Supplementrary Materials for Generalized Bargaining Mechanism

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Notation

The paper defines all notation used at the point it is introduced to reduce the need to go back to the notation section. In this section, we collect all the notation used in the paper in one section as a reference for the reader and introduce all new notation to be used in the proofs later in this document.

Used in the Paper

$\mathbb{Z}$ Integers starting at zero.

$\mathbb{Z}^+ \equiv \mathbb{N}$ Natural numbers (positive integers from 1).

$\mathbb{Z}^+$ Natural numbers (positive integers starting at 1).

$\mathbb{B}$ The set of booleans (True, False).

$\Re$ The set of real numbers.

$\mathbb{P}(x)$ The power-set of set x (all possible subsets of x).

$\mathbb{P}^i(x)$ Sets of cardinality i belonging to $\mathbb{P}(x)$.

AN Automated Negotiation.

AOP Alternating Offers Protocol.

SAOP Stacked Alternating Offers Protocol.

TAU Tentative Acceptance Unique Offers Protocol (Proposed).

SCS Slow Concession Strategy (Proposed).

PNM Perfect Negotiation Mechanism.

OPNM Outcome-Perfect Negotiation Mechanism.

$\mathcal{S}$ Negotiation scenario.

$\mathcal{M}$ Negotiation Protocol.

$\mathcal{A}^{\mathcal{S}}$ Negotiators for Scenario $\mathcal{S}$.

$\mathbb{S}$ Strategies associated with negotiators for Scenario $\mathcal{S}$.

n_A The number of negotiators/agents/preferences in a negotiation.

$\mathcal{D}^{\mathcal{S}}$ Negotiation Domain for scenario $\mathcal{S}$ (Outcome Space + Preferences).

Ω Outcome space (all possible agreements).

Ω_x^v The set of valid outcomes that can be offered by agent x.

Ω_x^i The set of outcomes constituting the offer of agent x at step i. In all mechanisms considered in this work, this set is either empty (written as ϕ) or contains a single offer ω_x^i

Ω_{yx}^i The set of outcomes constituting the response of agent x at step i to the offer from agent y. In all mechanisms considered in this work, this set is either empty (written as ϕ) or contains a single response $a_{yx}^i \equiv \mathbb{1}[\Omega_{yx} i \neq \{\}]$

n_o The cardinality of Ω (size of the outcome space).

ϕ Represents disagreement as an output of a negotiation, ending negotiation as an offer from an agent, and the empty set otherwise.

Ω^+ Outcome space or disagreement ϕ (all possible negotiation outcomes).

$\Omega_x^\top$ Best set of outcomes for negotiator x (none of these outcomes is better than any other in the set and any of them is better than any outcome not in the set).

$\Omega_x^\perp$ Worst set of outcomes for negotiator x (none of these outcomes is worse than any other in the set and any of them is worse than any outcome not in the set).

$\mathcal{F}^{\mathcal{S}} x$ Preferences of negotiator x in scenario $\mathcal{S}$.

u_x Utility function of negotiator x.

$\omega_1 \succcurlyeq_x \omega_2$ Outcome ω_1 is not worse for negotiator x than outcome ω_2 given the preferences $\mathcal{F}^{\mathcal{S}} x$.

$\omega_1 \succ_x \omega_2$ Outcome ω_1 is better for negotiator x than outcome ω_2 given the preferences $\mathcal{F}^{\mathcal{S}} x$.

$\omega_1 \approx_x \omega_2$ Outcome ω_1 is neither better not worse for negotiator x compared with outcome ω_2 given the preferences $\mathcal{F}^{\mathcal{S}} x$.

$\mathcal{I}^{\mathcal{S}}$ Negotiator's Information Sets representing all information available to all negotiators in scenario $\mathcal{S}$.

$\mathrm{I}^{\mathcal{S}} x(\mathcal{F}^{\mathcal{S}})$ Information available to negotiator x in scenario $\mathcal{S}$ about preferences of all negotiators $\mathcal{F}^{\mathcal{S}}$ in that scenario.

$\mathbb{D}$ A set of domains.

$\mathcal{D}_x^{\mathcal{S}}$ All the information available to negotiator x in scenario $\mathcal{S}$.

$\overline{\Omega}_x \subseteq \Omega$: The subset of the outcome space Ω satisfying: $\omega \succcurlyeq_x \phi$.

$\overline{\Omega}^{\mathcal{S}}$: All possible rational agrements: $\bigcap_{x\in\mathcal{A}^{S}} \overline{\Omega}_x$.

$\underline{\Omega}_x \subseteq \Omega$: The subset of the outcome space Ω satisfying: $\phi \succ_x \omega$.

$\underline{\Omega}^{\mathcal{S}}$: All irrational agreementst that should never occur defined as: $\bigcup_{x\in\mathcal{A}^{S}} \underline{\Omega}_x$.

$\widehat{\Omega}^{\mathcal{S}}$: Win-win deals defined as the set of rational outcomes strictly better than disagreement: $\left\{\omega \in \overline{\Omega}^{\mathcal{S}} \mid \forall x \in \mathcal{A}^{\mathcal{S}} \rightarrow \omega \succ_x \phi\right\}$.

$\mathcal{P}$: The pareto frontier defined as the set of rational outcomes that cannot be improved for one negotiator without making at least one other negotiator worst off: $\psi \in \mathcal{P} \subseteq \overline{\Omega}^{\mathcal{S}} \leftrightarrow \neg\exists\omega \in \Omega \setminus \{\psi\} \mid \exists x \in \mathcal{A}^{\mathcal{S}} \mid \omega \succ_x \psi \wedge \forall x \in \mathcal{A}^{\mathcal{S}} \setminus \{x\}\, \omega \not\succ_x \psi$.

$\omega_*^{\mathcal{S}}$ The outcome of a negotiation using scenario $\mathcal{S}$.

$\mathcal{S}()$ The outcome of a negotiation using scenario $\mathcal{S}$ (same as $\omega_*^{\mathcal{S}}$).

$\mathcal{T}^{\mathcal{S}}i$ The negotiation trace showing all offers and responses (in order) in scenario $\mathcal{S}$ up to round i.

$\mathbb{T}^{\mathcal{S}}i$ The domain of all possible traces.

f The Filter used in representing a negotiation protocol.

σ The Evaluation Strategy used in representing a negotiation protocol.

$\triangleright$ Continue negotiation (returned by the evaluation policy to signal that the negotiation should go on).

s A negotiation strategy (consists of an offering policy and an acceptance policy).

π_x The Offering Policy of negotiator x.

ρ_x The Acceptance Policy of negotiator x.

Ω_x^* The Tentative Agreement Tuple associated with a specific negotiation thread x. In AOP and TAU, the evaluation rule is such that Ω_x^* has at most one element. We use ω_x^* as this element if it exists and set it to ϕ otherwise.

Used only in Supplementrary Materials

$\mathbb{P}_x^j$ An ordered tuple of j most recent unique offers proposed by negotiator x so far ordered from earliest to latest.

$\mathbb{A}_x^j$ An ordered tuple of j most unique recent offers accepted by negotiator x so far ordered from earliest to latest.

$\Omega_x^v(i)$ The set of valid outcomes that can be offered by agent x at round i.

$\hat{a}_x^i(\omega)$ The outcome ω will be accepted by negotiator x if it was offered at round i.

$\hat{a}_x(\omega)$ The outcome ω will be accepted by negotiator x at some time during the negotiation.

J_x^ω The first round at which agent x would make offer ω. We set $J_x^\omega = \infty$ if x will never make the offer during a negotiation.

Proofs and Theoretical Justifications

In the following subsections, we will use . to stand for any valid value (e.g. $TAU(.,\beta)$) stands for the TAU with any valid value for the parameter α). We will use TAU to mean $TAU(.,.)$.

