Assessment II
CS551K – Software Agent & Multiagent Systems
2020-2021

Instructions to students:

- Your solution should be one single PDF file which you should upload onto MyAberdeen by the established time/deadline. Please do not email us your solution.
- Your file should be named our PDF file should be named "CS551K-ASMNT1-Day-X-YourSurname-YourName-YourIDNo". For instance, "CS551K-ASMNT1-Day-1-Smith-John-999999.pdf", where Day 1-5 should be the day of the class test in question, and 999999 is your student ID. Please try to make your submission file less than 10MB as you may have issues uploading large files onto MyAberdeen.
- Indicate clearly in your submission which item each solution is for. If we cannot identify this, you may be marked down.

Auctions

1. Consider a first-price and sealed-bid auction with three agents and two items X and Y, i.e., $Ag = \{1,2,3\}$ and $\mathcal{Z} = \{X, Y\}$. The agents have to simultaneously submit their bids for the two items in closed envelopes and an object is allocated to the agent who submitted the highest bid for it (winner). In first-price auctions, the winners have to pay the amount equal to the highest bid. The corresponding valuation functions for the three agents are v_1, v_2 and v_3 such that:

	$\{X\}$	$\{Y\}$
v_1	2	0.7
v_2	2.5	2.1
v_3	2.2	2.3

The value 2 *in column* $\{X\}$ *and row* v_1 *means that* $v_1(\{X\}) = 2$. *Moreover*, for every $i \in Ag, v_i(\{X, Y\}) = v_i(\{X\}) + v_i(\{Y\})$.

a) Assume all the goods must be allocated, enumerate all the possible allocations in $alloc(\mathcal{Z}, Ag)$. Your answer should use of the notation used in the lectures. (**1 Mark**)
b) If each agent bids for his own valuation, what will be the allocation of goods? (**1 Mark**)
c) Compute the social welfare for all the possible allocations in $alloc(\mathcal{Z}, Ag)$. (**1.5 Marks**)
d) Let b $= \big((b_{1X}, b_{1Y}), (b_{2X}, b_{2Y}), (b_{3X}, b_{3Y})\big)$ be the vector of submitted bids. For example, $\big((1.5,0.3), (2.4,0.6), (1.6, 0.7)\big)$ means that agent 1 submitted a bid of 1.5 on item X and 0.3 on item Y whereas agent 2 submitted a bid of 2.4 on item X and 0.6 on item Y. The profit (or payoff) for item X with respect to a winner i is $b_{iX} - v_i(\{X\})$ *and* 0 *for the other players*. We say that b is a Nash Equilibrium iff for every $z \in \{X, Y\}$, for $i = \underset{a \in Ag}{\operatorname{argmax}}\, b_{az}$, it holds that $0 \leq \underset{j \neq i}{max}\, v_j(\{z\}) \leq \underset{j \neq i}{max}\, b_{jz} = b_{iz} \leq v_i(\{z\})$.
Provide a vector of submitted bids that is a Nash Equilibrium. (**1.5 Marks**)

2. Consider a combinatorial auction with *two agents* $Ag = \{1,2\}$ and four items V, W, X, Z. The bids of the two agents are modelled using XOR bids as follows:

Agent 1	$(\{V, W\},3)$ *XOR* $(\{W, X\},8)$ *XOR* $(\{X\},7)$
*Agent*2	$(\{W\},7)$ *XOR* $(\{X, Z\},5)$

a) What is the maximum social welfare possible? Enumerate all the possible allocations (when all goods are allocated) that can achieve this social welfare. (**1.5 Marks**)
b) What is the minimum social welfare possible? Enumerate all the possible allocations (when all goods are allocated) that can achieve this social welfare. (**1.5 Marks**)