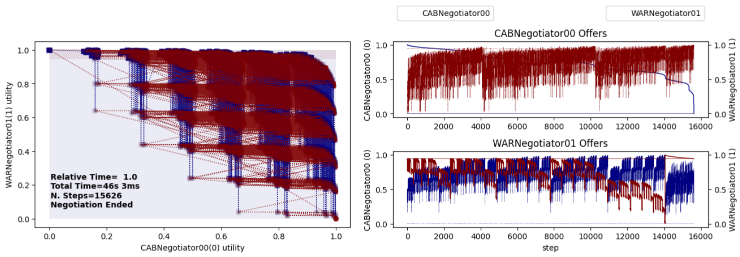


Figure 4: SCS against WAR cannot reach agreement in the Car domain with 10% rational outcomes even though win-win deals exist.

What Theorem 4 and Theorem 5 established is that Υ_{WAR} and Υ_{SCS} are exactly complete on Λ_{DN} which means that WAR is exactly complete when negotiating against itself and SCS is exactly complete when negotiating against itself. What happens if SCS is negotiating against WAR. In some cases, the negotiation will end in disagreement even when win-win deals exist. The reason is that WAR will start with offering irrational outcomes that will tend to have high utility for its partner running SCS. The partner will accept these offers and as a result cannot accept anything worse for itself later (because SCS uses AcceptBetter). Moreover, SCS will never accept these irrational outcomes. This is why the negotiation may end in disagreement. An example of this happening from the experiment reported in the paper is on the domain Car with 10% of the outcomes being rational for the second agent and all of them being rational for the first agent in which running SCS against WAR leads to no agreement even though win-win deals exist. Fig. 4 shows this negotiation.

Conjecture 1 (Υ_{SCS} Not Complete on Multilateral)**.** *SCS is not exactly complete on Λ_{DN} (i.e. only on Λ_{DN}^B).*

The proof of Theorem 4 depended crucially on the fact that SCS will not get stuck unable to accept any offer after it accepted an offer from its partner that is then rejected by that partner. In a multilateral negotiation, it may be the case that the partner will accept that offer later but another partner will not which means that an agreement may not be reachable. This is not a proof (and that is why this is a conjecture). We did not find any counter example in the 184 scenarios evaluated in this paper.

TAU(Υ_{SCS}) is Exactly Fair on Λ_{DN}^{RB}

Theorem 6 (Fairness). *TAU(Υ_{SCS}) is Exactly Fair for bilateral negotiations on Λ_{DN} with no irrational outcomes. Specifically, it finds an Ordinal Kalai Bargaining Solution in that case:*

$$\left|\overline{\Omega}^{\lambda}\right| > 0 \rightarrow \lambda_{TAU}(\Upsilon_{SCS}) \in \Omega_k \quad \forall \lambda \in \Lambda_{DN} \tag{61}$$

$$\textit{where } \Omega_k = \left\{\omega : \omega \in \operatorname*{argmax}_{\omega \in \mathcal{P}^{\lambda} \bigcup \{\phi\}} \min_{i \in \mathcal{N}} r_i(\omega)\right\}$$

Proof. From Theorem 4 (Completeness), we know that whenever there is disagreement, there are no Pareto-efficient outcomes and Eq. 61 holds trivially. We need to focus on the case when there is a proper agreement:

$$\omega_* \in \Omega$$

From Theorem 3 (Optimality), we know that — in this case — the agreement is Pereto-efficient.

$$\omega_* \in \mathcal{P}$$

The offering policy of SCS is to order rational outcomes in descending order of value and offer them in sequence repeating the last one. This assigns each outcome a unique integer giving the step it is offered (i.e. $R_i(\Omega_i^{\top}) \equiv R_i(\omega^T) = 0 \forall \omega^T \in \Omega_i^{\top}, R_i(\phi) = \left|\overline{\Omega}_i\right|$). We can model the fact that SCS never offer irrational outcomes by assigning all irrational outcomes an infinite value inf (From Theorem 1, none of these outcomes will ever be reached). Let's call this assigned integer the offering order $R_i(\omega)$.

The definition of the value function r can be related to this offering order by:

$$r_i(\omega) \approx \frac{R_i(\phi) - R_i(\omega)}{R_i(\phi) - R_i(\Omega_i^{\top})} \quad \forall \omega \in \overline{\Omega}_i$$

$$r_i(\omega) \approx \frac{|\Omega| - R_i(\omega)}{|\Omega| - 0} \quad \forall \omega \in \overline{\Omega}_i$$

$$r_i(\omega) \approx \frac{|\Omega| - R_i(\omega)}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i$$

The reason this is not a full equality is that outcomes with the same value for the agent have different consecutive offering orders but their value is the same. This will not affect the logic of this proof though. What really matters for our purposes is the following fact:

$$f\left(r_i(\omega)\right) = \frac{|\Omega| - R_i(\omega)}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i \tag{62}$$

where $f(x)$ is a monotonically non-decreasing mapping. It assigns the same rank to consecutive outcomes with the same utility with the property

$$f(x_1) > f(x_2) \implies x_1 \geq x_2$$

For an agreement to be reached at step T in TAU, the same outcome must be offered by all agents and the agreement is reached at the step in which the last agent offers it:

$$\omega_* \in \Omega_i^T \qquad \forall i \in \mathcal{N}$$

$$\therefore T = \max_{i \in \mathcal{N}} R_i(\omega_*)$$

$$\therefore \omega_* \in \left\{\omega : \omega \in \operatorname*{argmin}_{\omega \in \mathcal{P}} \max_{i \in \mathcal{N}} R_i(\omega)\right\}$$

Note that in the last step we had to take into account the fact that multiple outcomes may be the same for all agents.

$$\therefore \omega_* \in \left\{\omega : \omega \in \operatorname*{argmin}_{\omega \in \mathcal{P}} \max_{i \in \mathcal{N}} c - c \times f\left(r_i(\omega)\right)\right\}$$

where $c = |\Omega|$. But c does not depend on ω or the agent index:

$$\therefore \omega_* \in \left\{\omega : \omega \in \operatorname*{argmin}_{\omega \in \mathcal{P}} \max_{i \in \mathcal{N}} -f(r_i(\omega))\right\}$$

$$\therefore \omega_* \in \left\{\omega : \omega \in \operatorname*{argmin}_{\omega \in \mathcal{P}} \max_{i \in \mathcal{N}} -r_i(\omega)\right\}$$

because f is monotonically non-decreasing

$$\therefore \omega_* \in \left\{\omega : \omega \in \operatorname*{argmin}_{\omega \in \mathcal{P}} -\min_{i \in \mathcal{N}} r_i(\omega)\right\}$$

$$\therefore \omega_* \in \left\{\omega : \omega \in \operatorname*{argmax}_{\omega \in \mathcal{P}} \min_{i \in \mathcal{N}} r_i(\omega)\right\} = \Omega_k$$

□

The proof did not explicitly use the assumption that all outcomes are rational but this is implied in using the Kalai Bargaining Solution which is defined for this case Kalai (1977). The standard Kalai Bargaining solution for cardinal preferences is defined by:

$$\Omega_k^* = \left\{\omega : \omega \in \operatorname*{argmax}_{\omega \in \mathcal{P}} \min_{i \in \mathcal{N}} u_i(\omega) - u_i(\phi)\right\}$$

which is not the same as the Ordinal Kalai Bargaining Solution we define here as:

$$\Omega_k = \left\{\omega : \omega \in \operatorname*{argmax}_{\omega \in \mathcal{P}} \min_{i \in \mathcal{N}} r_i(\omega)\right\}$$

except for the case when all outcomes are rational.

TAU(Υ_{WAR}) is Exactly Fair on Λ_U^B

Theorem 7 (Fairness). *TAU(Υ_{SCS}) is Exactly Fair for bilateral negotiations on Λ_U. Specifically, it finds an Ordinal Kalai Bargaining Solution in that case:*

$$\left|\overline{\Omega}^{\lambda}\right| > 0 \rightarrow \lambda_{TAU}(\Upsilon_{SCS}) \in \Omega_k \quad \forall \lambda \in \Lambda_{DN} \tag{63}$$

$$\textit{where } \Omega_k = \left\{\omega : \omega \in \operatorname*{argmax}_{\omega \in \mathcal{P}^{\lambda} \bigcup \{\phi\}} \min_{i \in \mathcal{N}} r_i(\omega) - r_i(\phi)\right\}$$

Proof. From Theorem 4 (Completeness), we know that whenever there is disagreement, there are no Pareto-efficient outcomes and Eq. 63 holds trivially. We need to focus on the case when there is a proper agreement:

$$\omega_* \in \Omega$$

From Theorem 4 (Optimality), we know that — in this case — the agreement is Pereto-efficient.

$$\omega_* \in \mathcal{P}$$

The offering policy of WAR is to first offer all irrational outcomes then to order rational outcomes in descending order of value and offer them in sequence repeating the last one. This assigns each outcome a unique integer giving the step it is offered (i.e.). We modeled the fact that WAR offers all irrational outcomes first by assigning adding the number of irrational outcomes to the step at which all rational outcomes are offered (From Theorem 1, none of these outcomes will ever be reached). Let's call this assigned integer the offering order $R_i(\omega)$.

This leads to the following two equations:

$$R_i(\Omega_i^\top) = R_i(\omega^T) = |\underline{\Omega}_i| \, \forall \omega^T \in \Omega_i^\top$$

$$R_i(\phi) = |\Omega|$$

The definition of the value function r can be related to this offering order by:

$$r_i(\omega) \approx \frac{R_i(\phi) - R_i(\omega) + n_{ir}}{R_i(\phi) - R_i(\Omega_i^\top) + n_{ir}} \quad \forall \omega \in \overline{\Omega}_i$$

where $n_{ir} = |\underline{\Omega}_i|$

$$\therefore r_i(\omega) - r_i(\phi) \approx \frac{|\Omega| - R_i(\omega) - |\underline{\Omega}_i|}{|\Omega| - |\underline{\Omega}_i| + |\underline{\Omega}_i|} - r_i(\phi) \quad \forall \omega \in \overline{\Omega}_i$$

$$\therefore r_i(\omega) - r_i(\phi) \approx \frac{|\overline{\Omega}_i| - R_i(\omega)}{|\Omega|} - \frac{|\overline{\Omega}_i| - |\Omega|}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i$$

$$\therefore r_i(\omega) - r_i(\phi) \approx \frac{|\overline{\Omega}_i| - R_i(\omega)}{|\Omega|} + \frac{|\underline{\Omega}_i|}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i$$

$$\therefore r_i(\omega) - r_i(\phi) \approx \frac{|\Omega| - R_i(\omega)}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i$$

$$\therefore r_i(\omega) - r_i(\phi) \approx 1 - \frac{R_i(\omega)}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i$$

$$\therefore f\left(r_i(\omega) - r_i(\phi)\right) = 1 - \frac{R_i(\omega)}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i$$

$$\therefore f\left(r_i^*(\omega)\right) = 1 - \frac{R_i(\omega)}{|\Omega|} \quad \forall \omega \in \overline{\Omega}_i \tag{64}$$

where $r_i^*(\omega) \equiv r_i^*(\omega) - r_i(\phi)$. $f(x)$ is a monotonically non-decreasing mapping. It assigns the same rank to consecutive outcomes with the same utility with the property

$$f(x_1) > f(x_2) \implies x_1 \geq x_2$$

Eq. 64 has the same form as Eq. 62. Applying the same argument exactly using r^* instead of r, we get at:

$$\therefore \omega_* \in \left\{ \omega : \omega \in \underset{\omega \in \mathcal{P}}{\operatorname{argmax}} \min_{i \in \mathcal{N}} r_i^*(\omega) \right\}$$

$$\therefore \omega_* \in \left\{ \omega : \omega \in \underset{\omega \in \mathcal{P}}{\operatorname{argmax}} \min_{i \in \mathcal{N}} \left(r_i(\omega) - r_i(\phi)\right) \right\} = \Omega_k$$

□

WAR weakly dominates SCS on Λ_{DN}^B

Theorem 8 (Weak Domination)**.** *WAR weakly dominates SCS for TAU on Λ_{DN}^B.*

$$\begin{aligned} &\mathcal{P}_\Upsilon(i \approx WAR, ...) = \phi \\ &\vee \hat{u}_i(\mathcal{P}_\Upsilon(i \approx WAR, ...)) \geq \hat{u}_i(\mathcal{P}_\Upsilon(i \approx SCS, ...)) \\ &\forall i \in \mathcal{N}^\lambda \forall \lambda \in \Lambda_{DN}^B \end{aligned} \tag{65}$$

Proof. Let's assume — without loss of generality — that agent j uses SCS and run two negotiations N_{WAR}^λ and N_{SCS}^λ where agent i is using WAR and SCS respectively. From now one, we mark a value v that belongs to N_σ^λ as $z\ |_\sigma$ (i.e. v given that agent i uses strategy σ). For example $\omega_*\ |_{SCS}$ is the agreement when agent i plays SCS against j. [7]

Case 1: No Irrational Outcomes for i

Consider first any scenario $\lambda \in \Lambda_{DN}$ with no irrational outcomes for i (i.e. $\overline{\Omega}_i = \Omega$). In this case SCS and WAR offer exactly in the same way. The only difference is that SCS accepts all rational outcomes while WAR accepts only outcomes better than the best it accepted so far.

Because WAR and SCS have exactly the same offering strategy in this case, the offers exchanged until the step $\min(T\ |_{WAR}, T\ |_{SCS})$ will be exactly the same. We now consider the three alternatives for the negotiation length:

1. $T\ |_{WAR} < T\ |_{SCS}$: The agreement cannot be worse for i because it offers its outcomes in descending order of their value (i.e. $\omega_*\ |_{WAR} \geq \omega_*^\lambda\ |_{SCS}$)
2. $T\ |_{WAR} = T\ |_{SCS}$: The agreement must be the same (i.e. $\omega_*\ |_{WAR} = \omega_*^\lambda\ |_{SCS}$)
3. $T\ |_{WAR} > T\ |_{SCS}$: The agreement worse for i in principle. This cannot happen though. For the negotiation to take longer in this case, ω_* must have been offered by both agents and must have been accepted by j (because its acceptance strategy does not change). This means that to reach $T\ |_{WAR}$, ω_* must have been rejected at step $T\ |_{optim}$ by WAR. Nevertheless, ω_* must be acceptable for SCS (otherwise it would have not been an outcome). But the acceptance strategy of WAR accepts any outcome acceptable by SCS which is a contradiction.

We have shown that in the first two subcases $\omega_*\ |_{WAR} \succcurlyeq_i \omega_*\ |_{SCS}$ and the third subcase cannot happen.

$$\text{Case 1} \implies \omega_*\ |_{WAR} \succcurlyeq_i \omega_*\ |_{SCS}$$

Case 2: Some Irrational Outcomes for i We will consider the same three subcases based on the relative length of the two negotiations assuming that there are n_i irrational outcomes for agent i:

1. $T\ |_{WAR} < T\ |_{SCS}$: The agreement cannot be worse for i because it offers its outcomes in descending order of their value (i.e. $\omega_*\ |_{WAR} \geq \omega_*^\lambda\ |_{SCS}$)
2. $T\ |_{WAR} = T\ |_{SCS}$: The agreement cannot be worse for i because it will be offering an outcome n_i steps higher in its ordered outcome set and j will be offering the same outcome it always offers at this step. (i.e. $\omega_*\ |_{WAR} \geq \omega_*^\lambda\ |_{SCS}$).
3. $T\ |_{WAR} > T\ |_{SCS}$: The agreement worse for i in principle. We will consider this case in more details.

From the three subcases above, we conclude that the only viable possibility to have a worse outcome for i in N_{WAR} is to have a longer negotiation:

$$\omega_*\ |_{WAR} \prec_i \omega_*\ |_{SCS} \implies T\ |_{WAR} > T\ |_{SCS}$$

We again have three subcases to consider based on the difference between these two times:

1. $T\ |_{WAR} - T\ |_{SCS} < n_i$: i will be offering an outcome $n_i - (T\ |_{WAR} - T\ |_{SCS})$ steps higher in its ordered outcome list and j will be offering an outcome worse for itself than $\omega_*\ |_{SCS}$. There are two possibilities depending on who offered the outcome that was accepted as the negotiation outcome:

[7] j always plays SCS in this proof.

(a) Agent j: Because $\omega_* |_{WAR} \preceq_j \omega_*^\lambda |_{SCS}$, $\omega_*^\lambda |_{SCS}$ must have been offered earlier by j in N_{WAR}. Because, $\omega_* |_{SCS}$ is rational for i, it must have been accepted at that point by i. Assuming $\omega_* |_{WAR} \prec_i \omega_* |_{SCS}$, then $\omega_* |_{SCS}$ must also have been offered by i in N_{WAR}. Moreover, given that i offered n_i irrational outcomes first, the step at which it offered $\omega_* |_{SCS}$ in N_{WAR} must be larger than the step it offered that outcome in N_{SCS} which means that must have been accepted by j. We just showed that $\omega_* |_{SCS}$ must have been offered and accepted by both agents in N_{WAR}. This implies that $\omega_* |_{WAR} = \omega_* |_{SCS}$. Therefore, $\omega_* |_{WAR} \approx_i \omega_* |_{SCS}$.

(b) Agent i: Because $\omega_* |_{WAR}$ is higher in the ordered outcome list than $\omega_* |_{SCS}$, it cannot be worse. Therefore, $\omega_* |_{WAR} \succcurlyeq_i \omega_* |_{SCS}$.

Taken together this shows that $T |_{WAR} - T |_{SCS} < n_i \implies \omega_* |_{WAR} \approx_i \omega_* |_{SCS}$

2. $T |_{WAR} - T |_{SCS} = n_i$: i will be offering an outcome $n_i - (T |_{WAR} - T |_{SCS}) = 0$ steps higher in its ordered outcome list which means it is offering the same outcome (i.e. $\omega_* |_{WAR} = \omega_*^\lambda |_{SCS}$). Therefore, $\omega_* |_{WAR} \approx_i \omega_* |_{SCS}$.

3. $T |_{WAR} - T |_{SCS} > n_i$: i will be offering an outcome $(T |_{WAR} - T |_{SCS}) - n_i$ steps lower in its ordered outcome list and j will be offering an outcome worse for itself than $\omega_* |_{SCS}$. There are two possibilities depending on who offered the outcome that was accepted as the negotiation outcome:

(a) Agent j: Following the same argument for this condition above we reach the same conclusion. Therefore, $\omega_* |_{WAR} \succcurlyeq_i \omega_* |_{SCS}$.[8]

(b) Agent i: Because $\omega_* |_{WAR} \prec_i \omega_* |_{SCS}$, $\omega_*^\lambda |_{SCS}$ must have been offered earlier. The same reasoning as in the previous case applies from there. $\omega_* |_{WAR} \succcurlyeq_i \omega_* |_{SCS}$. But that is a contradiction which means that this case cannot happen.

We have shown that in all possible situations, $\omega_* |_{WAR} \succcurlyeq_i \omega_* |_{SCS}$. □

Why would anyone care about this notion of weak domination? In some practical situations, it is possible to repeat a failed negotiation. In such cases, knowing that s_1 weakly dominates s_2 — in the sense given above — suggests that we should start by using s_1 and if an agreement is not reached, we can switch to s_2. In our specific case, this suggests attempting WAR first then if and only if we could not reach agreement we can switch to SCS.

WAR is a PBE of TAU on Λ_{DN}^B

To prove that WAR is a negotiation equilibrium for TAU, we proceed in the following steps: In Section we rigorously define the induced incomplete information game corresponding to TAU on a set of scenarios and define its information sets. Section defines the beliefs of agents in the game. Section describes *consistent* Bayesian belief update given that the partner is using WAR and shows that assuming no information about the partner makes this process much simpler than if partial information about partner preferences (e.g. a linear aggregation utility function) is available. Finally, Section completes the proof by showing that WAR is a Perfect Bayesian Equilibrium for the induced game making it — by definition — a negotiation equilibrium.

[8] The only difference is that we do not need to use the assumption that $\omega_* |_{WAR} \prec_i \omega_* |_{SCS}$ because it is guaranteed in this case.

The Induced Game Let n_s be the number of scenarios is a scenario set Λ, n_o^λ be the cardinality of the outcome space for $\lambda \in \Lambda$ and n_A^λ be the number of agents in that scenario. Moreover, let's define Λ^Ω as the subset of Λ that share the same outcome space Ω. n_o^Ω is defined correspondingly as the number of outcomes in Ω which is shared by all members of Λ^Ω.

In this section, we only consider the bilateral negotiations case (i.e. $n_A = 2$). For simplicity of exposition we assume that all scenarios have the same outcome size n_o and will generalize that back in the following proposition.

Given a set of scenarios Λ, Nature moves first and once by selecting a scenario from this set of scenarios based on some probability distribution. Nature never moves again. In this paper, we are focused on the case of a general negotiator (i.e. one that can negotiate in any scenario) with no information about the partner which means that all scenarios are sampled with equal probability.

Information sets correspond to decision points of agents. There is one information set for each offering and each acceptance decision by each agent. Because TAU always terminates in $n_o + 1$ rounds (Theorem 1), the longest path in the game tree is of length $4n_o + 4 + 1$ (1 for nature's move and a factor 4 for the two offers and two acceptance decisions per round). Each offering node has a branching factor of $n_o + 1$ and each acceptance node has a branching factor of 2 while the root node (Nature's) has a branching factor of n_s. If we consider the two levels corresponding to an offer and its response in a round together, we get a branching factor of $2n_o + 2$ with a tree of height $1 + 2(n_o + 1)$ leading to a total of $n_s\,(2n_o + 2)^{2(n_o+1)}$ nodes.

The only incomplete information in the game is which scenario are we playing which means that the nodes of each decision level in the game (i.e. each decision made by each agent) are distributed into information sets of size n_s each[9]. Each information set corresponds to one possible negotiation *trace* defined as the complete history of the decisions made in the negotiation (common knowledge).

We can summarize this information in the following proposition

Proposition 5 (Game). *TAU induces a game G^Λ for any set of bilateral scenarios Λ. The game G^Λ is an incomplete information game with at most $\max_{\lambda\in\Lambda} 5 + 4n_o^\lambda$ containing a total of $\sum_{\lambda\in\Lambda} \left(2n_o^\lambda + 2\right)^{2\left(n_o^\lambda+1\right)}$. These nodes decompose into information sets corresponding to specific negotiation traces. Each information set has exactly n_s nodes (all at in the same level).*

Modeling Beliefs in the Induced Game The belief of an agent in the induced game at an information set is a probability distribution over all possible nodes in that set. This is a distribution over n_s values which is in general infinite in size. In this paper, we are only interested in analyzing games played using TAU against WAR. This will simplify this distribution a lot. WAR does not use any cardinal information in its decision (i.e. it is fully characterized by full ordering it uses for offering and the rational outcome set). Moreover, the negotiation outcome (payoffs) will be determined completely by the order in which offers were proopsed and the binary acceptance decisions. This means that all preference allocations for any given scenario that lead to the same ordering of outcomes for all agents will correspond to the same payoffs

[9] We choose the common practice of having the largest possible information sets and leaving it to agent beliefs to assign zero probabilities to irrelevant nodes. For example, in the first node, we have $\sum_{\lambda in \Lambda} K_\lambda$ nodes, where K_λ are all possible preference assignments for scenario λ. Nevertheless, the agent about to move knows its own preferences and the outcome space which assigns zero probability to most of these nodes in its belief.

and may be collapsed together. Moreover, equality does not matter when WAR is playing itself as there is a full ordering imposed by the fact that if two outcomes are the same for an agent, they must be offered according to the predefined (yet arbitrary) common ordering defined by the mechanism.

