McGill University
Faculty of Science

Midterm Examination

Math 317 – Numerical Analysis

Examinar: Tiago Salvador

Date: October 27th, 2016

Time: 8:35 AM - 9:55 AM

Student name (last, first)	**Student number (McGill ID)**

This exam contains a total of 6 **pages** (including this cover page) and **5 questions**.

INSTRUCTIONS

- Print your full name and student number clearly on each page
- Answer all 5 questions direcly on the exam; show your work
- If you need more space, use the back of the pages
- This a is a **closed** book exam
- Calculators, notes, formula sheets are **not** permitted.

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
Total:	50	

1. (10 marks) Compute the Lagrange interpolation polynomial for $\{(0,1),(1,2),(2,3),(3,-2)\}$ and express the polynomial in the simplest form.

2. (10 marks) A clampled cubic spline S for a function f is defined on $[-1, 1]$ by

$$S(x) = \begin{cases} S_0(x) = 1 - 6(x+1) + 3(x+1)^2 - 2(x+1)^3 & \text{if } x \in [-1, 0), \\ S_1(x) = a + bx + cx^2 + dx^3 & \text{if } x \in [0, 1]. \end{cases}$$

Given $f'(-1) = f'(1)$, find the constants a, b, c and d.

3. (a) (5 marks) Find the constants a, b, c such that the finite difference of the first derivative

$$D_h f(x_0) := af(x_0 - h) + bf(x_0) + cf(x_0 + h)$$

has the highest degree of accuracy possible.

(b) (5 marks) Assume $f \in C^3(\mathbb{R})$. Using the Taylor expansion, find a formula for the error of $D_h f$ and express it in the simplest form.

4. Consider the weighted integral $I(f) := \int_{-1}^{1} f(x)x dx$.

   (a) (7 marks) Find the constants a, b, c such that the 3-point quadrature

   $$I_h(f) := af(-1) + bf(0) + cf(1)$$

   has the highest degree of accuracy with respect to $I(f)$.

   (b) (3 marks) Find the degree of accuracy of $I_h(f)$ from part (a).

5. Consider the fixed point iteration $x_{n+1} = g(x_n)$ where

$$g(x) = x + \mu(2 - e^x)$$

with $\mu \neq 0$.

(a) (2 marks) Find the fixed point x^* of g.

(b) (5 marks) Show that the fixed point iteration converges locally if $\mu \in (0, 1)$.

(c) (3 marks) Find the value μ for which the fixed point iteration converges at least quadratically.