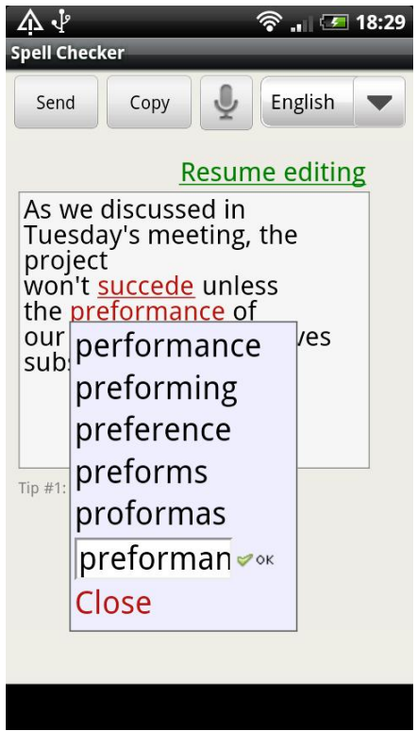
CSE202
Design and Analysis of Algorithms

Week 12 — String Algorithms 2
Tries & Compression

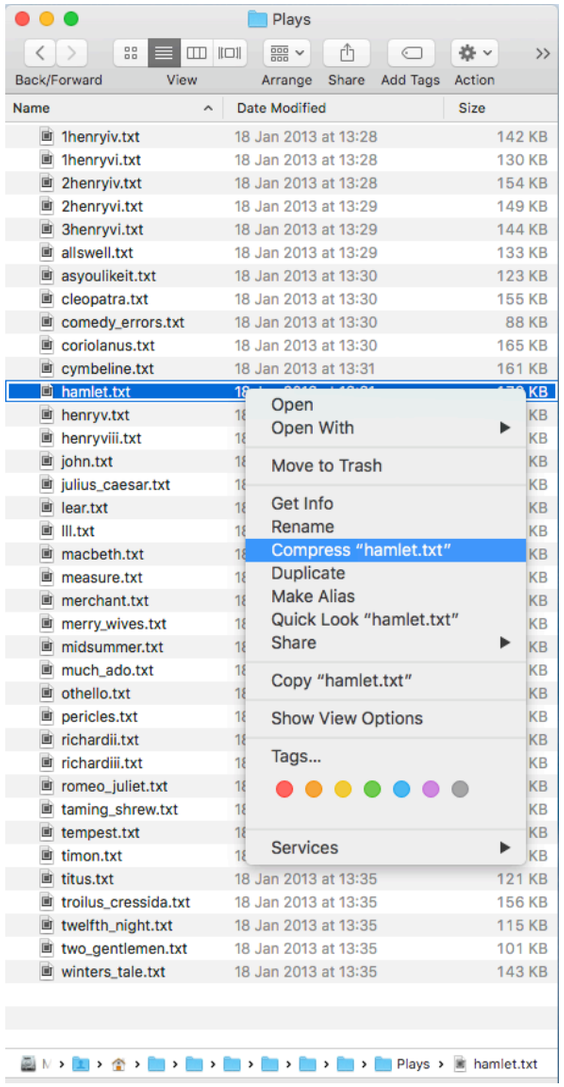
Dictionaries & Compression

Dictionaries & Compression

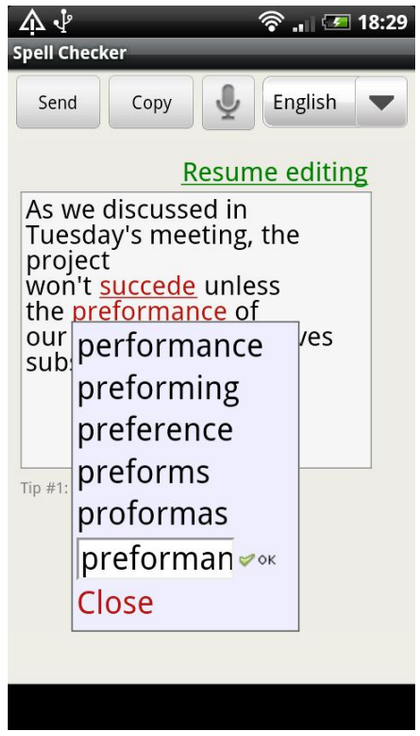


check existence
auto-complete

Dictionaries & Compression



shrink text by
a factor 2.5,
without any loss



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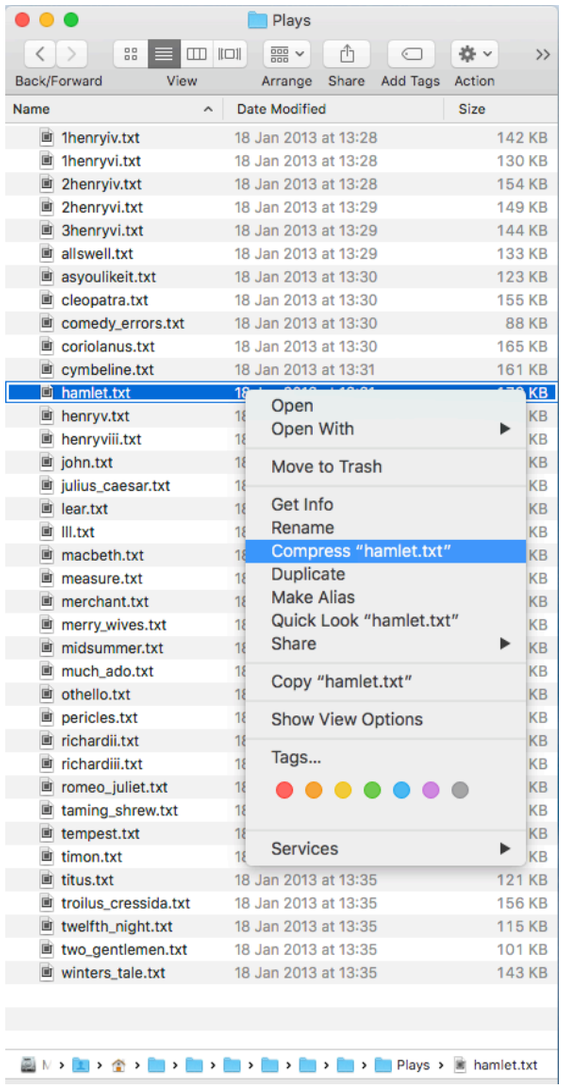

