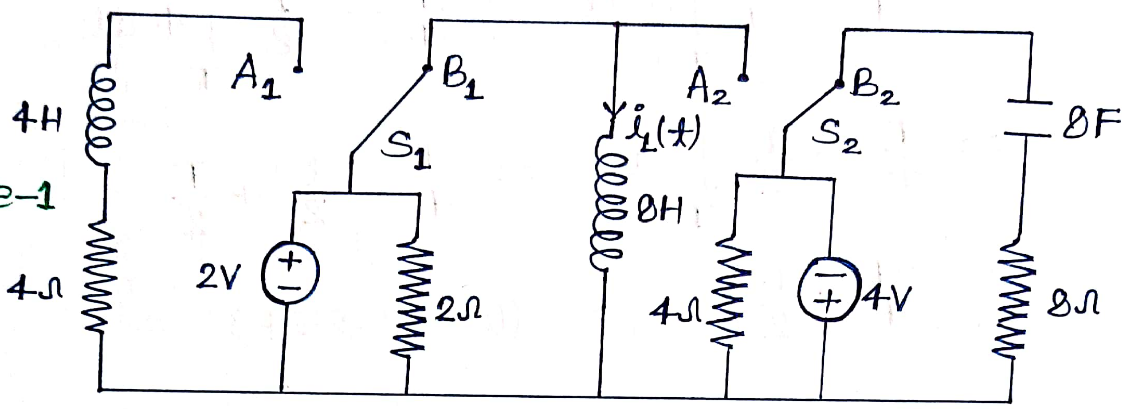
ASSIGNMENT-2
SOLUTION

SOL(1)

Case(I): $0 \leq t < T$; switch S_1 is connected to position B_1 & switch S_2 is connected to position B_2.

Figure-1



initial current through inductor $= i_L(0^-) = 0A$ (Given)

$\therefore i_L(0) = i_L(0^+) = i_L(0^-) = 0A$

Here inductor start charging by 2 volt source.

$L = 8H, \quad V = 2$ volt

$$\therefore i_L(t) = \frac{1}{L}\int_0^t V \cdot dt = \frac{1}{8}\int_0^t 2\, dt = \frac{1}{4}\int_0^t dt = \left(\frac{t}{4}\right) \quad \text{— (a)}$$

$\therefore$ at $t = T$; $i_L(T) = \frac{T}{4}$

(2.25 P)

Case(II): $T \leq t < 2T$; switch S_1 is connected to position A_1 & switch S_2 is connected to position A_2.

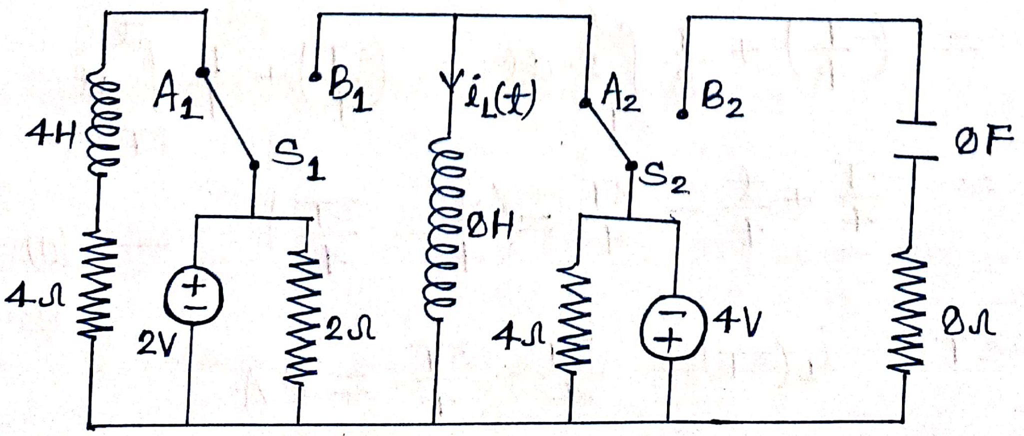


Figure-2

initial current through inductor $= i_L(T) = \frac{T}{4}$

Here inductor start charging in opposite direction by 4V source.

$L = 8H, \quad V = 4$ volt

$$\therefore i_L(t) = \frac{T}{4} - \frac{1}{L}\int_T^t V \cdot dt = \frac{T}{4} - \frac{1}{8}\int_T^t 4\, dt$$

$$= \frac{T}{4} - \frac{1}{2}\int_T^t dt = \frac{T}{4} - \frac{t}{2} + \frac{T}{2}$$

$$= \left(\frac{3T}{4} - \frac{t}{2}\right) \quad \text{— (b)} \quad (2.25\,P)$$

$$\therefore \text{at } t = 2T; \quad i_L(2T) = \frac{3T}{4} - \frac{2T}{2} = \left(\frac{-T}{4}\right)$$

Case (III) :- $2T \leq t < 3T$; switch S_1 is connected to position B_1 & switch S_2 is connected to position B_2.

