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Practical-1

Limits and continuity.

1) $$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$$

$$= \lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\right]$$

$$= \lim_{x\to a}\left[\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}\right]$$

$$= \lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \frac{1}{3}\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \frac{1}{3}\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3}\times\frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}} = \frac{1}{3}\times\frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}}$$

2) $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$= \lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$= \lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y}+\sqrt{a})}$$

$$= \lim_{y\to 0}\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$= \frac{1}{\sqrt{a+0}\left(\sqrt{a} + \sqrt{a}\right)}$$

$$= \frac{1}{\sqrt{a}\,(2\sqrt{a})}$$

$$= \frac{1}{2a}$$

3) $$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3}\sin x}{\pi - 6x}$$

substitute $x - \frac{\pi}{6} = h$

$$x = h + \frac{\pi}{6}$$

where $h \to 0$

$$= \lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{6}\right) - \sqrt{3}\sin\left(h + \frac{\pi}{6}\right)}{\pi - 6\left(h + \frac{\pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h + \pi}{6}\right)}$$

$$= \lim_{h \to 0} \frac{\cos h \cdot \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \cdot \frac{1}{2}\right)}{\pi - 6h + \pi}$$

$$= \lim_{h\to 0} \frac{\cos\frac{\sqrt{3}}{2}h - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}}{2}h}{-6h}$$

$$= \lim_{h\to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$= \lim_{h\to 0} \cancel{\sin 4h} \frac{\sin h\frac{4}{2}}{-6h}$$

$$= \lim_{h\to 0} \frac{\sin h}{3h} = \frac{1}{3}\lim_{h\to 0}\frac{\sin h}{\cancel{\sin} h} = \frac{1}{3}.$$

4) $\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$

By rationalizing Numerator & Denominator both

$$= \lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$= \lim_{x\to\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$= \lim_{x\to\infty} \frac{8}{2}\frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$= 4\lim_{x\to\infty}\frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

after applying limit

we get

$= 4.$

5) ① $F(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$ } at $x = \pi/2$

$F(\pi/2) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore F(\pi/2) = 0$

F at $x = \pi/2$ is define

a) $\lim_{x \to \frac{\pi}{2}^+} F(x) = \lim_{x \to \pi/2^+} \frac{\cos x}{\pi - 2x}$

By substituting Method

$x - \frac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$

$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{\pi - 2\left(\frac{2h + \pi}{2}\right)}$

$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{2}\right)}{-2h}$

$\lim_{h \to 0} \frac{\cos h . \cos \frac{\pi}{2} - \sin h . \sin \frac{\pi}{2}}{-2h}$

$\lim_{h \to 0} \frac{\cos h . 0 - \sin h}{-2h}$

$\lim_{h \to 0} \frac{-\sin h}{-2h}$

$\frac{1}{2} \lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{1}{2}$

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b) $\lim_{x \to \pi/2^-} F(x) = \lim_{x \to \pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$= \lim_{x \to \pi/2^-} \frac{2 \sin x . \cos x}{\sqrt{2 \sin^2 x}}$

$= \lim_{x \to \pi/2^-} \frac{2 \cos x}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}} \lim_{x \to \pi/2^-} \cos x$

LHL = RHL

$\therefore$ F is not continuse at $x = \pi/2$

② $F(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \le x \le 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \le x < 9$

at $x=3$ & $x=6$.

→ at $x=3$

* $F(3) = \frac{x^2-9}{x-3} = 0$

F at $x=3$ define

* $\lim_{x\to 3^+} F(x) = \cancel{\lim_{x \to 3^+}} x+3$

$F(3) = x+3 = 3+3 = 6$

F is define at $x=3$

$\lim_{x \to 3^+} F(x) = \lim_{x\to 3^+} (x+3) = 6$

$\lim_{x\to 3^-} F(x) = \lim_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)} = (x+3)$

$\therefore$ LHL = RHL

F is continuse at $x=3$

→ for $x=6$

$F(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{27}{9} = 3$

$\lim_{x\to 6^+} \frac{x^2-9}{x+3}$

$= \lim_{x\to 6^+} \frac{(x-3)(x+3)}{(x+3)}$

$= \lim_{x\to 6^+} (x-3) = 6-3 = 3$

$= \lim_{x\to 6^-} x+3 = 3+6 = 9$

$\therefore$ LHL $\neq$ RHL

Function is not continuous.

6) (1) $F(x) = \frac{1-\cos 4x}{x^2}$ $\quad x<0$
$\quad = k$ $\quad x=0$ $\Big\}$ at $x=0$

F is continuous at $x=0$

$= \lim_{x\to 0} F(x) = F(0)$

$= \lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$

$= \lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$

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$$= 2 \lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$$

$$= 2 \lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$= 2\,(2)^2 = k$$

$$\Rightarrow k = 8$$

② $F(x) = (\sec^2 x)^{\cot^2 x}$ $x \neq 0$
$= k$ $x = 0$ $\}$ at $x=0$

$\rightarrow F(x) = (\sec^2 x)^{\cot^2 x}$

$$= \lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$= \lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$$

$$= e \quad \ldots \left[\lim_{x\to 0} (1+px)^{1/px} = e\right]$$

$k = e$

③ $F(x) = \dfrac{\sqrt{3} - \tan x}{\pi - 3x}$ $x \neq \pi/3$
$= k$ $x = \pi/3$ $\}$ at $x = \frac{\pi}{3}$

$\rightarrow x - \frac{\pi}{3} = h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$$F\left(\frac{\pi}{3}+h\right) = \frac{\sqrt{3} - \tan\left(\frac{\pi}{3}+h\right)}{\pi - 3\left(\frac{\pi}{3}+h\right)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3\left(\frac{\pi}{3}+h\right)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3}\cdot\tan h}}{\pi - \pi - 3h}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{3}-\left(\tan\frac{\pi}{3}+\tanh\right) * \left(1-\tan\frac{\pi}{3}.\tanh\right)}{1-\tan\frac{\pi}{3}.\tanh}}{-3h}$$

$$= \lim_{h\to 0} \cancel{\sqrt{3} - (\sqrt{3}.\tanh) * (1 -}$$

$$= \lim_{h\to 0} \frac{\frac{\sqrt{3}\left(1-\tan\frac{\pi}{3}.\tanh\right) - \tan\frac{\pi}{3}+\tanh}{1-\tan\frac{\pi}{3}.\tanh}}{-3h}$$

$$= \lim_{h\to 0} \frac{\frac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}.\tanh)-(\sqrt{3}+\tanh)}{1-\tan\pi/3.\tanh}}{-3h}$$

$$= \lim_{h\to 0} \frac{-4\tanh}{-3h(1-\sqrt{3}\tanh)}$$

$$= \lim_{h\to 0} \frac{4\tanh}{3h(1-\sqrt{3}\tanh)}$$

$$= \frac{4}{3}\lim_{h\to 0}\frac{\tanh}{h} * \lim_{h\to 0}\frac{1}{(1-\sqrt{3}\tanh)}$$

$$= \frac{4}{3}\frac{1}{1-\sqrt{3}(0)}$$

$$= \frac{4}{3}$$

7) (1) $F(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x=0$

$$F(x) = \frac{1-\cos 3x}{x\tan x}$$

$$\lim_{x\to 0} \frac{2\sin^2 3/2x}{x\tan x}$$

$$\lim_{x\to 0} = \frac{\dfrac{2\sin^2\frac{3x}{2}}{x^2} \times x^2}{\dfrac{x-\tan x}{x^2} \times x^2}$$

$$= 2\lim_{x\to 0} \left(\frac{3}{2}\right)^2 = \frac{2\times 9}{42} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \qquad g = f(0)$$