We repeat the main definitions from the paper that we need for proofs here:

Definition 1. Let, i be the current step and n_o be the size of the outcome space, and Ω_x^i be the offer from agent x at step i , the Tentative Acceptance Unique Offers Protocol (TAU) is defined as $\text{GBM}(1, 1, \rho_o, \chi_{at}|\chi_{rr}, f_{TAU}, \sigma_{TAU})$ where:

$$f_{TAU}(x) = \left\{P_x^{last}\right\} \cup (\Omega \setminus P_x) \tag{1}$$

$$\gamma_{TAU}(x) = \Omega_x^*(i) = \begin{cases} \Omega_x^*(i-1) \circ \omega_x^i, & \Omega_{yx} \neq \{\}\, \forall y \in \mathcal{A} \setminus \{x\} \\ \Omega_x^*(i-1) & otherwise. \end{cases} \tag{2}$$

$$\sigma_{TAU} = \begin{cases} \phi & \exists x \in \mathcal{A} \mid \Omega_x^i = \phi \vee i > |\Omega| \\ \{\omega\} & \omega \in \Omega_x^* \forall x \in \mathcal{A} \\ \triangleright & otherwise. \end{cases} \tag{3}$$

Definition 2. SCS's offering policy: Let's define $\succ_l$ as a shared full ordering of the outcome space and the set of best valid outcomes not offered yet as:

$$\overline{\Omega}_x^v(i) \equiv \left\{\omega \in \Omega_x^v(i) \cap \overline{\Omega}_x \mid \omega \succcurlyeq_x \psi \quad \forall\psi \in \Omega_x^v(i)\right\}$$

The offering policy of SCS can then be expressed as follows:

$$\pi_{SCS}(i) = \Omega_x^i = \sup_{\succ_l}(\overline{\Omega}_x^v(i)) \tag{4}$$

Offers are given in descending order according the Agent's preferences. Ties are broken using any predefined shared common lexical ordering on the outcomes. It does not matter what this ordering is. What only matters is that it is shared between all negotiators. Without this shared ordering, exact optimality is lost. It is important that agents not following this lexical ordering rule can never gain any utility because of it which means this rule is incentive compatible.

Definition 3. SCS's selection policy:

$$\rho_{SCS}(i) = \Omega_{yx} = \begin{cases} \left\{\omega | \omega \succ_x \omega_y^* \forall\omega \in \Omega_y^i\right\} & \omega_y^* \neq \phi \\ \Omega_y^i \cap \overline{\Omega}_x & \omega_y^* = \phi \end{cases} \tag{5}$$

We will use the following properties of offers directly deducible from Eq. 4

- For SCS, a better offer is always offered before a worse offer:

$$\psi \succ_x \omega \wedge \omega_x^i = \omega \implies \psi \in \mathbb{P}_x^i \tag{6}$$

- SCS never offers ϕ under TAU (because it can always repeat) which means it never chooses to explicitly end the negotiation.

$$\neg\exists i \in \mathbb{Z} \mid \omega_x^i = \phi \tag{7}$$

- SCS never repeats an offer before all offers not worse than it are tried:

$$\omega_x^i = \omega_x^{i-1} \implies \omega \in \mathbb{P}_x^{i-2} \quad \forall \omega \in \Omega \mid \omega \succ_x \omega_x^i \tag{8}$$

- SCS never repeats an offer before trying all its rational outcomes:

$$\omega_x^i = \omega_x^{i-1} \implies \omega \in \mathbb{P}_x^{i-2} \qquad \forall \omega \in \overline{\Omega} \tag{9}$$

Because win-win deals are a subset of rational outcomes:

$$\omega_x^i = \omega_x^{i-1} \implies \omega \in \mathbb{P}_x^{i-2} \qquad \forall \omega \in \widehat{\Omega} \tag{10}$$

Considering the selection policy of SCS (Eq. 5), we can directly deduce the following properties:

- If SCS is willing to accept an offer, it will accept any better offer

$$\omega \succ_x \psi \wedge \hat{a}_x^i(\psi) \implies \hat{a}_x^j(\omega) \qquad \forall j \leq i \tag{11}$$

The reason that this holds is that SCS will only accept an offer if it is better than everything that was offered before. This means that it will accept any better offer at the same or any earlier round.

- If SCS rejects an offer, it must have received an offer not worse for itself earlier:

$$\omega_y^i = \omega \wedge \neg a_{yx}^i \implies \exists \psi \in \mathbb{P}_y^{i-1} \mid \psi \not\prec_x \omega \tag{12}$$

- If SCS actually accepts an offer at round i, it will reject any worse offer coming later:

$$\omega_y^i = \omega \wedge a_{yx}^i \implies \neg \hat{a}_x^j(\omega) \quad \forall j > i \forall \omega \in \Omega \tag{13}$$

We can also directly see the following properties of running TAU with SCS for all negotiators from the definitions above:

- If an outcome is offered by both negotiators and is acceptable to both negotiators by some time step i, the negotiation ends with this outcome as an agreement:

$$\exists \omega \in \Omega \mid \omega \in \mathbb{P}_x^i \wedge \hat{a}_x^j(\omega) \forall j < i \forall x \in \mathcal{A} \implies \omega_* = \omega \tag{14}$$

- For an outcome to become the agreement of a negotiation, it must be offered by all negotiators and be acceptable by each of them before and at the round it received it:

$$\omega_* = \omega \implies \omega \in \mathbb{P}_x^{i_x} \wedge \hat{a}_x^j(\omega) \forall j < i_x \forall x \in \mathcal{A} \tag{15}$$

TAU Always Terminates

In the paper, Section "Tentative Acceptance Unique Offers Protocol", we claimed that TAU always terminates.

Theorem 1. TAU always terminates.

Proof. From Eq. 1, the size of valid set is the difference between the outcome space size and the number of unique offers for every negotiator:

$$n_v \equiv \min_{x \in \mathcal{A}} |\Omega_x^v| = \min_{x \in \mathcal{A}} n_o - |\mathbb{P}_x^\infty| \leq n_o - \max_{x \in \mathcal{A}} |\mathbb{P}_x^\infty| \tag{16}$$

Let $n_v(i)$ be the size of the valid set of outcomes from all negotiators as defined above at round i.

From Eq. 3, for any round i if all negotiators repeat their last offer, the negotiation terminates:

$$\omega_x^i = \omega_x^i \forall x \in \mathcal{A} \Rightarrow terminates(TAU, i).$$

From this we conclude that if the negotiation is not terminated at round i, there is at least one negotiator not repeating at this round:

$$\neg terminates(TAU, i) \Rightarrow \exists y \in \mathcal{A} \mid \omega_y^i \neq \omega_y^i$$

$$\therefore \neg \text{terminates}(TAU, i) \Rightarrow \exists y \in \mathcal{A} \mid \qquad |\omega_x^\infty|_i > |\omega_x^\infty|_{i-1} \tag{17}$$

From Eq. 17 and Eq. 16,

$$\therefore \neg terminates(TAU, i) \Rightarrow n_v(i) < n_v(i-1)$$

Which means that if a negotiation did not end at round i, the size of the valid set of outcomes n_v will go down.

$$\therefore \lim_{i \to \infty} n_v(i) = 0$$

Negotiators will eventually run out of unique outcomes to offer and the negotiation will terminate.

$\therefore$ TAU will always terminate. □

Empirical support comes from the fact that in all cases in the main and preliminary experiments. TAU terminated.

TAU is Exactly Rational

Theorem 2. TAU is Exactly Rational.

Proof. We will prove that TAU is exactly rational when SCS is used.

$\because$ From Eq. 5, SCS will only accept an offer if it is better than disagreement.

$\because$ From Eq. 3, an agreement can only be reached if all agents accept it.

$\therefore$ An outcome that is worse than disagreement for any SCS negotiator cannot be the output of a negotiation.

$$\therefore \omega_*^{\mathcal{S}} \neq \phi \implies \omega_* \in \overline{\Omega} \tag{18}$$

$\therefore$ If all negotiators use SCS, TAU is Exactly Rational.

$\therefore$ TAU is Exactly Rational. □

Empirical support comes from the fact that in all cases in the preliminary and main experiments, TAU never ended with an agreement with a utility lower than the reserved value of any negotiator involved. This can be seen in Fig. 1 (main paper) and Table ?? in this document from the fact that Pareto Optimality was always one implying that all outcomes are on the Pareto-frontier which contains only rational outcomes by definition.

