Yassin Dinana
52094480

Assessment 2
Part 1

Class test 6

Q1:

a) *Assume all the goods must be allocated, enumerate all the possible allocations in alloc(Z, Ag). Your answer should use the notation used in the lectures*.

	{X}	{Y}
v_1	2	0.7
v_2	2.5	2.1
v_3	2.2	2.3

Ag (Agents) = {1,2,3}. –Meaning we have three agents in the problem.
$\mathcal{Z} = \{X, Y\}$
$v_1(\{X\}) = 2$, $v_1(\{Y\}) = 0.7$, $v_1(\{X, Y\}) = 2.7$
$v_2(\{X\}) = 2.5$, $v_2(\{Y\}) = 2.1$, $v_2(\{X, Y\}) = 4.6$
$v_3(\{X\}) = 2.2$, $v_3(\{Y\}) = 2.3$, $v_3(\{X, Y\}) = 4.5$

$alloc(\mathcal{Z}_1, Agent) = (\{X\}, \{Y\}, \{\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_2, Agent) = (\{Y\}, \{X\}, \{\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_3, Agent) = (\{\}, \{X\}, \{Y\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_4, Agent) = (\{\}, \{Y\}, \{X\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_5, Agent) = (\{X\}, \{\}, \{Y\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_6, Agent) = (\{Y\}, \{\}, \{X\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_7, Agent) = (\{X,Y\}, \{\}, \{\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_8, Agent) = (\{\}, \{X,Y\}, \{\}, v_1, v_2, v_3)$
$alloc(\mathcal{Z}_9, Agent) = (\{\}, \{\}, \{X,Y\}, v_1, v_2, v_3)$

b) <u>If each agent bids for his own valuation, what will be the allocation of goods?</u>

By looking at the table in part A, every auctioneer would try and gain the most profit, by visualising all the states in part A, it can concluded that the highest allocation is:

$$alloc(\mathcal{Z}_3, Agent) = (\{\ \}, \{X\}, \{Y\}, v_1, v_2, v_3)$$

Yielding at expected return of 4.8.

c) <u>Compute the social welfare for all possible allocations in alloc(Z, Ag)</u>

$sw(\{X\}, \{Y\}, \{\ \}, v_1, v_2, v_3) = 2 + 2.1 = 4.1$
$sw(\{Y\}, \{X\}, \{\ \}, v_1, v_2, v_3) = 0.7 + 2.5 = 3.2$
$sw(\{\ \}, \{X\}, \{Y\}, v_1, v_2, v_3) = 2.5 + 2.3 = 4.8$
$sw(\{\ \}, \{Y\}, \{X\}, v_1, v_2, v_3) = 2.1 + 2.2 = 4.3$
$sw(\{X\}, \{\ \}, \{Y\}, v_1, v_2, v_3) = 2 + 2.3 = 4.3$
$sw(\{Y\}, \{\ \}, \{X\}, v_1, v_2, v_3) = 0.7 + 2.2 = 2.9$
$sw(\{X,Y\}, \{\ \}, \{\ \}, v_1, v_2, v_3) = 2 + 0.7 = 2.7$
$sw(\{\ \}, \{X,Y\}, \{\ \}, v_1, v_2, v_3) = 2.5 + 2.1 = 4.6$
$sw(\{\ \}, \{\ \}, \{X,Y\}, v_1, v_2, v_3) = 2.2 + 2.3 = 4.5$

Q2:

All allocation that can be applied by looking at both agents in the question:

$alloc(Z_1, Ag) = (\langle \{V\}, \{W, X, Z\} \rangle, v_1, v_2)$	$alloc(Z_8, Ag) = (\langle \{W, X, Z\}, \{V\} \rangle, v_1, v_2)$
$alloc(Z_2, Ag) = (\langle \{V, W\}, \{X, Z\} \rangle, v_1, v_2)$	$alloc(Z_9, Ag) = (\langle \{W, X\}, \{Z, V\} \rangle, v_1, v_2)$
$alloc(Z_3, Ag) = (\langle \{V, W, X\}, \{Z\} \rangle, v_1, v_2)$	$alloc(Z_{10}, Ag) = (\langle \{V, X\}, \{Z, W\} \rangle, v_1, v_2)$
$alloc(Z_4, Ag) = (\langle \{W\}, \{V, X, Z\} \rangle, v_1, v_2)$	$alloc(Z_{11}, Ag) = (\langle \{X\}, \{Z, V, W\} \rangle, v_1, v_2)$
$alloc(Z_5, Ag) = (\langle \{Z\}, \{V, W, X\} \rangle, v_1, v_2)$	$alloc(Z_{12}, Ag) = (\langle \{X, Z\}, \{V, W\} \rangle, v_1, v_2)$
$alloc(Z_6, Ag) = (\langle \{Z, V\}, \{W, X\} \rangle, v_1, v_2)$	$alloc(Z_{13}, Ag) = (\langle \{X, Z, V\}, \{W\} \rangle, v_1, v_2)$
$alloc(Z_7, Ag) = (\langle \{Z, V, W\}, \{X\} \rangle, v_1, v_2)$	$alloc(Z_{14}, Ag) = (\langle \{W, Z\}, \{X, V\} \rangle, v_1, v_2)$

All possible social welfare after calculating it using the table given for both agents in the question:

$sw(alloc(Z_1, Ag) = (\langle \{V\}, \{W, X, Z\} \rangle, v_1, v_2)) = 0 + 7 = 7$	$sw(alloc(Z_8, Ag) = (\langle \{W, X, Z\}, \{V\} \rangle, v_1, v_2)) = 8 + 0 = 8$
$sw(alloc(Z_2, Ag) = (\langle \{V, W\}, \{X, Z\} \rangle, v_1, v_2)) = 3 + 5 = 8$	$sw(alloc(Z_9, Ag) = (\langle \{W, X\}, \{Z, V\} \rangle, v_1, v_2)) = 8 + 0 = 8$
$sw(alloc(Z_3, Ag) = (\langle \{V, W, X\}, \{Z\} \rangle, v_1, v_2)) = 8 + 0 = 8$	$sw(alloc(Z_{10}, Ag) = (\langle \{V, X\}, \{Z, W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z_4, Ag) = (\langle \{W\}, \{V, X, Z\} \rangle, v_1, v_2)) = 0 + 5 = 5$	$sw(alloc(Z_{11}, Ag) = (\langle \{X\}, \{Z, V, W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z_5, Ag) = (\langle \{Z\}, \{V, W, X\} \rangle, v_1, v_2)) = 0 + 7 = 7$	$sw(alloc(Z_{12}, Ag) = (\langle \{X, Z\}, \{V, W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z_6, Ag) = (\langle \{Z, V\}, \{W, X\} \rangle, v_1, v_2)) = 0 + 7 = 7$	$sw(alloc(Z_{13}, Ag) = (\langle \{X, Z, V\}, \{W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z_7, Ag) = (\langle \{Z, V, W\}, \{X\} \rangle, v_1, v_2)) = 3 + 0 = 3$	$sw(alloc(Z_{14}, Ag) = (\langle \{W, Z\}, \{X, V\} \rangle, v_1, v_2)) = 0 + 0 = 0$

a)

$sw(alloc(Z, Ag) = (\langle \{V, X\}, \{Z, W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z, Ag) = (\langle \{X\}, \{Z, V, W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z, Ag) = (\langle \{X, Z\}, \{V, W\} \rangle, v_1, v_2)) = 7 + 7 = 14$
$sw(alloc(Z, Ag) = (\langle \{X, Z, V\}, \{W\} \rangle, v_1, v_2)) = 7 + 7 = 14$

b) The only allocation that minimizes the welfare is the one that has no values:

$sw(alloc(\mathcal{Z}, Ag)) = (\langle \{W, Z\}, \{X, V\} \rangle, v_1, v_2)) = 0 + 0 = 0$