Because the outcome space Ω^λ is common knowledge for all agents, the induced game decomposes into a set of independent games each sharing the same outcome space Ω (i.e. each game corresponds to on Λ^Ω set of scenarios). Beliefs over these games are disjoint and will be zero for all of these decomposed games except the one currently being played. From now on, we will drop the superscript Ω from Λ^Ω and n_o^Ω because all other members of the belief are zero all the time.

In summary, for the purposes of this paper, we can model an agent's belief as a distribution over all possible permutations of the extended outcome space of our partner at every negotiation. There is always $(n_o+1)!$ such permutations which is always finite for Λ_{DN} that we are concerned with in this proof. We use Λ to represent the set of all possible permutations of $osplus$ and λ to be one such permutation. Moreover, we define λ_ω as all permutations ending with ω, λ^ω as all permutations starting with ω.

Proposition 6 (Belief). *The induced game G^Λ induced by the bilateral discrete scenarios with no information Λ_{DN} when agent j is playing WAR has information sets for which the number of nodes with nonzero probabilities is $(n_o+1)!$ corresponding to all possible permutations of Ω^+ where n_o is the number of outcomes of the outcome space (known at the first round).*

$$\lambda \in \Lambda_{DN} \wedge j \approx WAR \implies \mathbb{B}_i \equiv \{p(\lambda) \forall \lambda \in \Lambda\}$$

Bayesian Belief Update We are only considering belief update compatible with knowing that our partner is using WAR because that is the only update relevant for *checking* that WAR is a pure PBE for the induced game.

We also need the following Lemma about its three stages:

Lemma 3 (WAR's Repetition). *WAR never ends a negotiation by sending ϕ and never repeats until round n_o+1 where n_o is the size of the outcome space.*

$$j \approx WAR \wedge \omega_j^k = \omega_j^{k-1} \implies k = n_o$$

Proof. The first phase of WAR involves sending all irrational outcomes $\underline{\Omega}_i$ and the second phase all rational outcomes $\overline{\Omega}_i$. But these are all possible outcomes: $\Omega = \overline{\Omega}_i \cup \underline{\Omega}_i$.

$\therefore$ The second phase (Rational) ends only when all outcomes in the outcome space are offered and there are n_o of them.

$\therefore$ The third phase (Repeat) can only start at round n_o+1. □

Consider a negotiation between i and j on a scenario from Λ_{DN} in which j uses WAR. We augment the belief of i with another data structure called its Partner's Offer Rationality Distribution $\mathbb{K}_i$ which is defined as a distribution over the rationality of every past or potential offer from j:[10]

$$\mathbb{K}_i^k \equiv p(\omega_j^k \succcurlyeq_j \phi) \qquad \forall k \in [1, n_o]$$

Three special properties of this distribution will have an important role for the proposed belief update rule:

Ignorance A partner rationality belief is in the Ignorance state iff $\mathbb{K}_i^k \notin \{0,1\} \quad \forall k \in [1, n_o]$

[10] We ignore round n_o+1 because it does not matter as the negotiation will end with disagreement anyway once this round is reached.

RationalAfter(k) A partner rationality belief is in the Ignorance state iff $\mathbb{K}_i^k = 1 \quad \forall k \le j \le n_o$

IrrationalBefore(k) A partner rationality belief is in the Ignorance state iff $\mathbb{K}_i^k = 0 \quad \forall 1 \le j \le k$

The final component of the belief state for i is the Learned Inequality Set $\mathcal{O}_i$ consisting of a set of inequalities that it learns from the negotiation trace.

The full belief state of i at step k consists of the tuple $\langle \mathbb{B}_i, \mathbb{K}_i \mathcal{O}_i \rangle$. Hereafter, we use p_k to indicate a probability as in i's belief state at step k.

It is important to note at this point that WAR's behavior does not allow i to ever have a probability that an equality holds other than zero or one. That is because the only information available to it is the order at which offers are given which can in some cases be used to infer inequality and acceptance decisions which can be used to infer inequality between an outcome and disagreement. Notice also that elements of $\mathcal{O}_i$ are always strict inequalities because they are intended to model the order in which offers are given by j rather than the underlying preferences that may have some equalities. Moreover, it is not possible to infer equality of any two outcomes from the behavior of WAR because $\omega_1 \approx_j \omega_2$ can only be inferred from knowing that $\omega_1 \succcurlyeq_j \omega_2$ and $\omega_1 \preceq_j \omega_2$, but TAU does not allow repetition which make it impossible to learn both facts.

Initial Belief The belief of i is initialized by assigning a probability value for every permutation of the extended outcome space as described above. We are only concerned with the case with no information ($\mathrm{I}^\lambda i = \mathcal{F}^\lambda i$). In this case, all permutations are equally likely and they all get a probability of $\frac{1}{(n_o+1)!}$. Moreover:

$$p_0(R_j(\omega_1) > R_j(\omega_2)) = p_0(R_j(\omega_2) > R_j(\omega_1)) = \frac{1}{2} \qquad \forall \omega_1, \omega_2 \in \Omega^+$$

where $R_j(\omega)$ is the round at which j will offer ω (See the proof for Theorem 8).

Therefore, i must assign a probability $\frac{n_o!}{(n_o+1)!} = \frac{1}{n_o}$ to the proposition that agent j's offers will be rational because this only happens if all outcomes are rational which means that ϕ comes last in the permutation of Ω^+. This implies that i starts its Partner's Offer Rationality Distribution in the Ignorance state. Moreover, the $|\mathcal{O}| = 0$ as all permutations are equally likely implying by symmetry that there are no known inequalities.

$$\begin{aligned} \mathbb{K}_i^0 &= \left\{ \frac{1}{n_o} \forall \omega \in \Omega \right\} \\ \mathcal{O}_i^0 &= \{\} \\ \mathbb{B}_i^0 &= \left\{ \frac{1}{n_o!} \forall \lambda \in \Lambda \right\} \end{aligned} \tag{66}$$

Update Process When j acts, i uses its current belief as the prior and then updates the probability distribution over permutations in the following step.

Firstly, it determines its the new partner rationality belief: (1) If j offers an outcome it already rejected earlier, then we know that it is in the Irrational phase. (2) If j offers an outcome that it accepted earlier then we know that it is in the Rational phase. (3) If j accepts an offer that it offered earlier, then we know that it is in the Rational phase at least since it made that offer. (4) If j rejects an offer that it offered earlier, we know that it was in the Irrational phase up to the point it made that offer. Using these rules, i determines its current state. These are the four main if-statements in Algorithm 1

Algorithm 1 Partner's Offer Rationality Distribution Update: UpdateOR

Input: $\omega_i^k, \omega_j^k, a_{ij}^k, \mathcal{T}^k$
Output: $\mathbb{K}_i$

if $\exists j < k : \omega_i^j = \omega_j^k \wedge \neg a_{ji}^j$ **then** ▷ Offers rejected
  Set IrrationalBefore(k)
  $p_k(\omega_j^k \succcurlyeq_j \phi) \leftarrow 0 \qquad \forall 1 \leq k \leq k$
end if
if $\exists j < k : \omega_i^j = \omega_j^k \wedge a_{ji}^j$ **then** ▷ Offers accepted
  Set RationalAfter(k)
  $p_k(\omega_j^k \succcurlyeq_j \phi) \leftarrow 1 \qquad \forall n_o \geq k \geq k$
end if
if $a_{kj} \wedge j < k : \omega_j^j = \omega_i^k$ **then** ▷ Accepts offered
  Set RationalAfter(j)
  $p_k(\omega_j^k \succcurlyeq_j \phi) \leftarrow 1 \qquad \forall n_o \geq k \geq j$
end if
if $\neg a_{kj} \wedge j < k : \omega_j^j = \omega_i^k$ **then** ▷ Rejects offered
  Set IrrationalBefore(j)
  $p_k(\omega_j^k \succcurlyeq_j \phi) \leftarrow 0 \qquad \forall 1 \leq k \leq j$
end if
Renormalize $\mathbb{K}_i^k$: $(p_i(a) \leftarrow \frac{p_i(a)}{\sum_a p_i(a)})$
return $\mathbb{K}_i^k, \mathbb{B}_i^k$

Algorithm 2 Learned Inequality Set Update: UpdateInequalities

Input: $\mathbb{K}_i^k, \mathcal{O}_i^{k-1}, \omega_i^k, \omega_j^k, a_{ij}^k, \mathcal{T}^k$
Output: $\mathcal{O}_i$

$\mathcal{O}_i^k \leftarrow \mathcal{O}_i^{k-1}$
if a_{kj} **then** ▷ Accepts
  $\mathcal{O}_i^k \leftarrow \mathcal{O}_i^k \bigcup \{\langle \omega_i^k, \phi \rangle\}$
end if
if $\neg a_{kj}$ **then** ▷ Rejects
  $\mathcal{O}_i^k \leftarrow \mathcal{O}_i^k \bigcup \{\langle \phi, \omega_i^k \rangle\}$
end if
if $\exists j \in [1, k]$: Rational(j) **then**
  $\mathcal{O}_i^k \leftarrow \mathcal{O}_i^k \bigcup \{\langle \omega_i^k, \phi \rangle\}$
  $j \leftarrow \min_{1 \leq k \leq n_o}$ Rational(k) ▷ Earliest rational offer
  for $j \leq k \leq k$ **do**
    $\mathcal{O}_i^k \leftarrow \mathcal{O}_i^k \bigcup \{\langle \omega_j^k, \omega_j^l \rangle \quad \forall k < l \leq k\}$ ▷ Subsequent offers are worse
  end for
end if
return $\mathcal{O}_i^k$

Algorithm 3 Information Set Belief Update: UpdateISBelief

Input: **Input:** $\mathbb{K}_i^k, \mathcal{O}_i^k, \mathbb{B}_i^{k-1}$
Output: $\mathbb{B}_i^k$

for $\omega_1, \omega_2 \in \mathcal{O}_i^k$ **do**
  for $\lambda \in \Lambda$ **do** ▷ Remove incompatible permutations
    if $R_j(\omega_1) \mid_\lambda > R_j(\omega_2) \mid_\lambda$ **then**
      $p_i(\lambda) \leftarrow 0$
    end if
  end for
end for
Renormalize $\mathbb{B}_i^k$: $(p_i(a) \leftarrow \frac{p_i(a)}{\sum_a p_i(a)})$
return $\mathbb{B}_i^k$

Algorithm 4 Belief Update Process: UpdateBelief

Input: $\mathbb{K}_i^{k-1}, \mathbb{B}_i^{k-1}, \mathcal{O}_i^{k-1}, \omega_i^k, \omega_j^k, a_{ij}^k, \mathcal{T}^k$
Output: $\mathbb{K}_i^k, \mathbb{B}_i^k, \mathcal{O}_i^k$

$\mathbb{K}_i^k, \mathbb{B}_i^k \leftarrow$ UpdateOS($\omega_i^k, \omega_j^k, a_{ij}^k, \mathcal{T}^k$)
$\mathcal{O}_i^k \leftarrow$ UpdateInequalities($\mathbb{K}_i^k, \mathcal{O}_i^{k-1}, \omega_i^k, \omega_j^k, a_{ij}^k, \mathcal{T}^k$)
$\mathbb{B}_i^k \leftarrow$ UpdateISBelief($\mathbb{K}_i^k, \mathcal{O}_i^k, \mathbb{B}_i^k$)
return $\mathbb{K}_i^k, \mathbb{B}_i^k, \mathcal{O}_i^k$

Secondly, if the state switches to Rational, i examines the negotiation history and assigns all outcomes that were offered at the point the switch is determined to have happened to the set of rational outcomes. If the state switches to Irrational, all offers from i so far are assigned to the set of irrational outcomes. This is the first operation in every if-statement in Algorithm 1.

Thirdly, the Learned Inequality Set is updated given the information we have so far. (1) if an offer is rejected by j, it must be irrational, and (2) if it is accepted, it must be rational. These correspond to the first two if-statements in Algorithm 2. Finally, (3) if i knows a step j at which j is in its rational phase (i.e. Rational(j)), then it can add an equality for any two subsequent offers of j stating that the earlier is better than the later. This is the third if-statement in Algorithm 2.

Finally, the distribution over permutations is updated according to Algorithm 3 by assigning zero probability to all permutations that are incompatible with the current set of rational and irrational outcomes.

The overall process is decribed in Algorithm 4 The process and arguments described above uses all the information available to i from the behavior of j under the assumption that j is using WAR which leads to the following proposition

Proposition 7 (Consistency)**.** *The belief update process described by Algorithm 4 form a consistent belief with any strategy profile that has agent j playing WAR on the Λ_{DN} scenario set.*

WAR is a PBE for the Induced Game First of all, we need the following lemma about WAR's decision making in information sets.

Lemma 4 (WAR cannot be affected)**.** *WAR's offers and responses (i.e. all its decisions) are independent of the negotiation trace given its preferences. In other words, there is no way to influence the behavior of WAR when negotiating with it.*

Proof. Offers of WAR are purely determined by the relative ordering of outcomes in its preferences and the common ordering defined for the negotiation. Both are independent of any events that happen during the negotiation.

Responses of WAR depend only on the rationality of the outcome which is — again — independent of any events that happen during the negotiation. □

Let agent j be using WAR in a bilateral negotiation on a scenario from Λ_{DN} with the TAU protocol:

Lemma 5 (Uniform Belief)**.** *At any step of the negotiation, agent i's belief is uniform over all nodes in the active information set that have nonzero probabilities.*

$$p_k(\lambda) \in \left\{0, \frac{1}{n_p^k}\right\} \qquad \forall \lambda \in \Lambda$$

where $n_p^k = |\{\lambda : p_k(\lambda) \neq 0\}|$

Proof. Agent i's belief is initialized according to Eq 66. In this state, the lemma follows trivially.

$$\therefore p_0(\lambda) \in \left\{0, \frac{1}{n_p^k}\right\} \qquad \forall \lambda \in \Lambda$$

After every update of the belief state using Algorithm 3, each permutation will either have been assigned a zero probability or be divided by the sum of all probabilities that are nonzero (The re normalization step).

$$\because p_{k-1}(\lambda) \in \left\{0, \frac{1}{n_p^{k-1}}\right\} \qquad \forall \lambda \in \Lambda$$

For permutations assigned zero probability, the Lemma follows trivially again. For those not assigned a zero probability, their probability will be updated only by the re normalization step.

$$\therefore p_k(\lambda) = p_{k-1}(\lambda) \times \frac{n_p^{k-1}}{n_p^k}$$

$$\therefore p_k(\lambda) \in \left\{0, \frac{1}{n_p^k}\right\} \qquad \forall \lambda \in \Lambda$$

□

Lemma 6 (Unknown Ranking of New Outcomes). *Let $\omega \in \Omega$ be an outcome that is not yet offered by i:*

$$\omega \notin \Omega_i^k \implies p_i(\omega \succ_j \phi) = p_i(\phi \succ_j \omega). \qquad (67)$$

Let $\omega_1, \omega_2 \in \Omega$ be two members of the outcome space that are not yet offered by j:

$$\omega_1, \omega_2 \notin \Omega_i^k \implies p_i(\omega_1 \succ_j \omega_2) = p_i(\omega_1 \succ_j \omega_2). \quad (68)$$

Proof. Agent i's belief is initialized according to Eq 66. In this state, the lemma follows trivially. From Lemma 5, all permutations that have nonzero probability are equally probable.

Let $\omega \in \Omega$ be an outcome that is not yet offered by i. The only cases in which an inequality involving ϕ is added are in lines 3, 6, 9 of Algorithm 2 corresponding in each case to an offer from i. Crucially, WAR never offers ϕ so line 12 in the algorithm will not add an inequality with ϕ on either side. Moreover, all cases in which the belief (p_i) changes in Algorithm 1 (lines 3, 7, 11, 15) involve in the condition for running them an offer from i. There are no other places in which p_i changes in Algorithm 4 (except for the re normalization step which has no effect for our purposes). This proves Eq. 67.

Let $\omega_1, \omega_2 \in \Omega$ be two members of the outcome space that are not yet offered by j. By the same argument in the previous paragraph, there are no places in which p_i changes in Algorithm 4 for these outcomes. This proves Eq. 68.

□

Theorem 9 (Equilibrium). *Υ_{WAR} is a negotiation equilibrium of TAU for bilateral negotiations on Λ_{DN}.*

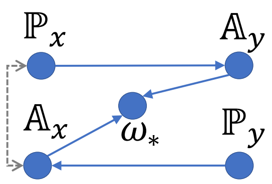


Figure 5: The final agreement is fully determined by the sequence of acceptances of all partners that are in turn fully determining by the sequence of offers from each partner.

Proof. We assume that agent j uses WAR and consider its partner i. Our goal is to show that the best response of i is to use WAR as well. Given that we established the consistent belief update rule earlier, this concludes the proof.

Firstly we define the expected utility of i [11] using strategy π (represented as $i \approx \pi$):

$$\mathrm{EU}(\pi) = \sum_{\lambda \in \Lambda^{DN}} \sum_{\omega \in \Omega_\lambda^+} p\left(\omega \mid i \approx \pi\right) \hat{u}_i(\omega), \qquad (69)$$

where $p\left(\omega \mid i \approx \pi\right) \equiv p[\omega_*^\lambda = \omega : i \approx \pi]$ is the probability that the scenario λ will end with the outcome ω if agent i uses strategy π and $\Omega_\lambda^+ = \Omega^\lambda \cup \{\phi\}$.

$$\therefore \mathcal{B}(SCS) = \operatorname*{argmax}_\pi \mathrm{EU}(\pi)$$

We know that WAR offers and accepts only rational outcomes. This means that no matter what π is:

$$p\left(\omega \mid i \approx \pi\right) = 0 \quad \forall \omega \notin \overline{\Omega}_y$$

$$\therefore \mathrm{EU}(\pi) = \sum_{\lambda \in \Lambda^{DN}} \sum_{\omega \in \overline{\Omega}_y^\lambda} p\left(\omega \mid i \approx \pi\right) \hat{u}_i(\omega), \qquad (70)$$

$$\therefore \mathcal{B}(WAR) = \operatorname*{argmax}_s \sum_{\lambda \in \Lambda^{DN}} \sum_{\omega \in \overline{\Omega}_y^\lambda} p\left(\omega \mid i \approx WAR\right) \hat{u}_i(\omega) \qquad (71)$$

$$\therefore \mathcal{B}(WAR) = \operatorname*{argmax}_s \sum_{\lambda \in \Lambda^{DN}} \mathrm{EU}^\lambda(s) \qquad (72)$$

where $\mathrm{EU}^\lambda(\pi) = \sum_{\omega \in \overline{\Omega}_y^\lambda} p\left(\omega \mid i \approx \pi\right) \hat{u}_i(\omega)$

Also we know from Theorem 1 that the negotiation for scenario λ will end at some finite time step $T_s \in \mathbb{N}^+$ for any strategy s. This means that if we can show that there is no strategy that can outperform WAR against itself for any scenario, we have shown that WAR is a best response.

We will now focus on a single negotiation scenario $\lambda \in \Lambda^{DN}$ in agent j uses WAR and agent i uses $\hat{s}$.

Case 1: Changing the acceptance policy ($\hat{s} = (\eta_{\hat{s}}, \rho_{\hat{s}} \neq \rho_{WAR})$):

Let's consider a strategy $\hat{s} = (\eta_{\hat{s}}, \rho_{\hat{s}} \neq \rho_{WAR})$. From Lemma 4, we can conclude that the only way for $\hat{s}$ to influence the final agreement ω_*^λ is by changing $\mathbb{A}_i$ (because it cannot influence Ω_j or $\mathbb{A}_j$ and Ω_i is independent of the acceptance policy given the offering policy which we will consider later).

Because $\rho_{\hat{s}} \neq \rho_{WAR}$, an outcome ω must exist that is either accepted by $\hat{s}$ and rejected by WAR or the other way around. Let ψ be the best outcome for i that it accepted so far:

$$\psi = \operatorname*{argmax}_{\omega \in \Omega_j^*(k)} \hat{u}_i(\omega)$$

The only way this decision can affect the expected utility is if either $\hat{\omega}$ or ψ ends up being the agreement:

$$\mathrm{EU}^\lambda(\hat{s}) \neq \mathrm{EU}^\lambda(WAR) \implies \omega_*^\lambda \in \{\hat{\omega}, \psi\}, \qquad (73)$$

1-1) $\hat{\omega}$ is rejected by $\hat{s}$ and accepted by WAR when offered by j at step k.

The only way this can affect expected utility is by preventing $\hat{\omega}$ from becoming an agreement (i.e. $p\left(\hat{\omega} \mid i \approx \hat{s}\right) = 0$ and $p\left(\hat{\omega} \mid i \approx WAR\right) = 1$). Most importantly, this decision does not affect $p\left(\omega \mid i \approx \hat{s}\right)$ for any outcomes except $\hat{\omega}$ (Lemma 4).

From Eq. 71 and the fact that all utilities and probabilities are positive:

[11] If preferences are given as a partial ordering instead of a utility function. We can define the utility of an outcome $\hat{u}_i(\omega)$ as its rank in that ordering with $\hat{u}_i(\omega) = 0 \forall \omega \in \Omega_i^\perp$ and $\hat{u}_i(\omega) \leq n_o \forall \omega \in \Omega_i^\top$.

$$\therefore \text{Case 1-1} \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR) \tag{74}$$

1-2) $\hat{\omega}$ is accepted by $\hat{s}$ and rejected by WAR when offered by j at step k.

Because WAR only rejects irrational outcomes,

$$\therefore \hat{u}_i(\hat{\omega}) \leq \hat{u}_i(\phi) \tag{75}$$

If the negotiation ends with ψ, there is no change in expected utility and if it ends by $\hat{\omega}$, the expected utility cannot increase because WAR against itself will never lead to an irrational outcome (Theorem 2).

$$\therefore \text{Case 1-2} \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR) \tag{76}$$

From Eq 74 and Eq 76 (and the fact that these are all the possible options for Case 1):

$$\therefore \hat{s} = (\eta_{\hat{s}}, \rho_{\hat{s}} \neq \rho_{WAR}) \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR) \tag{77}$$

$\therefore$ No matter what offering policy is used, it is not possible to find a better response to WAR that has a different acceptance policy. Now we need to show that fixing the acceptance policy as ρ_{WAR}, no offering policy can improve upon η_{WAR}.

