2022530 → Syed BE - Ass - 2

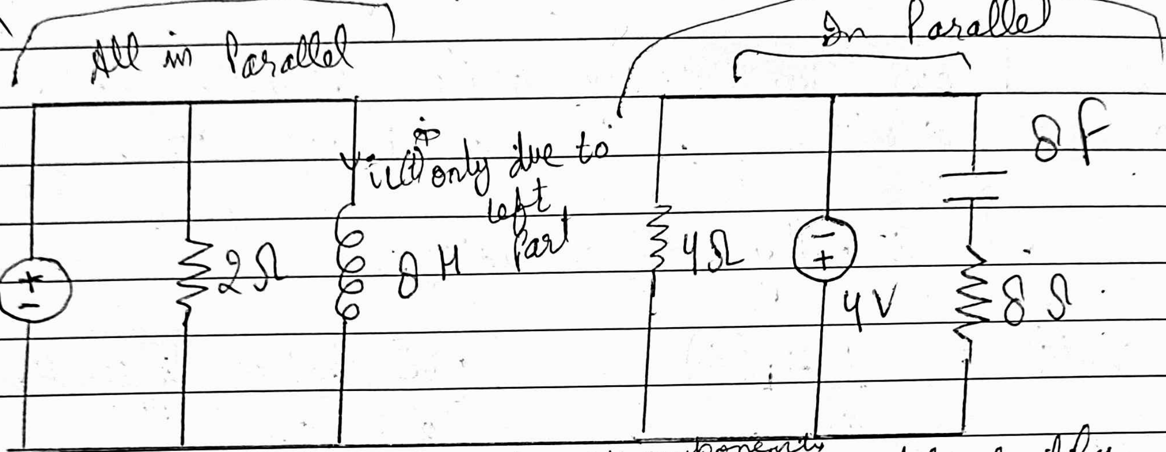
## Ques 1

As, initially the current across inductor = 0 & presuming $C_V = 0$.

so, when the $S_1$ is connected to $B_1$ & $S_2$ connected to $B_2$.

This is circuit for even to odd period

the circuit would look like this.




( Here Both the circuits components work independently as Both have no in & out current

~~We know, $v_c(0^-) = v_c(0^+) =$~~

The time interval given :

$2nT \le t < (2n+1)T$, $n = 0, 1, 2, 3 \ldots$

$(2n+1)T \le t < (2n+2)T$, $n = 0, 1, 2, \ldots$

By observing above,

0T — T 2T — 3T 4T — 5T ... } $S_1 \leftrightarrow B_1$, $S_2 \leftrightarrow B_2$

T — 2T 3T — 4T ... } $S_1 \leftrightarrow A_1$, $S_2 \leftrightarrow B_2$

Thus we can say there is alternate switching at each time interval T.

~~at t=~~

In the interval
$0^{+} \le t \le T$

$i_L(0^-) = i_L(0^+) = 0.$ ~~We st. know in the steady~~

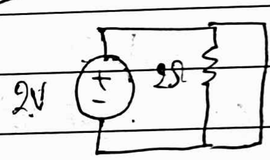
But as time passes, current will start to pass through inductor ~~as, in the stea~~ by short circuit.

$\because$ we know in steady state inductor behaves as a shor circuit. & we know, $V_L = L\frac{di}{dt}$

As, 2V directly to inductor.

$$V_L(t) = L\frac{di_L(t)}{dt}$$ , ~~This is a differential eq. of $i_L$~~

$$\int 2 = 8\int \frac{di_L(t)}{dt}$$



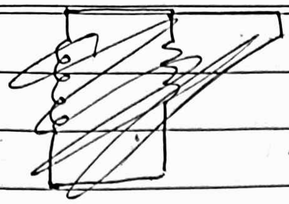
By integrating. $\Rightarrow$ $[t]_0^t$

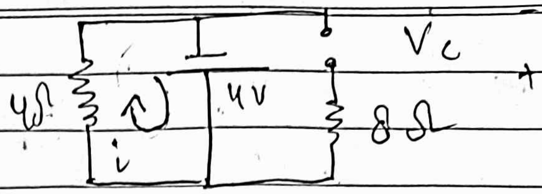
$$\int 2dt = \int 8\, di_L(t) \Rightarrow (2t)_0^t = 8[i_L]_{i_L(0)}^{i_L(t)}$$

