Relevant Eqn for Source-Free RLC Circuit

Type	Condition	Criteria	α	ω_0	Response
1) Parallel / Series	overdamped	$\alpha > \omega_0$ OR ($\xi > 1$)	$(1/2RC)$ / $(R/2L)$	$\frac{1}{\sqrt{LC}}$	$A_1 e^{s_1 t} + A_2 e^{s_2 t}$, where $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega^2}$
2) Parallel / Series	Critically Damped	$\alpha = \omega_0$ OR $\xi = 1$	$(1/2RC)$ / $(R/2L)$	$\frac{1}{\sqrt{LC}}$	$e^{-\alpha t}(A_1 t + A_2)$
3) Parallel / Series	Underdamped	$\alpha < \omega_0$ OR $0 < \xi < 1$	$(1/2RC)$ / $(R/2L)$	$1/\sqrt{LC}$	$e^{-\alpha t}(B_1 \cos \omega_d t + B_2 \sin \omega_d t)$ where, $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Ans (1)

Step (I): at $t = 0^-$

$V_c(0^-) = 50$ Volt , capacitor is charged.

Step (II): at $t = 0^+$

$V_c(0^+) = V_c(0^-) = 50$ Volt

Step (III): at $t = \infty$

$V_c(\infty) = 0$

$\therefore V_c(t) = [V_c(0^+) - V_c(\infty)]e^{-t/\tau} + V_c(\infty)$

Here $\tau = RC = 1600$ msec

$V_c(t) = 50 \cdot e^{-(10^5 t)/16}$

$\therefore V_c(160\mu sec) = 18.39$ volt

Ans (2)

Part (a): After $t=0$ sec, the current source has turned itself off and R_2 is shorted. We are left with a parallel RLC circuit comprised of R_1, a 4H inductor, and 1 μF capacitor.

1st Method: We may now calculate (for $t>0$)

$$\alpha = \frac{1}{2RC} = \frac{1}{2\times R_1 \times 10^{-6}}$$

& $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4\times 10^{-6}}} = \frac{1}{2\times 10^{-3}}$$

Therefore, to establish a critically damped response in the circuit for $t>0$, we need to set —

$$\alpha = \omega_0$$

$$\therefore R_1 = 1\,K\Omega$$

2nd Method:

For parallel R-L-C circuit,

Damping Ratio, $\xi = \frac{1}{2R}\sqrt{\frac{L}{C}}$

& for critical damping, $\xi = 1$

Therefore, $$1 = \frac{1}{2R_1}\sqrt{\frac{4}{10^{-6}}}$$

$$\therefore R_1 = 1\,K\Omega$$

Part (b): We note that at $t=0^-$, the current source is ON. Inductor & Capacitor can be treated as short & open circuit respectively.

Here $V_c(0^+) = V_c(0^-) = V_c(0) = 100$ Volt (given)

$$0.5 \times \left(\frac{R_1}{R_1+R_2}\right) \times R_2 = 100$$

$\because R_1 = 1 K\Omega$ (already evaluated)

after solving, $R_2 = 250\Omega$

Part (c): Response of Parallel R-L-C circuit in critically damped condition —

$$V_c(t) = (C_1 + C_2 t) e^{-\alpha t} \quad \text{— (1)}$$

$$\alpha = \frac{1}{2R_1C} = 500$$

$\because V_c(0^+) = 100$ (given)

$\therefore C_1 = 100$

Now, $$\frac{d V_c(t)}{dt} = C_1(-\alpha) \cdot e^{-\alpha t} + C_2\left[t(-\alpha) \cdot e^{-\alpha t} + e^{-\alpha t}\right]$$