Case 2: Changing the offering policy only: $\hat{s} = (\eta_{\hat{s}}, \rho_{\hat{s}} = \rho_{WAR})$

Let's consider a strategy $\hat{s} = (\eta_{\hat{s}} \neq \eta_{WAR}, \rho_{WAR})$. $\hat{s}$ must offer some outcome $\hat{\omega}$ at round k while WAR would have offered $\omega^* \mid_{WAR}$. The situation is more complicated than when we changed acceptance policy because the offering policy of i can affect the final agreement in two ways (Fig 5):

2-A) by affecting the acceptance policy changing $\mathbb{A}_j^{T_{\hat{s}}}$ for the partner.

2-B) by affecting the offering policy changing $\Omega_j^{T_{\hat{s}}}$ for the partner. This can only happen if the negotiation length $T_{\hat{s}}$ is greater than what it would have been if WAR was used instead of $\hat{s}$ (T_{WAR}) because Ω_j is independent of the behavior of i (Lemma 4).

Because WAR offers all rational outcomes in descending order of their value for itself, $\hat{\omega}$ cannot be better than any outcome offered by WAR up to step k

$$\therefore \hat{u}_i(\hat{\omega}) \leq \hat{u}_i(\omega) \quad \forall \omega \in \Omega_i^{:k}$$

Moreover, it cannot be better than $\omega^* \mid_{WAR}$

$$\therefore \hat{u}_i(\hat{\omega}) \leq \hat{u}_i(\omega^* \mid_{WAR}) \tag{78}$$

We need to consider three sub-cases depending on when did the negotiation end:

2-1) If the negotiation ends before k then $\hat{s}$ and WAR behave in the same way:

$$T_{\hat{s}} < k \implies \mathrm{EU}^{\lambda}(\hat{s}) = \mathrm{EU}^{\lambda}(WAR) \wedge T_{WAR} = T_{\hat{s}}$$

2-2) If the negotiation ends at step k when $\hat{s}$ is used by i (i.e. $T_{\hat{s}} = i$), then we have three possibilities:

2-2-1) The negotiation ended with the outcome i accepted. In this case, it will be the same for $\hat{s}$ and WAR because both use ρ_{WAR}:

$$T_{\hat{s}} = k \wedge \omega_* = \omega_j^k \wedge a_{ii} \implies \mathrm{EU}^{\lambda}(\hat{s}) = \mathrm{EU}^{\lambda}(WAR)$$

2-2-2) $\hat{\omega}$ is the agreement which means that WAR accepted $\hat{s}$ but this implies that it would have accepted any outcome $\omega \succcurlyeq_i \omega_{\hat{s}}$ including $\omega^* \mid_{WAR}$ (Eq. 11). From Eq. 78 and Eq. 71, we therefore conclude that:

$$T = k \wedge \omega_* = \hat{\omega} \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR)$$

2-2-3) The negotiation ends with disagreement. Because η_i is the only policy different from the case when i uses WAR, this can only change the situation if i ends the negotiation by sending ϕ or by repeating its last offer. In both cases, i receives its reserved value. Given that TAU with WAR is Exactly Rational (Theorem 2), the expected utility cannot go up:

$$T = k \wedge \omega_* = \phi \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR)$$

Combing cases 2-2-1, 2-2-2, 2-2-3, we have:

$$T = k \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR)$$

2-3) The negotiation ends after round k.

If the negotiation ends after round k, we have two possibilities based on the time at which the negotiation ends ($T_{\hat{s}}$) relative to what would have happened if η_{WAR} was used (T_{WAR}) instead of $\eta_{\hat{s}}$:

2-3-1) Shorter or same length negotiation: $k < T_{\hat{s}} \leq T_{WAR}$.

In this case, offering $\hat{\omega}$ can only change the final agreement by changing the set of outcomes accepted by partner j:

$$\therefore T_{\hat{s}} \leq T_{WAR} \wedge \mathrm{EU}^{\lambda}(\hat{s}) \neq \mathrm{EU}^{\lambda}(WAR) \implies \mathbb{A}_j^{T_{\hat{s}}} \neq \mathbb{A}_j^{T_{WAR}}$$

because $\eta_{\hat{s}}$ has no effect on the order of outcomes in Ω_j.

Moreover, the only way $\mathbb{A}_j^{T_{\hat{s}}} \neq \mathbb{A}_j^{T_{WAR}}$ can affect $\mathrm{EU}^{\lambda}(\hat{s})$ is by changing the final agreement which implies that WAR accepts some outcome offered by i at step $j > i$ that it would not have accepted if i did not offer $\hat{\omega}$ at step i but we know that $\hat{\omega}$ is rejected by j because the negotiation did not end at step i.

$$\begin{aligned}
\therefore \mathbb{A}_j^{T_{\hat{s}}} \neq \mathbb{A}_j^{T_{\hat{s}}} &\implies \exists i < j \leq T_{\hat{s}} : \hat{a}_j^{T_{\hat{s}}}(\omega_i^j) \\
&\wedge \omega_* = \omega_i^j \\
&\implies \omega^* \mid_{WAR} \succ_i \omega_i^j \\
&\implies \hat{u}_i(\omega^* \mid_{WAR}) > \hat{u}_i(\omega_i^j) \\
&\implies \hat{a}_j^{T_{\hat{s}}}(\omega_i^j) \\
&\implies \hat{a}_j^{T_{\hat{s}}}(\omega^* \mid_{WAR})
\end{aligned}$$

where we used Eq. 11 in the last step. But this implies that if agent i did offer $\omega^* \mid_{WAR}$ at any point during the negotiation, it would have been accepted changing the agreement from ω_i^j to $\omega^* \mid_{WAR}$ which we have already shown to have a higher utility.

$$\begin{aligned}
\therefore T_{\hat{s}} \leq T_{WAR} \wedge \mathrm{EU}^{\lambda}(\hat{s}) &\neq \mathrm{EU}^{\lambda}(WAR) \\
\implies \mathrm{EU}^{\lambda}(\hat{s}) &< \mathrm{EU}^{\lambda}(WAR)
\end{aligned} \tag{79}$$

$$\therefore k < T_{\hat{s}} \leq T_{WAR} \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR)$$

2-3-2) Longer negotiation: $k < T_{\hat{s}} \wedge T_{\hat{s}} > T_{WAR}$.

To force negotiation extension, i (employing $\hat{s}$) cannot offer the agreement WAR would have reached (because the acceptance policies of both agents and the offering policy of j did not change):

$$\omega_* \mid_{WAR} \notin \Omega_i^{T_{WAR}} \mid_{\hat{s}}$$

$$\omega_* \mid_{WAR} \in \Omega_i^{T_{WAR}} \mid_{WAR}$$

In this case, the change in the agreement must come through a change in a sequence of acceptances (Fig. 5):

$$\begin{aligned}
\therefore T_{\hat{s}} > T_{WAR} \wedge \mathrm{EU}^{\lambda}(\hat{s}) &\neq \mathrm{EU}^{\lambda}(WAR) \\
\implies \exists z \in \mathcal{N} &: \mathbb{A}_z^{T_{\hat{s}}} \neq \mathbb{A}_z^{T_{WAR}}
\end{aligned} \tag{80}$$

If $z \neq i$, we have the same case as in 2-3-1) and we have shown that this cannot lead to an improved expected utility which implies that the change must come from agent i:

$$\therefore T_{\hat{s}} > T_{WAR} \wedge \mathrm{EU}^{\lambda}(\hat{s}) \neq \mathrm{EU}^{\lambda}(WAR) \implies \mathbb{A}_i^{T_{\hat{s}}} \neq \mathbb{A}_i^{T_{WAR}}$$
$$\implies \Omega_j^{T_{\hat{s}}} \neq \Omega_j^{T_{WAR}}$$

From Lemma 4, we know that the first T_{WAR} outcomes in $\Omega_j^{T_{\hat{s}}}$ cannot be changed.

$$\begin{aligned} &\therefore T_{\hat{s}} > T_{WAR} \wedge \mathrm{EU}^{\lambda}(\hat{s}) \neq \mathrm{EU}^{\lambda}(WAR) \\ &\implies \exists j \in \mathbb{N}^{+} : T_{WAR} < j \leq T_{\hat{s}} \wedge \omega_* = \omega_j^j \end{aligned} \tag{81}$$

Intuitively, this means that the only way for i to improve its utility using $\hat{s}$ over what it could have achieved using WAR is to extend the negotiation forcing its partners to propose more outcomes and guarantee that the negotiation ends with agreement on one of these newly proposed outcomes.

In summary, i needs to have ω_i^i be a *distracting outcome*. This outcome must be dominated by the final agreement for WAR (i.e. $\omega_i^i \prec_i \omega_* \mid_{WAR}$) before T_{WAR} (i.e. $i < T_{WAR}$). From the first requirement we know that $\omega_i^i \notin \Omega_i^i$ and from the second we know that $\omega_* \mid_{WAR} \notin \Omega_i^i$:

$$\therefore \omega_i^i, \omega_* \mid_{WAR} \notin \Omega_i^i$$

From Lemma 6, we conclude that

$$\therefore p_i(\omega_i^i \succ_j \omega_* \mid_{WAR}) = p_i(\omega_* \mid_{WAR} \succ_j \omega_i^i)$$

Therefore, it is impossible to find $\omega_i^i \prec_j \omega_*$ and in this case also there is no way to improve the expected utility of the agent

$$\therefore k < T_{\hat{s}} \wedge T_{\hat{s}} > T_{WAR} \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR)$$

Which concludes the proof for Cass 2-3

$$\therefore T > i \implies \mathrm{EU}^{\lambda}(\hat{s}) \leq \mathrm{EU}^{\lambda}(WAR)$$

From cases 2-1, 2-2, 2-3, varying the offering policy also does not lead to a better response to WAR.

$$\therefore WAR \in \mathcal{B}(WAR)$$

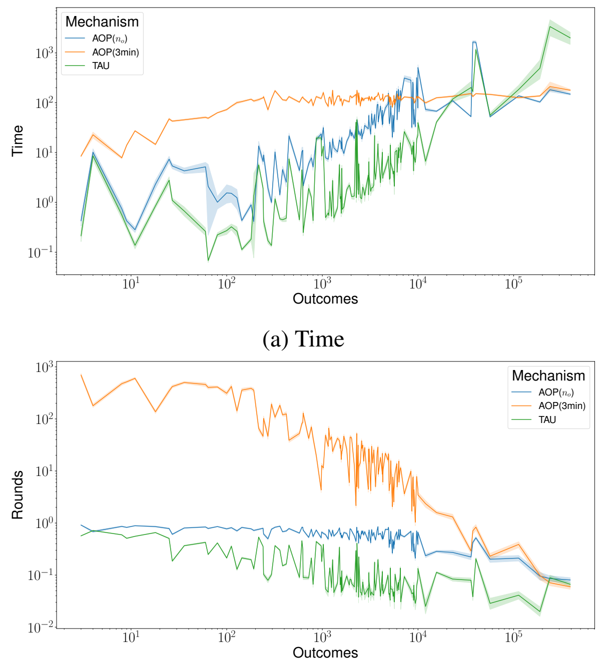
□

Extended Discussion

Scalability Analysis

TAU have an intrinsic time-limit which scales linearly with the size of the outcome-space $O(n_o)$. This means that it scales *exponentially* with the number of issues. Moreover the simplest implementation of the proposed adapter needs $O(n_o \log n_o$ operations for sorting the outcome space. On the other hand, the proposed adapter have the advantage that it is never tempted to repeat offers unnecessarily (thanks to TAU disallowing it). Most AOP strategies — on the other hand — do waste a large fraction of negotiation time just repeating offers.

Fig. 6 shows the effect of outcome-space size on the actual wall time (Fig. 6a) and number of rounds used relative to the outcome-space size (Fig. 6b). In terms, of real time, TAU outperformed both AOP methods for outcome spaces up to a tens of thousands of outcomes. This may be due to never repeating offers. For outcome spaces larger than this, and up to fifty thousand outcomes, both approaches start to require almost the same time. For outcome spaces larger than a hundred thousand, AOP outperforms TAU. This is most likely due to outcome space sorting implemented by the adapter. Fig. 6b shows the same comparison in terms of the number of rounds needed to end the negotiation. In this case, the proposed approach seems to almost always outperform AOP.



(a) Time

(b) Number of Rounds

Figure 6: Scalability with outcome space size

Notes about timing

The description of TAU is agnostic on the timing of offers as long as all responses to every offer are processed before another offer from the same agent is allowed.

If TAU is implemented serially using round-robin (as usually done for AOP), coming later in the ordering imposed entails a slight advantage as theagent may need to concede slightly less. Consider a bilateral zero-sum negotiation with two outcomes that are both rational to both negotiators. The agent that starts the second round will have to concede to its less favorite outcome which becomes the agreement. To avoid this asymmetry, we can either run all offering policies in parallel and serialize the evaluation of agreements after reception of responses randomly or just randomly assign an order of offering at every round.

It is interesting to note that AOP suffers from a bias toward agents that come earlier in the negotiation if they can determine the last round and have some information about the likelihood that each outcome is rational for their partner. In such cases, the first agent to offer in the last round becomes a de-facto dictator. The usual way to handle this issue is to run negotiation limited by real time (e.g. 3min) instead of the number of rounds but this solution reduces the risk instead of eliminating it. For TAU, the randomization proposed in this section completely eliminates any ordering effect.

Related Work

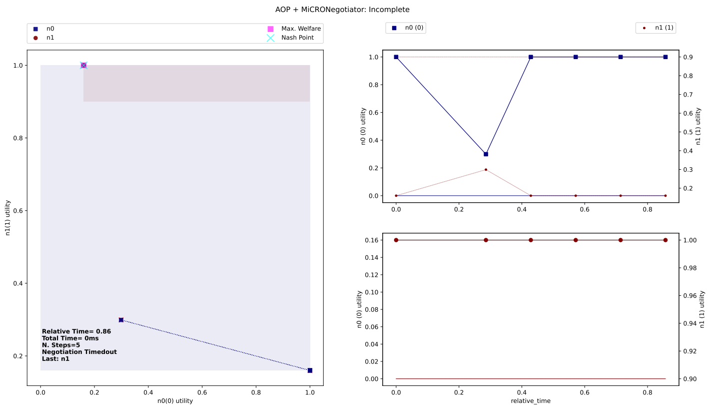
Optimality and Fairness in AOP

It is not very hard to come up with a strategy assignment rule that renders AOP exactly rational, complete and optimal. Consider the following pair of strategies: **Strong**, and **Weak**. Strong offers its best outcome continuously until the partner starts repeating offers at which point it starts conceding (i.e. offers the best outcome it did not offer yet breaking ties in any stable order) until no more outcomes as good as disagreement are available then it ends the negotiation. Weak starts with its best offer and concedes until no offers as good as disagreement are available then repeats its last offer forever. Strong and Weak accept any offer as good as their next offer.

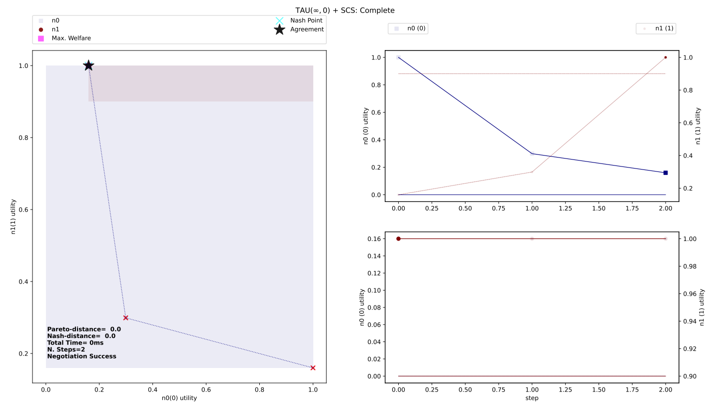
It is straightforward to show that for any bilateral negotiation, assigning one agent the **Strong** strategy and the other the **Weak** strategy, conditions for rationality optimality and completeness are met as long as the time limit imposed by AOP allows Strong to run to completion. The main problem here is that the agreement reached will always be biased toward the agent running the Strong strategy.

Moreover, these strategies do not form a negotiation equilibrium. Specifically, Weak is not the best response to Strong. For example, conceding partially (e.g. top 10 outcomes) then repeating, will force Strong to start conceding leading to a better outcome for Weak.

MiCRO Strategy



(a) MiCRO reaches no agreement



(b) SCS reaches agreement

Figure 7: A simple example of running MiCRO under AOP and SCS under TAU for the NiceOrDie ANAC scenario. MiCRO fails to reach agreement while SCS succeeded helped by the protocol

MiCRO de Jonge (2022) is the most related AOP strategy to SCS. When MiCRO plays against itself, its offering strategy (which is the main component of MiCRO) is *exactly* the same as SCS with only one crucial difference. MiCRO can run out of outcomes to offer if the scenario is not balanced. The paper presents an example. Fig. 7 provides another example taken from the ANAC domain NiceOrDie.

Given the design of AOP, MiCRO does not have the option of repeating offers at the end here because this can be exploited. A partner can just repeat its offers early and MiCRO will not know the difference. This is why the design of the protocol matters. In AOP, repeating an offer does not entail a penalty. In TAU, repeating an offer will force the agent to never change it again. That is why TAU native strategies (like SCS and WAR) can assume that an agent repeating its offers has already run out of concessions to make and can concede themselves.

The offering policy of WAR, on the other hand, is different from MiCRO because of the added irrational phase. Again, MiCRO cannot employ such tactic as offering an irrational outcome under AOP can lead to immediate acceptance which is irrational for the agent to ever do (except if there is learning between negotiations which we do not consider in this work). TAU, by redefining agreement as requiring explicit offering and acceptance from *all* agents avoids this issues allowing agents to better explore the outcome space.

It can easily to be shown that MiCRO is not a best response to itself when the agent has some specific information about its partner. The general idea is to use some of the rounds early in the negotiation to offer outcomes that are known (or have high probability) of being worse than the next best outcome for MiCRO independent of their utility for MiCRO's opponent (distracting offers). These will be considered concessions and will force MiCRO to concede more than against itself.

In many practical situations, it will be easy to find outcomes that are — with high probability — not the next best for MiCRO. Consider a negotiation between a buyer using MiCRO and a seller knowing this fact (with the seller starting the negotiation). No matter how many other issues are involved in the negotiation, it is always the case that the buyer prefers lower prices. This means that the seller knows that any outcome that has any price less than the highest price will not be in $\Omega^{\top}_{buyer}$ and can confidently use the second highest price with any other value for the rest of the issues as a distracting first offer. It can then use MiCRO for the rest of the negotiation. This can either lead to a better or the same agreement for the seller but never a worse one.

It is instructive to see how much information we need to know about partner's preferences to find a better response for WAR. For MiCRO, we only need to know one outcome that is not on its set of best outcomes to find a better response. WAR starts the negotiation by offering all of its irrational outcomes which means that using *time-wasters* will not be effective as the opponent is already doing the same.

For the buyer-seller example discussed, the seller cannot just use any offer with a non-maximal price as its first offer (as it did with MiCRO) because this outcome will be accepted by WAR if it is better than disagreement and may become the negotiation outcome which means it should either be irrational for the buyer or a good outcome for the seller. Outcomes with low prices will likely not satisfy the first criterion and outcomes with high prices will likely not satisfy the second criterion. There is no way to know which outcomes are in $\underline{\Omega}_{WAR}$ without knowing a lot of information about its market position (and any outside options it may have).

MCP and Zeuthen

A major difference between TAU and MCP is that MCP defines concession by referring to the partner utility function which implies that agent preferences are common knowledge. No such assumption is implied by TAU. Moreover, MCP does not allow repeated offers with the same utility while TAU allows these repetitions as long as the outcome itself changes. MCP assumes that all offers are sent in parallel but TAU does not require this assumption.

Zeuthen is a strategy for MCP that resembles SCS. Nevertheless, The Zeuthen strategy uses the partner utility function which makes it *impractical*. Moreover, WAR has the extra phase of irrational outcome offering which does not exist in Zeuthen (and cannot exist given the constraints of the MCP protocol).

In summary, TAU artificially resembles MCP but it solves a different problem and in a different way. Moreover, this paper is not even claiming the novelty of either TAU or SCS as both are already published by Mohammad (2023b). The contributions of this paper are clearly stated as the theoretical analysis of TAU, the novel WAR strategy and the novel adapter.

Myerson-Satterthwaite no-go theorem

A known result in mechanism design literature is the Myerson-Satterthwaite no-go theorem Myerson and Satterthwaite (1983) which states that there is no mechanism that allows a buyer and a seller with private valuation of an indivisible good (i.e. private preferences) to always reach an efficient agreement when such agreement is possible without external subsidy. This situation can be modeled as a negotiation with two issues (price and good ownership) and automated negotiation can be considered a mechanism for trying to reach an agreement. Does this conflict with the results reported in this paper? No. The theorem is concerned only with a specific set of domains (one continuous and one binary issue, and utility functions with linearly separable price and ownership components in which the value of money and ownership are monotonically increasing). Moreover, the theorem only states that there are some choices of distributions over values for any mechanism that will not lead to agreement. In our case, we are interested in a very specific distribution (uniform distribution over the scenario set) but a much wider set of scenarios which means that the theorem does not apply.

Limitations and Extensions

Ordinal Kalai Bargaining Solution

WAR and SCS were shown to find a *fair* agreement in the sense of finding the Ordinal Kalai Bargaining Solution. Can they guarantee finding the Kalai Bargaining Solutions itself? No. The reason is that these strategies have no access to the true utility value of any outcomes. They only use the partial ordering induced by the utility function if one is defined. As such, it is not possible for them to find the exact Kalai Bargaining solution. This limitation though is alleviated partially in practice as in most cases the Ordinal Kalai Bargaining Solution is close to the Kalai Bargaining Solution.

Effect of Privacy Valuation

As described in the paper, information revelation is not usually considered when evaluating automated negotiation strategies and protocols. Nevertheless, we believe that it is an important factor that needs to be taken into account in any such evaluation. In this work, the proposed approach is *guaranteed* to have a level of information revelation that is at least the same as the baselines and is expected — and found to be — to be higher. If we only consider the Advantage accrued to the agent, the proposed approach appears even better than in the results reported so far. In the paper, we used the same weight for Privacy and Advantage when calculating the agent score arguing that this is an upper limit on the expected valuation of privacy in realistic situations. Fig. 8 shows the effect of privacy weight on the improvement achieved by the proposed method in agent score for different dataset. For the largest main experiment, the agents need to value their privacy more than three times their utility/advantage/profit in order to prefer using AOP over the proposed approach. The smallest such cross-over was 1.2 for year 2015 and the largest was 8 for year 2010. In all cases agents valuing their advantage at least as much as their privacy will choose the proposed approach as stated in the paper.

Continuous Outcome Spaces

Continuous outcome-spaces provide a challenge to the proposed approach in two ways. Regarding the protocol, it is trivial to achieve the required constraint of non-repetition by simply changing offers by a small fraction that (assuming continuity of the utility function) does cause relatively small change in utility. The adaptation algorithm described in the paper needs to find the *nearest* outcome to the one received from the adapted strategy in utility. This is not even defined in continuous outcome spaces. An extension of TAU that can define some limit on similarity between offers may be able to extend the proposed method to continuous and hybrid outcome spaces but it is not clear whether this will still result in similar improvements to what we reported in this paper or not.

