MATH 318, Assignment 1

Due date: September 23, in class

1. (4 points) For each statement below choose if it is true or false:
   (A) $1 \in 1$, (B) $1 \subseteq 1$,
   (C) $1 \in \{1, 2\}$, (D) $1 \subseteq \{1, 2\}$.

2. (2 points) Find all sets x satisfying $\bigcup x = \emptyset$. Justify your answer.

3. (1) (2 points) Compute the transitive closure T of the following relation R on $\{1, 2, 3, 4\}$:
   $$R = \{(1, 2), (2, 3), (3, 1), (4, 4)\}.$$
   (2) (1 point) Is T an equivalence relation? Justify your answer.
   (3) (1 point) If T is an equivalence relation, then compute the equivalence class of 1.

4. (3 points) For each of the relations below decide if it is an equivalence relation. Justify your answers.
   (1) E on $\mathbb{N}$ defined as follows: $x \; E \; y$ if $x < y$,
   (2) E on $\mathbb{N}$ defined as follows: $x \; E \; y$ if $x^2 = y^2$,
   (3) E on $\mathbb{R}$ defined as follows: $x \; E \; y$ if $x - y \notin \mathbb{Q}$,

5. (2 points) Consider the relation R on $\{1, 2, 3, 4\}$ defined by $(x, y) \in R$ if $x < y$.
   (1) Compute $R \circ R$.
   (2) Compute $R \circ R^{-1}$.

6. (1 point) How many binary relations are there on the set $\{1, 2, 3\}$?

7. (1 point) How many reflexive binary relations are there on the set $\{1, 2, 3\}$?

8. (1 point) How many binary relations which are both reflexive and symmetric are there on the set $\{1, 2, 3\}$?

9. (2 points) How many equivalence relations are there on the set $\{1, 2, 3\}$?