$\because$ $$\frac{dV_c(t)}{dt} = \frac{I_c}{C}$$

$$\Rightarrow \frac{dV_c(0^+)}{dt} = \frac{I_c(0^+)}{C} = \frac{-0.5}{10^{-6}} = (-0.5 \times 10^6)$$

$$= (-5 \times 10^5)$$

after solving –

$$C_2 = (-45 \times 10^4)$$

$$\therefore V_c'(t) = \left(100 - 45 \times 10^4 t\right) e^{-500t}$$

$$\therefore V_c(1\,msec) = \left(100 - 45 \times 10^4 \times 10^{-3}\right) \cdot e^{-500 \times 10^{-3}}$$

$$\simeq -212 \text{ Volt}$$

Ans ③: $I_c = 12\,A$

$I = 13\,A$

Ans ④:

$$I_1 = \begin{cases} 200\,mA, & t < 0 \\ -240 \cdot e^{-50{,}000t} \cdot mA, & t \geq 0 \end{cases}$$

$$I_L = \begin{cases} 360\,mA, & t < 0 \\ 360 \cdot e^{-50{,}000t} \cdot mA, & t \geq 0 \end{cases}$$

Ans ⑤:

(a) $i_L(0^-) = 1\,A$

(b) $V_c(0^-) = 48\,V$

(c) $i_R(0^+) = 2\,A$

(d) $i_c(0^+) = -3A$

(e) Here $\alpha = \frac{1}{2RC} = 5$

$$\omega_o = \frac{1}{\sqrt{LC}} = 4.89$$

$\because \alpha > \omega_o \Rightarrow$ overdamped condition

Response of overdamped parallel R-L-C circuit

$$V_c(t) = C_1 \cdot e^{S_1 t} + C_2\, e^{S_2 t}$$

where, $S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$

$$S_1 \cong (-4), \quad S_2 \cong (-6)$$

$$\therefore V_c(t) = C_1 e^{-4t} + C_2 e^{-6t} \quad \text{—— ①}$$

$$V_c(0^+) = V_c(0^-) = 48 = C_1 + C_2 \quad \text{—— ⓐ}$$

$$\frac{dV_c(0^+)}{dt} = \frac{I_c(0^+)}{C} = \frac{-3}{(1/240)} = C_1(-4) + C_2(-6)$$

$$4C_1 + 6C_2 = 720 \quad \text{—— ⓑ}$$

after solving — $C_1 = (-216)$, $C_2 = (264)$

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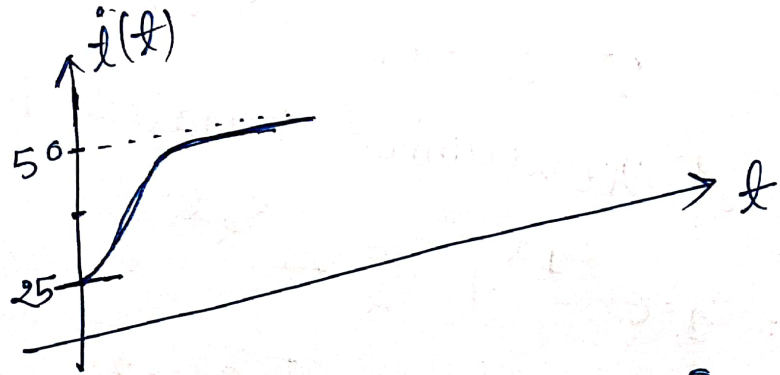
$\therefore V_c(t) = -216\, e^{-4t} + 264\, e^{-6t}$

$\therefore V_c(0.2) = -17.54$ Volt

Ans ⑦: $\tau = \frac{L}{R_{eq}} = 2$ sec

$i(t) = 25 + 25\left(1 - e^{-0.5t}\right) u(t)$ A



Ans ⑧:

$$V_c(t) = \begin{cases} 100\text{V}, & t < 0 \\ 20 + 80\, e^{-t/1.2}, & t \geq 0 \end{cases}$$

$$i(t) = \begin{cases} 0.1923 \text{ A}, & t < 0 \\ 0.1 + 0.4 \cdot e^{-t/1.2}, & t > 0 \end{cases}$$