Repeating Negotiations

It is customary when running tournaments between negotiation strategies (e.g. in the official ANAC competition) to repeat each session multiple times (usually ten) to average over stochasticity in strategies' behavior. In this paper, we opted to run each session once due to the computational cost entailed by repetition given the huge number of sessions we ran with some of which taking hours because we did not limit the time allowed in the rounds-limited sessions. Nevertheless, this should not affect our results as we are not comparing the underlying strategies themselves. Our goal is to compare the proposed protocol and adapter with AOP. This calls for having a larger number of negotiation scenarios (940 in our experiment) and larger combinations of strategies ($2,529$ in our experiment).

Learning between Negotiations

Strategies for ANAC 2017 and 2018 were designed to learn from repeated encounters with their partners. In our evaluations, each strategy encounters each partner once in the same scenario. This disables learning for these strategies. This means that any comparison between the underlying strategies for these years is unfair and we do not conduct any such comparison.

Evaluation and Reproduction

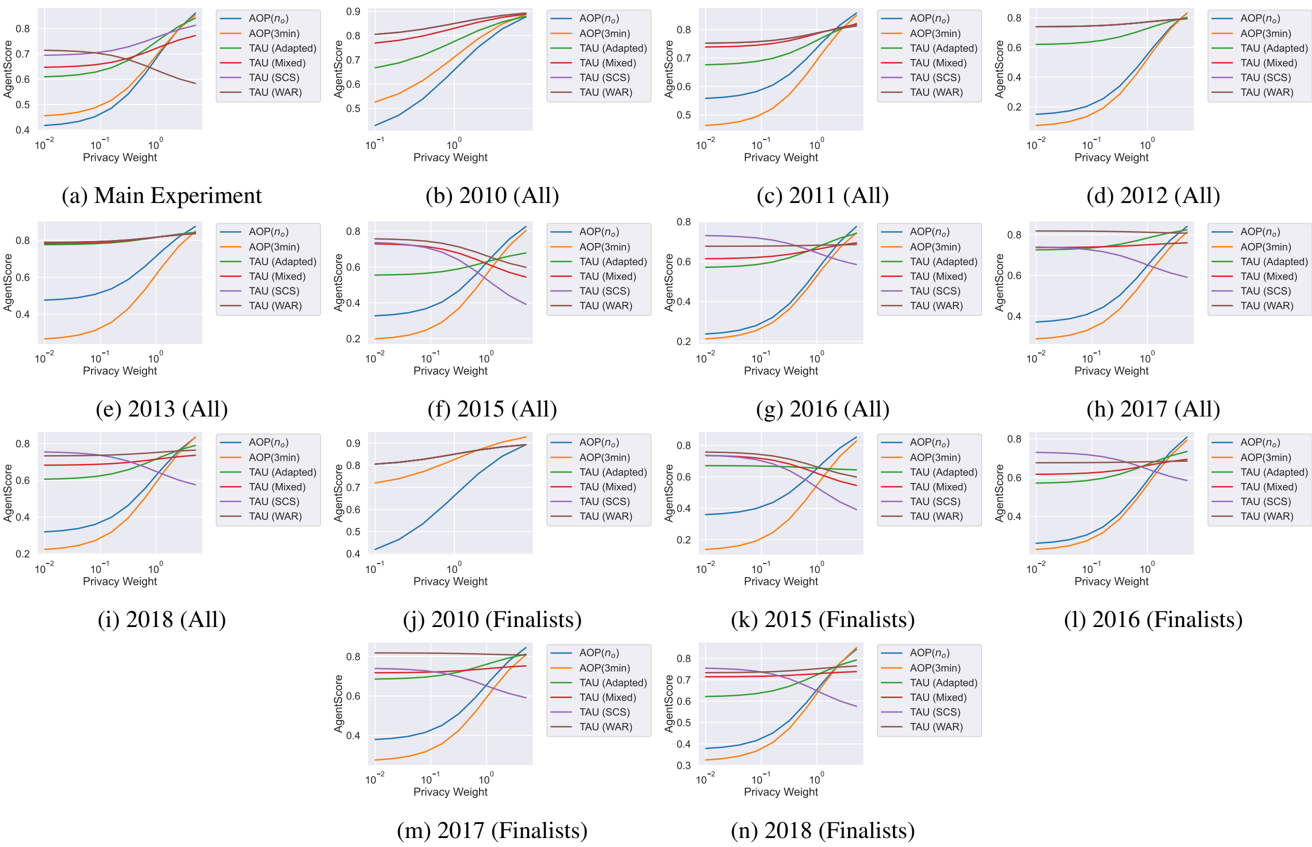
This section provides detailed information about the experiments reported in the paper as well as more experiments that further support the claims given in the paper.

Reproduction

We provide the complete source code used in this work including the code for the proposed protocol, adaptation algorithm, and for running all the experiments and generating all the figures in the paper and this appendix. All the negotiation scenarios used in the experiment are available online as part of the GENIUS platform with the exception of domains for years 2017-2022 which we collected manually with help from the ANAC organizers. We provide all of these domains in the supplementary materials. The raw results for this experiment are over $240M$ in size and cannot be shared in this supplementary materials with the submission but will be made available to the research community in case of acceptance. Moreover, a copy of the domains used after being prepared by the *prepare.py* script and with all statistics included will be made available on the same repository.

The requirements file provided with the code is pinned to the exact version of the libraries used in this work to make it possible to regenerate our results even if future versions of the libraries used changed their API or behavior. A README file is provided to explain the (hopefully simple) method for running all scripts. We provide automated high level scripts (experimentall.sh, peryearall.sh, ijcaigames.sh, ijcaipaperfigs.sh) that can run the experiments and generate results with no arguments to simplify the reproduction process.

Unfortunately, our experiments are slow (because they use what we believe is the largest dataset of automated negotiation scenarios available) and may take several weeks to complete even on



(a) Main Experiment (b) 2010 (All) (c) 2011 (All) (d) 2012 (All)
(e) 2013 (All) (f) 2015 (All) (g) 2016 (All) (h) 2017 (All)
(i) 2018 (All) (j) 2010 (Finalists) (k) 2015 (Finalists) (l) 2016 (Finalists)
(m) 2017 (Finalists) (n) 2018 (Finalists)

Figure 8: Effect of weight valuation on the improvement on agent score.

a powerful server. For this reason, we provide an argument ”–outcomelimit” to limit the size of outcomes to be used that can be passed to the python scripts to generate the results for a small set of the scenarios.

If accepted, we will host this appendix with the code, results and domains on a publicly open GitHub repositories.

We plan to submit a pull request with the proposed methods to the community-driven automated negotiation package NegMAS allowing other researchers to use the proposed protocol, strategy (WAR), and adapter directly.

The Evaluation Approach

Most research in automated negotiation targets new strategies for existing protocols. Evaluating new strategies is easy. Either use a tournament like experiment or some form of Empirical Game Theoretic analysis. In our case, we are proposing a new protocol which means that there are no existing strategies to compare with. Furthermore, the main other protocol we are comparable with has no known best strategy in any sense (even though MiCRO comes close to be the best baseline available). Our goal in this paper is to evaluate TAU itself as well as the proposed strategy and adapter.

In this paper we opted to evaluate the proposed approach by imagining agents deciding to either use AOP with some pre-existing strategy or switching to the proposed protocol and adaptation method. We assume that the agent will want to switch only if its expected *agent score* is higher upon switching. This must happen for *every* i engaging in the negotiation. If we could show that agents will prefer to switch to the proposed approach (which we claim we can), the next step is to evaluate the new approach in terms the *designer score* and ideally show that it does improve all aspects of this score independently and in aggregation (which we claim is shown by the experiment reported in the paper and further supported by the extra results reported in this appendix).

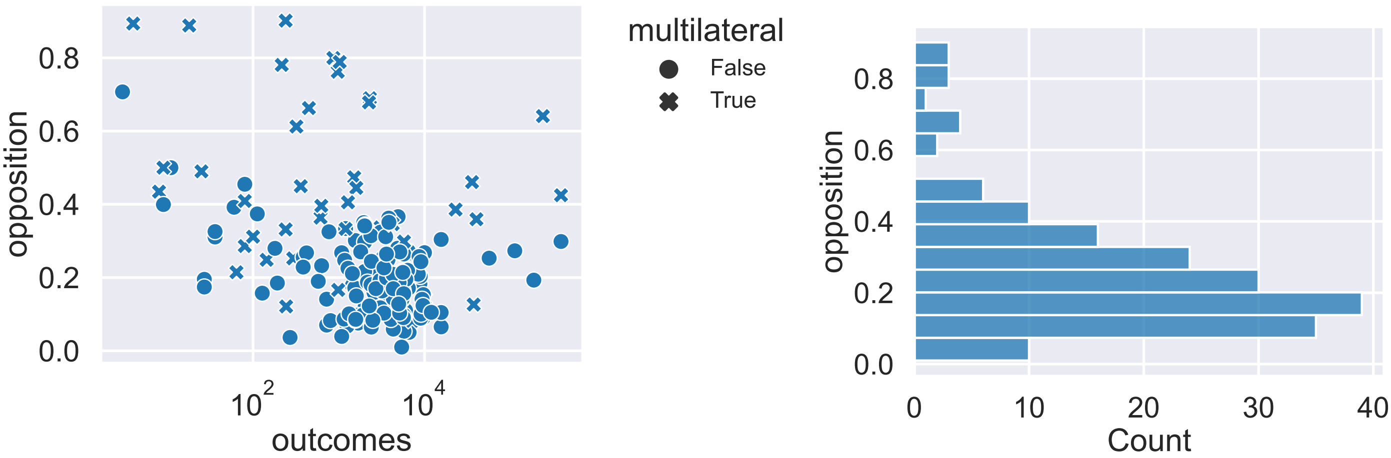
Experimental Setup

The first experiment compared the proposed approach against AOP for *almost* all available ANAC scenarios (193 of them) when using widely used baselines and top performing strategies (11 leading to 121 combinations). Nine of these scenarios appeared twice in the dataset because they were used unmodified in two ANAC years leading to 184 *unique* scenarios.

Scenario Choice We used all scenarios from the ANAC competition available for us that had less than five million outcomes, use linear aggregation utility functions (because most AOP strategies expected that), and are available publicly through the GENIUS platform or were made available to us by the organizers of the competition (for years 2018 to 2022). The only year that was skipped was 2014 (nonlinear). The only scenario ignored was AgentHp2 from year 2016 ($n_o = 5,674,801$) because it was too large compared with all the others.

A common measure of competitiveness in automated negotiation is the **opposition level** defined as the Euclidean distance between an imaginary outcome representing each agent receiving its maximum utility and the Pareto-frontier. Fig. 9 shows the distribution of opposition levels against outcome space size for the 193 ANAC scenarios used in this work. In the first experiment, we varied the reserved value of the second agent — as reported in the paper — to increase the variety of situations tested which led to four extra scenarios per ANAC domain (the distribution for the complete set is shown in Fig. 10. In the remaining experiments — and to make sure that ANAC conditions are reproduced as faithfully as possible — we did not modify the reserved values.

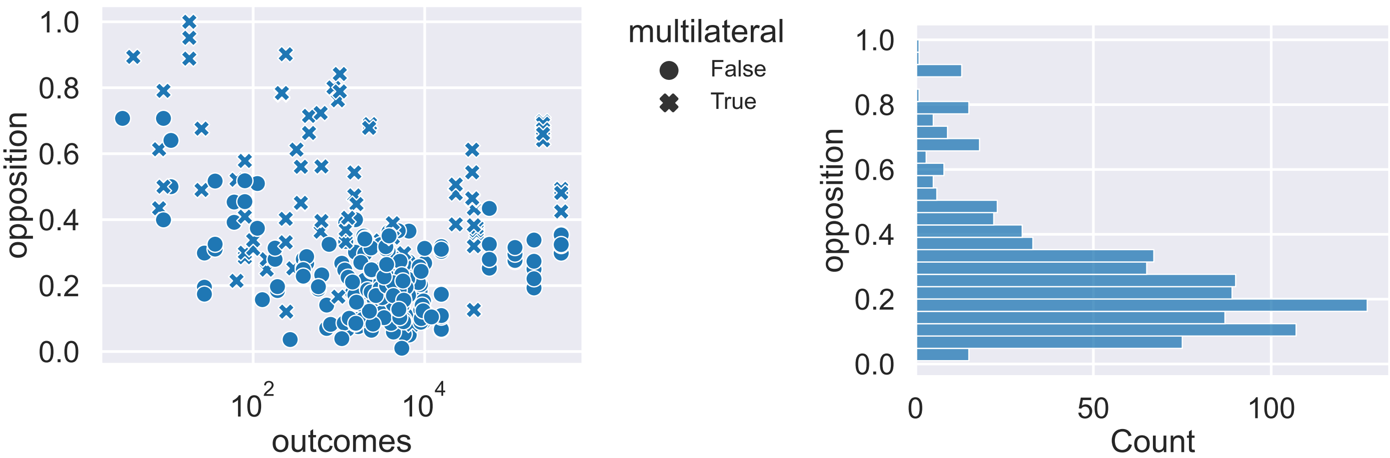
Nuances of the Experiments Few AOP strategy combinations raised exceptions on some scenarios. In such cases we repeated the negotiation up to 3 times until no exceptions are raised and reported the results. If the strategy combination still fails, we consider the negotiation as a failure and assign all agents their re-



(a) Against Number of Outcomes

(b) Histogram

Figure 9: Distribution of opposition levels against number of outcomes for the 193 scenarios used in our experiments before being augmented by varying the reservation value of the second agent.



(a) Against Number of Outcomes

(b) Histogram

Figure 10: Distribution of opposition levels against number of outcomes for the 965 scenarios used in our experiments. These are the scenarios used in the first experiment reported in the paper

served values. Table 2 reports the fraction of runs that did not complete due to exceptions which is a tiny fraction (less than 0.015%.

Utility Normalization Negotiation scenarios with unnomalized utility functions cannot be directly compared. For example a domain with a utility range from a million and a billion will swamp another with a unit utility range. For this reason, and because some AOP strategies expected normalized utility functions, we normalized all utility functions to range between zero and one. There are some subtleties in this normalization. Firstly, should the reservation value also be guaranteed to be between zero and one? Because that was not the assumption of any strategies, we scaled reserved values but allowed them to be negative. Should we first shift then scale the utility function or the other way around? There is no systematic difference for strategies here, and we chose the first option as it works for utility functions will all negative values. Notice that the specifics of the normalization method do not affect the pareto-frontier and scaling does not affect the Nash bargaining solution or the Ordinal Kalai solution but does affect the Kalai bargaining solution.

Evaluating Fairness

The concepts of maximum welfare and Pareto-optimality are almost agreed upon in automated negotiation research but there are multiple conceptions of fairness based on the Bargaining Solution assumed (Nash, Kalai, Kalai-Smorodinsky) as discussed in the paper. We defined fairness in the paper by the similarity to the nearest bargaining solution.

As indicated in the paper, there are three main solutions to the bargaining solutions that are usually used as measures of *fairness*:

Nash Bargaining Solution Ω_n This was the earliest of the three solutions and defined by Nash Jr (1950) as the unqiue Nash Equilibrium for the Nash Bargaining Game that satisfies Perto-optimality, symmetry, scale-invariance and independence of irrelevant alternatives (IIA):

$$\Omega_n \equiv \underset{\omega \in \mathcal{P}}{\operatorname{argmax}} \prod_{i \in \mathcal{N}^\lambda} \hat{u}_i(\omega) - \hat{u}_i(\phi)$$

Kalai Bargaining Solution Ω_k Defined by Kalai (1977) as the unique Nash Equilibrium when scale-invariance is dropped and IIA and resource monotonicity axioms are kept:

$$\Omega_k^* \equiv \underset{\omega \in \mathcal{P}}{\operatorname{argmax}} \min_{i \in \mathcal{N}^\lambda} \hat{u}_i(\omega) - \hat{u}_i(\phi)$$

Kalai-Smorodinsky Bargaining Solution Ω_{ks} Defined by Kalai and Smorodinsky (1975) as the unique Nash Equilibrium when IIA axiom is replaced by the resource monotonicity axiom:

$$\frac{\hat{u}_1(\Omega_{ks}^*) - \hat{u}_1(\phi)}{\hat{u}_2(\Omega_{ks}^*) - \hat{u}_2(\phi)} = \frac{\max_\psi(\hat{u}_1(\psi)) - \hat{u}_1(\phi)}{\max_\psi(\hat{u}_2(\psi)) - \hat{u}_2(\phi)} \wedge \Omega_{ks}^* \in \mathcal{P}$$

Note that this definition is only valid for bilateral negotiations with continuous outcome spaces. For bilateral negotiations with discrete outcome spaces, we can find the nearest Pereto-efficient outcome:

$$\Omega_{ks} \equiv \min_{\omega \in \mathcal{P}} d\left(\omega, \Omega^*_{ks}\right)$$

where $d\left(a, b\right)$ is the Euclidean distance between a and b and Ω_{ks} is defined only for bilateral negotiations. We do not use this bargaining solution in our work but include it here for the sake of completeness.

We can define an ordinal version of these bargaining solution by using the relative rank (defined in the paper) instead of the utility function. This is specially useful for strategies that do not use utility values directly and rely completely on the rank information like MiCRO, WAR and SCS.

Empirical Evaluation Details

In this section, we provide more information about the empirical evaluation experiments presented in the paper. Table 2 shows a summary of the content of these datasets. All AOP strategies used are available in the GENIUS platform and are accessible from NegMAS. For each dataset, we ran every combination of strategies using AOP with round and time limits and using the proposed TAU protocol and adapter. If we have n_π strategies and n_s scenarios we run $n_s\left(2n_\pi^2 + \left(n_\pi + 2\right)^2\right)$ negotiations. The table shows also the number of cases in which the negotiation did not complete due to one of the agents throwing an exception or entering an infinite loop[12]. Only 0.01% negotiations failed. Specifically, AgentHP2 (year 2016) failed in all attempts on scenarios NewDomain and EnergyLarge (largest domain of year 2015), while XianFaAgent failed on TAU in all attempts on the largest domain of year 2015 (RobotKiller). We removed the failed scenario/strategy combinations from all conditions when calculating statistical significance[13].

There are three sets of experiments:

Main Experiment described in the first row of Table 2 in which we used the baseline and SOTA AOP mentioned in the paper (11 of them) as well as WAR, SCS and varied the reservation value of the second agent to increase the variation in the scenarios. This is by far the largest dataset we used. We did not use scenarios from year 2014 because they used nonlinear utility functions and most SOTA agents for AOP are not compatible with these scenarios.

Per-Year (All Strategies) The next eight datasets used all strategies submitted to ANAC of a specific year (available in GENIUS) on the ANAC scenarios from this year. No variation of reserved value was used in these experiments to match the situation for which the strategies were developed as much as possible. In this case, some *weak* agents were present to test cases in which exploitation is possible.

Per-Year (Finalists) The next eight datasets used only finalists from ANAC in a specific year on the ANAC scenarios from this year. No variation of reserved value was used in these experiments to match the situation for which the strategies were developed as much as possible. Because all agents were finalists in years 2011-2013, we did not repeat these experiments. In this case, all agents were *strong* in the sense that they passed the first round of filtering in ANAC.

[12] i.e. failed to propose/respond for more than $max(3600, n_o)$ seconds.

[13] Assigning them all the reserved value does not change any or the results reported in the paper.

The last set of experiments (finalists) used a subset of the scenarios and strategies used in the second set and we did not count them in the totals in Table 3.

Table 3 shows a summary for the final agent advantage, designer score, and speed for every dataset tested. The maximum value in every row is highlighted only if the difference to the other two conditions is statistically significant after Bonferroni's multiple-comparisons correction. As the table shows, the proposed method outperforms AOP with round and time limits in every evaluation metric in every dataset. The supplementary materials include the results of statistical tests using both Wilcoxon's rank test and dependent t-test. This table differs from the version provided in the paper in reporting the raw scores instead of scores relative to AOP(3min), reporting the standard deviation and reporting the speed for all datasets.

Detailed Results

This section gives more details about all the experiments reported in the paper. In this summary-box we summarize the most important points in this section:

- Only WAR and SCS appeared as pure Nash Equilibria in the main experiment. Moreover, only WAR appeared as a pure Nash equilibrium for all datasets.
- In all replicator dynamics experiments, AOP strategies were driven to extinction at the first fraction of the simulation.
- WAR and SCS achieved exact completeness and optimality in all bilateral scenarios and most multilateral scenarios and achieved no less than 0.99 fairness even when the conditions of the theorems proven above were not satisfied.
- TAU with the adapter outperforms the corresponding AOP strategy in all evaluation criteria except privacy which only shows mild reduction.
- TAU-native strategies outperform AOP adapted strategies in all evaluation criteria except privacy for bilateral negotiations. Multilateral negotiation results are less conclusive.
- All rational agents should prefer TAU over AOP (with the adapter in multilateral negotiations and with WAR in bilateral negotiations) for all datasets tested.

In the following subsections, we provide more detailed results for every experiment reported in the main paper.

Replicator Dynamics

The paper reported replicator dynamics analysis of on the main dataset which is by-far the largest datasets. Fig. 11 reports these results again. It is clear that WAR is the only strategy that monotonically increases its fraction in the pool of agents. Fig. 12 (13) reports the same results for all bilateral (multilateral) datasets. The following is clear from these replicator dynamics:

- All AOP strategies are driven to extinction pretty fast in all datasets. Only TAU strategies have any non-zero fraction of the population by the end of the simulation.
- TAU-native strategies out-competed adapted strategies for all datasets except year 2017 for which they are driven to extinction.

Table 2: The datasets used in our experiments.