TAU is Exactly Optimal

Theorem 3 (Optimal). TAU is Exactly Optimal.

Proof. We will assume again that both negotiators are using SCS. We need to consider too cases.

Case 1: $\left|\widehat{\Omega}\right| = 0$. In this case, there are no outcomes that dominate disagreement.

$\because$ TAU is Exactly Rational (Theorem 2), it will either lead to disagreement or to some agreement $\omega_* \succcurlyeq_x \phi \forall x \in \mathcal{A}$. This is the optimal result.

$$\left|\widehat{\Omega}\right| = 0 \implies optimal(TAU) \tag{19}$$

Case 2: $\left|\widehat{\Omega}\right| > 0$. There are some win-win outcomes which means that the Pareto-frontier is not empty. We now need to show that: $\left|\widehat{\Omega}\right| > 0 \implies \omega_* \in \mathcal{P}$

We will use a proof by contradiction. Assume that $\omega_* \notin \mathcal{P}$ and that agreement was reached at round T:

$$\therefore \exists \psi \neq \omega_* \mid \psi \succ_x \omega_* \forall x \in \mathcal{A}$$

$$\because \psi \succ_x \omega_* \text{ from assuming non Pareto optimality}$$

$$\therefore J_x^{\psi} < J_x^{\omega_*} \text{ SCS always offers from best to worst}$$

ψ was offered by all negotiators sometime during the negotiation. From Eq. 11:

$$\because \hat{a}_x^T(\omega_*) \wedge \psi \succ_x \omega_* \quad \forall x \in \mathcal{A}$$
$$\therefore \hat{a}_x^i(\psi) \quad \forall i < T \forall x \in \mathcal{A}$$

But this implies that ψ must have been acceptable all the time by all negotiators and was actually offered by all of them which would have lead to immediate agreement:

$$\omega_* = \psi$$

But this is a contradiction as we assumed $\omega_* \neq \psi$.

$$\therefore \omega_* \neq \phi \implies \omega_* \in \mathcal{P}$$

□

Empirical support comes from the fact that in all cases in the preliminary and main experiments, TAU found agreements on the Pareto-frontier. This can be seen in Table 1 and Fig. 1 from the fact that Pareto Optimality was always one which implies that all outcomes are on the Pareto-frontier.

On the other hand, TAUNR has a Pareto Optimality less than 1 in the cases with opponent reserved values of 0.5 and 0.9 in main experiment as shown in Fig. 1 which implies that it is NOT exactly optimal (a single counter example is needed here).

TAU is Exactly Complete

Theorem 4 (Completeness). TAU is Exactly Complete.

Proof. We will – again – assume that both negotiators are using SCS. For TAU to be incomplete, we must have at least one win-win deal in a negotiation that ends either with disagreement or an agreement not in the set of win-win deals. Formally:

$$\neg complete(TAU) \implies \left|\widehat{\Omega}\right| > 0 \wedge \omega_* \notin \widehat{\Omega}$$

$\therefore$ we only need to consider negotiations with $\left|\widehat{\Omega}\right| > 0$ and show that in all such negotiations $\omega_* \neq \phi \wedge \omega_* \in \widehat{\Omega}$.

$\because$ The negotiation always terminates (Theorem 1) after a finite number of rounds T.

$\therefore$ One of the top two conditions in Eq 3 is satisfied at T.

Let's consider first the case where a negotiation ends with an agreement: $\omega_* \neq \phi$

From 18, we know that the agreement is rational:

$$\omega_* \neq \phi \implies \omega_* \in \overline{\Omega}$$

and from the agreement assumption and Eq. 15, the agreement was offered and was acceptable by all negotiators.

$$\omega_* \neq \phi \implies \omega_* \in \mathbb{P}_x^T \wedge \hat{a}_x^j(\omega) \forall j < T \forall x \in \mathcal{A} \tag{20}$$

Now consider a rational outcome that is not a win-win deal: $\psi \in \overline{\Omega} \setminus \widehat{\Omega}$

$$\because \psi \in \overline{\Omega} \setminus \widehat{\Omega}$$
$$\therefore \omega \succ_x \psi \quad \forall \omega \in \widehat{\Omega} \tag{21}$$

From Eq. 6, and Eq 21, we can conclude that all win-win deals were offered during the negotiation by all negotiators:

$$\omega_* \notin \widehat{\Omega} \implies \omega_* \in \mathbb{P}_x^T \quad \forall \omega \in \widehat{\Omega} \quad \forall x \in \mathcal{A} \tag{22}$$

From Eq. 11, and Eq 21, we can conclude that all win-win deals were offered during the negotiation by all negotiators:

$$\omega_* \notin \widehat{\Omega} \implies \hat{a}_x^j(\omega) \quad \forall j < T \forall \omega \in \widehat{\Omega} \forall x \in \mathcal{A} \tag{23}$$

Substituting Eq. 22, Eq. 24 into Eq. 14, we get:

$$\begin{aligned} \omega_* \notin \widehat{\Omega} &\implies \hat{a}_x^j(\omega) \wedge \omega_* \in \mathbb{P}_x^T \quad \forall j < T \forall \omega \in \widehat{\Omega} \forall x \in \mathcal{A} \\ &\implies \omega_* \in \widehat{\Omega} \end{aligned} \tag{24}$$

We have proven a contradiction.

$$\therefore \left|\widehat{\Omega}\right| > 0 \wedge \omega_* \neq \phi \implies \omega_* \in \widehat{\Omega} \tag{25}$$

We now have one final possibility to check: $\left|\widehat{\Omega}\right| > 0 \wedge \omega_* = \phi$. Can this happen?

From Eq 3 disagreement ($\omega_* = \phi$) can happen only in one of three ways:

1. One negotiator ends the negotiation: $\exists x \in \mathcal{A} \mid \omega_x^T = \phi$. This cannot happen because of Eq. 7.
2. A negotiator breaks the rules and repeates too early: $\left(\omega_x^T \in \mathbb{P}_x^\infty \wedge |\mathbb{P}_x^\infty| \leq \beta\right)$. This cannot happen because $\beta = 0$.
3. All negotiators are repeating offers: $\forall x \in \mathcal{A} \mid \omega_x^T = \omega_x^{T-1}$. We need to show that this is also cannot happen.

We only need to show that it is never the case that $\left|\widehat{\Omega}\right| > 0 \wedge \exists T \in \mathbb{Z} \mid \omega_x^T = \omega_x^{T-1} \quad \forall x \in \mathcal{A}$.

We will do a proof by contradiction again.

To arrive at round T, all rational outcomes for x must have been offered by the agent:

$$\omega_x^T = \omega_x^{T-1} \implies \omega \in \mathbb{P}_x^{T-2} \quad \forall \omega \in \overline{\Omega}_x \tag{26}$$

This is true for both negotiators which means:

$$\omega_x^T = \omega_x^{T-1} \forall x \in \mathcal{A} \implies \omega \in \mathbb{P}_x^{T-2} \quad \forall \omega \in \overline{\Omega}_x \forall x \in \mathcal{A} \tag{27}$$

This means that all possible rational outcomes for both negotiators have been offered.

$$\because \widehat{\Omega} \subseteq \overline{\Omega} \subseteq \overline{\Omega}_x \forall x \in \mathcal{A}$$

$$\therefore \omega \in \mathbb{P}_x^T \quad \forall \omega \in \widehat{\Omega} \quad \forall x \in \mathcal{A} \tag{28}$$

Now consider negotiator x that rejects win-win deal $\omega \in \widehat{\Omega}$. From Eq. 12, we know that a better or equivalent offer for x must have been proposed earlier:

$$\omega_y^i = \omega \wedge \neg a_{yx}^i \implies \exists \psi \in \mathbb{P}_y^{i-1} \cap \widehat{\Omega} \mid \psi \succcurlyeq_x \omega \tag{29}$$