$\therefore$ f is not continuere at $x=0$

Redefine function

$$F(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim_{x\to 0} F(x) = f(0)$

f has removable discontinuity at $x=0$

(2) $F(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^\circ}{x^2} & x \neq 0 \\ = \dfrac{\pi}{6} & x = 0 \end{cases}$ at $x=0$

$$= \lim_{x\to 0} (e^{3x}-1)\sin\left(\frac{\pi x}{180}\right) \Big/ x^2$$

$$= \lim_{x\to 0} \frac{e^{3x}-1}{x} \cdot \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= \lim_{x\to 0} \frac{3\cdot e^{3x}-1}{3x} \cdot \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= 3 \lim_{x \to 0} \frac{e^{3x} - 1}{3x} \cdot \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$= 3 \log_e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

F is continuose at $x=0$

8) $F(x) = \frac{e^{x^2} - \cos x}{x^2}$ $x = 0$

is continuous at $x=0$

$\therefore$ Given

F is continouous at $x=0$

$$\lim_{x \to 0} F(x) = F(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = F(0)$$

$$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$= \lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$= \log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$$

$$= \log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$$

Multiply with 2 on num & Denominator

$$= 1 + 2 \times \frac{1}{4} = \frac{3}{2} = f(0)$$

9) $F(x) = \dfrac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $\therefore x \neq \pi/2$

$F(0)$ is continuous at $x = \pi/2$

$$= \lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$= \lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$$

$$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$$

$$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$$

$$F(\pi/2) = \frac{1}{4\sqrt{2}}$$

Practical-2

Derivatives

Q.1) show that the Following function defined from R to R are differentiable

1) $\cot x$ 2) $\operatorname{cosec} x$ 3) $\sec x$

Q.2) If $F(x) = 4x+1$ $x \le 2$

$= x^2 + 5$ $x > 0$ at $x = 2$ then

find F is differentiable or not

Q.3) If $F(x) = 4x+7$ $x < 3$

$= x^2 + 3x$ ~~F~~ $x \ge 3$ at $x = 3$ then

Find F is differentiable or not

Q.4) If $F(x) = 8x - 5$ $x \le 2$

$= 3x^2 - 4x + 7$ $x < 2$ at $x=2$ then

Find F is differentiable or not

→ Q.1

1) $\cot x$

$F(x) = \cot x$

$$= \lim_{x \to a} \frac{F(x) - F(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{1/\tan x - 1/\tan a}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}$$

Put $x-a=h$, $x=a+h$ as $x\to a$, $h\to 0$

$$DF(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h)\tan a}$$

$\because \tan A - \tan B = \tan(A-B)(1+\tan A \cdot \tan B)$

$$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan a + \tan(a+h))}{h \times \tan(a+h)\tan a}$$

$$= \lim_{h\to 0} \frac{-\tan h}{h} \times \frac{1+\tan a \tan(a+h)}{\tan(a+h)\tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a} = \frac{-1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$$DF(a) = -\cos^2 a$$

$\therefore$ f is differentiable $\forall a \in R$

2) $\operatorname{cosec} x$

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$$

$$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$$

put $x-a=h$ $x=a+h$ as $x \to a$ $h \to 0$

$$DF(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \cdot \sin(a+h)}$$

$$DF(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin h/2}{h/2} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h/2)}$$

$$= \frac{-1/2 \times 2\cos\left(\frac{2a}{2}\right)}{\sin(a+0)}$$

$$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

3) $\sec x$

$F(x) = \sec x$

$$DF(a) = \lim_{x \to a} \frac{F(x) - F(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

put $x-a=h$

$x=a+h$

as $x \to a$, $h \to 0$

$$DF(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h\to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}} \times -\frac{h}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= -1/2 \times \frac{-2\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a.$$

Q.2

LHD:

$$Df(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x+1 - (4\times 2+1)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x\to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$$Df(2^-) = 4$$

RHD:

$$Df(2^+) = \lim_{x\to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x\to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(x+2)(x-2)}{(x-2)}$$

$$= 2+2 = 4$$

$Df(2^+) = 4$

RHD = LHD

F is differentiable at $x = 2$

Q. 3

RHD:

$$Df(3^+) = \lim_{x \to 3^+} \frac{F(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 3x + 1 - (3^2 + 3 \times 3 + 1)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x^2 + 6x - 0x - 18}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x - 3}$$

$$= \lim_{x \to 3^+} \frac{(x+6)(x-3)}{(x-3)} = 3+6 = 9$$

$Df(3^+) = 4$

$$= \lim_{x \to 3^+} \frac{f(x) - f(3)}{x - 3}$$

$$= \lim_{x \to 3^-} \frac{4x + 2 - 14}{x - 3}$$

$$= \cancel{\lim_{x \to 3^-}} \frac{4(x-3)}{(x-3)}$$

$Df(3^-) = 4$

RHD $\neq$ LHD

F is not differentiable at $x = 3$

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Q.4

$$F(2) = 8\times 2-5 = 16-5 = 11$$

RHD:

$$DF(2^+) = \lim_{x\to 2^+} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x+7-11}{x-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-4x-4}{4-2}$$

$$= \lim_{x\to 2^+} \frac{3x^2-6x+2x-4}{4-2}$$

$$= \lim_{x\to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$$

$$= \lim_{x\to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$$

$$= 3\times 2+2 = 8$$

$$DF(2^+) = 8$$

LHD:

$$DF(2^-) = \lim_{x\to 2^-} \frac{f(x)-f(2)}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-5-11}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8x-16}{x-2}$$

$$= \lim_{x\to 2^-} \frac{8(x-2)}{x-2)}$$

$$= 8$$

$$DF(2^-) = 8$$

LHD = RHD

f is differentiable at $x=2$

Practical - 3

Application of Derivative.

1) Find the intervals in which function is increasing or decreasing.

