Elementary Numerical Analysis (MATH-317) Midterm 1
3 problems, total points = 30, total time = 45 minutes.

Question 1)

a) You are given the polynomial

$$P(x) = x^4 - 2x^3 - 3x^2 + x - 4 = (x - 1.5)Q(x) + b_0.$$

Use **Horner's method** to find $P(1.5)$ and $Q(x)$ with as few additions and multiplications as possible. How many of these operations did you perform?

b) You are given that

$$f(x) = (x-1)^2(x)(x+2),$$

and you plan to use the Bisection Method over the interval $[-3, 0.5]$ to find a root of $f(x) = 0$. Can you say to which root the method will converge? Why?

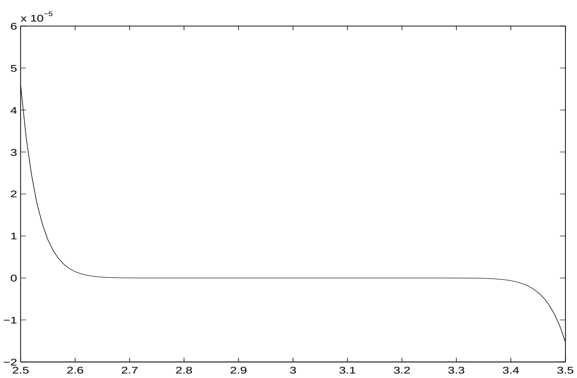
Question 2) The goal of this problem is to find the root of

$$f(x) = (x-3)^{15} * (x-4) = 0$$

over the interval $[2, 3.5]$. To do this, Newton's method was implemented. Given below are $|p_n - p|$ for $n = 0, 1, 2, 3, 4$. The starting point was $p_o = 2$. The true solution is $p = 3$.

n	|pn-p|
0	1.0000
1	0.9355
2	0.8751
3	0.8185
4	0.7655

a) Define rate of convergence, and order of convergence.
b)What is the apparant **rate** of convergence (looking at the numbers above)? What is the apparant **order** of convergence and how does this compare to the *usual* order of convergence for Newton's method? What could have gone wrong?
c) Write the formula for Newton's method in this case.
d) Figure (1) is a graph of $f(x)$ between $[2.5, 3.5]$.



Which, if any, of the following algorithms
Secant, False Position, Müller's Method.
would you recommend for locating the root? Explain your reasons briefly.

e) Given $f(x)$, how would you recommend we find the root $x = 3$?

Question 3

a) We want to approximate the roots of

$$\frac{1}{3}x^2 - \frac{123}{4}x + \frac{1}{6} = 0$$

using 3-digit rounding arithmetic. The true solutions are $x_1 = 0.005420372688, x_2 = 92.24457963$.

Either: predict **without calculation** which root —x_1 or x_2 — will be computed with smaller relative error using 3-digit rounding and the quadratic formula. Explain your reasoning.
Or: use **3-digit arithmetic** and the quadratic formula to approximate the roots. Find the relative errors for both roots. Which root — x_1 or x_2 - did you compute more accurately using the quadratic formula. Why?

b) Do one of the following problems (i) or (ii):

- **i.** Find the second Taylor Polynomial $P_2(x)$ for $f(x) = e^x \cos(x)$ about $x_0 = 0$, as well as the remainder term $R_2(x)$.
- **ii)** Suppose $f(x) = (x - p)^m \phi(x)$, $\phi(p) \neq 0$, and f''' is continuous on an open interval containing p. Show that the following fixed point method has $g'(p) = 0$:

  $$g(x) = x - \frac{mf(x)}{f'(x)}.$$

  What is the order of convergence for this fixed point method?