Dataset	n_π	n_c	Domains	n_s	Negotiations	Type	n_f,	n_τ	n_{ao}	Years
1 (Main)	11	411	184	736	302,496	both	0	0	0	2010 - 2022*
2 (2010-All)	7	179	3	3	537	bilateral	0	0	0	2010
3 (2011-All)	8	228	8	8	1,824	bilateral	0	0	0	2011
4 (2012-All)	8	228	15	15	3,420	bilateral	0	0	0	2012
5 (2013-All)	7	179	14	14	2,506	bilateral	0	0	0	2013
6 (2015-All)	24	1,828	15	15	2,742	multilateral	50	0	50	2015
7 (2016-All)	16	836	15	15	1,254	multilateral	6	4	2	2016
8 (2017-All)	18	1,048	10	10	1,048	multilateral	0	0	0	2017
9 (2018-All)	20	1,284	4	4	5,136	multilateral	0	0	0	2018
10 (2010-Finalists)	4	68	3	3	204	bilateral	0	0	0	2010
11 (2015-Finalists)	8	228	15	15	3,420	multilateral	0	0	0	2015
12 (2016-Finalists)	10	344	15	15	5,160	multilateral	0	0	0	2016
13 (2017-Finalists)	10	344	10	10	3,440	multilateral	0	0	0	2017
14 (2018-Finalists)	13	563	4	4	2,252	multilateral	0	0	0	2018
Total	164	7,768	184	867	380,835	both	56	4	52	2010 - 2022*

* Except year 2014. n_π/n_c: N. Strategies/Conditions n_s: N. Scenarios n_f: N. Failed n_τ Adapter failures n_{ao}: AOP failures

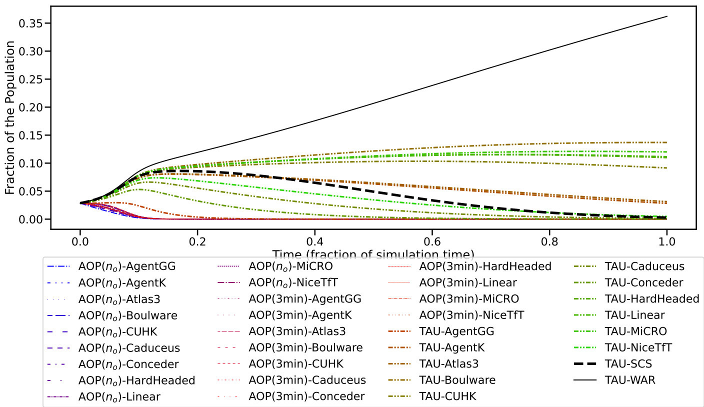


Figure 11: Frequency of different strategies: Main Dataset

- WAR out-competed SCS in all bilateral datasets except 2013 (they behaved identically on 2010 datasets).
- SCS out-competed WAR in all multilateral datasets except 2018 (finalists).

ANAC 2010-2022 (Main Dataset)

For this experiment, we use six baselines: Time-based Conceder, Linear and Boulware strategies, the behavior-based Nice Tit for Tat (NTfT) strategy Baarslag, Hindriks, and Jonker (2013), and the recently proposed MiCRO de Jonge (2022). We also use the following set of SOTA strategies for AOP: Caduceus Gunes, Arditi, and Aydogan (2017) , Atlas3 Mori and Ito (2017)(winners of multilateral competitions without uncertainty or learning in 2015 and 2016), AgentK by Kawaguchi, Fujita, and Ito (2013), Hardheaded by van Krimpen, Looije, and Hajizadeh (2013), CUHK by Hao and Leung (2014), and AgentGG (most recent winners of bilateral competitions without uncertainty or learning at ANAC in 2010, 2011, 2012, 2022). All 121 combinations of these strategies were used.

Fig. 15 analyzes the advantage of different protocol-strategy combinations. Fig. 15a shows the advantage achieved by each strategy the 13 tested. A common finding that is repeated for all datasets is that TAU (Adapted) *always* outperforms both AOP variations for each individual strategy confirming the finding from equilibria analysis and replicator dynamics that all rational agents will chose TAU over AOP for this dataset. Moreover, the highest performance was achieved when WAR is run against itself or against adapted strategies. Fig. 15b reports the advantage of the top-2 strategies for each protocol variation. Note that these results represent the average performance of each strategy against all other strategies under the same protocol. The figure also shows the average score for comparison. WAR achieves the highest advantage (0.72) compared with 0.66 for SCS and 0.55 for the best AOP strategy (Boulware) under AOP.

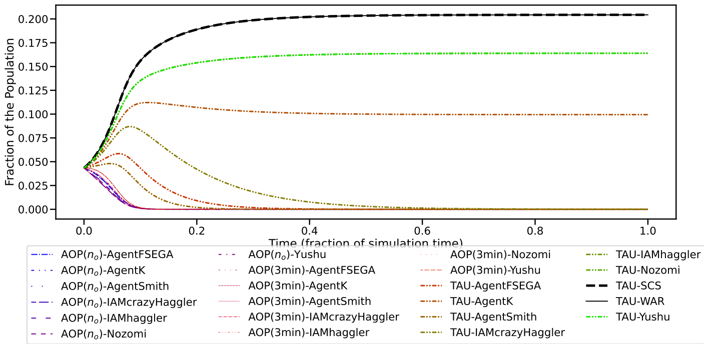
Fig. 16 shows the two other factors in agent score Mohammad (2023a): Partner Welfare and Privacy. We see that switching to TAU does increase partner welfare by 23% while decreasing privacy by 8% on average. Adding this to the 17% average increase in advantage, it is clear that agent score is higher for TAU except at unrealistically high weights for privacy (i.e. valuing privacy more that four times as utility). Fig. 16c shows that the effect of switching to TAU using the proposed adapter is small for all strategies considered. WAR has the lowest privacy level (0.71) among the top strategies which is much lower than average. This is expected as WAR does not only offer rational outcomes like most other strategies but it offers all irrational outcomes by design.

Fig. 17 shows the average performance of each condition in terms of designer score, speed and each individual factor of the designer score. As reported in the paper, all TAU conditions outperform AOP variations in *all* metrics. Moreover, it is clear that WAR and SCS achieve 100% completeness and optimality as expected from the theoretical analysis reported in the paper. Fairness – on the other hand – is 99% for WAR and 97% for SCS. The only cases in which WAR was not exactly fair are EnergyLarge and KillerRobot scenarios. Both did not belong to Λ_U^B. The highest average designer score was achieved by SCS (0.93) when playing against itself. This is mostly due to its higher welfare compared with WAR. An interesting finding is that all strategy conditions achieve at least 91% total fairness. Fig. 14 shows the top fairness score achieved for each protocol according to the Nash, Kalai and ordinal versions of them. As expected WAR achieves the highest ordinal Kalai fairness. MiCRO achieves the highest Nash fairness when used in AOP.

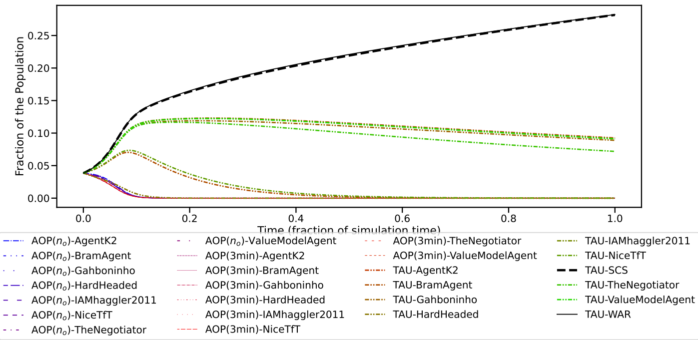
Fig. 18 shows the designer score and its factors for the top performing strategy combination for each protocol as well as the average score per protocol. Here, we find that MiCRO when playing AOP against itself, Linear or Conceder can achieve a designer score equal to WAR (0.9) but SCS still slightly outperform everyone at a designer score of 0.93 against itself. In terms of speed,

Table 3: Summary of all results. The best value per dataset and performance measure is **bold**

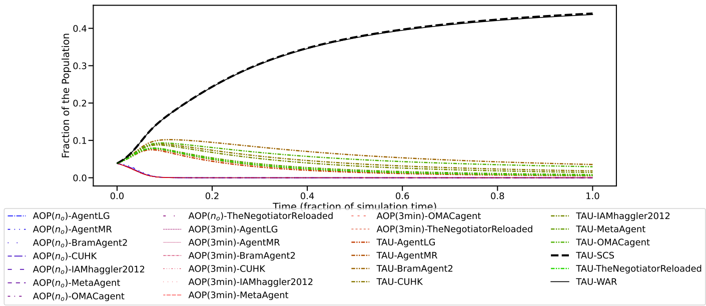
Dataset	Session Statistic Measure	AOP(n_o) mean	AOP(n_o) std	AOP(3min) mean	AOP(3min) std	TAU (Adapted) mean	TAU (Adapted) std	TAU (Mixed) mean	TAU (Mixed) std	TAU (SCS) mean	TAU (SCS) std	TAU (WAR) mean	TAU (WAR) std
final	Advantage	0.41	0.42	0.45	0.42	0.61	0.31	0.65	0.29	0.69	0.23	**0.72**	0.24
	Designer	0.50	0.43	0.54	0.42	0.80	0.33	0.85	0.27	**0.93**	0.10	0.90	0.11
	Speed	5.31	20.45	0.83	9.86	9.19	25.43	14.42	36.64	33.19	61.94	23.03	**62.20**
2010	Advantage	0.38	0.40	0.49	0.38	0.64	0.31	0.76	0.18	**0.80**	0.06	**0.80**	0.06
	Designer	0.41	0.43	0.53	0.41	0.81	0.39	0.94	0.21	**0.99**	0.01	**0.99**	0.01
	Speed	0.29	0.38	0.01	0.04	3.33	4.70	7.26	9.17	51.28	46.35	51.39	**52.13**
2010finalists	Advantage	0.37	0.39	0.70	0.24	**0.80**	0.05	**0.80**	0.05	**0.80**	0.06	**0.80**	0.06
	Designer	0.42	0.46	0.79	0.26	**0.99**	0.01	**0.99**	0.01	**0.99**	0.01	**0.99**	0.01
	Speed	0.26	0.42	0.00	0.00	6.88	5.83	10.66	9.22	51.28	46.35	51.39	**52.13**
2011	Advantage	0.55	0.39	0.46	0.41	0.67	0.29	0.74	0.21	0.75	0.19	**0.75**	0.19
	Designer	0.61	0.42	0.49	0.44	0.80	0.35	0.88	0.25	**0.90**	0.22	**0.90**	0.22
	Speed	0.32	1.13	0.07	0.27	8.94	23.33	17.51	27.08	127.88	185.29	134.73	**219.70**
2012	Advantage	0.14	0.31	0.07	0.22	0.62	0.31	0.74	0.12	**0.74**	0.12	**0.74**	0.12
	Designer	0.15	0.32	0.07	0.23	0.80	0.37	0.97	0.03	**0.97**	0.02	**0.97**	0.02
	Speed	0.08	0.25	0.00	0.01	2.36	6.28	5.52	16.56	91.71	86.16	79.95	**91.77**
2013	Advantage	0.47	0.43	0.26	0.38	0.78	0.17	0.78	0.16	**0.79**	0.15	**0.79**	0.15
	Designer	0.49	0.43	0.27	0.39	0.94	0.16	0.96	0.10	**0.98**	0.03	**0.98**	0.03
	Speed	0.05	0.04	0.02	0.03	9.57	12.57	18.01	23.99	102.12	**127.49**	112.53	123.87
2015	Advantage	0.32	0.40	0.19	0.34	0.55	0.34	0.73	0.20	0.74	0.22	**0.76**	0.11
	Designer	0.31	0.37	0.18	0.31	0.65	0.40	0.83	0.19	0.81	0.12	**0.90**	0.10
	Speed	0.06	0.26	0.02	0.15	0.34	1.22	1.53	4.65	14.12	26.87	14.41	**32.58**
2015finalists	Advantage	0.35	0.41	0.13	0.30	0.67	0.26	0.74	0.19	0.74	0.22	**0.76**	0.11
	Designer	0.32	0.36	0.11	0.25	0.79	0.30	0.84	0.18	0.81	0.12	**0.90**	0.10
	Speed	0.12	0.66	0.01	0.02	0.64	1.90	2.08	6.30	14.12	26.87	14.41	**32.58**
2016	Advantage	0.23	0.35	0.21	0.34	0.57	0.31	0.61	0.30	**0.73**	0.22	0.68	0.16
	Designer	0.25	0.37	0.22	0.35	0.70	0.36	0.77	0.32	0.87	0.09	**0.89**	0.14
	Speed	0.59	1.49	0.24	1.06	2.31	3.47	2.93	4.75	33.98	48.05	68.89	**104.10**
2016finalists	Advantage	0.25	0.36	0.22	0.34	0.57	0.31	0.62	0.30	**0.73**	0.22	0.68	0.16
	Designer	0.29	0.39	0.24	0.36	0.73	0.37	0.77	0.32	0.87	0.09	**0.89**	0.14
	Speed	0.69	1.16	0.21	0.66	3.08	4.24	2.59	3.60	33.98	48.05	68.89	**104.10**
2017	Advantage	0.37	0.40	0.28	0.38	0.72	0.26	0.74	0.24	0.74	0.18	**0.82**	0.09
	Designer	0.36	0.39	0.27	0.37	0.79	0.30	0.80	0.26	0.78	0.16	**0.95**	0.06
	Speed	0.33	0.85	0.05	0.37	1.66	2.96	2.68	4.65	23.25	30.25	25.12	**32.10**
2017finalists	Advantage	0.37	0.40	0.27	0.38	0.68	0.31	0.72	0.27	0.74	0.18	**0.82**	0.09
	Designer	0.37	0.40	0.26	0.37	0.77	0.35	0.79	0.29	0.78	0.16	**0.95**	0.06
	Speed	0.34	0.78	0.01	0.02	1.36	2.64	2.48	4.42	23.25	30.25	25.12	**32.10**
2018	Advantage	0.31	0.39	0.22	0.35	0.60	0.31	0.68	0.25	**0.76**	0.22	0.73	0.16
	Designer	0.34	0.41	0.23	0.36	0.70	0.35	0.81	0.24	0.83	0.12	**0.92**	0.05
	Speed	0.27	1.38	0.15	1.32	0.68	1.52	1.57	3.89	34.26	**61.41**	23.86	45.98
2018finalists	Advantage	0.37	0.39	0.32	0.37	0.62	0.30	0.71	0.21	**0.76**	0.22	0.73	0.16
	Designer	0.40	0.41	0.33	0.39	0.72	0.32	0.85	0.17	0.83	0.12	**0.92**	0.05
	Speed	0.18	0.41	0.03	0.13	0.44	1.01	1.50	4.00	34.26	**61.41**	23.86	45.98



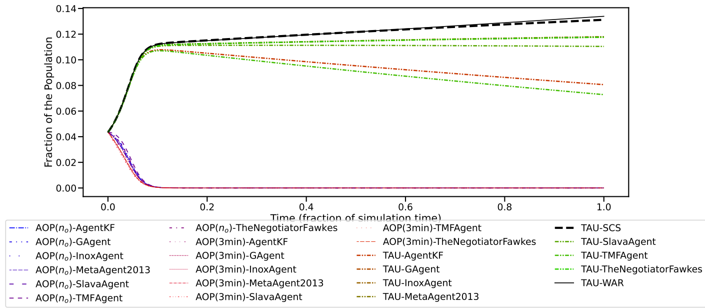
(a) Year 2010



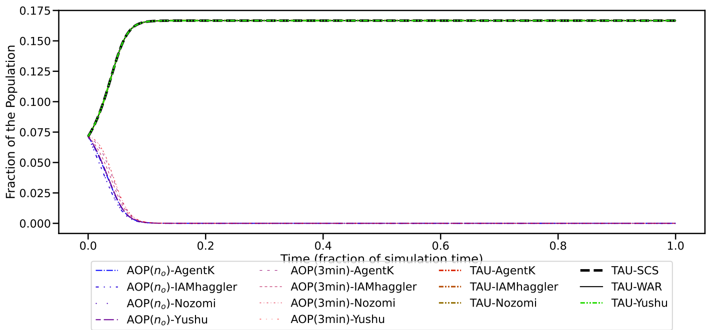
(b) Year 2011



(c) Year 2012



(d) Year 2013



(e) Year 2010 finalists

Figure 12: Evolution of strategy fraction over simulation times: Bilateral negotiations.

only AOP(Conceder) can slightly outperform WAR and SCS. These differences were not statistically significant. SCS achieves the highest agreement rate (completeness), welfare, and optimality with WAR and MiCRO coming close. WAR achieves the highest fairness level.

The remaining datasets show similar patterns to the ones reported in this section. In the following section, we report the results for each individual dataset presenting only differences from the pattern reported in this section.

ANAC 2010 - All (Finalists)

This dataset contained the strategies submitted to ANAC 2010 available in the GENIUS platform: AgentFSEGA, AgentK, AgentSmith, Nozomi, IAMcrazyHaggler, IAMhaggler, Yushu. The finalists for this year were AgentK, Yushu, Nozomi and IAMhaggler.

The scenarios used were: Travel (188,160 outcomes), EnglandZImbabwe (576 outcomes) and ItexvsCypress (180 outcomes).

Fig. 19 shows the results for year 2010 dataset. Again, TAU outperforms AOP in all evaluation dimensions when using the proposed adapter, SCS or WAR. The performance ofSCS and WAR is identical which is expected given the fact that all scenarios were all-rational in this year.

Fig. 20 shows the results when using the finalist agents only with almost the same pattern of performance. The main difference is that AgentK when run using the proposed adapter can now achieve the same advantage and designer score as TAU-native strategies.

ANAC 2011 - All

This dataset contained the strategies submitted to ANAC 2011 available in the GENIUS platform: HardHeaded, Gahboninho, IAMhaggler2011, BramAgent, AgentK2, TheNegotiator, NiceTitForTat, ValueModelAgent. All these agents were finalists in this year.

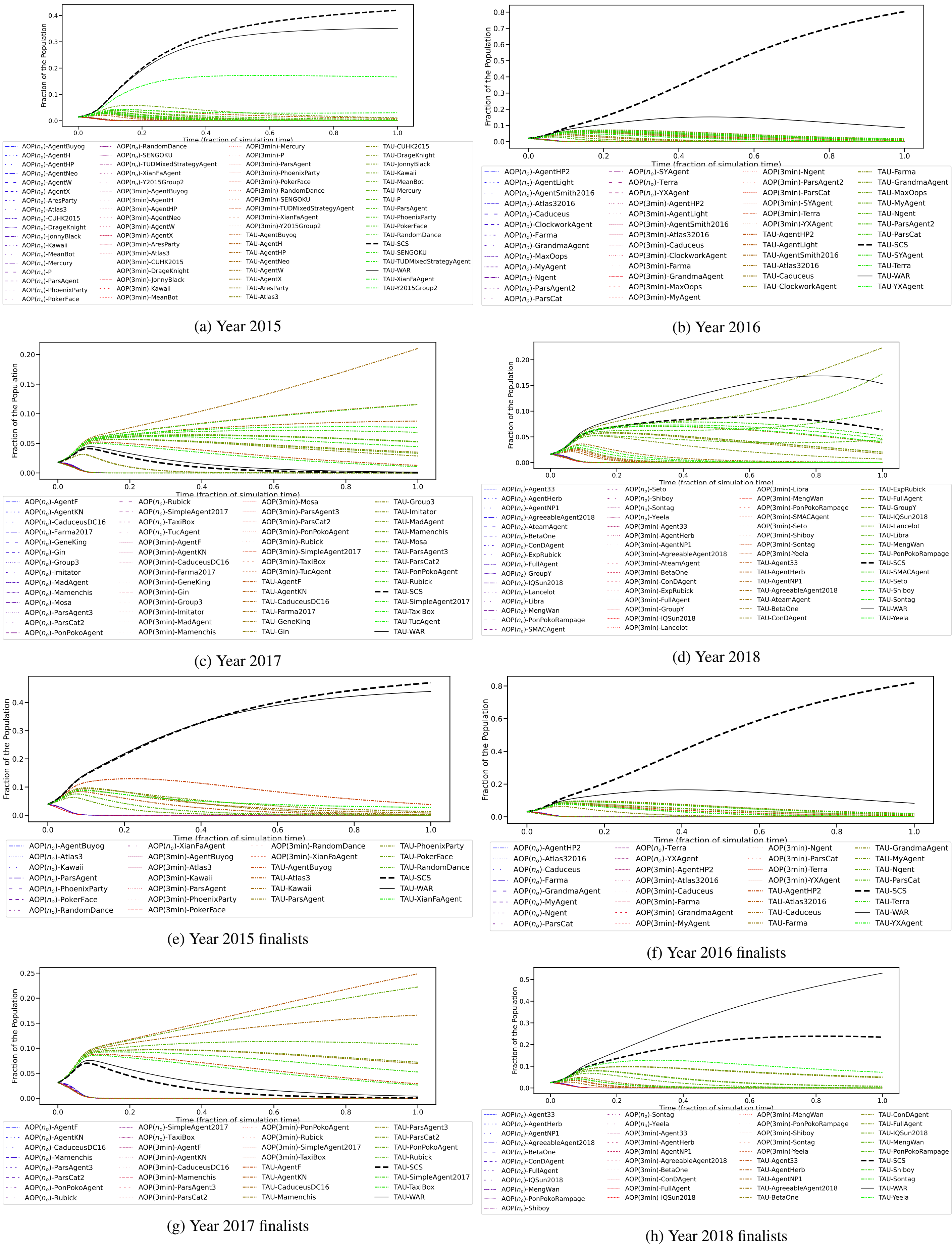
The scenarios used were: Energy (390625 outcomes), Car (15625 outcomes), Camera (3600 outcomes), Amsterdam (3024 outcomes), Grocery (1600 outcomes), Acquisition (384 outcomes), Laptop (27 outcomes), NiceOrDie (3 outcomes),

Fig. 21 shows the results for year 2011 dataset. The same pattern of scores found for year 2010 dataset can be seen.

ANAC 2012 - All

This dataset contained the strategies submitted to ANAC 2012 available in the GENIUS platform: CUHKAgent, OMACagent, TheNegotiatorReloaded, BramAgent2, MetaAgent, AgentLG, IAMhaggler2012, AgentMR. All these agents were finalists in this year.

The scenarios used were: Supermarket (112896 outcomes), EnergySmall (15625 outcomes), MusicCollection (4320 outcomes), Fitness (3520 outcomes), Phone (1600 outcomes), Barbecue (1440



(a) Year 2015

(b) Year 2016

(c) Year 2017

(d) Year 2018

(e) Year 2015 finalists

(f) Year 2016 finalists

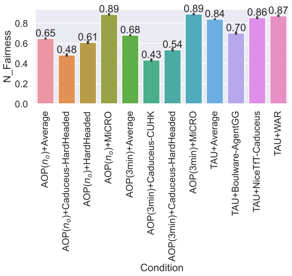
(g) Year 2017 finalists

(h) Year 2018 finalists

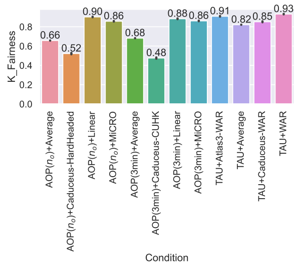
Figure 13: Evolution of strategy fraction over simulation times: Multilateral

outcomes), EnglandvsZimbabwe (576 outcomes), AirportSiteSelection (420 outcomes), HouseKeeping (384 outcomes), Acquisition (384 outcomes), Outfit (128 outcomes), Barter (80 outcomes), RentalHouse (60 outcomes), FlightBooking (36 outcomes), Fifty-Fifty (11 outcomes).

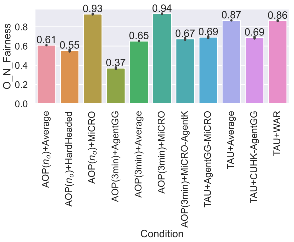
Fig. 22 shows the results for year 2011 dataset. The same pattern of scores found for year 2011 dataset can be seen.