Because ψ was offered first, it is also not worse than ω for agent y:

$$\because J_y^\psi < J_y^\omega$$

$$\therefore \psi \succcurlyeq_y \omega \wedge \psi \succcurlyeq_x \omega$$

Because $\omega \in \widehat{\Omega}$, we have:

$$\psi \in \widehat{\Omega}$$

This means that a negotiator can reject a win-win deal only if it accepted one earlier. Therefore no negotiator rejects all win-win deals.

$$\forall x \in \mathcal{A} \exists \omega_x \widehat{\Omega} \mid \omega_x \in \mathbb{A}_x^T$$

Let $Q_x \subset \widehat{\Omega}$ be the set of win-win deals rejected by negotiator x and $q_x^\top$ ($q_x^\perp$) be the best (worst) such outcome in Q_x for x. By definition

$$q_x^\top \succcurlyeq_x q_x^\perp \wedge q_y^\top \succcurlyeq_y q_y^\perp \tag{30}$$

Because a rejected offer must have a non-dominated offer accepted before it (Eq. 12), we have:

$$\exists \alpha_x \in \widehat{\Omega} - Q_x \mid \alpha_x \succcurlyeq_x q_x^\top \tag{31}$$

Because the negotiation did not end in agreement until step T and all win-win deals were offered by all negotiators (Eq. 28), we must have no win-win deal that is accepted by both negotiators:

$$\left|\widehat{\Omega} \setminus (Q_x \cup Q_y)\right| = 0 \tag{32}$$

From Eq. 31, Eq. 30, and E. 32 (and their symmetric counterparts), we get:

$$q_y^\top \succcurlyeq_x \alpha_x \succcurlyeq_x q_x^\top \tag{33}$$

By symmetry

$$q_x^\top \succcurlyeq_y \alpha_y \succcurlyeq_y q_y^\top \tag{34}$$

From Eq. 33 and Eq. 34 we get:

$$q_x^\top \approx_x q_y^\top \wedge q_x^\top \approx_y q_y^\top \tag{35}$$

In summary:

$$\left|\widehat{\Omega} \setminus (Q_x \cup Q_y)\right| = 0 \implies q_x^\top \approx_x q_y^\top \wedge q_x^\top \approx_y q_y^\top \tag{36}$$

Because of the lexical ordering rule in Eq. 4, we know that one of these two outcomes must have been received first by both negotiators. Moreover, by defintion, they both have been rejected. Therefore, there is another outcome that must have been accepted by both negotiators before them which must be a win-win deal and cannot be a member of either Q_x or Q_y:

$$\exists \psi \in \widehat{\Omega} \mid \psi \notin Q_x \wedge Q_y \tag{37}$$

From Eq. 38 and Eq. 37 we have:

$$\left|\widehat{\Omega} \setminus (Q_x \cup Q_y)\right| = 0 \implies \left|\widehat{\Omega} \setminus (Q_x \cup Q_y)\right| > 0 \tag{38}$$

which is a contradiction.

$$\therefore \omega_x^T = \omega_x^{T-1} \implies \omega_* \neq \phi$$

which is a contradiction.

$$\therefore \left|\widehat{\Omega}\right| > 0 \implies \omega_* \neq \phi \tag{39}$$

From Eq. 39 and Eq. 25, we get:

$$\therefore \left|\widehat{\Omega}\right| > 0 \implies \omega_* \in \widehat{\Omega} \tag{40}$$

TAU is Exactly Complete.

□

This is supported empirically in the paper by the fact that in both the preliminary and main experiments, the Agreement Rate of TAU was always 100% and that the agreement was always on the Pareto-frontier which implies being in the set of win-win deals (because every Pareto outcome is a win-win deal).

TAU with no repetition is NOT Exactly Complete

Theorem 5 (Incompleteness). TAU if modified to allow no repetition (called TAUNR hereafter) is not exactly complete.

Proof. We only need to find a single counter-example to prove this theorem. Consider two SCS negotiators A and B such that $\omega_1 \succ_A \phi \succ_A \omega_2 \not\succ_A \omega_3$, and $\omega_3 \succ_B \omega_2 \succ_B \omega_1 \succ_B \phi$. Negotiator A offer ω_1 then have to offer ϕ as it has nothing else to offer in the second round. This means that $\omega_A^2 = \phi$ and the negotiation will end in disagreement despite having a win-win outcome ω_1. □

Empirically, this shows in the main experiment in the fact that the agreement rate of TAUNR is not 1 when the opponent reserved value is set to 0.5 or 0.9 (it is around 0.85, 0.25 in these two cases).

It is interesting to note that this problem did not appear in the preliminary experiment. This may be due to the fact that the reserved values for all domains in this experiment (as used in ANAC) were either zero or very small and that the domains are balanced as shown by (de Jonge 2022). The definition of domain balance is beyond the scope of our paper and is not needed to understand anything in it. Please refer to (de Jonge 2022) for the exact definition and implications.

TAU is an Outcome-Perfect Negotiation Mechanism

Theorem 6 (OPNM). TAU is an Outcome-Perfect Negotiation Mechanism for scenarios with discrete outcome-spaces.

Proof. We have already shown that using SCS with TAU always terminates (Theorem 1), is Exactly Rational (Theorem 2), is Exactly Complete (Theorem 4), and is Exactly Optimal (Theorem 3).

∴ TAU is an Output-Perfect Negotiation Protocol. □

A Note about SCS

It can be shown that, under mild assumptions, SCS is a dominant strategy for TAU and is a best response to itself for the set of all possible discrete domains $\mathbb{D}^F$. This will be formally proven in an extended version of the paper. Due to lack of space, this result was omitted from the current paper. This feature of SCS is not needed for any of the conclusions reported in the current paper and is only mentioned here for the curious reader.

In the paper, we only alluded to the fact that we usually need more information about opponent's preferences than in the case of MiCRO to have any chance of finding a better response to SCS and provided a justifying example.

Examples

In this section we show some examples of TAU+SCS and AOP running to help intuiting how they work. All the examples shown in the section can be reproduced by running testtau.py and testaop.py provided in the supplementary materials.

Example on a Small Domain (TAU)

To reproduce, run

> python testtau.py small
> python testtau.py small –beta=-1

The resulting negotiation is visualized in Fig. 1. The left panel shows the negotiation in a 2D figure where the x-axis is the utiltiy of outcomes for the first negotiator and the y-axis is the utility for the second negotiator. Each outcome (no matter the dimensionality of the outcome-space) is repressented by a point in this space. The shaded areas are the areas above the reserved value for the corresponding negotiator. The Pareto-frontier, Nash Bargaining Solution, Maximum Welfare outcome and the agreement (if any) are all marked in the figure. Offers from the first negotiator are show in blue and from the second negotiator are shown in red. small transparent markers represent offers that were rejected and larger bold markers represent offers that ere accepted.

The right panel shows on top the offers of the first negotiator and on bottom the offers of the second negotiator. In both cases, we show the utility of the offer for the negotiator that offered it in continuous lines (blue for first negotiator and red for second negotiator). We also show that utility of the offer for the negotiator receiving it (not known to the negotiator offering it) in broken line. Markers have the same types and meaning as in the left panel. We also show the reserved value of each negotiator in a horizontal line in both graphs.

