MATH 318, Assignment 2

Due date: October 5, in class

Suppose $f : X \to Y$ is a function and $A \subseteq X$, $B \subseteq Y$

1. (3 points)
   (1) Show that if f is a surjection, then $f(f^{-1}(B)) = B$.
   (2) Is is always true that $f^{-1}(f(A)) = A$? Justify your answer.
   (3) Show that $f(A \cap f^{-1}(B)) = f(A) \cap B$.

2. (4 points) How many functions are there:
   (1) from 2 to 3,
   (2) from 5 to 1,
   (3) from 5 to 0,
   (4) from 0 to 5?

3. (3 points) Are the following sets equinumerous? Justify your answers.
   (1) $[0, 1)$ and $\mathbb{Q}$,
   (2) $[0, 1]^{\mathbb{N}}$ and $[0, \infty)$,
   (3) $[0, 1]^{\mathbb{N}}$ and $\mathbb{Q}^{\mathbb{N}}$?

4. (4 points) Are the following sets countable? Justify your answers.
   (A) $\mathbb{Z}^{\mathbb{N}}$,
   (B) $\mathbb{Z}^3 \cup \mathbb{Z}^7$,
   (C) $\bigcup_{n\in\mathbb{N}} \mathbb{Z}^n$,
   (D) $\mathbb{R} \times \mathbb{Q}$.

A *cycle* on a finite set X is a bijection $f : X \to X$ such that for some $x_0, \dots, x_n \in X$ we have $f(x_i) = x_{i+1}$ for $i < n$ and $f(x_n) = x_0$. If f is a cycle as above, then the set $\{x_0, \dots, x_n\}$ is called the *support* of the cycle f. Two cycles are called *disjoint* if their supports are disjoint

5. (2 points) Show that any bijection on a finite set is a composition of disjoint cycles.

A bijection $f : X \to X$ is an *involution* if $ff = \mathrm{id}_X$.

6. (4 points) Show that any bijection on any set (possibly infinite) is a composition of two involutions.