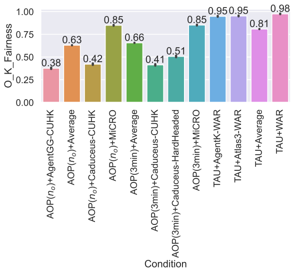
(a) Nash



(b) Kalai

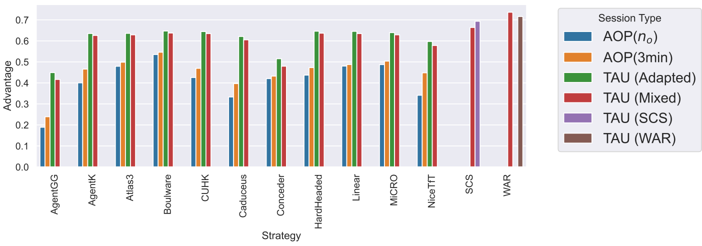


(c) Ordinal Nash

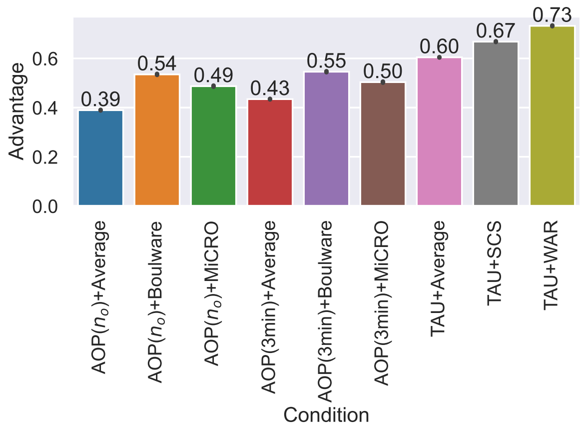


(d) Ordinal Kalai

Figure 14: Detailed Fairness Evaluation for the main dataset



(a) Advantage Summary



(b) Top strategies per protocol

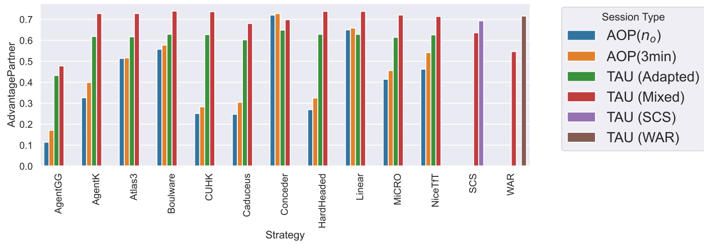
Figure 15: Analysis of Advantage for the main dataset.

ANAC 2013 - All

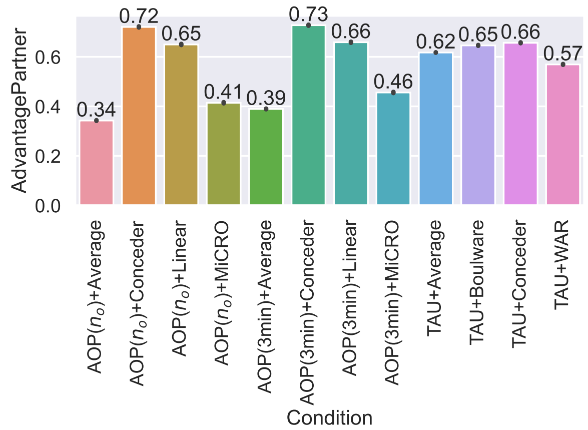
This dataset contained the strategies submitted to ANAC 2013 available in the GENIUS platform: AgentKF, TheFawkes, TMFAgent, MetaAgent2013, GAgent, InoxAgent, SlavaAgent. All these agents were finalists in this year.

The scenarios used were: Wholesaler (56700 outcomes), Kitchen (15625 outcomes), SmartPhone (12000 outcomes), Lunch (3840 outcomes), cameradomain (3600 outcomes), Animal (1152 outcomes), Icecream (720 outcomes), AcquisitionA (384 outcomes), DogChoosing (270 outcomes), Coffee (112 outcomes), DefensiveCharms (36 outcomes), planes (27 outcomes), Fifty2013 (11 outcomes), Ultimatum (9 outcomes).

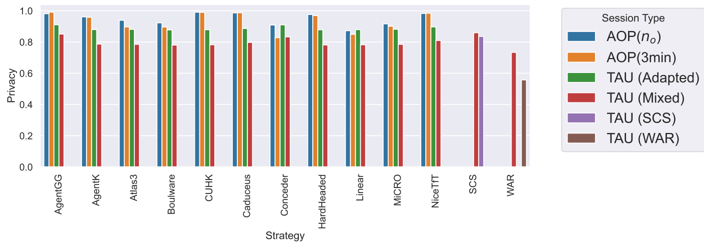
Fig. 23 shows the results for year 2011 dataset. The same pattern of scores found for year 2012 dataset can be seen with one exception. This is the first dataset so far for which a commination



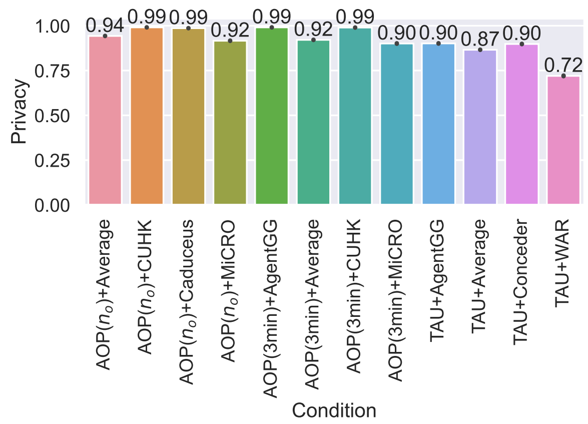
(a) Partner Welfare (Summary)



(b) Partner Welfare (Top Strategies)



(c) Privacy (Summary)



(d) Privacy (Top Strategies)

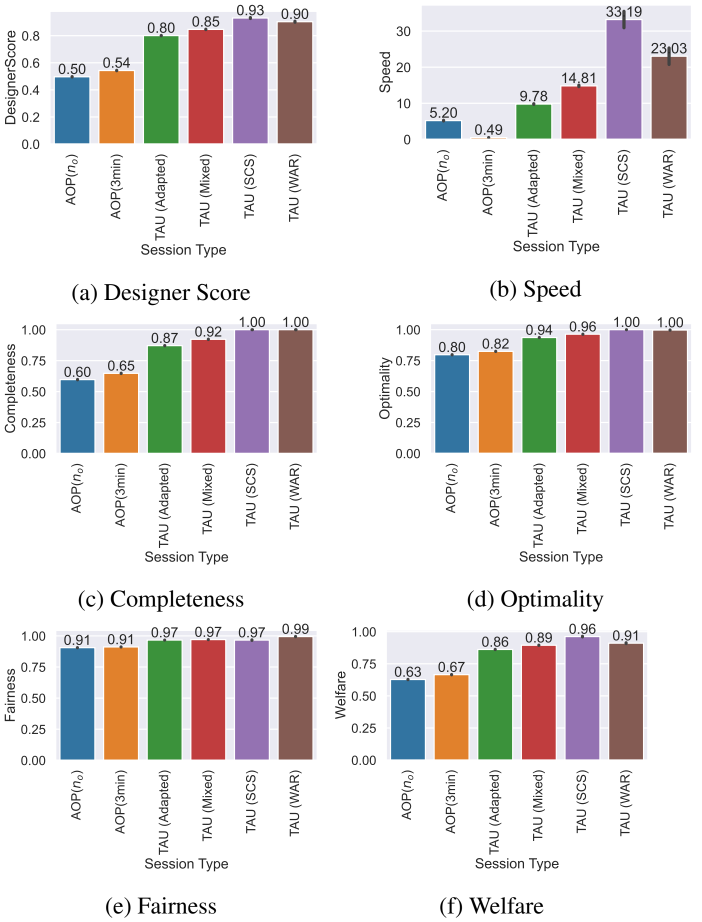
Figure 16: Partner Welfare and Privacy for the main dataset

of adapted strategies can achieve the same designer score as WAR and optim (AgentKF vs InoxAgent, and GAgent vs InoxAgent). It is interesting to note that InoxAgent could also achieve the same advantage as WAR for this dataset.

ANAC 2015 - All (Finalists)

This dataset contained the strategies submitted to ANAC 2015 available in the GENIUS platform: Atlas3, ParsAgent, RandomDance, Kawaii, AgentBuyong, PhoenixParty, XianFaAgent, PokerFace, AgentH, AgentHP, AgentNeo, AgentW, AgentX, AresParty, CUHKAgent2015, DrageKnight, Kawaii, Y2015Group2, JonnyBlack, MeanBot, Mercury, PNegotiator, SENGOKU, TUDMixedStrategyAgent.

The finalists for this year were: Atlas3, ParsAgent, Random-

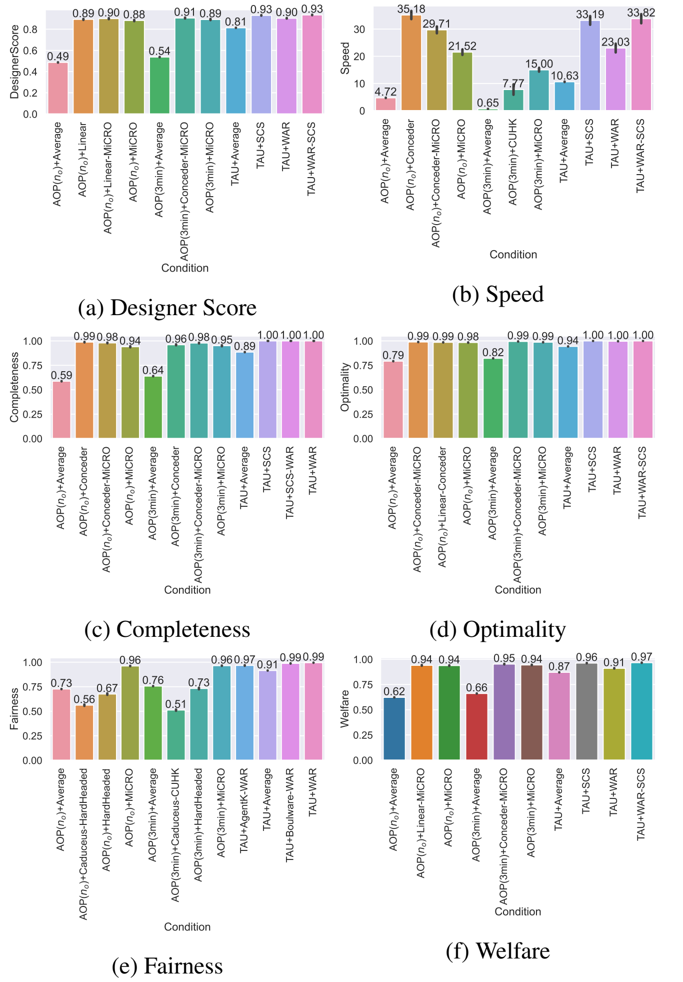


(a) Designer Score (b) Speed

(c) Completeness (d) Optimality

(e) Fairness (f) Welfare

Figure 17: Designer metrics on the main dataset.



(a) Designer Score (b) Speed

(c) Completeness (d) Optimality

(e) Fairness (f) Welfare

Figure 18: Top-2 scores of all strategy combinations and average score per protocol.

Dance, Kawaii, AgentBuyong, PhoenixParty, XianFaAgent, PokerFace.

The scenarios used were: KillerRobot (240000 outcomes), Politics (23040 outcomes), Symposium (2304 outcomes), University (2250 outcomes), Vacation (1600 outcomes), Dinner (1200 outcomes), Holiday (1024 outcomes), Tram (972 outcomes), BuildingConstruction (864 outcomes), ZoningPlan (448 outcomes), NewSporthal (243 outcomes), CarDomain (240 outcomes), CarPurchase (216 outcomes), BankRobbery (18 outcomes), Movie (4 outcomes).

This is the first purely multilateral dataset evaluated in this paper. Nevertheless, WAR and SCS still achieve 100% agreement rate and optimality. For the first time, an adapted strategy combination (AgentBuyog-XianFaAgent, ParsAgent-XianFaAgent) could outperform WAR and SCS by 1% in designer score but not in advantage. This was due to increased welfare. Fig. 24 shows these results. This is the first dataset for which running TAU native strategies against adapted strategies (TAU(Mixed)) achieves slightly higher designer score than SCS (0.84 compared with 0.81). The difference was not statistically significant. WAR still achieved the highest designer score of 0.9 in this case. The main cause of improved performance for Mixed runs was the increased fairness (0.94 compared with 0.87 for SCS).

Fig. 25 shows the results when focusing exclusively on finalist agents with the same pattern of results.

ANAC 2016 - All (Finalists)

This dataset contained the strategies submitted to ANAC 2016 available in the GENIUS platform: Caduceus, YXAgent, ParsCat, Farma, MyAgent, Atlas32016, Ngent, GrandmaAgent, AgentHP2, Terra, AgentLight, AgentSmith2016, ClockworkAgent, MaxOops, ParsAgent2, SYAgent.

The finalists for this year were: Caduceus, YXAgent, ParsCat, Farma, MyAgent, Atlas32016, Ngent, GrandmaAgent, AgentHP2, Terra.

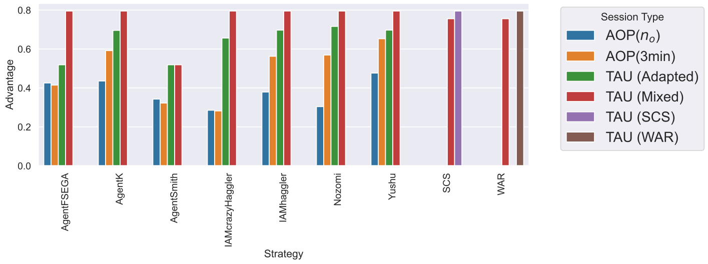
The scenarios used were: EnergyLarge (390625 outcomes), NewDomain (35840 outcomes), WindFarm (7200 outcomes), SmartEnergyGridLarge (4250 outcomes), KDomain (1280 outcomes), SmartEnergyGridMedium (984 outcomes), SmartEnergeGridSmall (625 outcomes), PEnergy (612 outcomes), DomainAce (320 outcomes), JapanTrip2016 (240 outcomes), SmartGridDomain (100 outcomes), SmartGrid (80 outcomes), TwF (25 outcomes), TriangularFight (9 outcomes), ElectricVehicle (8 outcomes).

Fig. 26 shows the summary results for this dataset. This was the first dataset for which an adapted strategy (MyAgent) outperformed WAR in terms of agent advantage (0.65 compared with 0.63). Moreover, SCS playing against AgentHP2 or Caduceus achieved higher designer score compared with WAR (0.94/0.93 compared with 0.89). Another interesting finding is that WAR's optimality was not 100% for the first time (0.96%) due to it achieving agreements that are not Pareto-optimal in the KillerRobot domain. This is another example of WAR not being optimal for Λ_{DN} in general.

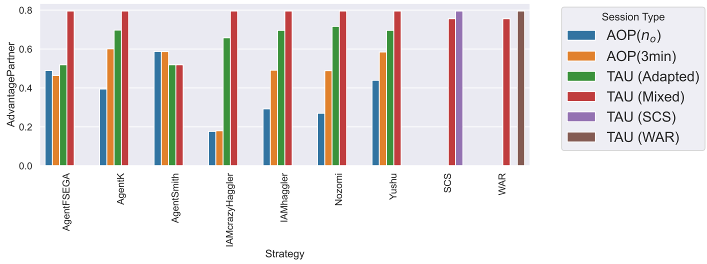
Fig. 27 shows the results when focusing exclusively on finalist agents with the same pattern of results.

ANAC 2017 - All (Finalists)

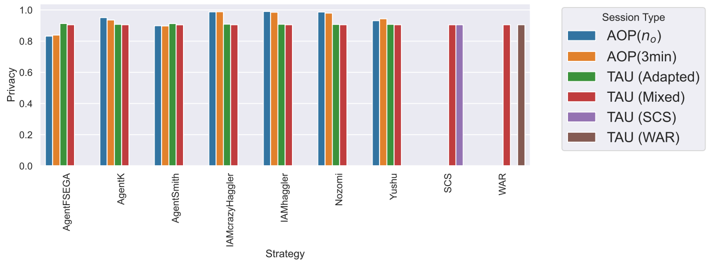
This dataset contained the strategies submitted to ANAC 2017 available in the GENIUS platform: PonPokoAgent, CaduceusDC16, ParsCat2, Rubick, ParsAgent3, AgentKN, AgentF, SimpleAgent2017, Mamenchis, Farma2017, GeneKing, Gin, Group3, Imitator, MadAgent, Mosa, TaxiBox, TucAgent.



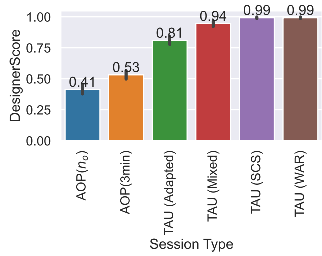
(a) Advantage



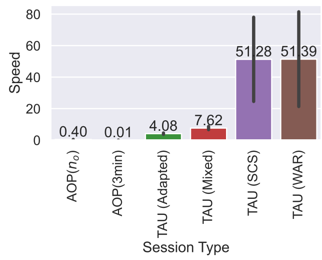
(b) Partner Advantage



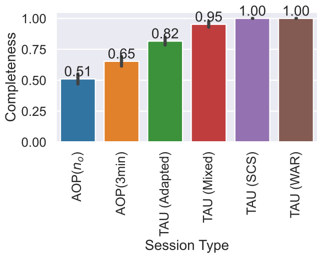
(c) Privacy



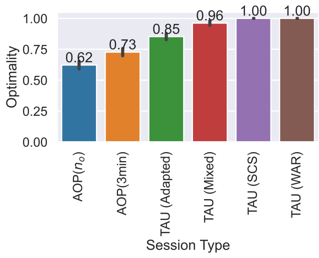
(d) Designer Score



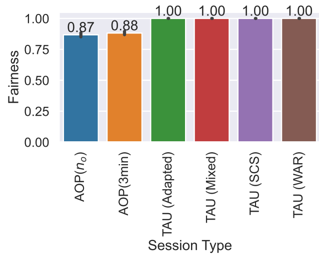
(e) Speed



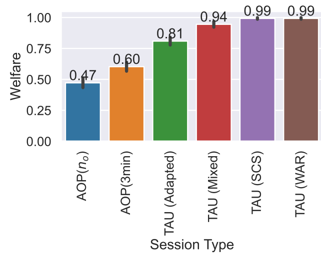
(f) Completeness



(g) Optimality

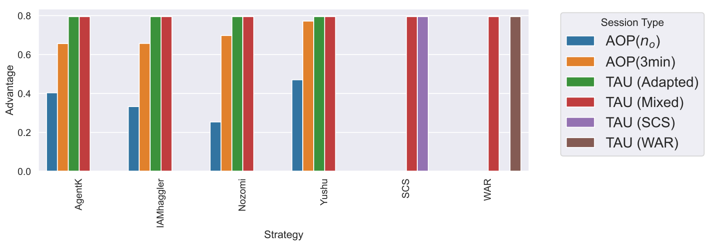


(h) Fairness

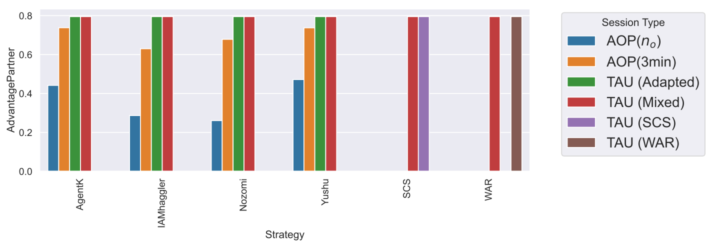


(i) Welfare

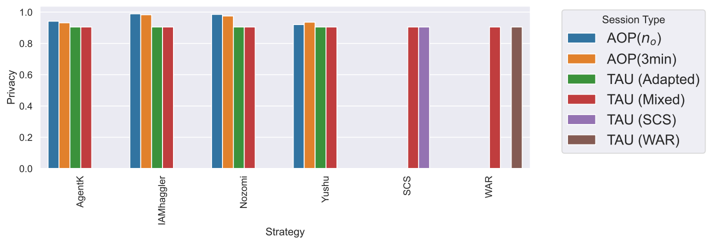
Figure 19: Results on the 2010 dataset.