Examining this example shows some features that are quite general for TAU with SCS:

- Both negotiators start offering from their best outcome down. The x-axis negotiator goes from right to left and the y-axis negotiator from top to bottom
- When two outcomes are in the same vertical (horizontal) line, the x-axis (y-axis) negotiator has no way to know that and may offer them in any order but will offer them all. Nevertheless, the y-axis (x-axis) negotiator will accept the one best for it (the one on the Pareto-frontier) and once it does that it rejects any other offers from the same vertical (horizontal) line. This is why Pareto optimal outcomes all appear bold in the figure and gives an intuitive understanding of why the protocol is Exactly optimal.
- Every outcome that is better for any negotiator than the final agreement gets a chance to be offered. As expected the Pareto Distance is zero. The whole negotiation took 13 rounds to finish (2ms)

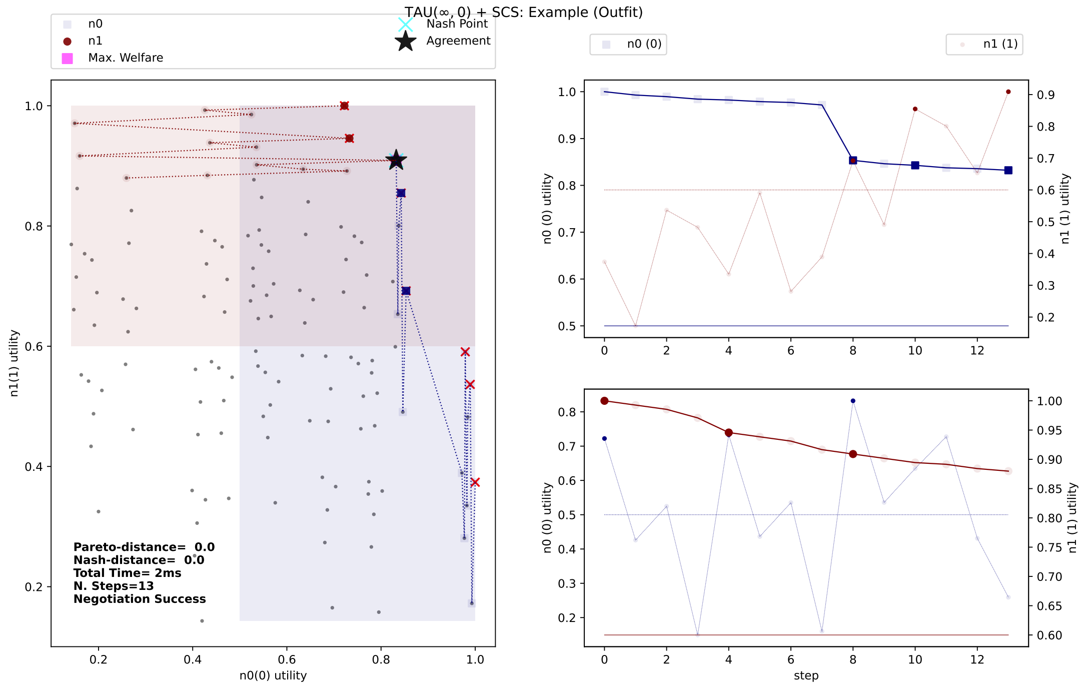

Figure 1: Example run of TAU+SCS on the Outfit Domain

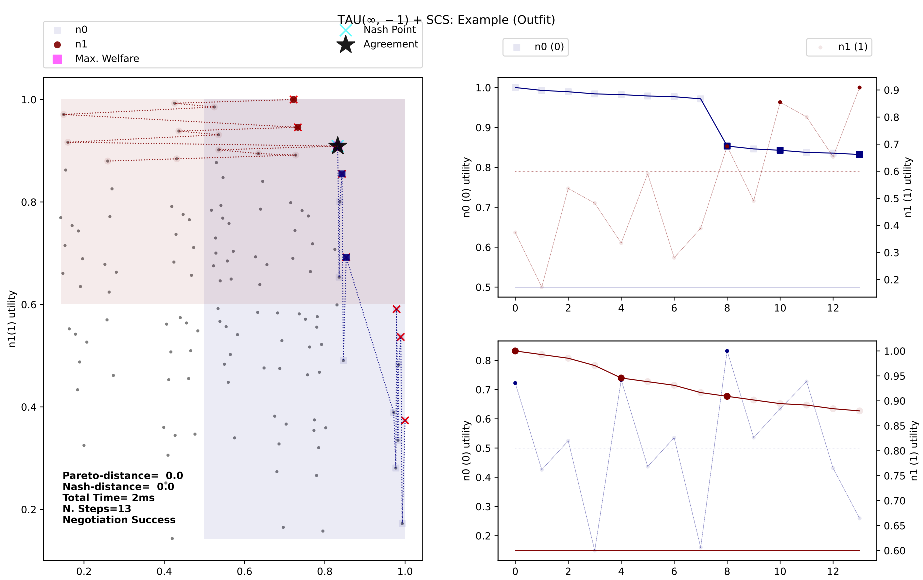

Figure 2: Example run of TAUNR+SCS on the Outfit Domain

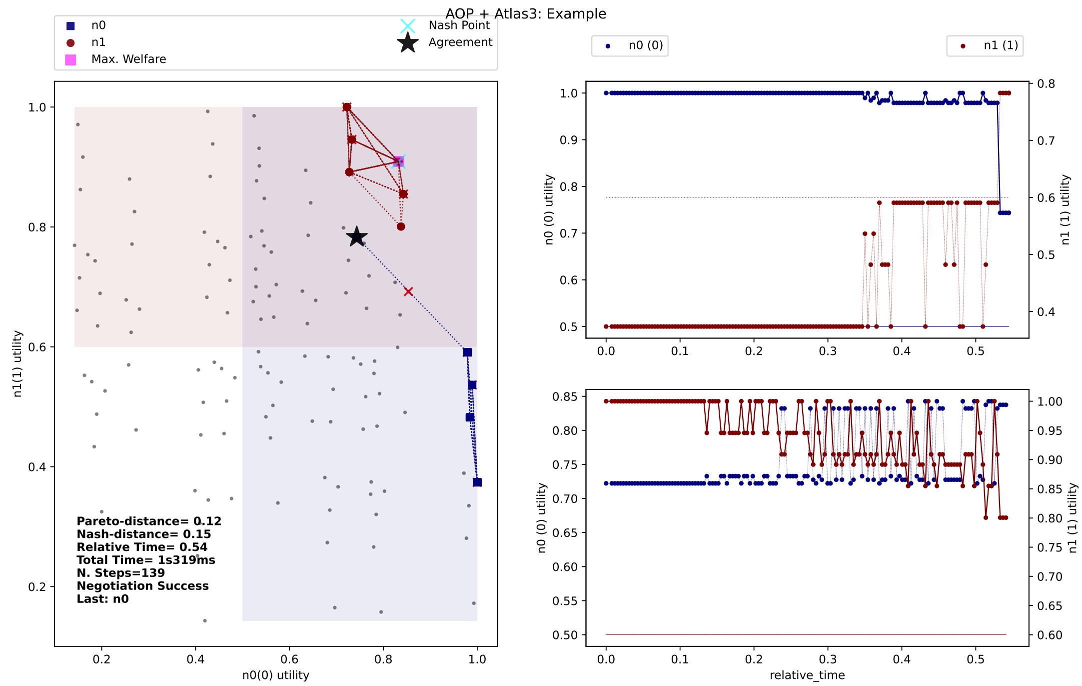

Figure 3: Example run of AOP+Atlas3 on the Outfit Domain

The behavior of TAUNR in this case (as shown in Fig. 2) is exactly the same because there was no need for repetition.

Example On a Small Domain (AOP+Atlas3)

To reproduce, run:

> python testsop.py small –strategy=Atlas3

For comparison, shows the behavior of one of the best state-of-the-art AOP strategies (Atlas3 (Mori and Ito 2017)). Examining this example shows some interesting behavior

- The negotiation started with a stretch offers in which each negotiator simple offered its best outcome repeatedly. This happened for around 15% of the negotiation time in this case but in some other cases this may constitute most of the negotiation time.
- Atlas3 uses opponent modeling to avoid offering outcomes not likely to be accepted near the end. It did not need to try every single outcome better than the agreement. This is a two-edged sword. On one hand, it reduces information revelation about the preferences of the negotiator (good). On the other, it may miss good outcomes. This is clearly the case in this example as the final agreement was not even on the Pareto-frontier (which as shown in both the main and preliminary experiments in the paper is common among AOP strategies).
- The negotiation took around 1.3 seconds which is around 65 times slower then TAU with SCS. This number should not be taken too seriously as implementation details may have some effect here. Nevertheless, the main experiment shows that the difference is orders of magnitude on average which is highly unlikely to be solely due to implementation details.