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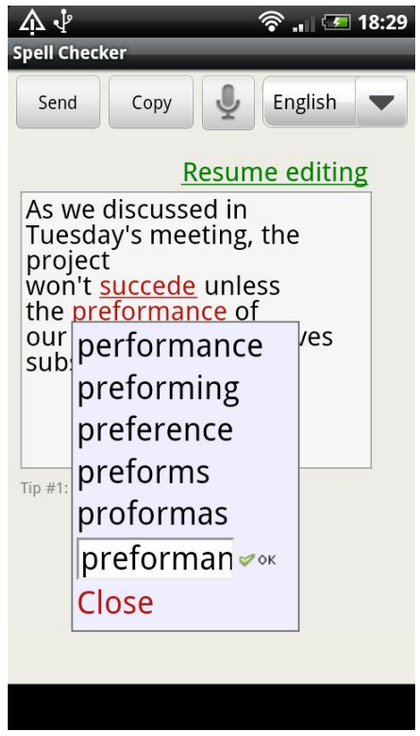

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Both rely on specific trees:
the tries

Lossless Compression Must (Discover and) Use Regularity

Alphabet Σ, with $|\Sigma| = R$.

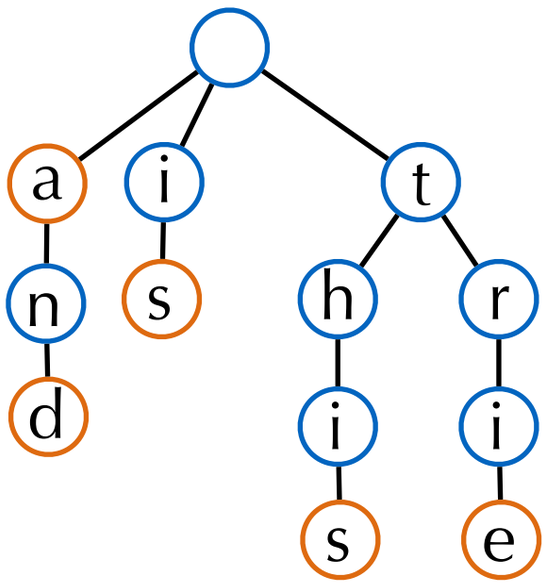
Not all R^n texts of length n can be reduced and still be distinct

An algorithm that compresses strings of length $2n$ by a factor 2 without loss can compress at most $1/R^n$ of them.