① $F(x) = x^3 - 5x + 11$

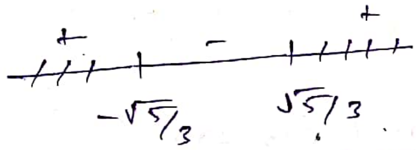
$\therefore\ F'(x) = 3x^2 - 5$

$\because F$ is increasing iff $F(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3})(x + \sqrt{5/3}) > 0$



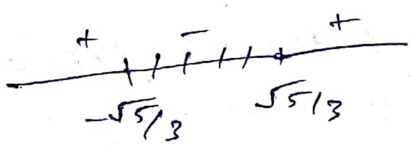
$x \in (-\infty, \sqrt{5/2}) \cup (\sqrt{5/3}, \infty)$

and F is decreasing iff $F'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore\ 3(x^2 - 5/3) < 0$

$\therefore\ (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$



$x \in (-\sqrt{5/3}, \sqrt{5/3})$

② $F(x) = x^2 - 4x$

$F'(x) = 2x - 4$

$\because$ $F(x)$ is increasing iff $F'(x) > 0$

$\therefore\ 2x - 4 > 0$

$2(x-2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and F is decreasing iff $F'(x) < 0$

$\therefore\ 2x - 4 < 0$

$2(x-2) < 0$

$\therefore x - 2 < 0$

or $x \in (-\infty, 2)$

3) $F(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

$+ \quad - \quad +$

$2 \quad 5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore 2(3x^2 + x - 10) < 0$

$\therefore 3x^2 + x - 10 < 0$

$\therefore 3x^2 + 6x - 5x - 10 < 0$

$\therefore 3x(x+2) - 5(x+2) < 0$

$\therefore (x+2)(3x-5) < 0$

$+ \quad - \quad +$

$-2 \quad 5/3$

$x \in (-2, 5/3)$

4) $F(x) = x^3 - 27x + 5$

$F'(x) = 3x^2 - 27$

$\because$ F is increasing if $F'(x) > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore (x - 3)(x + 3) > 0$

+ − +

−3 3

$x \in (-\infty, -3) \cup (3, \infty)$

and F is decreasing iff $F'(x) < 0$

$\therefore 3x^2 - 2 > 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore (x - 3)(x + 3) < 0$

+ − +

−3 3

$\therefore x \in (-3, 3)$

5) $F(x) = 2x^3 - 9x^2 - 24x + 64$

$F'(x) = 6x^2 - 18x - 24$

$\because$ F is increasing iff $F'(x) > 0$

$\therefore 6x^2 - 18x - 24 > 0$

$\therefore 6(x^2 - 3x - 4) > 0$

$x^2 - 3x - 4 > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$x(x - 4) + 1(x - 4) > 0$

$(x - 4)(x + 1) > 0$

− +

−1 4

$\therefore x \in (-1, 4)$

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$x \in (-\infty, -1) \cup (4, \infty)$

and f is decreasing iff $f'(x) < 0$

$\therefore 6x^2 - 18x - 24 < 0$

$\therefore 6(x^2 - 3x - 4) < 0$

$\therefore x^2 - 4x + x - 4 < 0$

$x(x-4) + 1(x-4)$

$(x-4)(x+1) < 0$

$-$ ++++++ $+$

-1 $\quad$ 4

$\therefore x \in (-1, 4)$

Q.2

1) $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward iff $f''(x) > 0$

$\therefore (6 - 12x) > 0$

$\therefore 12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$\because f''(x) > 0$

$\therefore x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

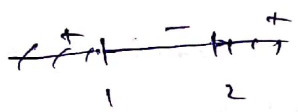
$\therefore 12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 3x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$



$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x \in (0, \infty)$

4) $y = 64 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$\therefore 12x - 18 > 0$

$\therefore 12(x - 18/12) > 0$

$\therefore x - 3/2 > 0$

$\therefore x > 3/2$

$\therefore x \in (3/2, \infty)$

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5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 - x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$\therefore f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 12(x + 2/12) > 0$

$\therefore x + 1/6 > 0$

$\therefore x > -1/6$

$\because f''(x) > 0$

$\therefore$ The exist interval

$x \in (-1/6, \infty)$

AL
21/12/19

Practical-4

Application of derivatives & Newton's Method.

Q.1) Find maximum and minimum value of following

1) $f(x) = \frac{x^2+16}{x^2}$

2) $f(x) = 3-5x^3+3x^5$

3) $f(x) = x^3-3x^2+1 \quad [-1/2, 4]$

4) $f(x) = 2x^3-3x^2-12x+1 \quad [-2, 3]$

Q.2) Find the root of the following equation by newton's (Take 4 iteration only) correct upto 4 decimal.

1) $f(x) = x^3-3x^2-55x+9.5$ (take $x_0 = 0$)

2) $f(x) = x^3-4x-9$ in $[2,3]$

3) $f(x) = x^3-1.8x^2-10x+17$ in $[1,2]$

Q.1

i) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$\therefore \quad 2x = \frac{32}{x^3}$

$\therefore \quad x^4 = 32/2$

$\therefore \quad x^4 = 16$

$\therefore \quad x = \pm 2$

$f'(x) = 2 + 96/x^4$

$f'(2) = 2 + 96/2^4$

$= 2 + 96/16$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore \quad f(2) = 2^2 + 16/22$

$= 4 + 16/4$

$= 8$

$f''(-2) = 2 + 96/(-2)^4$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum value at $x = 2$, and $x = -2$

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2) $f(x) = 3 - 5x^3 + 3x^5$

$\therefore f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$\therefore 15x^4 = 15x^2$

$\therefore x^2 = 1$

$\therefore x = \pm 1$

$\therefore f''(x) = -30x + 60x^3$

$f(1) = -30 + 60$

$= 30 > 0$ $\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$ $\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3 = 5$

$\therefore$ f has maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

3) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$\therefore 3x^2 - 6x = 0$

$\therefore 3x(x - 2) = 0$

$\therefore 3x = 0$ or $x - 2 = 0$

$\therefore x = 0$ or $x = 2$

$\therefore f''(x) = 6x - 6$

$\therefore f''(0) = 6(0) - 6$

$= -6 < 0$

$\therefore$ f has maximum value at $x = 0$.

$\therefore F(0) = (0)^3 - 3(0)^2 + 1 = 1$

$\therefore F''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

$\therefore$ f has minimum value at $x = 2$

$F(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

$\therefore$ F has maximum value 1 at $x = 0$ and F has minimum value -3 at $x = 2$

4) $F(x) = 2x^3 - 3x^2 - 12x + 1$

$\therefore F'(x) = 6x^2 - 6x - 12$

Consider, $F'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore 6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$\therefore x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$\therefore F''(x) = 12x - 6$

$\therefore F''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

$\therefore$ f has minimum value at $x = 2$

$\therefore F(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$= -19$

$$f''(-1) = 12(-1) - 6$$
$$= -12 - 6$$
$$= -18 < 0$$

$\therefore$ f has maximum value at $x = -1$

$$\therefore f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$$
$$= -2 - 3 + 12 + 1$$
$$= 8$$

$\therefore$ f has maximum value 8 at $x = -1$ and

f has minimum value -19 at $x = 2$

Q.2

1) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method.

$$x_{n+1} = x_n - f(x_n) / f'(x_n)$$
$$x_1 = x_0 - f(x_0) / f'(x_0)$$
$$\therefore x_1 = 0 + 9.5/55$$
$$\therefore x_1 = 0.1727$$
$$\therefore f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$
$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$
$$= -0.0829$$
$$\therefore f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$
$$= 0.0895 - 1.0362 - 55$$
$$= -55.9467$$
$$\therefore x_2 = x_1 - f(x_1) / f'(x_1)$$
$$= 0.1727 - 0.0829/55.9467$$
$$= 0.1712$$

$F(x_2) = (0.1712)^3 - 3(0.1712)^2 - \cancel{55}(0.1712) + 9.5$

$= 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$F'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= 0.0879 - 1.0272 - 55$