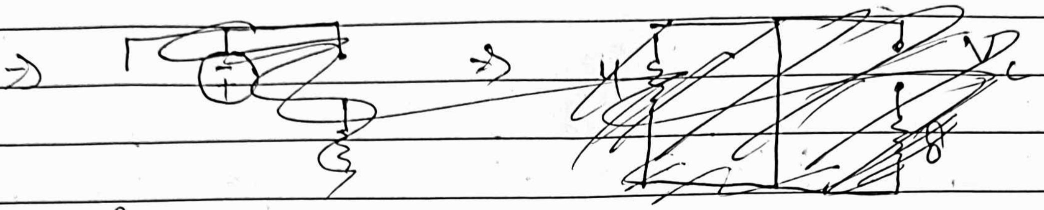
$$i_L(t) = \frac{t}{4}, \quad 0 \le t \le T \quad \& \; i_L(0^+) = i_L(0^-)$$

As, we can $i_L(t)$ through inductor is linear in time interval $0 \le t < T$

Now, in the same interval component 2: ~~two~~





$R_{eq} = 8\,\Omega$

By KVL $4 - 4i = 0$, $i = 1$

$i = 4/4 = 1$ Amp.

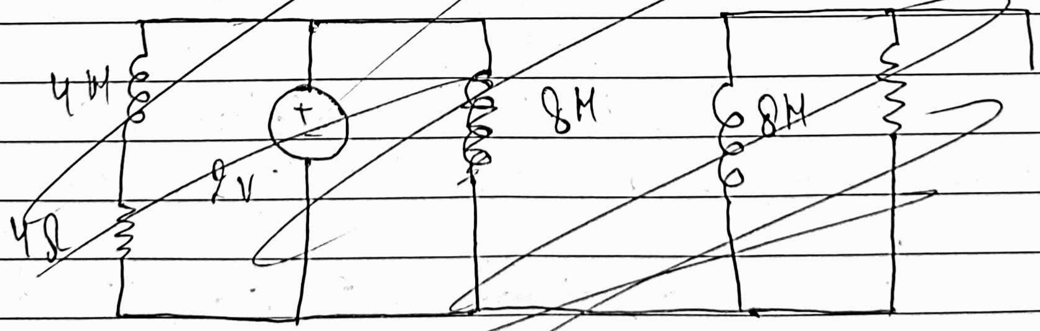
~~&~~

So, $V_c = 4\,V$ in the steady state.

~~we can~~

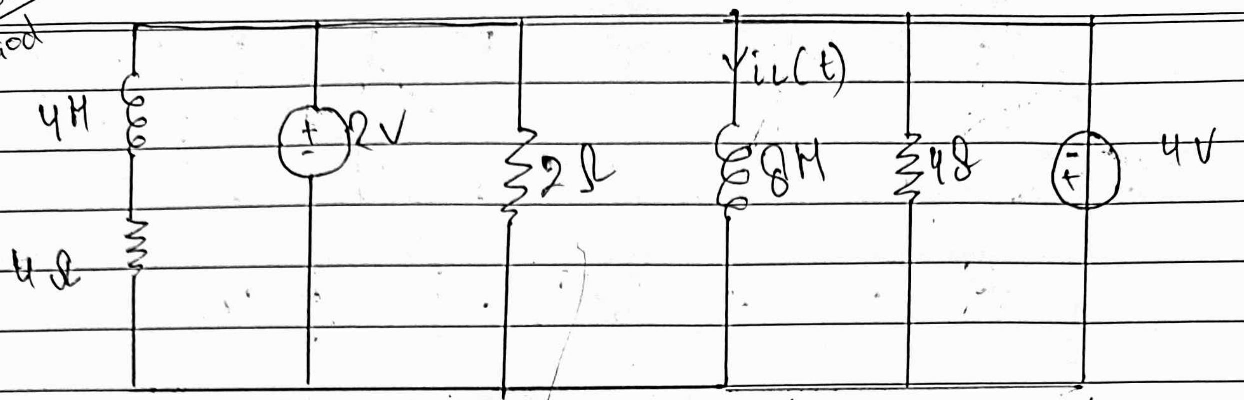
~~So, in the time interval, $0 - T$, we will find the ...~~

