MATH 318, Assignment 6

Due date (extended): November 23, in class

1. (1 point) Consider the language consisting of one symbol R for a binary relation. Let σ be the following sentence:

   $$\forall x\, \exists y\, \exists z\ (x \neq y \wedge y \neq z \wedge x \neq z \wedge x\ R\ y \wedge z\ R\ x).$$

   Does σ have a model? Justify your answer

A model is *finite* if its universe is finite. A model is *infinite* if its universe is infinite.

2. (2 point) Find a language L and a sentence σ such that σ has a model but does not have a finite model. Justify your answer.

3. (2 points) Find a language L and a sentence σ which has a model but does not have an infinite model.

Below, $\mathbb{N}_+$ stands for $\{1, 2, 3, \ldots\}$. Given a first-order language L and a sentence σ, the *spectrum* of σ is defined as $\mathrm{spec}(\sigma) = \{n \in \mathbb{N}_+ :$ there exists a model $M \models \sigma$ such that the universe of M has n elements$\}$.

4. For each set X below find a language L and a sentence σ in L such that $\mathrm{spec}(s) = X$. Justify your answers
   (a) (1 point) $X = \mathbb{N}_+$,
   (b) (2 points) $X = \{n \in \mathbb{N}_+ : n \text{ is even}\}$,
   (c) (2 points) $X = \{n \in \mathbb{N}_+ : n \text{ is a power of } 2\}$

5. (4 points) Consider the theory T of graphs. Show that there is no theory $T' \supseteq T$ such that the models of T' are exactly the connected graphs (i.e. the graphs for which for every pair of vertices there exists a path between them).

6. (4 points) Consider the theory T of linear orders. Show that there is no theory $T' \supseteq T$ such that the models of T' are exactly the well-orders.