

Question 3:

$$T_A = 7T_A\left(\frac{n}{2}\right) + n^2; T_B(n) = \alpha T_B\left(\frac{n}{4}\right) + n^2$$

first we solve the time complexity of $T_A(n)$.

$$a = 7, b = 2, K = \log_2 7 \approx 2.81, f(n) = n^2.$$

we can apply the first case of master's theorem.

$$\text{for } \varepsilon \approx 0.81 > 0; f(n) \text{ is } O(n^{\log_2 7 - \varepsilon}) = O(n^2).$$

$$\Rightarrow T_A(n) \in \Theta(n^{\log_2 7}) = \Theta(n^{2.81}).$$

To find the largest α for T_B that is asymptotically faster.

we set the values equal to each other, we solve α .

Then we deduct 1 from α .

$$K_A = \log_7 2 = \log_4 \alpha'$$

$$\Rightarrow \alpha' = 4.9 \rightarrow \alpha = 4.9 - 1 = 3.9.$$

hence The algorithm B runs asymptotically faster than T_A for $\alpha \leq 3.9$.