Now Now, to interval ~~0 to~~
$T - 2T$ $S_1$ to $A_1$ & $S_2$ to $A_2$



Now, $T \le t < 2T$

For all odd to even period




Here $i_L(t)$ only by Right component.
Both components work independently

∵ As, we know, ~~for second~~
$L\,di/dt = V$

$$\int 8\frac{di_L(t)}{dt} = \int -4$$

$$\Rightarrow -4[t]_T^t = 8\int_{T/4}^{i_L(t)} d\,i_L(t)$$

$$\Rightarrow T - t = 2i_L(t) - T/2$$

$$i_L(t) = \frac{3T}{4} - t/2, \quad \text{for } T \le t < 2T$$

Now, ~~t ≥~~ $2T \le t < 3T$ follow the first diagram
Again inductor directly to power source. (2V)

$$\int 2 = \int 8\frac{di_L(t)}{dt} \Rightarrow \int 2dt = 8\int_{-T/4}^{i_L(t)} d_L(t)$$

∵ Here, the current of
$i_L(T^-) = i_L(T^+)$
$i_L(2T^-) = i_L(2T^+)$

$$= t - 2T = 4(i_L(t) + T/4)$$

$$i_L(t) = t/4 - 3T/4$$

So we observe, that $i_L(2T) = i_L(t) = \frac{3T}{4} - 2T/2$
$= -T/4$

so for each end Point,
$\because$ end current of one interval serves as the starting current for next interval.

Again for $3T \leq t \leq 4T$.

$$\int -4 = \int \frac{8\, di_L(t)}{dt} = \int_{3T}^{t} -4\,dt = 2\int_{0}^{i_L(t)} di_L(t)$$

$$i_L(t) = \frac{3T}{2} - t/2$$

Here similarly, $i_L(3T^-) = i_L(3T^+)$

Again for $4T \leq t < 5T$,

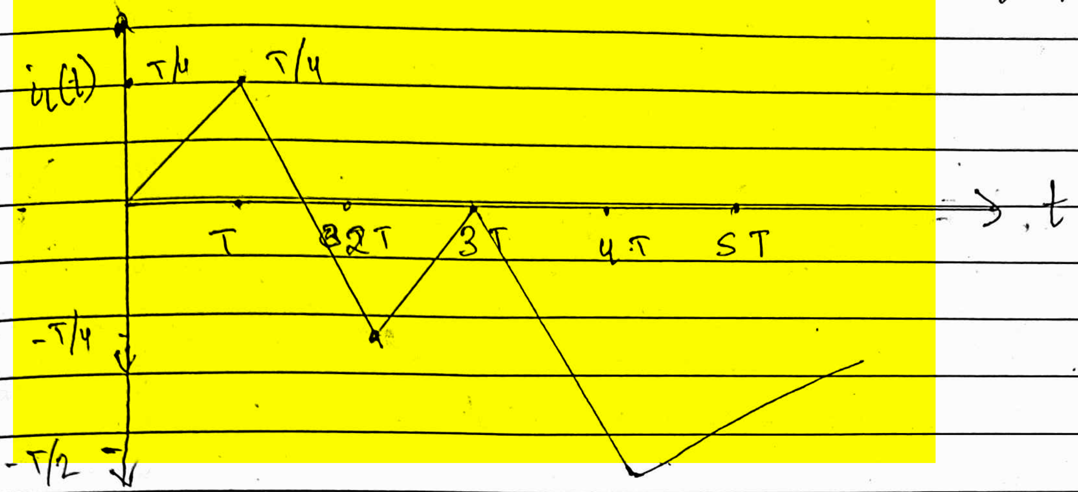
so, $2 = \frac{8\, di_L(t)}{dt} \Rightarrow \int_{4T}^{t} dt = 4\int_{-T/2}^{i_L(t)} di_L(t)$

$\Rightarrow$ ~~$t - 4T = 4i$~~
$\Rightarrow t - 4T = 4i_L(t) + 2T$
$= i_L(t) = t/4 - \frac{6T}{4}$

so, $i_L(t) = \begin{cases} t/4, & 0 \leq t < T \\ 3T/4 - t/2 & T \leq t < 2T \\ t/4 - 3T/4 & 2T \leq t < 3T \\ 3T/2 - t/2 & 3T \leq t < 4T \\ t/4 - \frac{6T}{4} & 4T \leq t \leq 5T \end{cases}$

.... and so on

Now, we can also plot a graph.