I. Tries (aka Digital Search Trees)

Tries

Children are sorted by alphabetical order

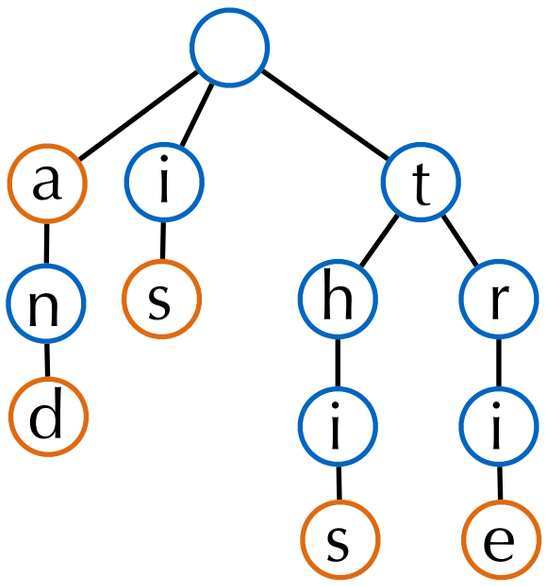


Trie built from the strings
“and”, “this”, “is”,
“a”, “trie”

Stores pairs (string, value)

Tries

Children are sorted by alphabetical order



Trie built from the strings
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```
class Node:

    def __init__(self,sizeAlpha,val=None):
        self.val = val # None except at end of string
        self.child = [None]*sizeAlpha
```

Stores pairs (string, value)

Tries

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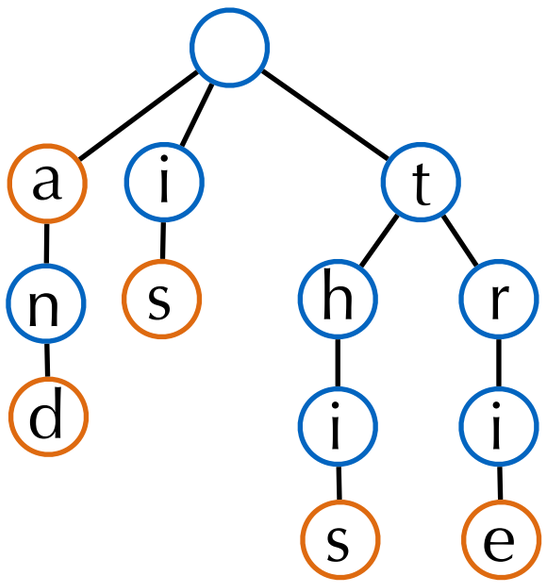

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```
class Node:

    def __init__(self,sizeAlpha,val=None):
        self.val = val # None except at end of string
        self.child = [None]*sizeAlpha
```

```
class Trie:

    def __init__(self, sizeAlpha=256):
        self.root = None
        self.sizeAlpha = sizeAlpha

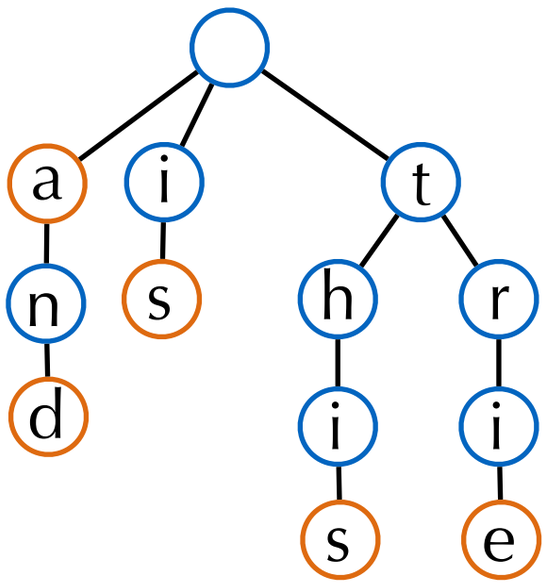
    def insert(self,str,val,index=0):
        self.root=self._insert(self.root,str,val,index)

    def find(self,str,index=0):
        return self._find(self.root,str,index)

    def longestprefix(self,str,index=0):
        return self._longestprefix(self.root,str,index)
```

