Question 2:

a)

$T(n) = 25\,T\left(\frac{n}{5}\right) + n.$

$a = 25, \quad b = 5, \quad K = \log_5(25) = 2; \quad f(n) = n.$

we can apply the 1st case of the master's thm.

for $\varepsilon = 1$; $\quad f(n) = \cancel{f(n^{2-\varepsilon})} = O(n^{2-\varepsilon}) = O(n).$

$\rightarrow T(n)$ is $\Theta(n^2)$

b) $T(n) = 2.T\left(\frac{n}{3}\right) + n.\log n.$

$a = 2, b = 3, K = \log_3 2 \simeq 0.63 < 1, f(n) = n\log n$

we apply the 3rd case of master theorem.

$n\log n$ is $\Omega(n)$.

i $\quad \Omega(n^{\log_3 2 + \varepsilon}) = \Omega(n)$ for $\varepsilon \simeq 0.73$

ii $\quad \frac{n}{3}\log\left(\frac{n}{3}\right) \leq \frac{1}{3} n\log n \quad C = \frac{1}{3}.$ (graphing) \\ last page

$\Longrightarrow T(n)$ is $\Theta(n\log n).$

c) $T(n) = T\left(\frac{3n}{4}\right) + 1.$

we can apply the second case of the master theorem.

$a = 1, \ b = \frac{3}{4}, \ f(n) = 1, \ K = \log_{\frac{4}{3}} 1 = 0.$

$f(n)$ is $\Theta(n^0 \log^0 n) = \Theta(1) \quad (P = 0)$

$\Longrightarrow T(n)$ is $\Theta(\log^{P+1} n) = \Theta(\log n).$