Now, we can use Laplace transform to Transform to unit step function.

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

$$\text{or } i_L(t) = \frac{t}{4} u(t) - \frac{t}{4} u(t - T)$$

$$+ \left(\frac{3T}{4} - \frac{t}{2}\right)\left(u(t-T) - u(t-2T)\right)$$

$$+ \left(\frac{t}{4} - \frac{3T}{4}\right)\left(u(t-2T) - u(t-3T)\right)$$

- - - and so on

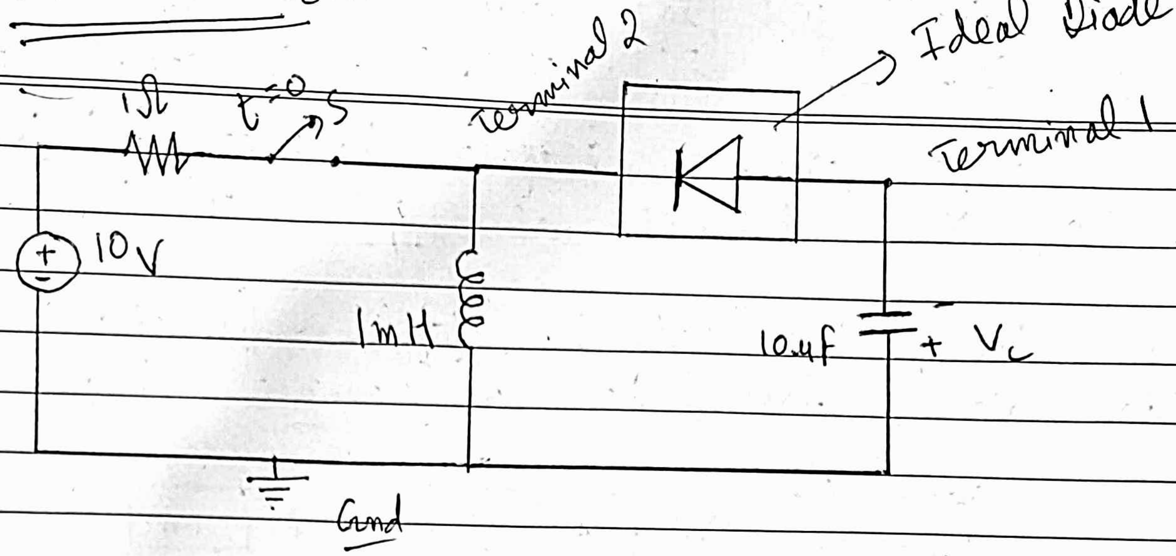
$\Rightarrow$ By solving the above Laplace transform we can change to the standard form

$$\text{or, } i_L(t) = \frac{t}{4} u(t) + 2 \sum_{n=1}^{\infty} (-1)^n (t - nT)\left(u(t - nT)\right)$$