Tries

Children are sorted by alphabetical order



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Exercise: auto-complete

```
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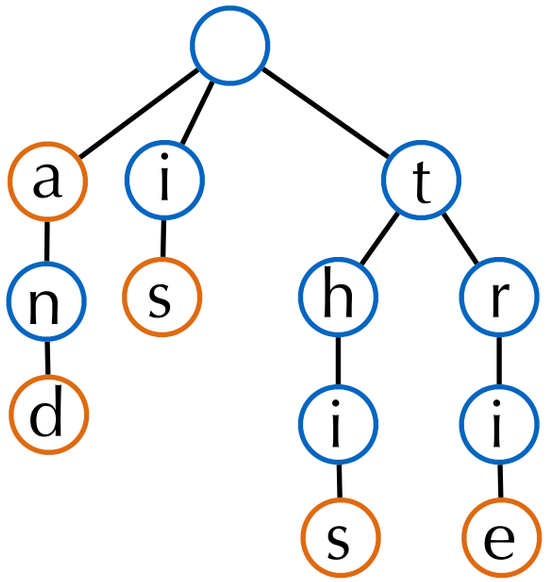
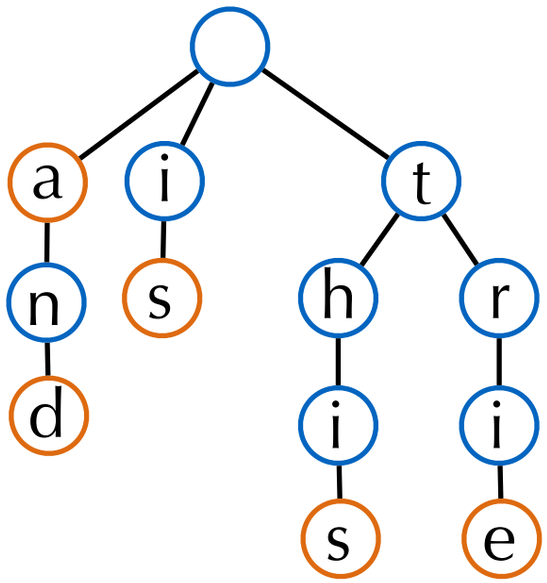
Find/Insert/LongestPrefix



Worst-case time *linear* in length of st (**optimal**).

Find/Insert/LongestPrefix



Worst-case time *linear* in length of st (**optimal**).

```
def _find(self,node,st,index):
    if node is None: return None
    if index == len(st): return node.val
    return self._find(node.child[ord(st[index])],st,index+1)
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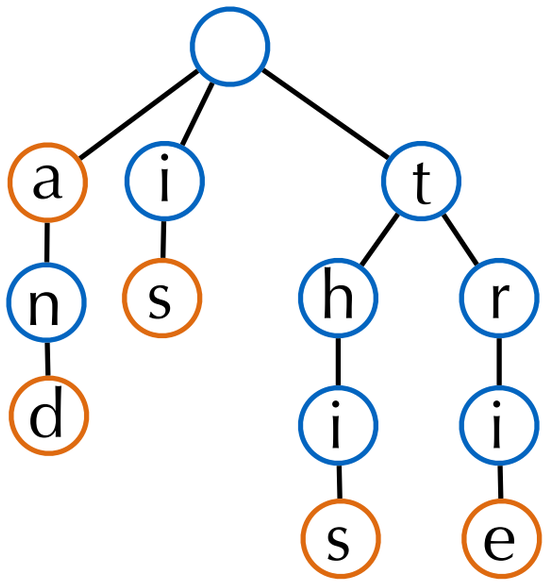
Find/Insert/LongestPrefix