$= -55.9393$

$\therefore x_3 = x_2 - f(x_2) / f'(x_2)$

$= 0.1712 + 0.0011 / 55.9393$

$= 0.1712$

$\therefore$ The root of equation is 0.1712

2) $F(x) = x^3 - 4x - 9 \quad [2, 3]$

$F'(x) = 3x^2 - 4$

$F(2) = 23 - 4(2) - 9$

$= 8 - 8 - 9$

$= -9$

$F(3) = 33 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6$

Let $x_0 = 3$ be the initial approximation

$\therefore$ By newton's Method

$x_{n+1} = x_n - f(x_n) / f'(x_n)$

$x_1 = x_0 - f(x_0) / f'(x_0)$

$= 3 - 6/23$

$= 2.7392$

$$f(x_1) = (0.7392)^3 - 4(2.7392) - 9$$
$$= 20.5528 - 10.9568 - 9$$
$$= 0.596$$

$$f'(x_1) = 3(2.7392)^2 - 4$$
$$= 22.5096 - 4$$
$$= 18.5096$$

$$x_2 = x_1 - f(x_1)/f'(x_1)$$
$$= 2.7392 - 0.596/18.5096$$
$$= 2.7091$$

$$f(x_2) = (2.7071)^3 - 4(2.7071)$$
$$= 19.8386 - 10.8284$$
$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$
$$= 21.9851 - 4$$
$$= 17.9851$$

$$x_3 = x_2 - f(x_2)/f'(x_2)$$
$$= 2.7071 - \frac{0.0102}{17.9851}$$
$$= 2.7071 - 0.0056$$
$$= 2.7015$$

$$f(x_2) = (2.7015)^3 - 4(2.7015) - 9$$
$$= 19.7158 - 10.806 - 9$$
$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4$$
$$= 21.8943 - 4$$
$$= 17.8943$$

$$x_4 = 2.7015 + 0.0901/17.8943$$
$$= 2.7015 + 0.0050$$
$$= 2.7065$$

3) $F(x) = x^3 - 1.8x^2 - 10x + 17 \quad [1, 2]$

$F'(x) = 3x^2 - 3.6x - 10$

$F(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= -1.8 - 10 + 17$

$= 6.2$

$F(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$

$= 8 - 7.2 - 20 + 17$

$= -2.2$

Let $x_0 = 2$ be initial approximation. By newton's Method.

$x_{n+1} = x_n - F(x_n) / F'(x_n)$

$x_1 = x_0 - F(x_2) / F'(x_2)$

$= 2 - 2.2 / 5.2$

$= 2 - 0.4230$

$= 1.577$

$F(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$

$= 3.9219 - 4.4764 - 15.77 + 17$

$= 0.6755$

$F'(x) = 3(1.577)^2 - 3.6(1.577) - 10$

$= 7.4608 - 5.6772 - 10$

$= -8.2164$

$x_2 = x_1 - F(x_1) / F'(x_1)$

$= 1.577 + 0.6755 / 8.2164$

$= 1.577 + 0.0822$

$= 1.6592$

$F(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$

$= 4.5677 - 4.9553 - 16.592 + 17$

$= 0.0204$

$$F'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$$
$$= 8.2588 - 5.97312 - 10$$
$$= -7.7143$$
$$x_3 = x_2 - F(x_2)/F'(x_2)$$
$$= 1.6592 + 0.0204/7.7143$$
$$= 1.6592 + 0.0026$$
$$= 1.6618$$

$$F(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 4.5892 - 4.9708 - 16.618 + 17$$
$$= 0.0004$$
$$F'(x_3) = 3(1.6618)^2 - 3.6(1.6618) - 10$$
$$= 8.2847 - 5.9824 - 10$$
$$= -7.6977$$
$$x_4 = x_3 - F(x_3)/F'(x_3)$$
$$= 1.6618 + \frac{0.0004}{7.6977}$$
$$= 1.6618.$$

Practical No - 5

Integration.

Q.1 Solve the following integration.

1) $$\int \frac{1}{\sqrt{x^2+2x-3}}\, dx$$

$$= \int \frac{1}{\sqrt{x^2+2x+1-4}}\, dx$$

$$= \int \frac{1}{\sqrt{(x+1)^2-4}}\, dx$$

Substitute $x+1 = t$

$dx = \frac{1}{t}\, dt$

where $t=1$ $\quad t = x+1$

$$\int \frac{1}{\sqrt{t^2-4}}\, dt$$

using

$$\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln\left(|x+\sqrt{x^2+a^2}|\right)$$

$$= \ln\left(|t+\sqrt{t^2-4}|\right)$$

$t = x+1$

$$= \ln\left(|x+1+\sqrt{(x+1)^2-4}|\right)$$

$$= \ln\left(|x+1+\sqrt{x^2+2x-3}|\right)+c$$

2) $\int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \dfrac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin(x)\,dx + \int 5x^{1/2}\,dx$

$= \dfrac{2x^3}{3} + 3\cos x + \dfrac{10\sqrt{x}}{3} + c$

$= \dfrac{2x^3 + 10\sqrt{x}}{3} + 3\cos x + c$

4) $\int \dfrac{x^3+3x+4}{\sqrt{x}}\,dx$

$= \int \dfrac{x^3+3x+4}{x^{1/2}}\,dx$

$= \int \dfrac{x^3}{x^{1/2}} + \dfrac{3x}{x^{1/2}} + \dfrac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \dfrac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \dfrac{4}{x^{1/2}}\,dx$

$= \dfrac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + c$

8) $\int t^{2} . \sin(2t^{4}) \, dt$

put $u = 2t^{4}$

$du = 2 \times 4t^{3}$

$$= \int t^{7} \times \sin(2t^{4}) \times \frac{1}{2.4t^{3}} \, du$$

$$= \int t^{4} \sin(2t^{4}) \times \frac{1}{2 \times 4} \, du$$

$$= \int t^{4} \sin(2t^{4}) \times \frac{1}{8} \, du$$

$$= \int \frac{t^{4} \times \sin(2t^{4})}{8} \, du$$

substitute t^{4} with $u/2$

$$= \int \frac{u/2 \times \sin(u)}{8} \, du$$

$$= \int \frac{u \times \sin(u)}{2} / 8 \, du$$

$$= \int \frac{u \times \sin(u)}{16} \, du$$

$$= \frac{1}{16} \int u \times \sin(u) \, du$$

$\int u \, dv = \cancel{vu - \int v \, du}$

~~where~~ $u = v$

$dv = \sin(u) \times du$

$du = 1 du$

$v = -\cos(u)$

$$= \frac{1}{16}\left(u \times (-\cos(u)) - \int -\cos(u)\,du\right.$$

$$= \frac{1}{16}\left(u\,(-\cos(u))\right) + \int \cos(u)\,du$$

$$= \frac{1}{16}\left(u \times (-\cos(u))\right)$$

Return the substitution $u = 2t^4$

$$= \frac{1}{16} \times \left(2t^4\left(-\cos(2t^4)\right) + \sin(2t^4)\right)$$

$$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$$

6) $\int \sqrt{x}\,(x^2 - 1)\,dx$

$$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$$

$$= \int x^{1/2} * x^2 - x^{1/2}\,dx$$

$$= \int x^{5/2} - x^{1/2}\,dx$$

$$= \int x^{5/2}\,dx$$

$$= -\int x^{1/2}\,dx$$

$$I_1 = \cancel{\frac{x^{5/2+1}}{5/2+1}} = \frac{x^{3/2}}{3/2} = \frac{2x^{7/2}}{7}$$