where u is step function

a → $100 \sin(10^4 t) = V_c(t),\ t \geq 0^+$
b → 100 volt $= V_c$

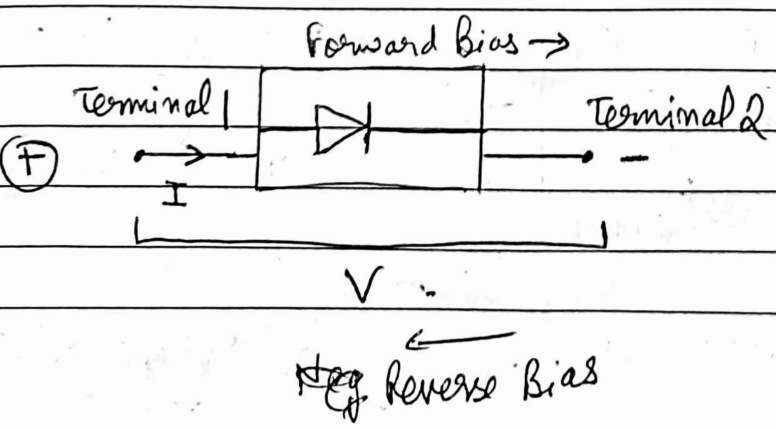
Ques - 2

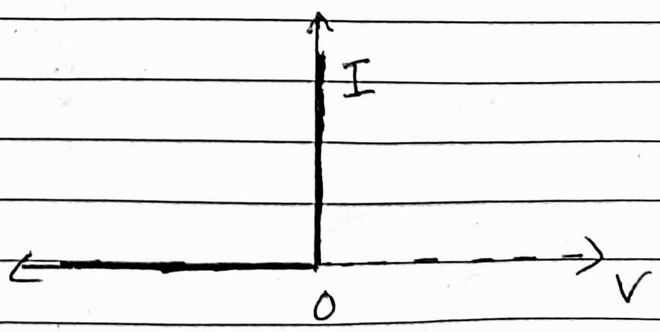



∵ The given electronic device is an Ideal diode as it can be inferred from the V-I characteristic curve
∵ when the diode is in forward bias
R = 0 & behaves like a short circuit.
∵ when the diode is in reverse bias.
R = ∞, like an open circuit.

~~When there is an application of positive voltage, in positive current increase~~

∵ when there is an application of ~~positive~~ voltage in the (+)ve direction (Terminal 1 to Terminal 2) (~~like open circuit~~) current increases. (see curve) (like short circuit)
∵ when there is an application of voltage in (−)ve direction (Terminal 2 to terminal 1), current does not increases & remains 0. (like open circuit)



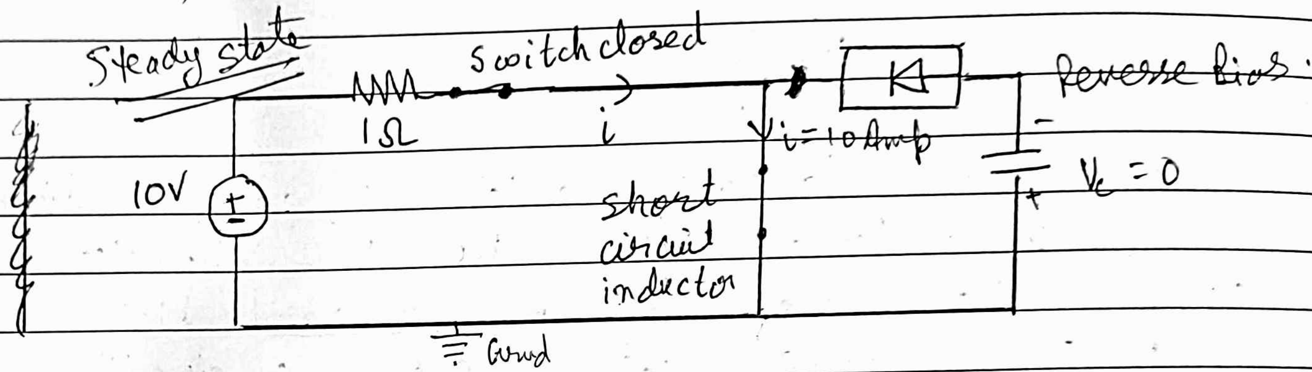




(V - I) characteristic curve.

when the circuit was left with switch closed. ie, from $(t \to -\infty \text{ to } 0)$ it was in steady state

$\because$ in steady state Inductor behaves as short circuit & capacitor behaves as open circuit.