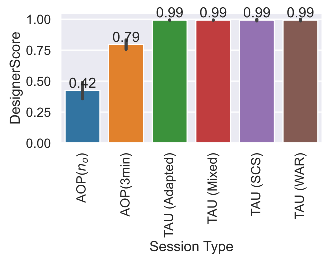
(a) Advantage



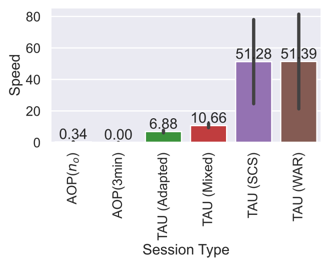
(b) Partner Advantage



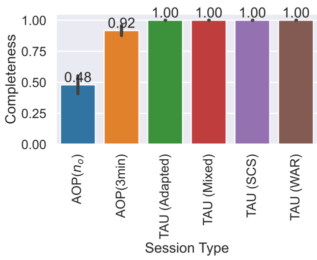
(c) Privacy



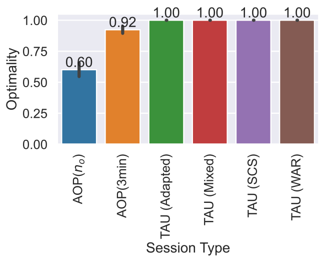
(d) Designer Score



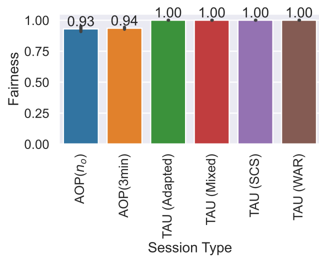
(e) Speed



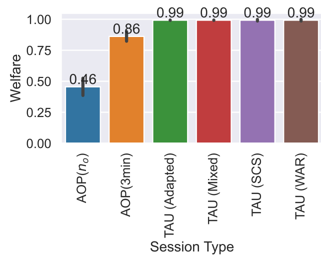
(f) Completeness



(g) Optimality



(h) Fairness

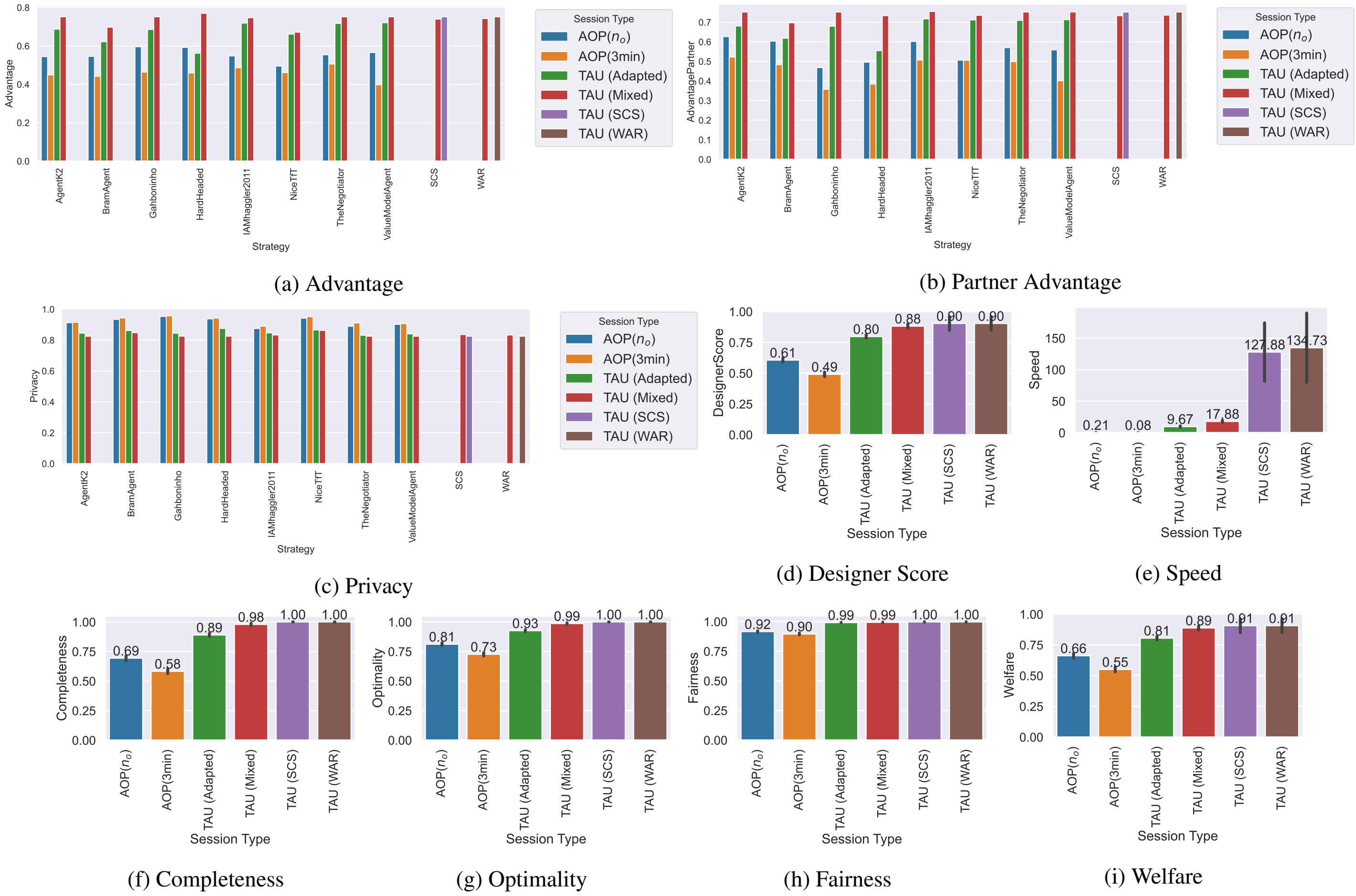


(i) Welfare

Figure 20: Results on the 2010finalists dataset.

The finalists for this year were: PonPokoAgent, CaduceusDC16, ParsCat2, Rubick, ParsAgent3, AgentKN, ParsCat2, AgentF, SimpleAgent2017, Mamenchis.

The scenarios used were: MyDomain (37440 outcomes), GeneJack (10000 outcomes), AlphabetMaze (7200 outcomes), Supermarket (5760 outcomes), Music (1512 outcomes), Tangxun (360

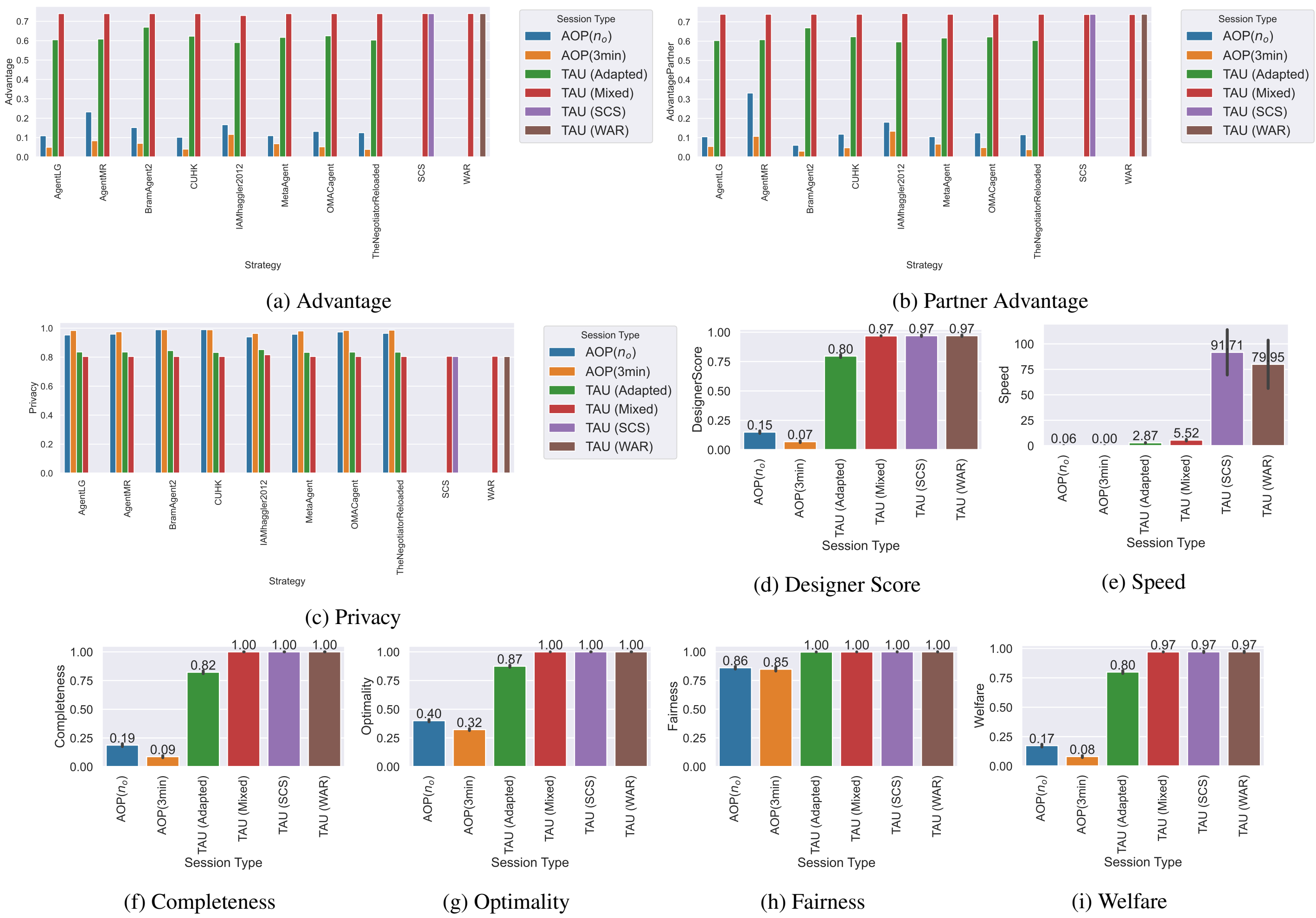


(a) Advantage (b) Partner Advantage

(c) Privacy (d) Designer Score (e) Speed

(f) Completeness (g) Optimality (h) Fairness (i) Welfare

Figure 21: Results on the 2011 dataset.

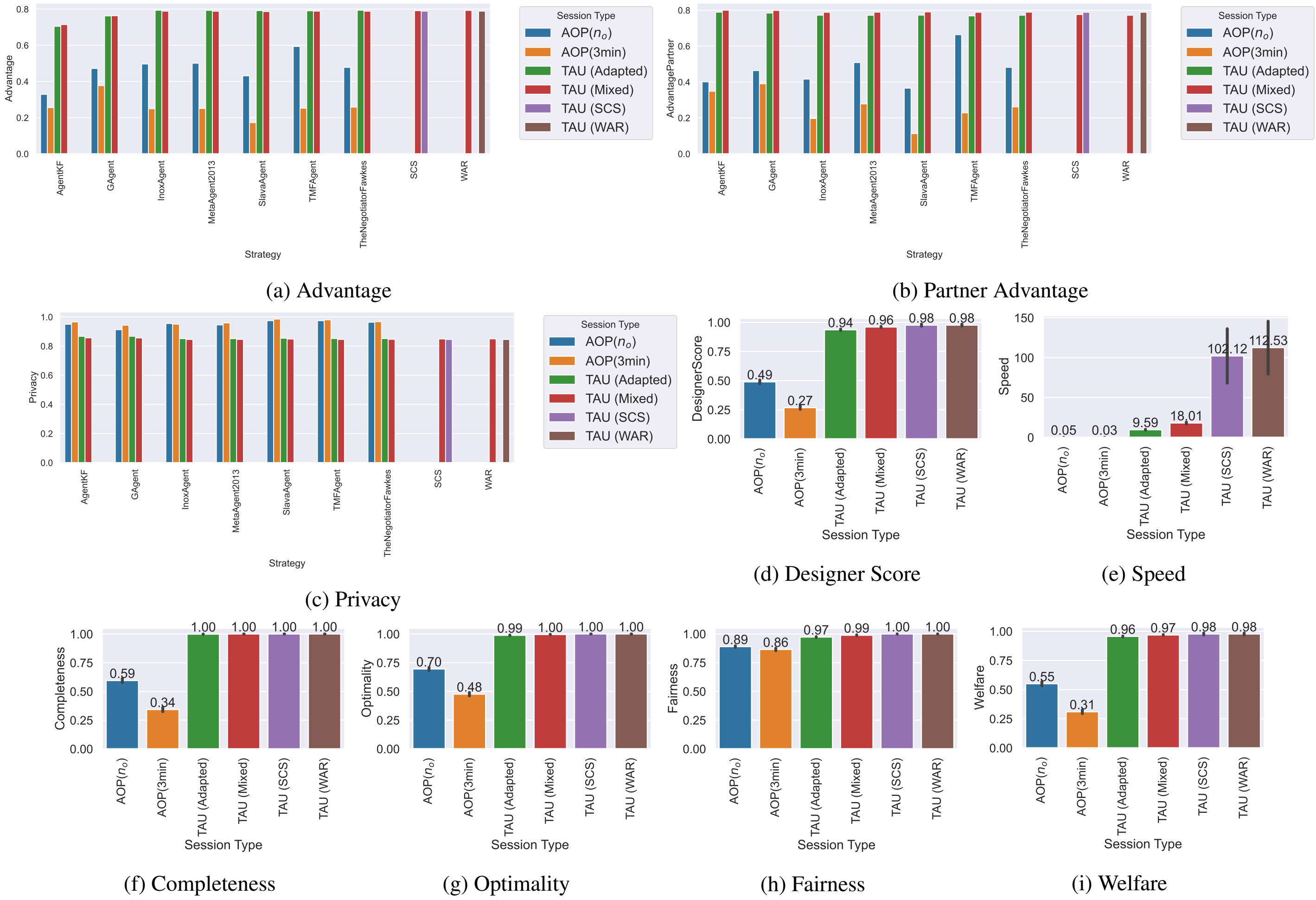


(a) Advantage (b) Partner Advantage

(c) Privacy (d) Designer Score (e) Speed

(f) Completeness (g) Optimality (h) Fairness (i) Welfare

Figure 22: Results on the 2012 dataset.



(a) Advantage (b) Partner Advantage

(c) Privacy (d) Designer Score (e) Speed

(f) Completeness (g) Optimality (h) Fairness (i) Welfare

Figure 23: Results on the 2013 dataset.

outcomes), MusicalInstuments (294 outcomes), MovieTime (144 outcomes), SmartGrid (80 outcomes), LunchTime (80 outcomes).

Fig. 28 shows the summary results for this dataset. This is the first dataset and only for which adapted strategies average higher designer scores than SCS (0.79 compared with 0.78). This difference was not statistically significant. TAU (Mixed) also outperformed SCS with a designer score of 0.8. WAR still achieved the highest designer score of 0.95. Again the reduced relative performance of SCS in terms of designer score on this dataset was due to reduced fairness. At least two adapted strategies (AgentKN and PonPokoAgent) achieved the same average advantage as WAR on this dataset (0.79). Two adapted strategy combinations managed to achieve the same designer score as WAR (0.95). These were AgentKN against Mamenchis and Mosa. Fig. 29 shows the results when focusing exclusively on finalist agents with the same pattern of results except that now AgentKN and PonPokoAgent managed to marginally achieve higher advantage compared with WAR (0.77 compared with 0.76) but that difference was not statistically significant.

ANAC 2018 - All (Finalists)

This dataset contained the strategies submitted to ANAC 2018 available in the GENIUS platform: MengWan, IQSun2018, PonpokoRampage, AgentHerb, FullAgent, BetaOne, AgreeableAgent2018, Shiboy, ConDAgent, Yeela, Sontag, Agent33, AgentNP1, ateamagent, ExpRubick, GroupY, Lancelot, Libra, Seto, SMACAgent.

The finalists for this year were: MengWan, IQSun2018, PonpokoRampage, AgentHerb, FullAgent, BetaOne, AgreeableAgent2018, Shiboy, ConDAgent, Yeela, Sontag, Agent33, AgentNP1.

The scenarios used were: IQSon (40320 outcomes), MengWan (3072 outcomes), Hamada (625 outcomes), BetaOne (64 outcomes).

Fig. 30 shows the summary results for this dataset. Selo adapted to TAU managed to slightly get higher advantage than WAR (0.72 compared with 0.71). This difference was not statistically significant. WAR against adapted Lancelot achieved a designer score of 0.94 compared with 0.92 for WAR mostly due to increased welfare. This difference also was not statistically significant. Fig. 31 shows the results when focusing exclusively on finalist agents with a similar pattern of results. This time no adapted strategy combination could achieve higher advantage compared with WAR and SCS and two different strategy combinations could again perform at the same level as WAR and SCS in terms of designer score achieving 0.92 (namely, AgentHerb or AgentNP1 against BetaOne).

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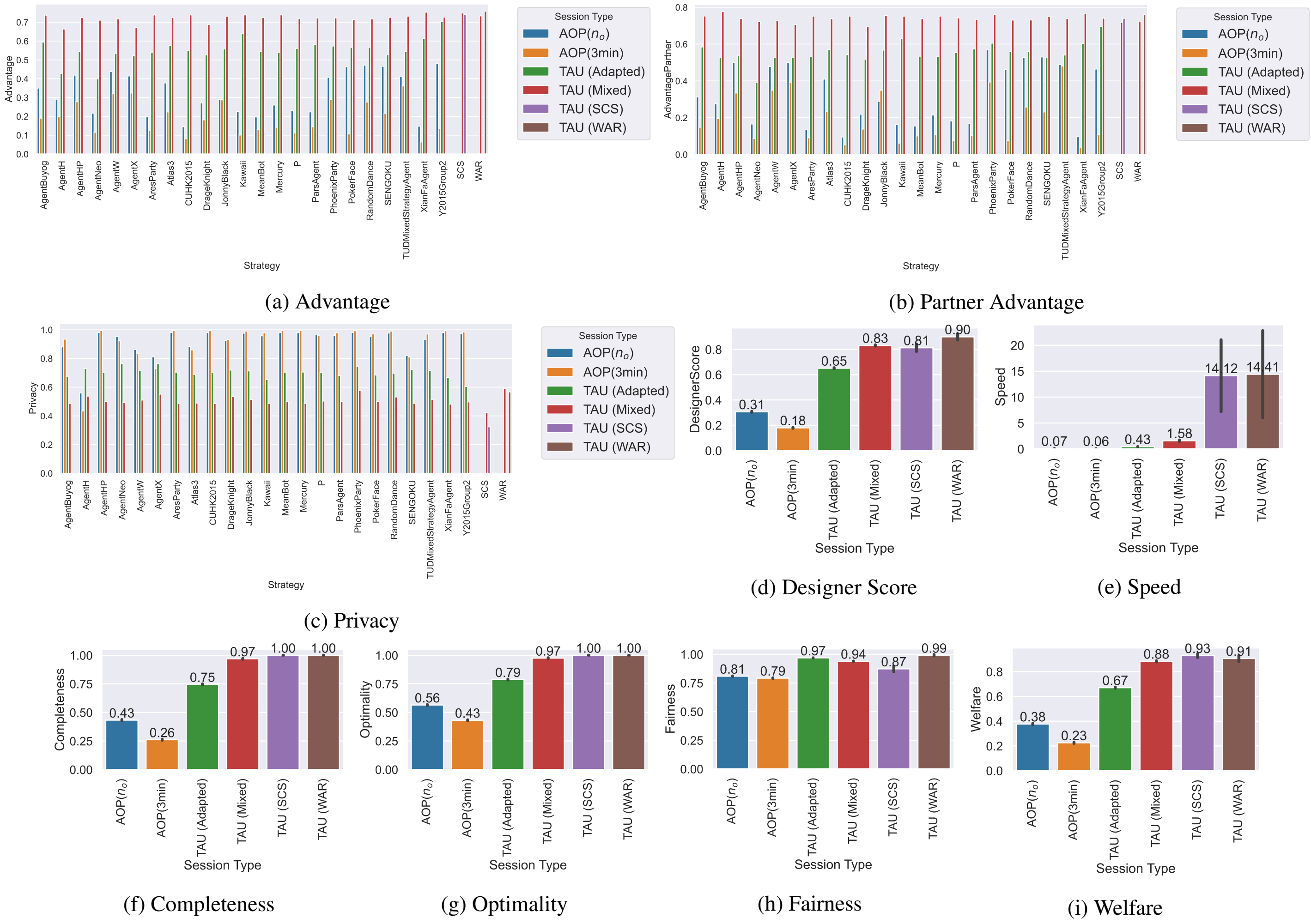


Figure 24: Results on the 2015 dataset.

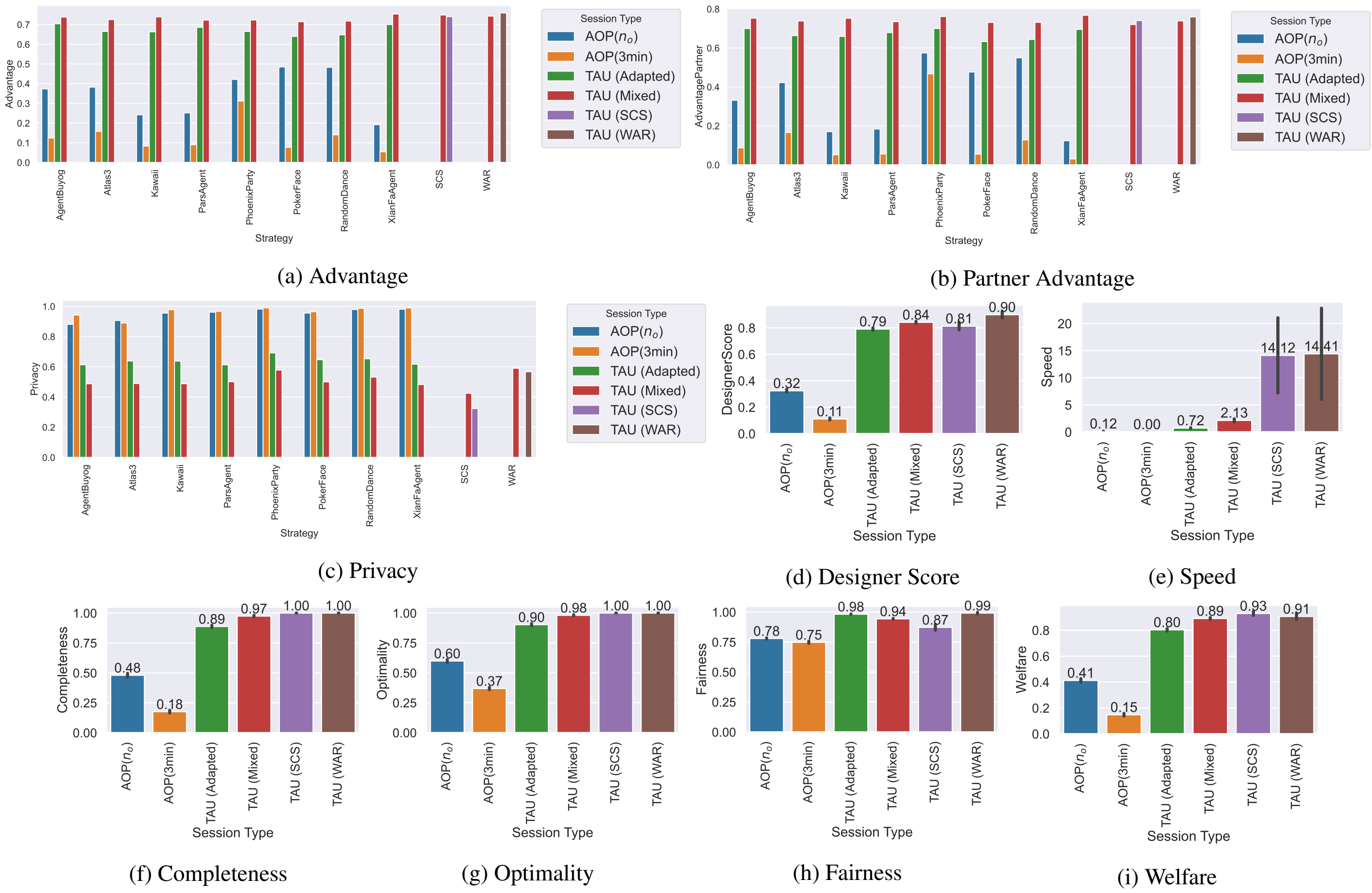
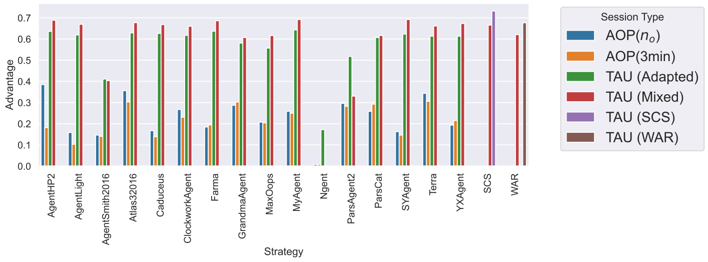
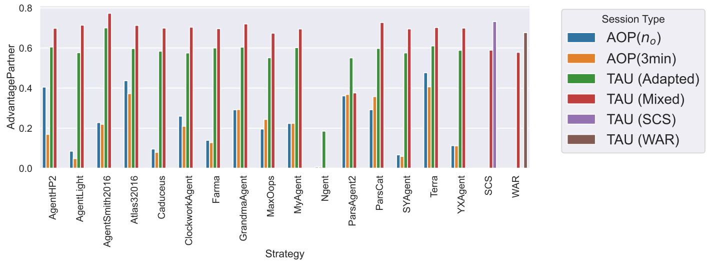


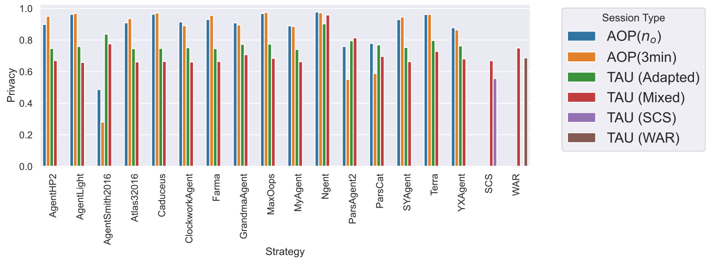
Figure 25: Results on the 2015finalists dataset.



