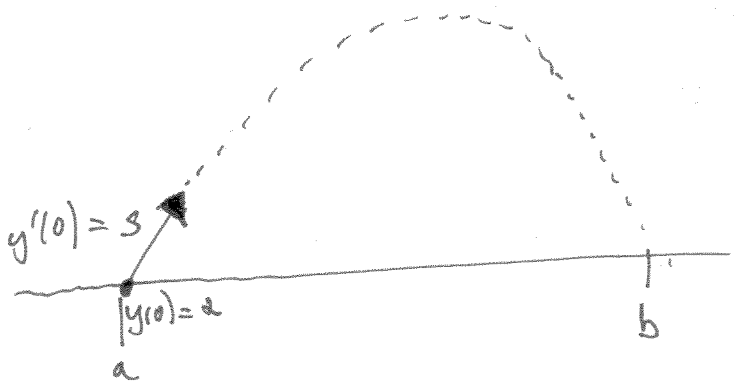
Today. Solving BVP with shooting ~~point~~ method.
We'll go through all the sub parts of the shooting ~~point~~ method

To start: consider IVP

$$y'' = 2y^3$$
$$y(a) = \alpha$$
$$y'(a) = s$$

$\longleftarrow$ s is some arbitrary parameter: the angle of ~~the~~ your "mortar"



Need to convert to system, to solve numerically

Let $\vec{Y} = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}$ with $Y_1 = y$, $Y_2 = y'$

Then $\vec{Y}' = \begin{pmatrix} Y_2 \\ 2Y_1^3 \end{pmatrix}$

Now use Euler's method to solve IVP: $\vec{Y}_{n+1} = \vec{Y}_n + (\Delta t)\vec{Y}_n'$

(or (Δx))

The solution depends on the choice of S, the slope.

Let's write

$y(b;s)$ — the sol'n at point b depends on S

Now consider BVP

$$u'' = 2u^3$$
$$u(a) = \alpha$$
$$u(b) = \beta$$

we can't solve this easily for u, but we can solve for $y(b;s)$.

↳ if we find some s^* such that

$$y(b;s^*) = \beta \quad (= u(b))$$

then we're done!

Define $\phi(s) = y(b;s) - \beta$

↳ Need to find the root of $\phi(s)$!

Ie, if $\phi(s^*) = 0$ then $y(b;s^*) = \beta$!

Use Newton's Method.

Remember:

$$s_{k+1} = s_k - \frac{\phi(s_k)}{\phi'(s_k)}$$

But wait... what is $\phi'(s_k)$? We don't know $\phi'(s) = \frac{d}{ds} y(b;s)$.

Define $z = \frac{\partial}{\partial s}(\phi(s)) = \phi'(s) = \frac{d}{ds} y(b;s)$

we can differentiate wrt x

$$\frac{d}{dx^2} z = \frac{\partial}{\partial x^2}\left(\frac{\partial}{\partial s} y(x;s)\right)$$

$$= \frac{\partial}{\partial s}\left(\frac{\partial}{\partial x^2} y(x;s)\right)$$

Suppose y solves ODE $y'' = f(x, y, y')$

$$= \frac{\partial}{\partial s}\left(f(x, y(x;s), y'(x;s)\right)$$

$$= f_y(x,y,y') \frac{d}{ds} y + f_{y'}(x,y,y') \frac{d}{ds} y'$$

$$= f_y(x,y,y') z + f_{y'}(x,y,y') z'$$

~~And with I.C. [illegible]~~

So $z \left(= \phi'(s)\right)$

Solves

$$\begin{cases} z'' = f_y(x,y,y')z + f_{y'}(x,y,y')z' \\ z(a) = 0 \\ z'(a) = 1 \end{cases}$$

for us, $f = 2y^3$

$f_y = 6y^2$

$f_{y'} = 0$

Ie
$$\begin{cases} z'' = 6y^2 z \\ z(a) = 0 \\ z'(a) = 1 \end{cases}$$

For every s_k of the newton solver we need to solve ODE

$$\begin{cases} y'' = 2y^3 \\ z'' = 6y^2 z \\ y(a) = \alpha \\ y'(a) = s_k \\ z(a) = 0 \\ z'(a) = 1 \end{cases}$$

eg. use Euler for systems.

Remember $z(b) = \phi'(s_k)$

So plug this in:

$$s_{k+1} = s_k - \frac{\phi(s_k)}{\phi'(s_k)}$$

$$= s_k - \frac{y(b; s_k)}{z(b)}$$