Example on a Difficult Domain

To reproduce, run: > python testtau.py difficult
> python testtau.py difficult –beta=-1
> python testsop.py difficult –strategy=micro
> python testsop.py difficult –strategy=Atlas3

We now try the same algorithms (plus MiCRO) on a difficult version of the same negotiation. Here we use the Outfit domain again (128 outcomes) but change reserved values to be 0.8, 0.9 which makes agreement much harder. For all of the methods that failed, the optimal win-win outcome was offered at some point but was not accepted and the chance was lost. The proposed method avoids this problem.

In this case, only TAU with SCS can find the agreement. All other methods fail. Fig. 4, 5, 6, 7 show these examples.

Example: MiCRO Incompleteness

To reproduce, run:

> python testsop.py incomplete
> python testtau.py complete

Fig. 9 shows the result of running MiCRO on the NiceOrDie domain (from the ANAC 2013 set). The fig-

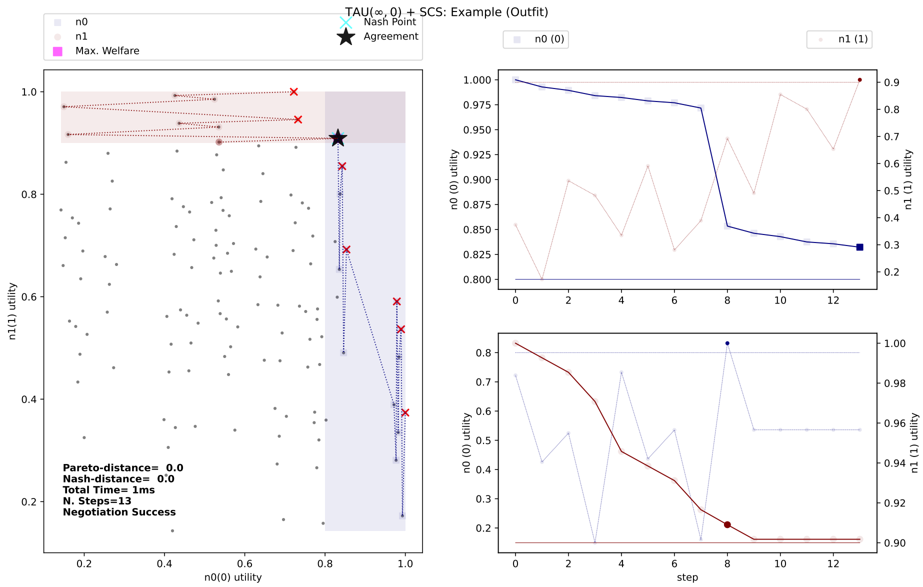

Figure 4: Example run of TAU+SCS on a difficult version of Outfit Domain

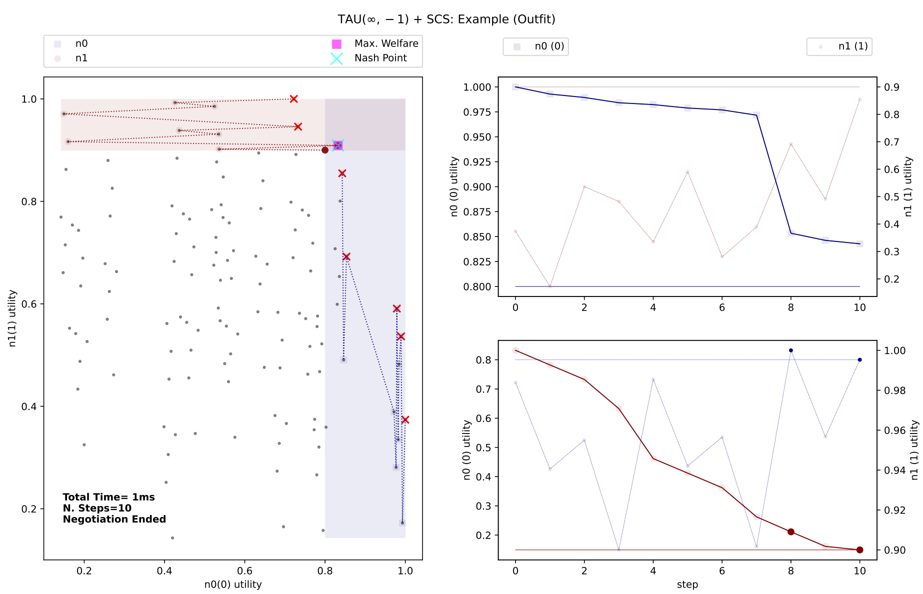

Figure 5: Example run of TAUNR+SCS on a difficult version of Outfit Domain

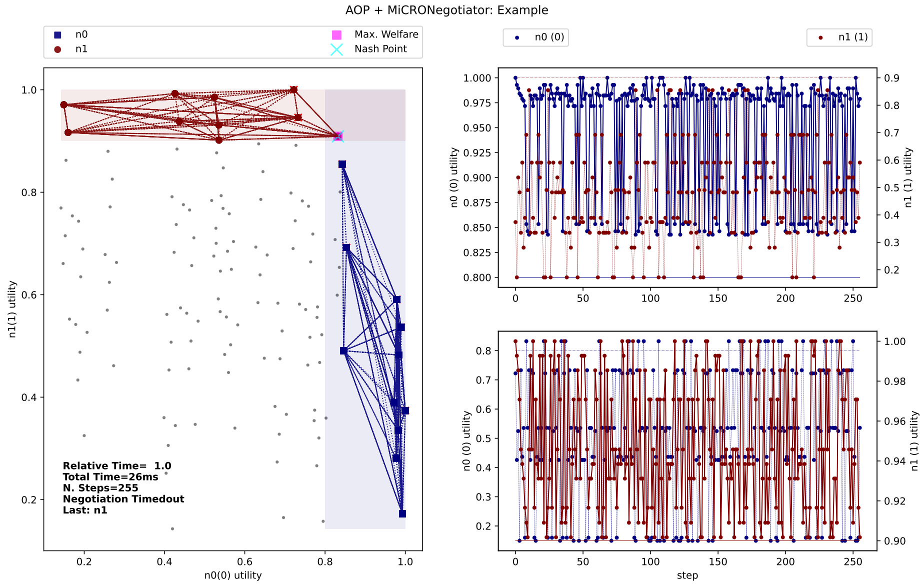

Figure 6: Example run of AOP+MiCRO on a difficult version of Outfit Domain

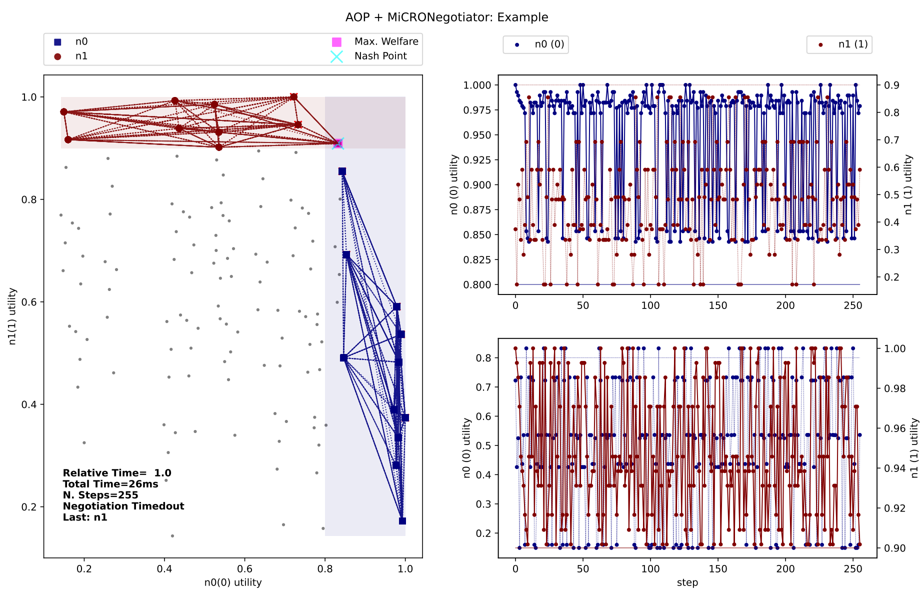

Figure 7: Example run of AOP+MiCRO on a difficult version of Outfit Domain

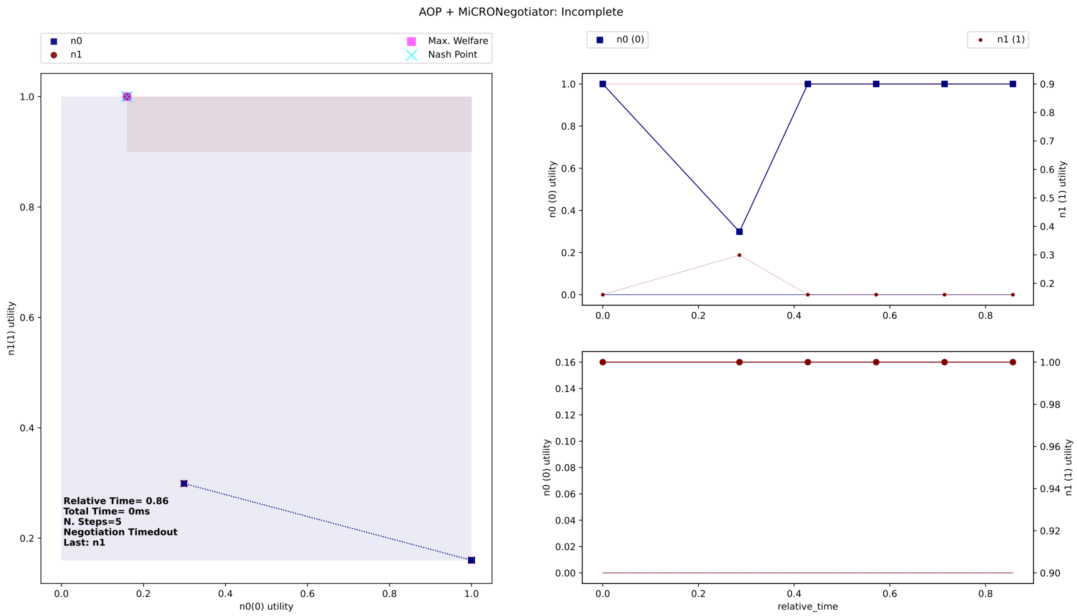

Figure 8: Example run of AOP+MiCRO on the NiceOrDie Domain showing incompleteness

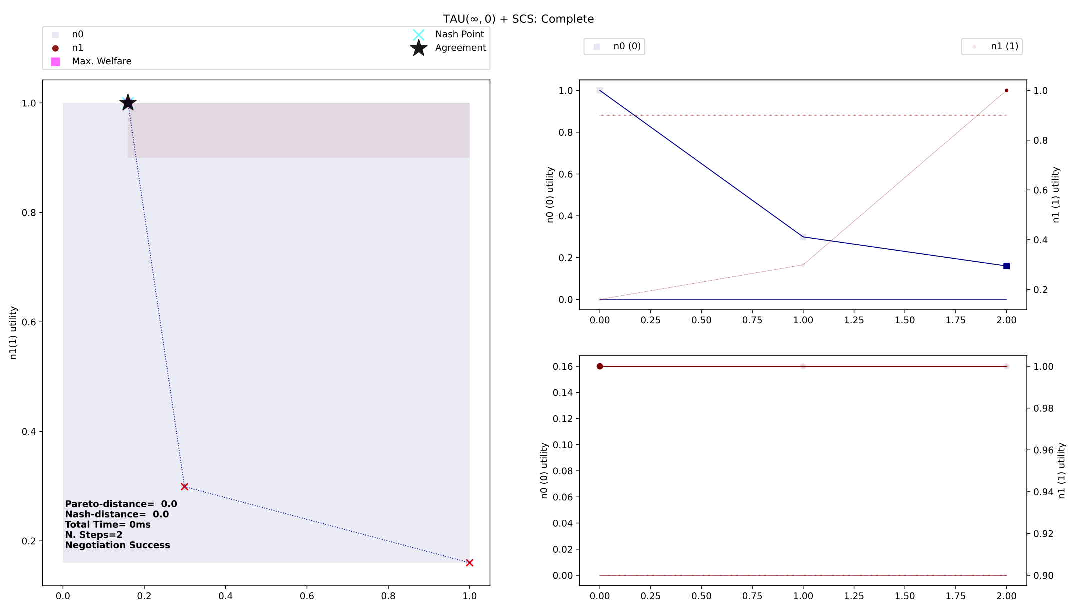

Figure 9: Example run of TAU+SCS on the NiceOrDie Domain showing agreement

ures shows another example that supports the example given in the paper to argue that MiCRO is not complete. In this case there are only three outcomes and one of them is a win-win deal. Nevertheless, MiCRO fails in getting an agreement. For comparison, Fig. 8 shows that the proposed protocol and strategy can get the win-win agreement in this case in two rounds.

Evaluation

Notes about Evaluation

Most research in automated negotiation targets new strategies for existing protocols. Evaluating new strategies is easy. Either use a tournament like experiment or some form of Empirical Game Theoretic analysis. In our case, we are proposing a new protocol which means that there are no existing strategies to compare with. Furthermore, the main other protocol we are comparable with has not known best strategy in any sense (even though MiCRO comes close to be the best baseline available). For this reason, we opted to evaluate our proposed protocol with an extremely simple strategy against AOP (the most widely used automated negotiation protocols) with the best strategies we could find for it. We believe that this comparison is the best that can be done under the circumstances.