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    if node is None: return -1
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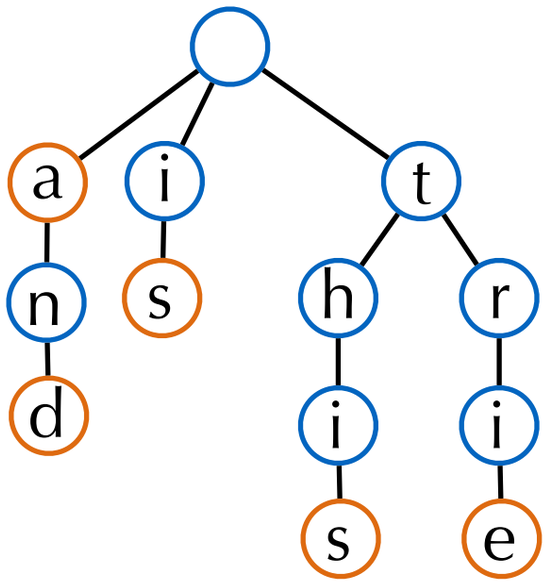

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    else:
        v = ord(st[index])
        node.child[v] = self._insert(node.child[v],st,val,index+1)
    return node
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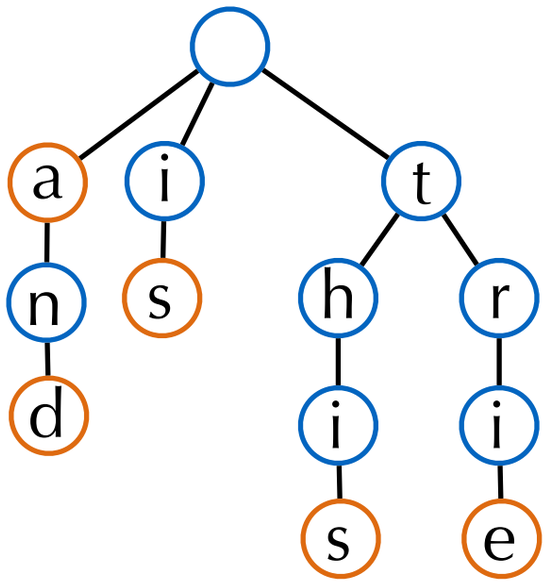

