# Practice Problems Midterm

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1. What does the binary number 1001001 correspond to in base 10 digits, i.e., human style.
2. How accurately do we need to know $e$ to be able to compute $e^{-1}$ with five correct decimals?
3. Find the third Taylor polynomial $P_3(x)$ for the function $f(x) = \sqrt{x+1}$ about $x_0 = 0$.
4. Consider the iteration with $g(x) = x + \frac{1}{2}(2 - e^x)$.
   (a) Show that the iteration has a fixed point $x^* = \log(2)$.
   (b) Show that the scheme satisfies all the conditions of the Fixed Point Theorem on the interval $[0, 1]$.
   (c) What is the order of convergence of the scheme? State the asymptotic error constant.
5. Given

| i | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| $x_i$ | 0 | 1 | 2 | 3 |
| $f(x_i)$ | 2 | 3 | 10 | 29 |

   construct the appropriate table of divided differences and hence state
   i. the polynomial of degree 2 which interpolates $f$ at $x_1$, $x_2$ and $x_3$.
   ii. the polynomial of degree 3 which interpolates $f$ at $x_0$, $x_1$, $x_2$ and $x_3$.
6. Explain Runge's phenomenon and how it can be fixed.
7. How does Hermite interpolation improve upon Lagrange interpolation?
8. Given the function $f(x) = \cos(\pi x)$ compute the Hermite interpolation polynomial with nodes $x_0 = 0$ and $x_1 = 1$.
9. A clamped cubic spline $S$ for a function $f$ is defined by

$$S(x) = \begin{cases} S_0(x) = 1 + b_0 x + 2x^2 - 2x^3 & x \in [0, 1) \\ S_1(x) = 1 + b_1(x-1) - 4(x-1)^2 - 7(x-1)^3 & x \in [1, 2] \end{cases}$$

   where $b_0$, $b_1$ are constants. Find $f'(0)$ and $f'(2)$.
10. Find the constants $a$, $b$, $c$ such that the finite difference of the first derivative

$$D_h f(x_0) := af(x_0 - h_1) + bf(x_0) + cf(x_0 + h_2)$$

   has the highest degree of accuracy possible.
11. Consider the integral $I(f) = \int_0^3 f(x)\,dx$. Find $a_0$, $a_1$, $a_2$, $a_3$ such that the quadrature

$$I_h(f) = a_0 f(0) + a_1 f(1) + a_2 f(2) + a_3 f(3).$$

   has the highest degree possible.
12. Determine constants $a$, $b$, $c$ and $d$ that will produce a quadrature formula

$$\int_{-1}^{1} f(x)\,dx \approx af(-1) + bf(1) + cf'(-1) + df'(1)$$

   that has at least degree of precision 3.