When the voltage will be applies (Electronic device) (ideal diode) would be in reverse bias




By KVL $10 - iR = 0$

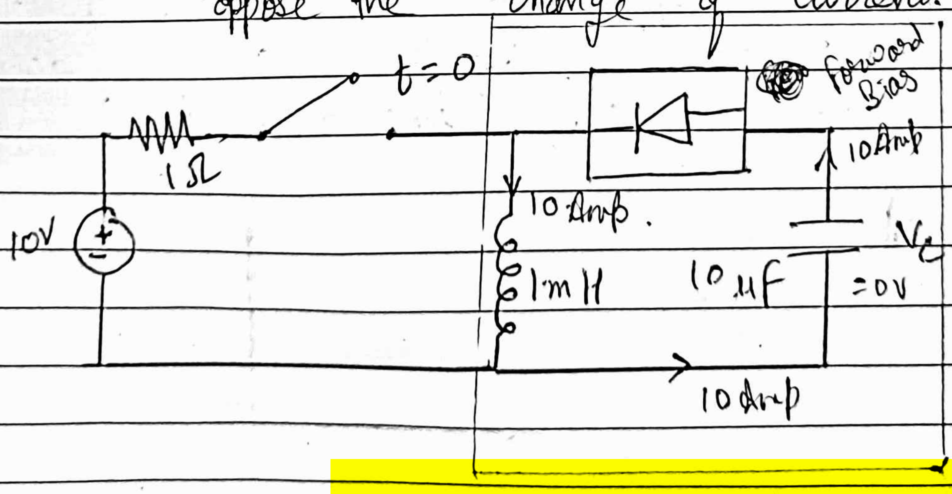
$i = 10/R = 10/1 = 10\,Amp.$

so, $V_c(0^-) = 0$ so, $i_L(0^-) = 10$ Amp

$$\boxed{V_c(0^-) = V_c(0^+) = 0}$$

switch open

Now, in the Transient state, inductor will oppose the change of current.




at $t \to 0^+$

Source free LC circuit

$V_c(0^-) = V_c(0^+) = 0$

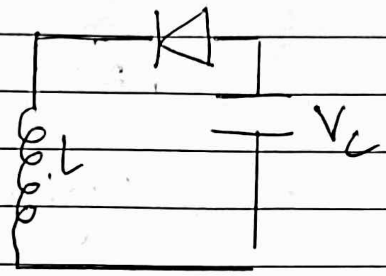
$i_L(0^-) = i_L(0^+) = 10$ Amp. [Property of Transient]

$$\boxed{i_L(0^-) = i_L(0^+) = 10 \text{ Amp}}$$ Property of LC circuit in transient state

This is like LC oscillation [but only half cycle possible]

∵ Now inductor will charge the inductor

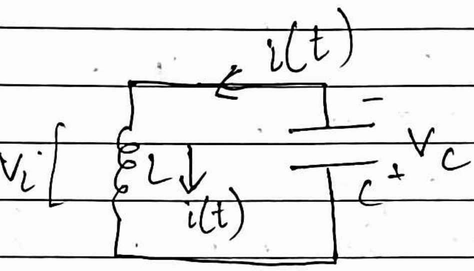
By KVL ~~in the~~




$\Rightarrow -V_L - V_C = 0$

$\Rightarrow -L\dfrac{d\,i(t)}{dt} - V_C = 0$

⇓




$\boxed{\because V_L = L\dfrac{di}{dt}}$ $\quad \boxed{\because q = CV \Rightarrow dq = C\,dV \quad i\,dt = dq}$

$\boxed{\because i(t) = C\dfrac{dV_C}{dt}}$

$\Rightarrow L\dfrac{di(t)}{dt} + \dfrac{q(t)}{C} = 0$

∵ As it has two energy storing devices so it is a second order circuit, so it has a differential eq of order 2

$\Rightarrow L\dfrac{di(t)}{dt} + \dfrac{1}{C}\int i\,dt = 0$ → This in integero differential eq.

$\Rightarrow L\dfrac{d^2 i(t)}{dt^2} = -\dfrac{1}{C}\,i$

$\Rightarrow \dfrac{d^2 i(t)}{dt^2} + \dfrac{1}{LC}\,i(t)$ } second order differential eq. for the LC series circuit

at time t ~~Here~~ $i(t)$ ~~denotes the current~~ in ~~the circuit or across~~ in inductor or ~~across~~ in capacitor

∵ Damping factor $\alpha = \frac{R}{2L}$ ∵ Natural freq. $= \omega_0 = \frac{1}{\sqrt{LC}}$

∵ Current $i(t)$ denotes the current passing through inductor or capacitor or circuit as $L$ & $C$ are in series

Now the characteristic eq.

$$D^2 + \frac{1}{LC} i(t) = 0 \Rightarrow \text{so, } D^2 = \frac{-1}{LC}$$

As, Negative here, so, $D = \pm \frac{i}{\sqrt{LC}}$ ] $i \rightarrow$ not current

here roots are complex conjugate of each others.