By using Figure-1,

initial current through inductor $= i_L(2T) = \left(\frac{-T}{4}\right)$

Here inductor start charging by 2V source.

$L = 8H, \quad V = 2$ volt

$$\therefore i_L(t) = i_L(2T) + \frac{1}{L}\int_{2T}^t V\, dt$$

$$= \left(-\frac{T}{4}\right) + \frac{1}{8}\int_{2T}^t 2 \cdot dt = \left(\frac{-T}{4}\right) + \frac{1}{4}\int_{2T}^t dt$$

$$= \frac{-T}{4} + \frac{t}{4} - \frac{2T}{4} = \left(\frac{t}{4} - \frac{3T}{4}\right) \quad \text{— (c)}$$

$(2.25\,P)$

$$\therefore \text{at } t = 3T; \quad i_L(3T) = \frac{3T}{4} - \frac{3T}{4} = 0\,A$$

Case(IV):- $3T \leq t < 4T$; switch S_1 is connected to position A_1 & switch S_2 is connected to position A_2.

By using Figure-2,

initial current through inductor $= i_L(3T) = 0\,A$

Here inductor start charging by 4 V source in opposite direction.

$$L = 8H, \quad V = 4 \text{ Volt}$$

$$\therefore i_L(t) = i_L(3T) - \frac{1}{L}\int_{3T}^{t} V\,dt$$

$$= 0 - \frac{1}{8}\int_{3T}^{t} 4\,dt = -\frac{1}{2}\int_{3T}^{t} dt$$

$$= -\frac{1}{2}(t - 3T) = \frac{3T}{2} - \frac{t}{2} \qquad \text{— (d)} \quad (2.25P)$$

$$\therefore \text{at } t = 4T;\ i_L(4T) = \frac{3T}{2} - \frac{4T}{2} = \left(\frac{-T}{2}\right)$$

Case(V):- $4T \leq t < 5T$; switch S_1 is connected to position B_1 & switch S_2 is connected to position B_2.

By using Figure-1,

initial current through inductor $= i_L(4T) = \left(\frac{-T}{2}\right)$

$$L = 8H, \quad V = 2 \text{ Volt}$$

$$\therefore i_L(t) = i_L(4T) + \frac{1}{L}\int_{4T}^{t} V\,dt = \left(\frac{-T}{2}\right) + \frac{1}{8}\int_{4T}^{t} 2\,dt$$

$$= \frac{-T}{2} + \frac{1}{4}\int_{4T}^{t} dt = \frac{-T}{2} + \frac{t}{4} - T = \frac{t}{4} - \frac{3T}{2} \qquad \text{— (e)}$$

$$\therefore i_L(5T) = \frac{5T}{4} - \frac{3T}{2} = \left(\frac{-T}{4}\right)$$

Now, $i_L(t) = eq^n(a) + eq^n(b) + eq^n(c) + eq^n(d) + eq^n(e) + \cdots$ TOC

$$\therefore\ i_L(t) = \frac{1}{4}r(t) - \frac{1}{2}r(t-T) + \frac{1}{4}r(t-2T) - \frac{1}{2}r(t-3T) + \frac{1}{4}r(t-4T) - \frac{1}{2}r(t-5T) + \cdots$$

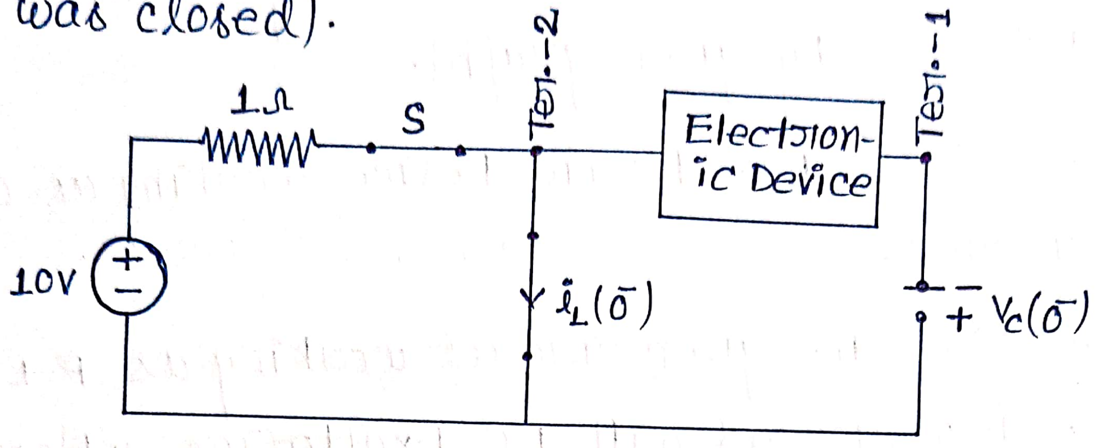
$$= \frac{1}{4}\left\{t\,u(t) + (t-2T)\cdot u(t-2T) + (t-4T)\,u(t-4T) + \cdots\right\} - \frac{1}{2}\left\{(t-T)\,u(t-T) + (t-3T)\,u(t-3T) + (t-5T)\,u(t-5T) + \cdots\right\}$$

$$= \frac{1}{4}\sum_{n=0}^{\infty}(t-2nT)\cdot u(t-2nT) - \frac{1}{2}\sum_{n=0}^{\infty}[t-(2n+1)T]\cdot u[t-(2n+1)T]$$