$$= \frac{2\sqrt{x^7}}{7} = \frac{2x^3\sqrt{x}}{7}$$

$$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$$

$$I = \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$$

7) $\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}} dx$

$= \int \frac{\cos x}{\sin x^{2/3}} dx$

put $t = \sin x$

$t = \cos x$

$= \int \frac{\cos x}{\sin x^{3/2}} \times \frac{1}{\cos x} dt$

$= \frac{1}{\sin x^{3/2}} dt$

$= \frac{1}{t^{2/3}} dt$

$I = \int \frac{1}{t^{2/3}} dt = \frac{-1}{(2/3-1)t^{2/3-1}}$

$= \frac{-1}{-1/3\, t^{2/3-1}} = \frac{1}{1/3\, t^{-1/3}} = \frac{t^{1/3}}{1/3} = 3t^{1/3}$

$= 3\sqrt[3]{t}$

$t = \sin x$

$= 3\sqrt[3]{\sin x} + c$

8) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} * \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} * \frac{1}{3x^2-6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt$

$= \int \frac{1}{3t}\,dt$

$= \frac{1}{3}\int \frac{1}{t}\,dt$

$= \frac{1}{3}\ln|t| + c$

$= \frac{1}{3}\ln|x^3-3x^2+1| + c$

9) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

$I = \int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

let $\frac{1}{x^2} = t$

$x^{-2} = t$

$\frac{-2}{x^3}\,dx = dt$

$$I = \frac{-1}{2}\int -\frac{2}{x^3}\sin\left(\frac{1}{x^2}\right)dx$$

$$= \frac{-1}{2}\int \sin t$$

$$= \frac{-1}{2}(-\cos t)+c$$

$$= \frac{1}{2}\cos t + c$$

Resubstitution $t = 1/x^2$

$$I = \frac{1}{2}\cos\left(\frac{1}{x^2}\right)+c$$

10) $\int e^{\cos^2 x}\cdot\sin 2x\,dx$

$$I = \int e^{\cos^2 x}\sin 2x\,dx$$

let $\cos^2 x = t$

$$-2\cos x\cdot\sin x\,dx = dt$$

$$-\sin 2x\,dx = dt$$

$$I = \int -\sin 2x\, e^{\cos^2 x}\cdot dx$$

$$= -\int e^t\,dt$$

$$= e^t + c$$

Resubstituting $t = \cos^2 x$

$$I = -e^{\cos^2 x} + c$$

04/01/2020

practical - 6

Application of integration & numerical integration

Q1 find the length of the following curve

1. $x = t\sin t$, $y = 1-\cos t$, $t \in [0, 2\pi]$

$x = t\sin t$ $\qquad$ $y = 1-\cos t$

$\frac{dx}{dt} = 1-\cos t$ $\qquad$ $\frac{dy}{dt} = \sin t$

$$L = \int_0^{2\pi} \sqrt{(1-\cos t + \sin t)^2 + (\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{1-2\cos t + 1}\, dt$$

$$= \int_0^{2\pi} \sqrt{2(1-\cos t)}\, dt$$

$$= \int_0^{2\pi} \sqrt{4\sin^2 t/2}\, dt$$

$$= 2\int_0^{2\pi} \sin t/2\, dt$$

$$\cancel{= 2\left[-\cos t/2\right]_0^{2\pi}}$$

$$= 2[-\cos\pi + \cos 0]$$

$$= 4$$

5.

2. $y = \sqrt{u-x^2}$ $x \in [-2, 2]$

$$l = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$y = \sqrt{u-x^2}$$

$$\therefore \frac{dy}{dx} = 2^2$$

$$= \int_0^2 + \left(\frac{-x}{\sqrt{u-x^2}}\right)^2 dx$$

$$= 2\int_0^2 \sqrt{\frac{1+x^2}{u-x^2}}\, dx$$

$$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$$

$$= 4\left[\sin^{-1}(x/2)\right]_0^2$$

$$= 2\pi$$

3. $y = x^{3/2}$ in $[0, 4]$

f of $(x) = 3/2$

$$[F(x)]^2 = 9/4\, x$$

$$l = \int_0^4 \sqrt{1 + (F'(x))^2}\, dx$$

$$= \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \int_0^4 \sqrt{\frac{4+9x}{4}}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2} \int_0^4 \frac{(4+9x)^{1/2+1}}{\sqrt{1/2+1}} dx$$

$$= \frac{1}{2} \int_0^4 \frac{(4+9x)^{1/2+1}}{\sqrt{1/2+1}} dx$$

$$= \frac{1}{27} \left[[4+0]^{3/2} - [4+36]^{3/2} \right]$$

$$= \frac{1}{27} \left[(4)^{3/2} - (40)^{3/2} \right]$$

4 = $x = 3\sin t \quad y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \quad \frac{dy}{dt} = -3\sin t$$

$$L = \int_0^{2\pi} \sqrt{3(\cos t)^2 + (-3\sin t)^2}\, dt$$

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5. $x = \frac{1}{6} y^3 + \frac{1}{2y}$ $\quad y \in [1,2]$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2} = \frac{y^4-1}{2y^2}$$

$$= \int_1^2 \sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 y^2 dy + \frac{1}{2}\int_1^2 y^{-2} dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[7/3 - \frac{1}{2}\right] = 17/12$$

Q.2 Using simpleson's Rule solve the following.

① $3\int_0^2 ex^2 dx$ with $n=4$

$a=0 \quad b=2 \quad n=4$

$$h = \frac{2-0}{4}$$

x	0	0.5	1	1.5	2
y	1	1.2840	2.7182	9.4877	54.5981
	y_0	y_1	y_2	y_3	y_4

By Simpson's Rule

$$\int_0^2 e^{x^2}\,dx = \frac{0.5}{3}\Big[(1+54.5981)+4(5.12890+9.4877$$
$$+2(2.7182+54.5981)\Big]$$

$$=\frac{0.5}{3}\left[55.5981+43.0868+114.63266\right]$$
$$=1.1279$$

② $\int_0^4 x^2\,dx$ $\quad h=\frac{4-0}{4}=1$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^4 x^2 = \frac{1}{3}\left[(0+0)+4(1+9)-(7(4))\right]$$
$$=21.333$$

③ $\int_0^{\pi/3} \sqrt{\sin x}\,dx \quad n=6$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$	$6\pi/18$	$8\pi/18$
y	0	0.4166	0.58	0.70	0.8087	0.8722	0.99

$$\int_0^{\pi/3}\sqrt{\sin x} = \frac{\pi}{4}\times 12.1163$$
$$=0.7049.$$

pratical - 7

Differential equation.