Here $i$ means symbol for complex roots. i.e. imaginary unit no. $\Rightarrow i = \sqrt{-1}$

[ $D = \alpha_1 = \frac{+i}{\sqrt{LC}}$ & $\alpha_2 = \frac{-i}{\sqrt{LC}}$

& $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1\times10^{-3} \times 10\times10^{-6}}} \Rightarrow \frac{1}{\sqrt{10^{-8}}} = \boxed{10^4 \text{ Hertz}}$

$\boxed{\alpha = \frac{R}{2L} = 0}$ as $R = 0$ & $\boxed{\omega_0 = 10^4 \text{ Hertz}}$

As, $\alpha < \omega_0$ so there is no damping so energy would be conserved & would oscillate.

As, initially after opening switch, $i_L(0^+) = 10$ Amp.

so, energy in the inductor (Magnetic energy) $= \frac{1}{2} L i^2 = \frac{1}{2} \times 10^{-3} \times 10 \times 10$

Initial energy of inductor. $\boxed{0.05 \text{ Joule}} \Leftarrow = \frac{10^{-1}}{2} = 0.05$ Joule

L → stores energy in the form of Magnetic field
& C → stores in the form of Inductor

As, this is LC oscillation, so, After some time & all energy would appear across Capacitor

$$= \cancel{\frac{1}{2}\frac{q^2}{C}} \quad \frac{q^2}{2C}$$

& As energy is conserved [R = 0]

$$\frac{1}{2}Li^2 = \frac{q^2}{2C}$$

$$0.05 \times 2 \times C = q^2 \Rightarrow q = \sqrt{0.05 \times 2 \times 10 \times 10^{-6}}$$

$$q = 0.001 \text{ coulomb.} \quad \& \quad q = CV, \quad \text{so } V = q/C$$

$$V_C = \frac{0.001}{10^{-5}} = 100 \text{ volt}$$

~~Here~~ Now
All energy is at capacitor
This is at when inductor has 0 current & C has Max voltage or $i_L = 0$ & $V_C \to$ Max

so, $\boxed{V_C = 100 \text{ Volt}}$

Also By the characteristic eq.

$$i(t) = e^{-\alpha t}[C_1 \cos \omega_d]$$

& $\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
& as $\alpha = 0$
$\omega_d = \omega_0$

$$= e^{-\alpha t}[C_1 \cos(\omega_0 t)]$$

$$= C_1 \cos \omega_0$$

$\alpha = 0$ so, $e^0 = 1$

Replace $C_1$ by A $= A \cos(\omega_0 t)$

where $C_1$ is Amplitude of oscillation or Replace $C_1$ by A

At

A → Amplitude

$\omega_0$ → Angular frequency of oscillation

~~As, $i_L(0^+) = i_C(0^+) = 10$ Amp~~

As $i_L(0^+) = i(0^+) = 10$ Amp.

$$i(t) = A\cos(\omega_0 t)$$

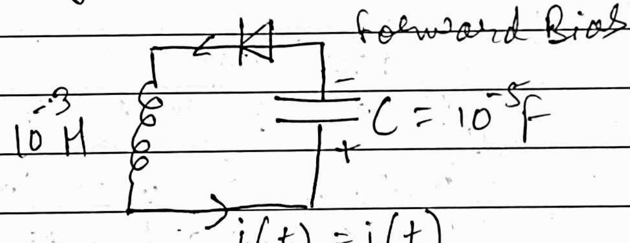
$$10 = A\cos(\omega_0 \times 0)$$

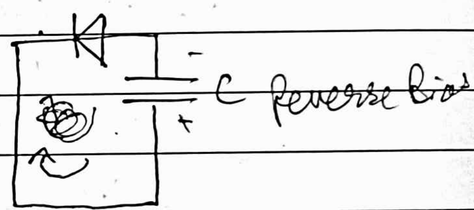
$$10 = A$$

so $\boxed{A = 10\ \cancel{\text{metres}}\ \text{Watts}}$

so, $i(t) = i_L(t) = 10\cos(\omega_0 t) = \boxed{10\cos(10^4 t)}$

As, current would not flow from inductor as then ideal diode would be in reverse bias, so there would be only one ~~cycle~~ half cycle or oscillation, ie, from L to C.