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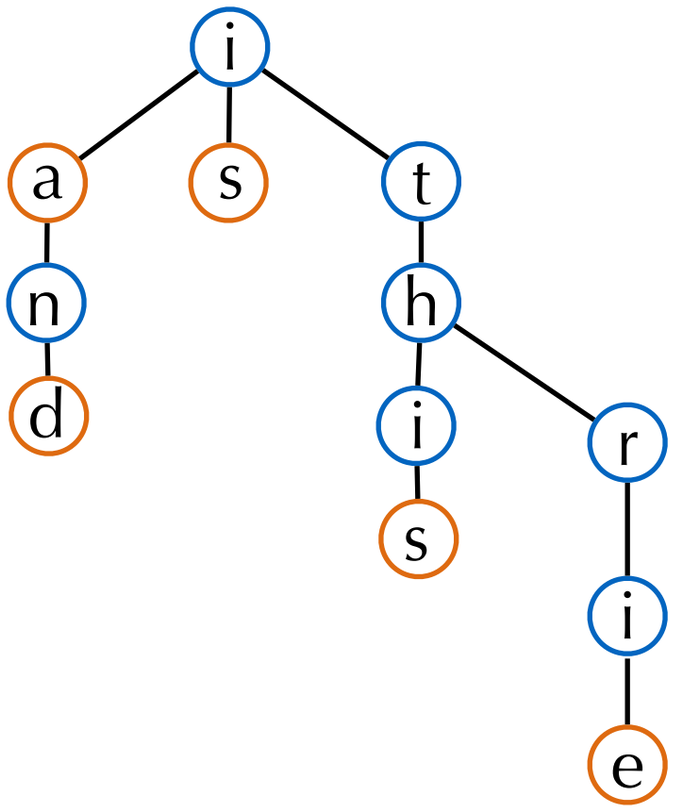
```

$R \times$ num. strings $\leq$ size(trie) $\leq R \times$ num. strings $\times$ average string length

Ternary Search Tries

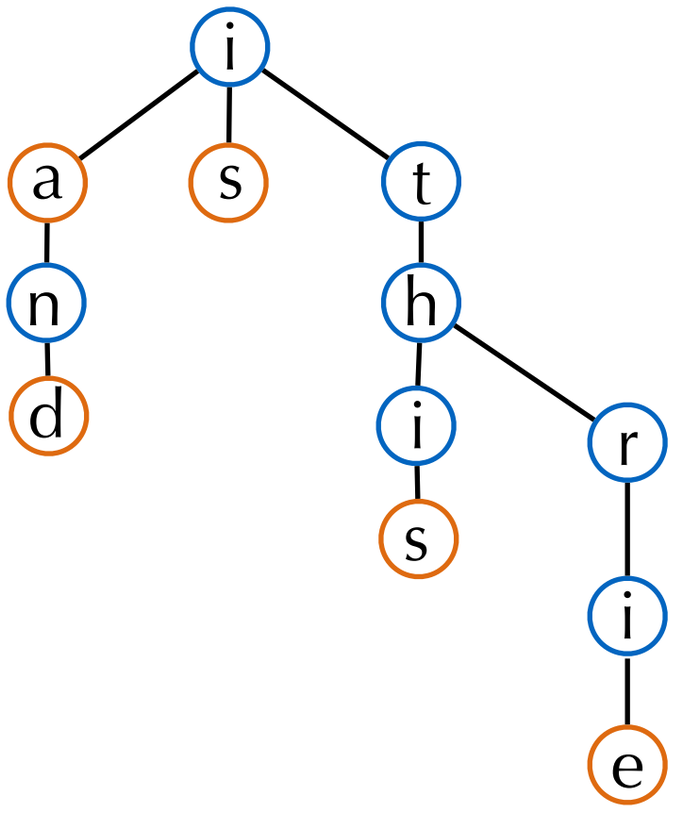


Save memory when
the alphabet is large
and the keys are long.

Each node has 3 children:
left for smaller in the alphabetic order
right for larger
middle for the next letters in the string

Hybrid between BST & Tries.

Ternary Search Tries



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Exercise:
find, insert

Hybrid between BST & Tries.

II. Lempel-Ziv-Welsh Compression

Create a Code with Longer and Longer Prefixes

Look for the longest prefix already in the code, and extend it

Code: a map $A^\star \to B^\star$, A & B two alphabets.

simplified example with $\Sigma = \{a, \ldots, z\}$

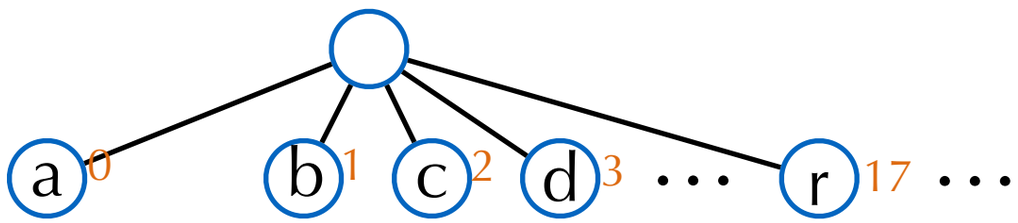
