

1) Composite midpoint rule

$$\int_a^b f dx = \sum_{k=0}^{N-1} f(x_{k+\frac{1}{2}})h + O(h^2)$$

$$x_{k+\frac{1}{2}} = \frac{x_k + x_{k+1}}{2}$$

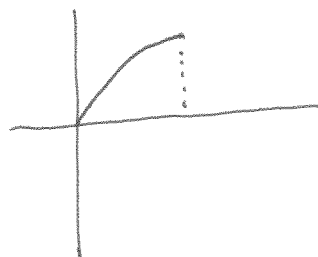
$$\begin{aligned} & \frac{h^3}{24} \sum_{k=0}^{N-1} f'''(\xi_k) \\ &= \frac{f'''(\xi)}{24} (b-a)h^2 \\ & \text{b/c } \sum_{k=0}^{N-1} f'''(\xi_k) = f'''(\xi) \\ & \quad \text{by I.V.T.} \end{aligned}$$

2) log log, with poly fit

eg)  $f = \sin(x)$ ,  $x \in [0, \frac{\pi}{2}]$

$$\int f = -\cos(x)$$

$$\int_0^{\frac{\pi}{2}} \sin(x) = 0 - (-1) = 1$$



$$3) \quad T'(t) = -k(T(t) - T_{env})$$

$$T(0) = T_0$$

exact sol'n

$$T(t) = (T_0 - T_{env})e^{-kt} + T_{env}$$

ux: Backward Euler (recall:  $f_{n+1} = f_n + h f'_{n+1}$ )

$$T_{n+1} = T_n + h(-k(T_{n+1} - T_{env}))$$

Don't know  $T_{n+1}$  - need to find  $T_{n+1}$  st

$$0 = T_n + h(-k(T_{n+1} - T_{env})) - T_{n+1}$$

I.e., think of  $T_{n+1}$  as a variable  $x$

$$0 = T_n + h(-k(x - T_{env})) - x$$

Need root solver: ux say Newton's method, as  $f_{\text{zero}}$

- plot vs true solution