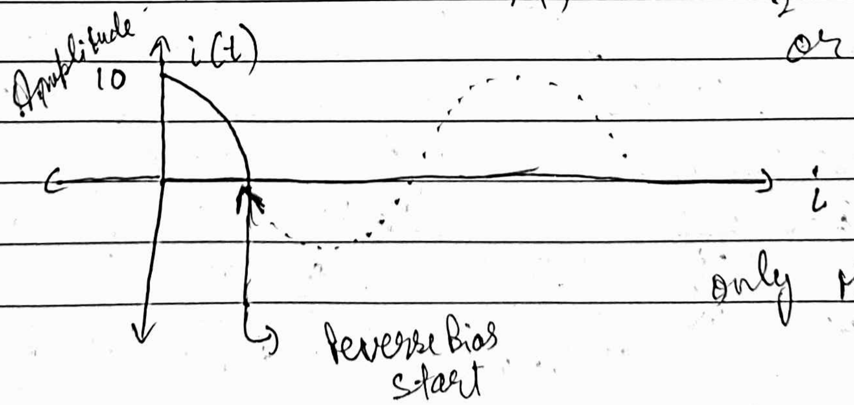


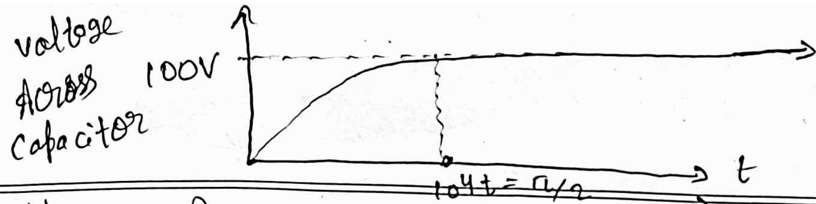

$i_L = 10$ Amp
$Q_C = 0$ Coul $V_C = 0$ Volt

$i_L = 0$ Amp
$Q = 0.001$ Coulomb
$V_c = 100$ Volt

so, when $10^4 t = \pi/2$, $t = \frac{\pi}{2} \times 10^{-4}$ secs
or 0.157 msec




only Half cycle Possible




steady state voltage $(V_c)$ at $10^4 t = \pi/2$

or $t = \frac{\pi}{2} \times 10^{-4}$ sec

Now By indegrating $i(t)$.

$\cancel{V_c(t) = \frac{1}{c}\int i\,dt}$ As, $V_c = \frac{1}{c}\int i\,dt$

so $V_c(t) = \frac{1}{c}\int i(t)\,dt$

$$= \frac{1}{c}\int 10\cos(10^4 t)\,dt$$

$$\cancel{= 10^5 \times 10 \int \cos(10^4 t)\,dt}$$

$$\boxed{C = 10\,\mu F}$$

$$\cancel{= 10^6 \sin(10^4 t)\,dt}$$

$$\cancel{10^5}$$

$$= \frac{1}{10 \times 10^{-6}} \times 10\,\frac{\sin(10^4 t)}{10^4}$$

$\because \int \cos(at) = \frac{\sin(at)}{a}$

$$\boxed{V_c(t) = 100\sin(10^4 t)}, \quad t \geq 0^+$$

Answer (a)

or say when $10^4 t = \pi/2$

And, at $t = 0.157$ ms, there is steady state

As, Here, LC oscillation would stop due to the presence of Diode ~~which~~ which will become Reverse Bias when capacitor will try to loose its charge, Hence this Half cycle can be said as steady state of Capacitor as Now voltage would not change so, when $10^4 t = \pi/2$

$V_c(t) = 100\sin(10^4 t) = 100\sin(\pi/2) = \boxed{100 \text{ Volt}} = V_c$

voltage Across capacitor steady state voltage Answer b