Another possibility was to try to run all possible strategies of AOP against each other for each domain and compare the average result against our proposed protocol with its simple strategy. Actually, this is kind of what (de Jonge 2022) did showing that MiCRO against itself dominates such comparisons and we use it in our evaluation experiments. Moreover, this evaluation method is not very informative because the average may go down due to the existence of lethal combinations of strategies that do not get any agreements for example. Moreover, most negotiation strategy are developed first by playing against themselves. Consider a decision maker deciding between using TAU or AOP. What they care about is the performance of the protocol (given the best strategies it can find for it). We believe that our evaluation methods provides enough evidence for this decision maker to make their decision even in the absence of other competitive strategies for TAU.

As a matter of practical necessity (to avoid infinite loops), In all experiments, we put an upper limit of $30min$ on any single negotiation which is much larger than what is needed by any negotiators. This was only hit by some of the state-of-the-art algorithms – most likely due to entering an infinite or practically infinite loop – in few cases (e.g. See the notes on the main experiment below).

This limit was never hit by the proposed protocol and strategy combination nor by MiCRO nor most of the state-of-the-art algorithms. This limit was never hit in the first experiment and was only hit by Atlas3 on three out of the 18 domains (Icecream, cameradomain, and planes) and only for the difficult case with reserved value of ‘0.9‘ for the second negotiator. These few failures may be due to bugs in the implementation or the Atlas3 strategy or the NegMAS-GENIUS bridge and they did not go away with multiple trials.

To err against our own proposal, we did not count the cases in which this limit is hit by any strategy when calculating its performance (including Time). This can only improve the performance of state-of-the-art algorithms compared with our proposed approach. Moreover, any such failures where not considered when calculating statistical significance (again exaggerating the performance of state-of-the-art algorithms against our proposal). Moreover, counting them would have decreased the performance of Atlas3 only slightly anyway.

All raw results of both experiments and all statistical test results are available in the supplementary materials.

All experiments were conducted using a fork of NegMAS 0.8 (Mohammad, Greenwald, and Nakadai 2019) platform with its NegMAS-GENIUS (Lin et al. 2014) for the official implementations of state-of-the-art (SOTA) AOP strategies on a MacBook Pro 2021 with an Apple M1 Pro CPU and 64GB of RAM.

As we focus on bilateral negotiations and most SOTA agents were developed with the assumption that the utility function is a linear aggregation, we used the domains for ANAC 2013 that were the latest set of domains satisfying both conditions (18 domains).