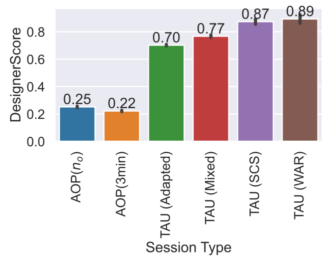
(a) Advantage



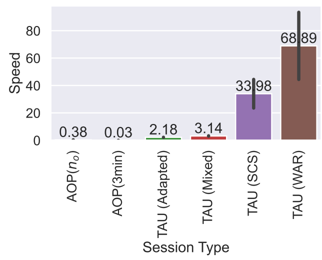
(b) Partner Advantage



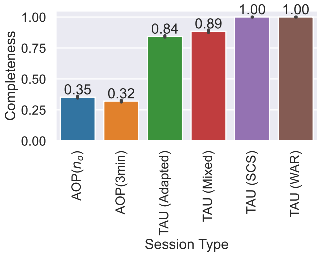
(c) Privacy



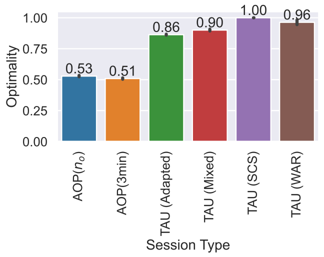
(d) Designer Score



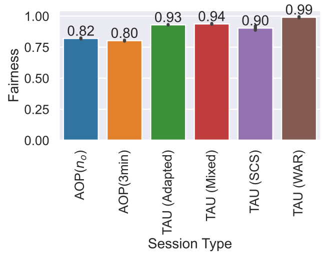
(e) Speed



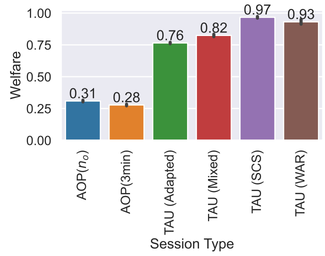
(f) Completeness



(g) Optimality

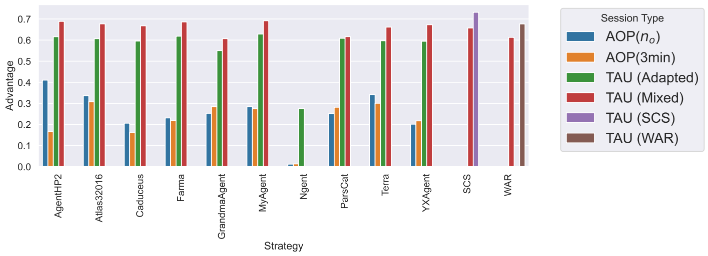


(h) Fairness

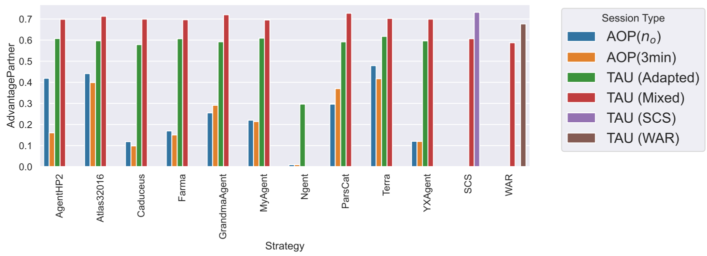


(i) Welfare

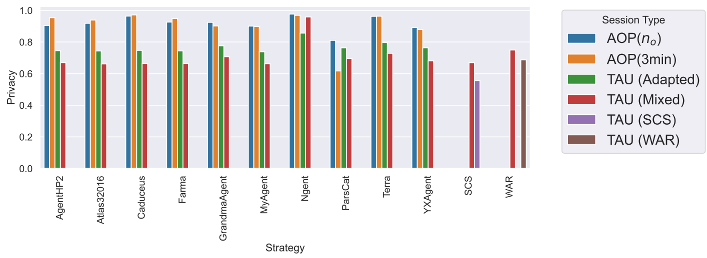
Figure 26: Results on the 2016 dataset.



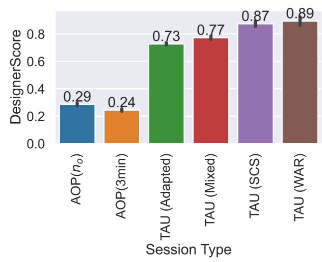
(a) Advantage



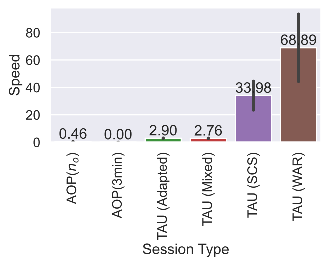
(b) Partner Advantage



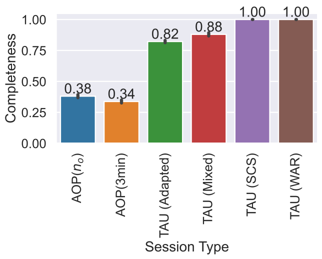
(c) Privacy



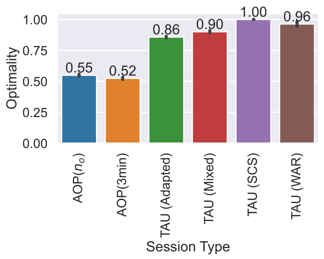
(d) Designer Score



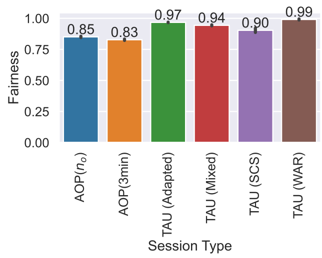
(e) Speed



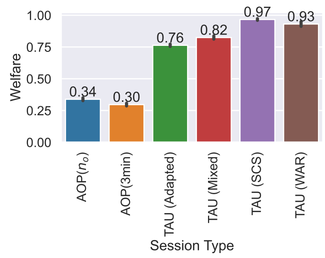
(f) Completeness



(g) Optimality



(h) Fairness



(i) Welfare

Figure 27: Results on the 2016finalists dataset.

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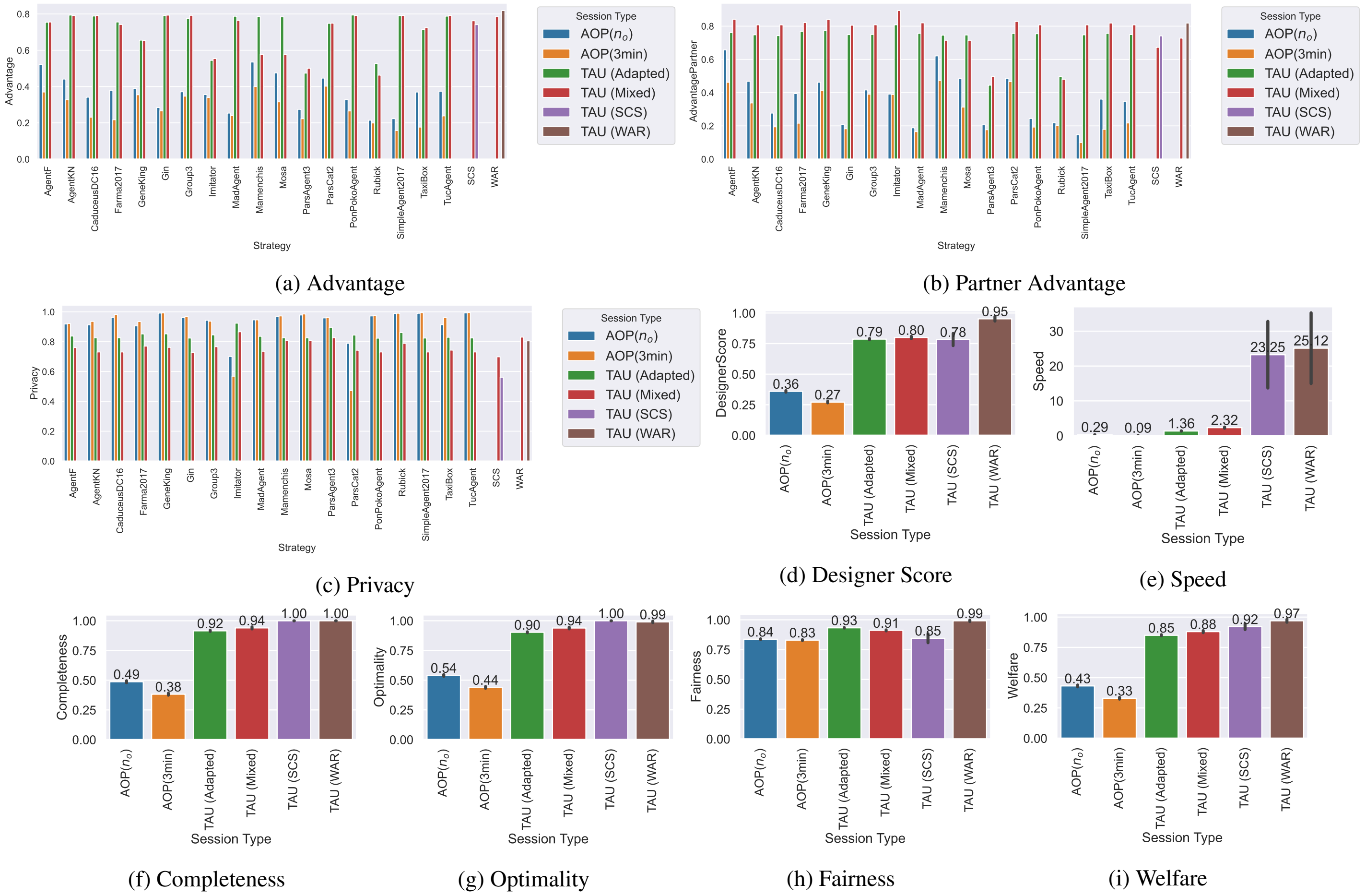
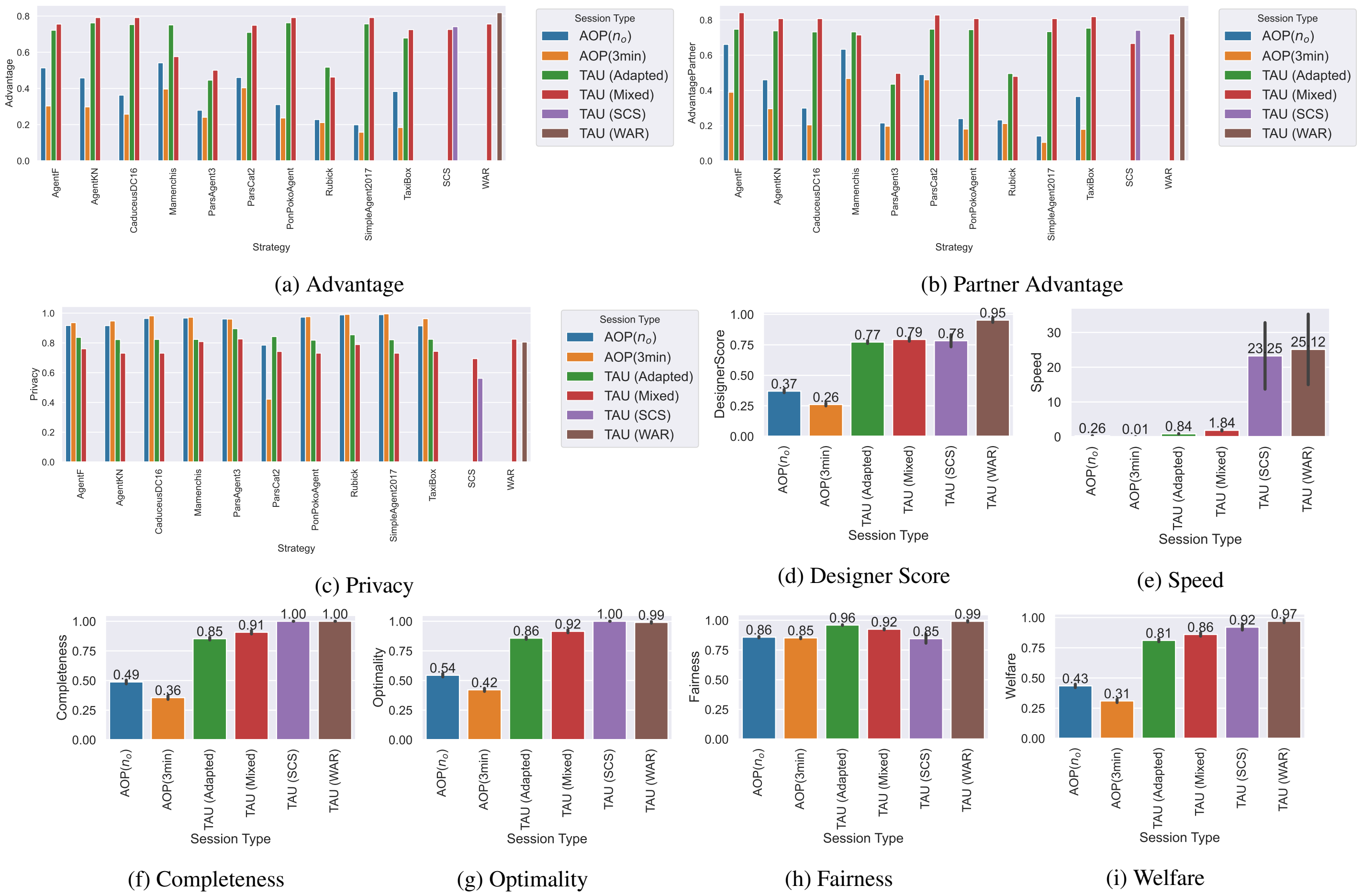


(a) Advantage (b) Partner Advantage

(c) Privacy (d) Designer Score (e) Speed

(f) Completeness (g) Optimality (h) Fairness (i) Welfare

Figure 28: Results on the 2017 dataset.



(a) Advantage (b) Partner Advantage

(c) Privacy (d) Designer Score (e) Speed

(f) Completeness (g) Optimality (h) Fairness (i) Welfare

Figure 29: Results on the 2017finalists dataset.

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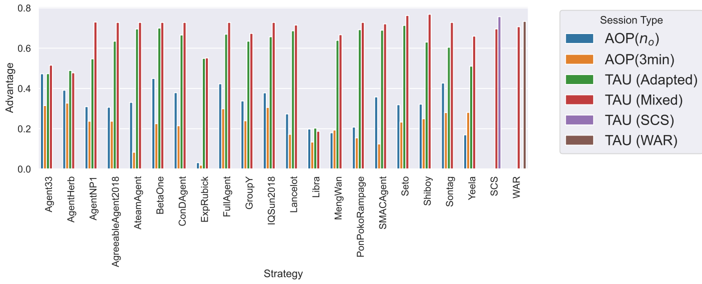
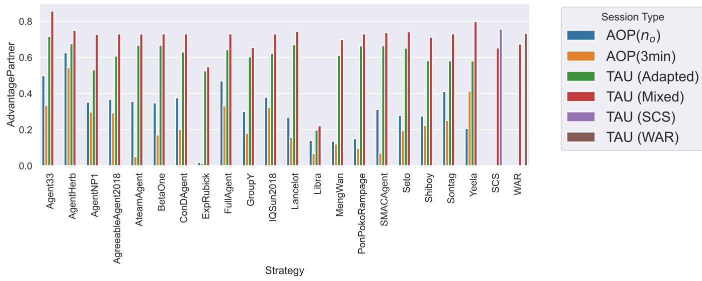
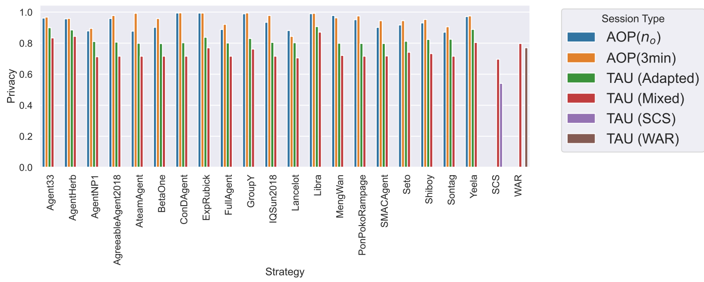


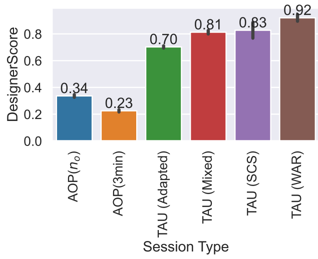
(a) Advantage



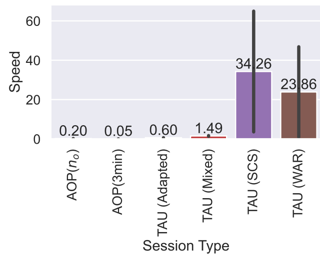
(b) Partner Advantage



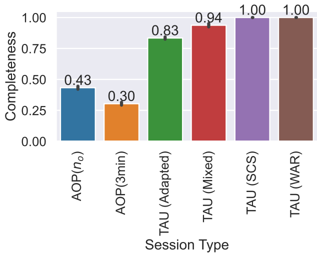
(c) Privacy



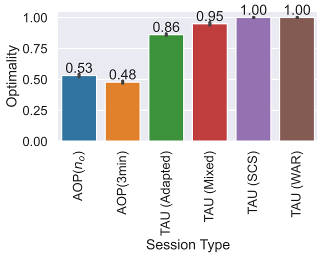
(d) Designer Score



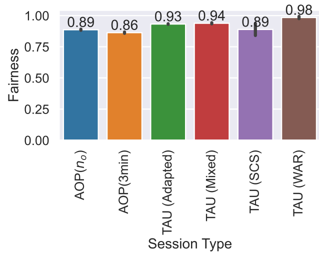
(e) Speed



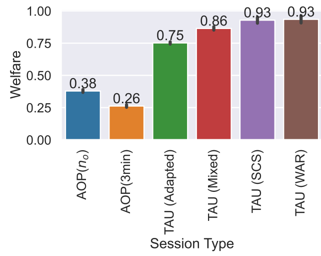
(f) Completeness



(g) Optimality

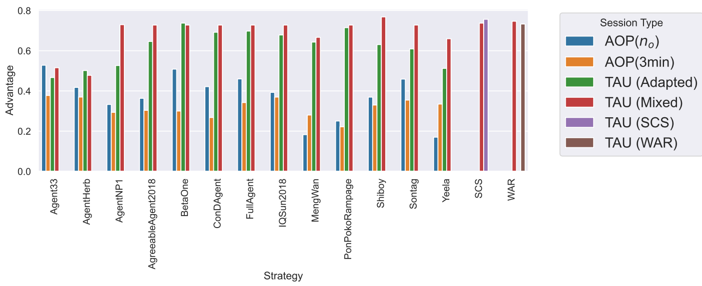


(h) Fairness

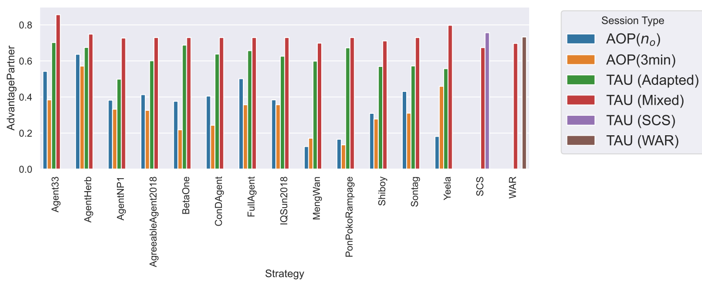


(i) Welfare

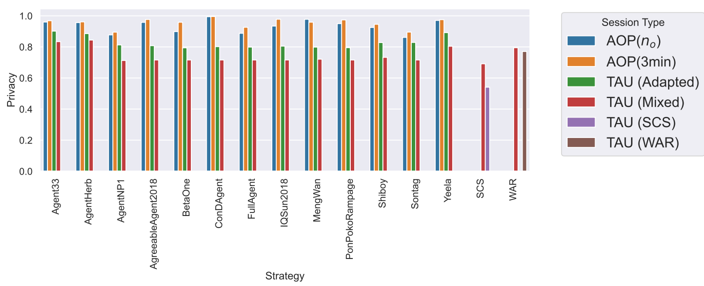
Figure 30: Results on the 2018 dataset.



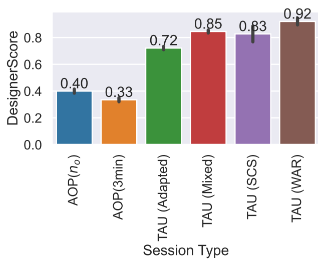
(a) Advantage



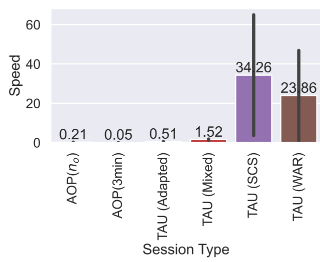
(b) Partner Advantage



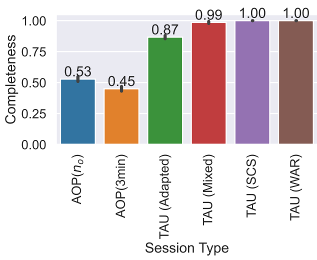
(c) Privacy



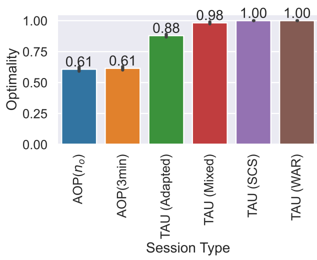
(d) Designer Score



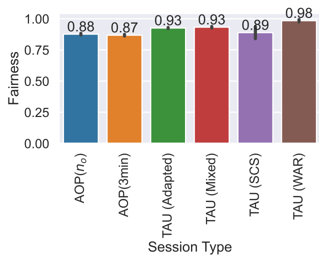
(e) Speed



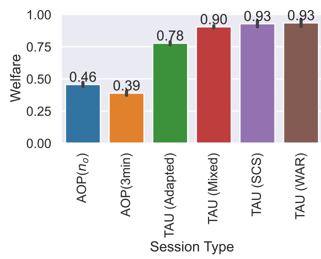
(f) Completeness



(g) Optimality



(h) Fairness



(i) Welfare

Figure 31: Results on the 2018finalists dataset.

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