MATH 318, Assignment 5

Due date: November 9, in class

We consider the deduction system for propositional logic with Modus Ponens as the inference rule and the following axiom schemes:

A1 $\varphi \to (\psi \to \varphi)$
A2 $(\varphi \to (\psi \to \chi)) \to ((\varphi \to \psi) \to (\varphi \to \chi))$
A3 $((\neg\varphi) \to (\neg\psi)) \to (\psi \to \varphi)$

Below, the symbols $\vee$ and $\wedge$ should be read as (any) expressions of the disjunction and conjuction using $\neg$ and $\to$.

1. (1 point) Let $\Gamma = \{p \wedge q, (\neg p) \vee q, p \vee r\}$. Is it true that $\Gamma \vdash r$? Justify your answer.

2. (1 point) Let $\Gamma = \{p \to q, q \to p, (\neg p) \wedge q\}$ Is it true that $\Gamma \vdash p$? Justify your answer.

In the problems below you are supposed to **find** formal deductions. This means it is not enough to show their existence (using the completeness theorem or the statement of the deduction theorem). However, in every problem you can reuse formal deductions that were shown in class as well as the formal deductions found in previous problems.

3. (1 point) Find a formal deduction showing that
$$\vdash (((\neg p) \to (\neg q)) \to q) \to (((\neg p) \to (\neg q)) \to p).$$

The variables α, β, γ denote formulas, you can use them in your deductions as in the axioms schemes.

4. Find formal deductions showing that:
   (a) (3 points) $\{\alpha \to \beta, \beta \to \gamma\} \vdash \alpha \to \gamma$,
   (b) (3 points) $\{\alpha \to \beta\} \vdash (\beta \to \gamma) \to (\alpha \to \gamma)$,
   (c) (3 points) $\vdash (\alpha \to \beta) \to ((\beta \to \gamma) \to (\alpha \to \gamma))$.

5. (3 points) Find a formal deduction showing that $\vdash \neg\neg\alpha \to \alpha$.

6. (3 points) Find a formal deduction showing that $\vdash \alpha \to \neg\neg\alpha$.