Q.1 Solve the following differential equation

① $x\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$p(x) = \frac{1}{x}$ $\quad Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x \, dx}$

$= x$

$y(IF) = \int Q(x)(IF)dx + c$

$= \int \frac{e^x}{x} \cdot x \cdot dx + c$

$= \int e^x \, dx + c$

$xy = e^x + c$

② $e^x \frac{dy}{dx} + 2e^{x}y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x}y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$p(x) = 2 \qquad Q(x) = e^{-x}$

$\int p(x)\,dx$

$IF = e^{\int 2\,dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int e^{-x} e^{2x}\,dx + c$

$= \int e^{x}\,dx + c$

$y \cdot e^{2x} = e^{x} + c$

3) $x\dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$x\dfrac{dy}{dx} = \dfrac{\cos x}{x} - 2y$

$\dfrac{dy}{dx} + \dfrac{2y}{x} = \dfrac{\cos x}{x^2}$

$p(x) = 2(x) \qquad Q(x) = \dfrac{\cos x}{x^2}$

$IF = e^{\int p(x)\,dx}$

$= e^{\int 2x\,dx}$

$= x^2$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \dfrac{\cos x}{x^2} \cdot x^2\,dx + c$

$= \int \cos x + c \qquad \therefore\ x^2 y = \sin x + c$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$$

$$p(x) = 3/x \qquad Q(x) = \sin x / x^3$$

$$p(x) = \int 3/x \, dx$$

$$= x^3$$

$$IF = e^{\int p(x)\,dx}$$

$$= x^3$$

$$y(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int \frac{\sin x}{x^3} \cdot x^3 \, dx + c$$

$$= \int \sin x + c$$

$$x^3 y = -\cos x + c$$

5) $e^x \frac{dy}{dx} + 2e^{2x} y = 2x$

$$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$$

$$P(x) = 2 \qquad Q(x) = \frac{2x}{e^{2x}} = 2xe^{-2x}$$

$$(IF) = e^{\int p(x)\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}$$

$$Y(IF) = \int Q(x)(IF)\,dx + c$$

$$= \int 2x e^{-2x} e^{2x} + c$$

$$y e^{2x} = \int 2x + c = x^2 + c$$

6) $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$

$$\sec^2 x \cdot \tan y \, dx = -\sec^2 y \tan x \, dy$$

$$\frac{\sec^2 x}{\tan x} dx = -\frac{\sec^2 x}{\tan y} dy$$

$$\log|\tan x| = -\log|\tan y| + c$$

$$\log|\tan x + \tan y| = c$$

$$\tan x + \tan y = e^c$$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

~~put $x-y+1=v$~~

$$x-y+1=v$$

$$1-\frac{dy}{dv} = \frac{dv}{dx}$$

$$1-\frac{dv}{dx} = \cancel{\sin^2 v}\,\frac{dy}{dx}$$

$$1-\frac{dv}{dx} = \frac{dy}{dx}\sin^2 v$$

$$\frac{dv}{dx} = 1 - \sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dx = \int dx$$

$$\tan v = x + c$$

$$\tan(x - y + 1) = x + c$$

8) $$\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 6}$$

Put $2x + 3y = v$

$$2 + \frac{3dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$$

$$= \frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$$

$$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$$

$$\frac{dv}{dx} = \frac{v - 1 + 2v + 4}{v+2}$$

$$= \frac{3v + 3}{v+2}$$

$$= \frac{3(v+1)}{v+2}$$

$$\int \frac{v+2}{v+1}\, dv = 3\, dx$$

$$\int \frac{v+1}{v}\, dx + \int \frac{1}{v+1} = \int 3\, dx$$

$$v + \log|v| = 3x + c$$

$$2x + 3y + \log|2x+3y+1| = 3x + c$$

$$3y = x + \log|2x+3y+1| + c$$

11/01/2021

practical-8

Euler's method.

1) $\frac{dy}{dx} = y + e^{x} - 2$

$y(0) = 2 \quad h = 0.5$

find $y(2)$

$f(x) = y + e^{x} - 2 \quad , x_0 = 0$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$y(2) = 9.815$

2) $\frac{dy}{dx} = 1+y^2$ $y(0)=0$, $h=0.2$ find $y(1)$

$x_0 = 0$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	1.04	0.408
2	0.4	0.408	1.1664	0.6412
3	0.6	0.6412	1.4111	0.9234
4	0.8	0.9234	1.8526	1.2939
5	1	1.2939		

$y(1) = 1.2939$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1)$

$x_0 = 0$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3512	0.7644	1.5057
5	1	1.5057		

$y(1) = 1.5057$

4) $\frac{dy}{dx} = 3x^2 + 1$ $y(1) = 2$ find $y(2)$

$h = 0.5$, $x_0 = 1$
&
$h = 0.25$

* for $h = 0.5$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

$* \; h = 0.25$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.50	4.4218	59.0569	19.3360
3	1.75	19.3360	1122.0429	299.9966
4	2	299.9966		

$$y(2) = 299.9966$$

5) $\dfrac{dy}{dx} = \sqrt{xy} + 2$

$y(1) = 1$ ~~find~~ $h = 0.2$

$x_0 = 1$ find $y(1)$

n	x_n	y_n	$F(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$$y(1) = 3.6$$

11/01/2020

practical-9

~~Circuits~~ Limits $ ~~potential~~ partial order derivatives.

Q1)

1) $$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$$

at $(-4,-1)$ Denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(-4)^3 - 3(-4) + (-4)^2 - 1}{-4(7)+5}$$

$$= \frac{-61}{9}$$

2) $$\lim_{(x,y)\to(-2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

at $(-2,0)$ Denominator $\neq 0$

$\therefore$ By applying limit

$$= \frac{(0+1)(4+0-8)}{2}$$

$$= -4/2 = -2$$

3) $$\cancel{\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}}$$

at $(1,1,1)$ Denominator ~~≠0~~ $=0$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$$

$$= \lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x^2}$$

on applying limits

$$= \frac{1+1(1)}{1^2}$$

$$= 1$$

Q.2

① $t(x,y) = xye^{x^2+y^2}$

$\therefore \quad t_x = \frac{\delta t}{\delta x}(x,y)$

$$= \frac{\delta(xye^{x^2+y^2})}{\delta x}$$

$$= ye^{x^2+y^2}(2x)$$

$$t_x = 2xye^{x^2+y^2}$$

$$t_y = \frac{\delta t(x,y)}{\delta y}$$

$$= \frac{\delta(xye^{x^2+y^2})}{\delta x}$$

$$= xe^{x^2+y^2}(2y)$$

$$t_y = \cancel{2ye}\ 2yxe^{x^2+y^2}$$

② $f(x,y) = e^{x}\cos y$

$$f_x = \frac{\delta f(x,y)}{\delta x}$$

$$= \frac{\delta(e^{x}\cos y)}{\delta x}$$

$$f_x = e^{x}\cos y$$

$$f_y = \frac{\delta f(x,y)}{\delta y}$$

$$= \frac{\delta(e^{x}\cos y)}{\delta y}$$

$$f_y = -e^{x}\sin y$$

③ $f(x,y) = x^{3}y^{2} - 3x^{2}y + y^{3} + 1$

$$f_x = \frac{\delta f(x,y)}{\delta x}$$

$$= \frac{\delta(x^{3}y^{3} - 3x^{2}y + y^{3}+1)}{\delta x}$$

$$f_x = 3x^{2}y^{2} - 6xy$$

$$f_y = \frac{\delta f(x,y)}{\delta y}$$

$$= \frac{\delta f(x^{3}y^{2} - 3x^{2}y + y^{3}+1)}{\delta y}$$

$$= 2x^{2}y - 3x^{2} + 3y^{2}$$

02

Q.3

(1) $t(x,y) = \frac{2x}{1+y^2}$

$$t_x = \frac{\delta t(x,y)}{\delta x}$$

$$= \frac{\delta t\,(2x/1+y^2)}{\delta x}$$

$$= \frac{1+y^2 \frac{\delta(2x)}{\delta x} - 2x \frac{\delta(1+y^2)}{\delta x}}{(1+y^2)^2}$$