For each experiment, we ran AOP with each strategy employed by both negotiators 3 times (leading to 54 negotiations for every strategy in every protocol variation). For TAU, we ran the two variations TAU and TAUNR with the proposed SCS. For AOP we ran negotiations with either a wall time limit of 3min (usually used in ANAC competitions) or $2n_o$ rounds (but not both). All differences were checked using factorial t-test (and Wilcoxon's nonparametric test) with Bonferroni's multiple-comparisons correction and differences reported hereafter are all statistically significant if not otherwise indicated

Definitions of Performance Metrics

We used the following measures of performance:

Utility: The average utility received by an agent above its reserved value relative to its maximum: $(u_x(\omega_*) - u_x(\phi)) / \max_{\omega \in \Omega^+}(u_x(\omega) - u_x(\phi))$.

Welfare: The average total utility received by both negotiators relative to the maximum possible welfare: $\sum_{x \in \mathcal{A}} (u_x(\omega_*) - u_x(\phi)) / \max_{\omega \in \Omega^+} \sum_{x \in \mathcal{A}} (u_x(\omega) - u_x(\phi))$.

Agreement Rate Fraction of negotiations with at least one rational outcome that end in agreement.

Pareto Optimality: One minus the Euclidean distance between the final agreement and the Pereto-frontier: $1 - \min_{\omega \in \mathcal{P}} \sqrt{\sum_{x \in \mathcal{A}} (u_x(\omega_*) - u_x(\omega))^2}$.

Nash Optimality: One minus the Euclidean distance between the final agreement and the Nash Bargain-

Table 1: Proposed Protocol and Strategy vs. AOP with baseline and SOTA strategies (n_o is outcome space size)

Condition	Welfare		AR	P. Optimality		N. Optimality		Rounds		Time	
	mean	std	mean	mean	std	mean	std	mean	std	mean	std
AOP($2n_o$)+AgentK	0.67	0.46	0.69	0.71	0.46	0.56	0.49	1.64	0.29	182.45	348.81
AOP($2n_o$)+Atlas3	0.95	0.11	1.00	0.99	0.03	0.83	0.21	0.56	0.38	64.30	133.07
AOP($2n_o$)+Boulware	0.91	0.23	0.94	0.96	0.11	0.82	0.27	1.45	0.27	5.05	18.18
AOP($2n_o$)+CUHK	0.48	0.49	0.50	0.54	0.50	0.35	0.52	1.89	0.46	104.84	196.64
AOP($2n_o$)+Caduceus	0.58	0.47	0.61	0.65	0.46	0.48	0.47	1.77	0.15	1720.65	5210.13
AOP($2n_o$)+HardHeaded	0.55	0.50	0.56	0.62	0.46	0.49	0.52	1.93	0.09	135.65	222.90
AOP($2n_o$)+MiCRO	0.96	0.11	1.00	1.00	0.00	0.86	0.22	0.18	0.18	0.16	0.30
AOP($2n_o$)+NiceTfT	0.79	0.36	0.83	0.89	0.23	0.69	0.33	1.77	0.62	3429.56	11955.94
AOP(3min)+AgentK	0.77	0.40	0.80	0.83	0.36	0.65	0.41	168.49	387.30	141.16	38.11
AOP(3min)+Atlas3	0.95	0.11	1.00	0.99	0.03	0.84	0.21	97.05	275.06	48.44	29.65
AOP(3min)+Boulware	0.94	0.12	1.00	0.99	0.04	0.84	0.22	14414.05	34216.39	127.30	20.96
AOP(3min)+CUHK	0.71	0.39	0.78	0.82	0.34	0.52	0.40	177.66	385.18	170.79	40.78
AOP(3min)+Caduceus	0.59	0.47	0.61	0.64	0.48	0.45	0.47	170.11	381.96	163.41	15.35
AOP(3min)+HardHeaded	0.82	0.37	0.83	0.90	0.23	0.76	0.34	176.98	385.44	173.16	8.47
AOP(3min)+MiCRO	0.96	0.11	1.00	1.00	0.00	0.86	0.22	0.18	0.18	0.15	0.29
AOP(3min)+NiceTfT	0.91	0.19	0.96	0.94	0.23	0.70	0.34	166.98	381.10	151.28	63.32
TAU+SCS	0.96	0.11	1.00	1.00	0.00	0.86	0.22	0.18	0.18	0.52	1.42
TAUNR+SCS	0.96	0.11	1.00	1.00	0.00	0.86	0.22	0.18	0.18	0.48	1.32

ing Solution[1]: $1 - \sqrt{\sum_{x \in \mathcal{A}} (u_x(\omega_*) - u_x(\omega_{nbs}))^2}$.

Rounds: The average number of offers exchanged relative to the outcome space size: $\left|\mathcal{T}^{\mathcal{S}}\right| / |\Omega|$.

Time: The average length of a negotiation in seconds.

Preliminary Experiment

We use three baselines for the preliminary experiment: Time-based Boulware strategy, The Nice Tit for Tat (NTfT) strategy by Baarslag, Hindriks, and Jonker (2013), and MiCRO by de Jonge (2022). To compare the proposed approach against using SOTA strategies with AOP, we also used the following set of strategies for AOP: Caduceus by Güneş, Arditi, and Aydoğan (2017), Atlas3 by Mori and Ito (2017)(most recent winners of non-repeated tracks with no uncertainty in 2015 and 2016), AgentK by Kawaguchi, Fujita, and Ito (2013), Hardheaded by van Krimpen, Looije, and Hajizadeh (2013), CUHK by Hao and Leung (2014) (most recent winners of non-repeated bilateral negotiation tracks at ANAC in 2010, 2011, 2012).

Table 1 shows the result of this experiment. TAU and AOP with MiCRO achieve the highest welfare (0.958 compared with 0.950 for Atlas3 — statistically insignificant) using the lowest number of rounds (0.181 compared with 0.564 for Atlas3 when round limited) and leads to 100% agreement rate and a Pareto Optimality of One (The only strategies achieving this). TAU and AOP with MiCRO were by far the fastest (Rounds and Time – statistically significant) with MiCRO being slightly faster in Time (statistically insignificant) due mainly to the simpler evaluation rule of AOP. Moreover, they had the highest Nash Optimality Solution despite using only outcome ordering (0.86 compared to 0.84 for Atlas3 — statistically insignificant). The high performance of MiCRO for these scenarios was also reported by (de Jonge 2022) and attributed to special characteristics of ANAC preferences (made even more special with having zero or very small reserved values for all negotiators in all scenarios rending any agreement better than disagreement).

[1]MiCRO and SCS has a disadvantage here as they have no access to the utility values, only the induced ordering.

Limitations and Extensions

In the paper, we mentioned that one of the main issues with TAU is that its complex evaluation rule makes it difficult to use for Human-Agent Negotiation. The main difficulty is that the evaluation rule of TAU is not memoryless (i.e. it requires keeping track of information between rounds). For AOP, the evaluation rule is memoryless which makes it much easier to use by people.

That is why we have the first parameter for TAU (the cardinality α) which limits the amount of memory needed to keep track of the past of the negotiation. In all our analysis we assumed that $\alpha = \infty$ which makes sense for Agent-Agent Negotiations that are not too long (i.e. on outcome-spaces that are not too large). For huge outcome spaces or when limited memory is available (or people are involved), we may set α to an appropriate finite number. This will most likely lead to loss of theoretical guarantees but we did not investigate this possibility yet. Is it possible to have a small α yet – empirically – keep good performance metrics (i.e. Pareto Optimality and Agreement Rate)?

As indicated in the paper, continuous outcome-spaces provide another challenge. An extension of TAU that can define some limit on similarity between offers may be able to extend the proposed method to continuous and hybrid outcome spaces but this will require further theoretical work to investigate the effects this may have on the guarantees of the protocol and its performance.

Finally, TAU is one of a family of protocols we call Generalized Bargaining Protocols that extend AOP keeping its main advantages (relative simplicity, offers coming only from negotiators, and being unmediated) while providing new opportunities for designing new protocols targeting specific applications. A future publication will focus on this set of protocols and investigate TAU within that framework.

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