$\rightarrow$ (1 P)

Sol(2):

Step(I): Analysis of circuit at $t=0^-$ (when switch was closed).



at $t=0^-$ (steady state),

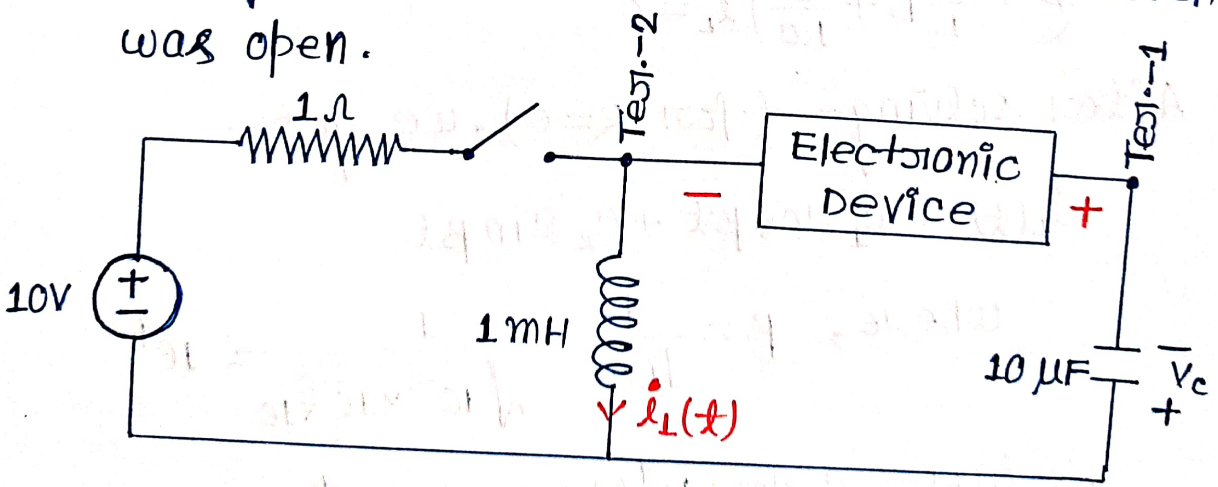
Inductor behave as a short circuit, Hence–

$$i_L(0^-) = \left(\frac{10}{1}\right) = 10\,A \rightarrow (1\,P)$$

Capacitor behave as a open circuit, Hence–

$$V_c(0^-) = 0 \text{ volt}$$

Step(II): Analysis of circuit at $t>0$ (when switch was open.



at $t=0^+$ (Transient State)

$i_L(0^+) = i_L(0^-)$

$i_L(0^+) = 10\,A$ [Inductor does not allow sudden change of current] $\rightarrow (1\,P)$

When Terminal-1 of Electronic Device is positive & Terminal-2 of Electronic Device is Negative then Electronic Device working as a Short Circuit (By given graph).

at $t>0$, Electronic Device working as a Short Circuit.

at $t>0$, the given circuit working as R-L-C series circuit with DC Excitation. where—

$R=0$ (Undamping condition)

General differential eq^n for series R-L-C circuit—

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = 0$$

$$D^2 i_L + \frac{R}{L} D i_L + \frac{i_L}{LC} = 0$$

$$\left(D^2 + \frac{R}{L} D + \frac{1}{LC}\right) i_L = 0$$

After solving— (for $R=0$), we get—

$$i_L(t) = C_1 \cos \beta t + C_2 \sin \beta t$$

where, $\beta = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-3} \times 10^{-6} \times 10}} = 10^4$

$$\therefore i_L(t) = C_1 \cos(10^4 t) + C_2 \sin(10^4 t) \quad \text{—— (1)}$$

$\because$ at $t=0^+$, $i_L(0^+) = i_L(0) = 10A$

& at $t=\infty$, $i_L(\infty) = 0\,A$

ive

$\therefore\ c_1 = 10\ \&\ c_2 = 0$

Hence, $i_L(t) = 10\cos(10^4 t)\ A$ $\rightarrow$ (5P)

$\therefore$ Voltage across capacitor, $V_c(t) = \frac{1}{C}\int i_L(t)\cdot dt$

$$V_c(t) = \left(\frac{1}{10\times10^{-6}}\right)\int 10\cdot\cos(10^4 t)\,dt = 100\cdot\sin(10^4 t)\ V$$

$\rightarrow$ (2P)

$\therefore$ Steady state magnitude voltage across capacitor $(t\rightarrow\infty) = 100$ Volt $\rightarrow$ (1P)