$$= \frac{2+2y^2}{(1+y^2)^2}$$

$$= 2/1+y^2$$

at $t(0,0)$

$$= 2/1+0 = 2$$

$$t_y = \delta\, t(2x, 1+y^2)$$

$$= 1+y^2 \frac{\delta(2x)}{\delta x} - 2x \frac{\delta(1+y^2)}{\delta x} \Big/ (1+y^2)^2$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

$$= \frac{-4xy}{(1+y^2)^2}$$

at $(0,0)$

$$= \frac{-4(0)(0)}{(1+0)^2} = 0$$

$$f(x,y) = \frac{y^2 - xy}{x^2}$$

$$f_x = \frac{x^2 \frac{\delta}{\delta x}(y^2 - xy) - (y^2 - xy)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{-x^2 y - 2x(y^2 - xy)}{x^4}$$

$$f_y = \frac{2y - x}{x^2}$$

$$f_{xx} = \frac{\delta}{\delta x}\left(\frac{-x^2 y - 2x(y^2 - xy)}{x^4}\right)$$

$$= \frac{x^4\left(\frac{\delta}{\delta x}(-x^2 y - 2xy^2 + 2x^2 y)\right) - (-x^2 y - 2xy + 2x^2 y)\frac{\delta}{\delta x}(x^4)}{(x^4)^2}$$

$$= \frac{x^4(-2xy - 2y^2 + 4xy) - 4x^3(-x^2 y - 2xy + 2x^2 y)}{x^6} \quad — (1)$$

$$f_{yy} = \frac{\delta}{\delta y}\left(\frac{2y - x}{x^2}\right)$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2} \quad — (2)$$

$$f_{xy} = \frac{\delta}{\delta y}\left(\frac{2y - x}{x^2}\right) \quad — 3$$

$$= \frac{2 - 0}{x^2} = \frac{2}{x^2}$$

$$f_{xy} = \frac{\delta}{\delta y}\,\frac{(-x^2y - 2xy^2 + 2x^3y)}{x^4}$$

$$= \frac{x^2 - 4xy + 2x^2}{x^4} \quad — ③$$

$$f_{yx} = \frac{\delta}{\delta x}\left(2y - \frac{x}{x^2}\right)$$

$$= \frac{x^2\frac{\delta}{\delta x}(2y - x) - (2y - x)\frac{\delta}{\delta x}(x^2)}{(x^2)^2}$$

$$= \frac{-x^2 - 4xy - 2x^2}{x^4} \quad — ④$$

from ③ & ④

$$f_{xy} = f_{yx}$$

2) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f_x = \frac{\delta}{\delta x}\left(x^3 + 3x^2y^3 - \log(x^2+1)\right)$$

$$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f_y = \frac{\delta}{\delta y}\left(x^3 + 3x^2y^2 - \log(x^2+1)\right)$$

$$= 6x^2y$$

$$t_{xx} = \frac{(6x + 6y^2) - (x^2+1)\frac{\delta}{\delta x}(2x) - 2x\left(\frac{\delta}{\delta x}(x^2+1)\right)}{(x^2+1)^2}$$

$$= \frac{6x + 6y^2 - (2(x^2+1) - 4x^2)}{(x^2+1)^2} \quad \text{— (1)}$$

$$t_{yy} = \frac{\delta}{\delta y}(6x^2 y)$$

$$= 6x^2 \quad \text{— (2)}$$

$$t_{xy} = \frac{\delta}{\delta y}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 0 + 12xy + 0$$

$$= 12xy \quad \text{— (3)}$$

$$t_{yx} = \frac{\delta}{\delta x}(6x^2 y)$$

$$= 12xy \quad \text{— (4)}$$

from (3) & (4)

$$t_{xy} = t_{yx}$$

3)

$$t(x,y) = \sin(xy) + e^{x+y} \quad (1)$$

$$t_x = y\cos(xy) + e^{x+y}$$

$$\cancel{= y\cos(xy) + e^{x+y}}$$

$$t_y = x\cos(xy) + e^{x+y}(1)$$

$$= x\cos(xy) + e^{x+y}$$

$$t_{xx} = \frac{\delta}{\delta x}\left(y\cos(xy) + e^{x+y}\right)$$

$$= -y\sin(xy)(y) + e^{x+y}(1)$$

$$= -y^2\sin(xy) + e^{x+y} \quad \text{— (1)}$$

3d $f_{yy} = \frac{\delta}{\delta y}\left(x\cos(xy) + e^{x+y}\right)$

$= -x\sin(xy)(x) + e^{x+y}(1)$

$= -x^2\sin(xy) + e^{x+y}$ — ②

$f_{xy} = \frac{\delta}{\delta y}\left(y\cos(xy) + e^{x+y}\right)$

$= -y^2\sin(xy) + \cos(xy) + e^{x+y}$ — ③

$f_{yx} = \frac{\delta}{\delta x}\left(x\cos(xy) + e^{x+y}\right)$

$= -x^2\sin(xy) + \cos(xy) + e^{x+y}$ — ④

from ③ & ④

$f_{xy} \neq f_{yx}$

Q.5

1) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{1^2+1^2} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}}(2x)$

$= \frac{x}{\sqrt{x^2+y^2}}$

$f_y = \frac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \frac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

f_y at $(1,1) = \frac{1}{\sqrt{2}}$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$
$$= \sqrt{2} + \frac{1}{\sqrt{2}}(\cancel{xy}^{x-1} + y - 1)$$
$$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$$
$$= \frac{x+y}{\sqrt{2}}$$

$f(x,y) = 1 - x + y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1 - \pi/2 + 0 \quad = 1 - \pi/2$

$f_x = 0 - 1 + y\cos x$

f_x at $(\pi/2, 0) = -1$

$f_y = 0 - 0 + \sin x$

f_y at $(\pi/2, 0) = \sin \pi/2$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$
$$= 1 - \pi/2 + (-1)(x - \pi/2) + 1(y-0)$$
$$= 1 - \pi/2 - x + \pi/2 + y$$
$$= 1 - x + y$$

$f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

$f_x = 1/x$

f_x at $(1,1) = 1$

$f_y = 1/y$

f_y at $(1,1) = 1$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= 0 + 1(x+1) + 1(y-1)$$

$$= x-1 + y-1$$

$$= x+y-2$$

practical 10

Q1 find directional derivative of the following fnct at given points & in direction of given vector

1) $f(x,y) = x + 2y - 3$ $\quad a = (1,-1)$ $\quad u = 3i - j$

$u = 3i - j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$D_u f(a) = \frac{1}{\sqrt{10}}$

