Math 318: Assignment 1 Solutions

Problem 1

Part (A)

False, since $1 \notin \{0\} = 1$.

Part (B)

True.

Part (C)

True.

Part (D)

False, since $0 \in 1$ but $0 \notin \{1, 2\}$.

Problem 2

We have the following equivalences:

$$\bigcup x = \varnothing \iff \bigcup x \subset \varnothing \iff \forall y \in x[y \subset \varnothing] \iff \forall y \in x[y \in \mathcal{P}(\varnothing)] \iff x \subset \mathcal{P}(\varnothing)$$

Now $\mathcal{P}(\varnothing) = \{\varnothing\}$, so the possible x are $\varnothing$ and $\{\varnothing\}$. $\square$

Problem 3

Part (1)

The transitive closure is the following:

$$T = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3), (4,4)\}$$

Part (2)

T is reflexive since $(k, k) \in T$ for $1 \leq k \leq 4$, is is symmetric since if $(k, l) \in T$ then $(l, k) \in T$, and it is transitive since it is a transitive closure. Thus T is an equivalence relation. $\square$

Part (3)

The equivalence class of 1 is $[1]_T = \{1, 2, 3\}$.

Problem 4

Part (1)

Since $1 \not< 1$, we have $1 \not\!E\, 1$ and thus E is not reflexive. Thus E is not an equivalence relation. $\square$

Part (2)

E is reflexive since for $x \in \mathbb{N}$, we have $x^2 = x^2$. Also E is symmetric since for $x, y \in \mathbb{N}$, we have that $x^2 = y^2$ iff $y^2 = x^2$. Finally, E is transitive since for $x, y, z \in \mathbb{N}$, if $x^2 = y^2$ and $y^2 = z^2$, we have $x^2 = z^2$. Thus E is an equivalence relation. $\square$

Part (3)

Since $0 - 0 = 0 \in \mathbb{Q}$, we have $0 \not\!E\, 0$ and thus E is not reflexive. Thus E is not an equivalence relation. $\square$

Problem 5

Part (1)

$R \circ R$ is the following:

$$R \circ R = \{(1,3), (1,4), (2,4)\}$$

Part (2)

$R \circ R^{-1}$ is the following:

$$R \circ R^{-1} = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$$

Problem 6

A binary relation is determined by selecting ordered pairs of elements of $\{1, 2, 3\}$, and there are $3^2 = 9$ of these, so there are 2^9 binary relations.

Problem 7

A reflexive binary relation is determined by selecting ordered pairs of distinct elements of $\{1, 2, 3\}$, and there are $3 \cdot 2 = 6$ of these, so there are 2^6 reflexive binary relations.

Problem 8

A reflexive symmetric binary relation is determined by selecting unordered pairs of distinct elements of $\{1, 2, 3\}$, and there are $\binom{3}{2} = 3$ of these, so there are 2^3 reflexive symmetric binary relations.

Problem 9

Equivalence relations are in correspondence with partitions, so it suffices to count the number of partitions of $\{1, 2, 3\}$. There are five partitions:

$$\{\{1,2,3\}\}, \{\{1,2\},\{3\}\}, \{\{2,3\},\{1\}\}, \{\{3,1\},\{2\}\}, \{\{1\},\{2\},\{3\}\}$$

Thus there are five equivalence relations.