2) $F(x) = y^2 - 4x + 1 \quad a = (3,4) \quad u = i + 5j$

$u = i + 5j$ is not a unit vector

$$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$$

unit vector along u is $\frac{u}{|u|} = \frac{1}{26}(1,5)$

$$= \left(\frac{1}{26}, \frac{5}{26}\right)$$

$$F(a) = F(3,4) = (4)^2 - 4(3) + 1 = 5$$

$$F(a+hu) = F(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$$

$$= F\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$$

$$F(a+hu) = \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$$

$$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{40h - 4h}{\sqrt{26}} + 5$$

$$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$$

$$D_uF(a) = \lim_{h \to 0} \frac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \cancel{h}\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right) / \cancel{h}$$

$$\therefore D_uF(a) = \frac{25h}{26} + \frac{36}{\sqrt{26}}$$

3) $2x+3y$ $\quad a=(1,2)$, $u=(3i+4j)$

Here $u=3i+4j$ is not a unit vector

$$|u| = \sqrt{(3)^2+(4)^2} = \sqrt{25} = 5$$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{5}(3,4) = \left(\frac{3}{5},\frac{4}{5}\right)$

$$f(a) = f(1,2) = 2(1)+3(2) = 8$$

$$f(a+hu) = f(1,2)+h\left(\frac{3}{5},\frac{4}{5}\right)$$

$$= f\left(1+\frac{3h}{5}, 2+\frac{4}{5}\right)$$

$$f(a+hu) = 2\left(1+\frac{3h}{5}\right)+3\left(\frac{2+4h}{5}\right)$$

$$= 2+\frac{6h}{5}+6+\frac{12h}{5}$$

$$= \frac{18h}{5}+8$$

$$D_uf(a) = \lim_{h\to 0}\frac{\frac{18h}{5}+8-8}{h} = 18/5$$

Q-2 find gradient vector for the following function at given,

1) $F(x,y) = x^y + y^x$ at $a=(1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y, \; x^y \log x + x y^{x-1})$

$F(1,1) = (1+0, \; 1+0)$

$= (1,1)$

2) $F(x,y) = (\tan^{-1} x) \cdot y^2$ $a=(1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x \right)$

$F(1,-1) = \left(\frac{1}{2}, \tan^{-1}(1)(-2) \right)$

$= \left(\frac{1}{2}, \frac{\pi}{4}(-2) \right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2} \right)$

3) $F(x,y,z) = xyz - e^{x+y+z}$ $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla F(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$F(1,-1,0) = ((-1)(0) - e^{1+(-1)+0}, (1)(0) - e^{1+(-1)+0}, (1)(-1) - e^{1+(-1)+0})$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q.3) find the equation of tangent & normal to each of the following using curves at given points.

$x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy} \cdot x$

$(x_0, y_0) = (1, 0)$ $\therefore x_0 = 1, y_0 = 0$

eqn of tangent

$F_x(x - x_0) + F_y(y - y_0) = 0$

$F_x(x_0, y_0) = \cos 0 \; 2(1) + e^0 0$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 - (\sin 0) + e^0 \cdot 1$

$= 0 + 1 \cdot 1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

— It is the required eqn of tangent

eqn of normal

$= ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1, 0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2)$

$\therefore x_0 = 2, y_0 = -2$

$f_x(x_0, y_0) = 2(2) - 2 = 2$

$f_y(x_0, y_0) = 2(-2) + 3 = -1$

eq^n of taget

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x - 2) + (-1(y + 2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$

→ It is required eq^n

eq^n of taget

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2(x - 2) + (-1(y + 2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$

→ It is reqired eq^n of taget

eq^n of Normal

$\Rightarrow ax + by + c = 0$

$bx + ay + d = 0$

$-1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 + 2(-2) + d = 0$

$-2 - 4 + d = 0$

$-6 + d = 0$

$\therefore d = 6$

Q 4 find the eq^n of tanget & normal line to each of the following surface

1) $x^2 - 2yz + 3y + xz = 7$ at $(2,1,0)$.

$$f_x = 2x - 0 + 0 + z$$

$$f_x = 2x + z$$

$$f_y = 0 - 2z + 3 + 0$$

$$= 2z + 3$$

$$f_z = 0 - 2y + 0 + x$$

$$= -2y + x$$

$$(x_0, y_0, z_0) = (2, 1, 0)$$

$$\therefore x_0 = 2, y_0 = 1, z_0 = 0$$

$$F_x(x_0, y_0, z_0) = (2)2 + 0 = 4$$

$$f_y(x_0, y_0, z_0) = 2(0) + 3 = 3$$

$$f_z(x_0, y_0, z_0) = -2(1) + 2 = 0$$

eq^n of tanget

$$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$$

$$4(x-2) + 3(y-1) + 0(z-0) = 0$$

$$4x - 8 + 3y - 3 = 0$$

$$4x + 3y - 11 = 0$$

→ This is required eq^n of tanget

eq^n of normal at $(4, 3, +1)$

$$\frac{x-x_0}{fx} = \frac{y-y_0}{fy} = \frac{z-z_0}{fz}$$

$$\frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{0}$$

$3xy^2 - xy + z = -4$ at $(1, -1, 2)$

$3xy^2 - x - y + z + 4 = 0$ at $(1, -1, 2)$

$fx = 3yz - 1 - 0 + 0 + 0$

$= 3yz - 1$

$fy = 3xz - 0 - 1 + 0 + 0$

$= 3xz - 1$

$fz = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$(x_0, y_0, z_0) = (1, -1, 2)$ $x_0 = 1, y_0 = -1, z_0 = 2$

$fx(x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$fy(x_0, y_0, z_0) = 3(+1)(2) - 1 = 5$

$fz(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$

→ This is reqrd eq^n of tangent

eq^n of normal at $(-7, 5, -2)$

$$\frac{x-x_0}{fx} = \frac{y-y_0}{fy} = \frac{z-z_0}{fz}$$

$$\frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

Q. Find the local maxima & minima for the follwg function.

i) $F(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0$

$= 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$f_x = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad - (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad - (2)$

Multiply eqn 1 with 2

$\therefore 4x - 2y = -4$

$2y - 3x = 4$

$x = 0$

Substitute value of x in eqn (1)

$2(0) - y = -2$

$-y = -2$

$\therefore y = 2$

$\therefore$ critical points are (0, 2)

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Scanned with CamScanner

Here $r > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$\therefore$ f has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$0 + 4 - 0 + 0 - 8$

$= -4$

2) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$\therefore 8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0 \quad - ①$

$f_y = 0$

$3x^2 - 2y = 0 \quad - ②$

multiply eqn ① with 3

② with 4

$12x^2 + 9y = 0$

$12x^2 - 8y = 0$

$\underline{- \qquad + \qquad}$

$y = 0$

Substitute value of y in eq ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r = 0$

$rt - s^2 = 0(-2) - (0)^2$

$= 0 - 0 = 0$

$r = 0$ & $rt - s^2 = 0$

(nothing to say)

3) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0 \quad \therefore 2x + 2 = 0$

$x = \frac{-2}{2} \qquad \therefore x = -1$

$f_y = 0$

$-2y + 8 = 0$

$y = \frac{-8}{2} = 4$

$\therefore y = 4$

$\therefore$ Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= 73$

AK
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