`abracadabracadabracadabra`

$A = \{a, \ldots, z\}$
$B = \{0, \ldots, 9\}$

a—z 0—25

input 25 characters in Σ

output 16 characters in $\mathbb{N}$



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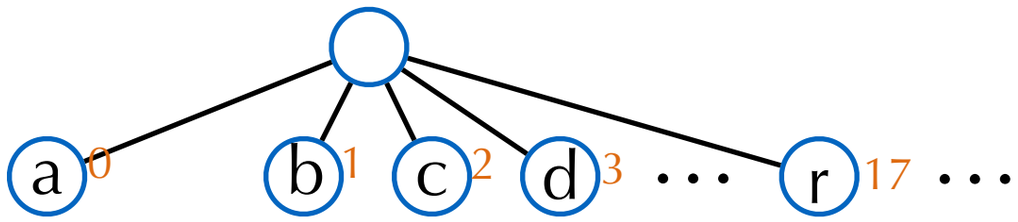

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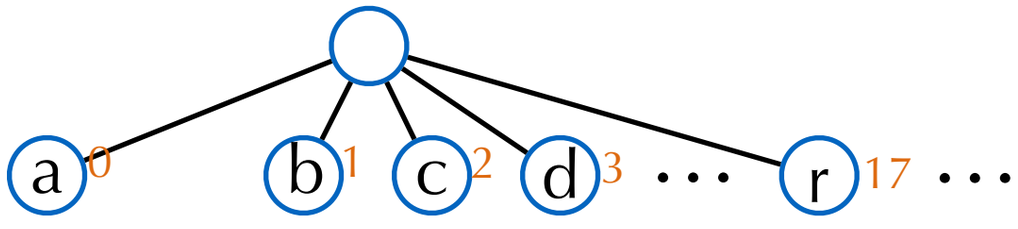

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0

ab 26

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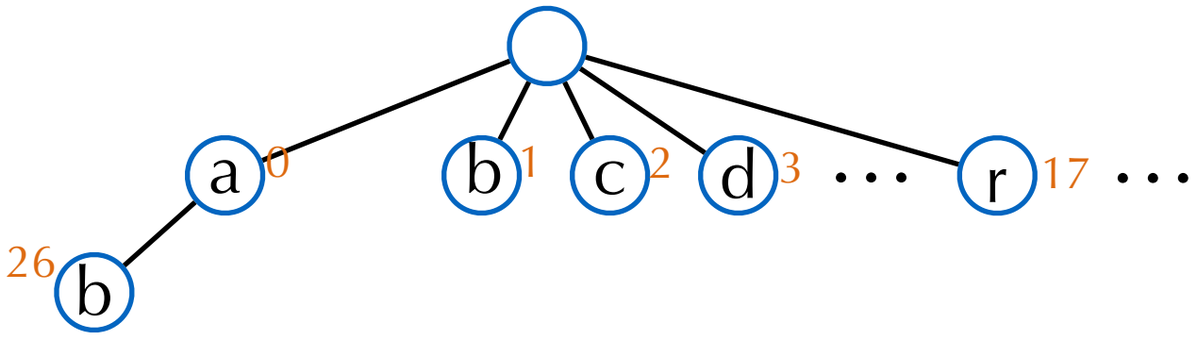

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ab|ra|cadabra|cadabra

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a—z 0—25

0 ab 26

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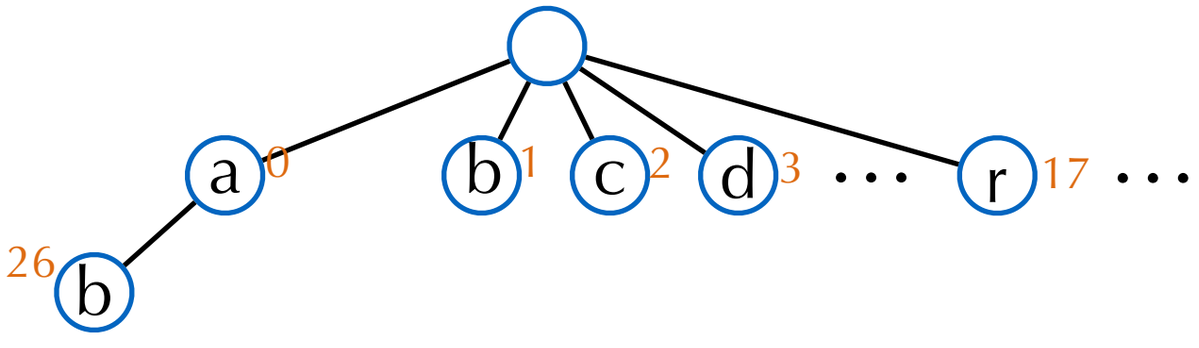
output 16 characters in $\mathbb{N}$



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0,1 ab 26

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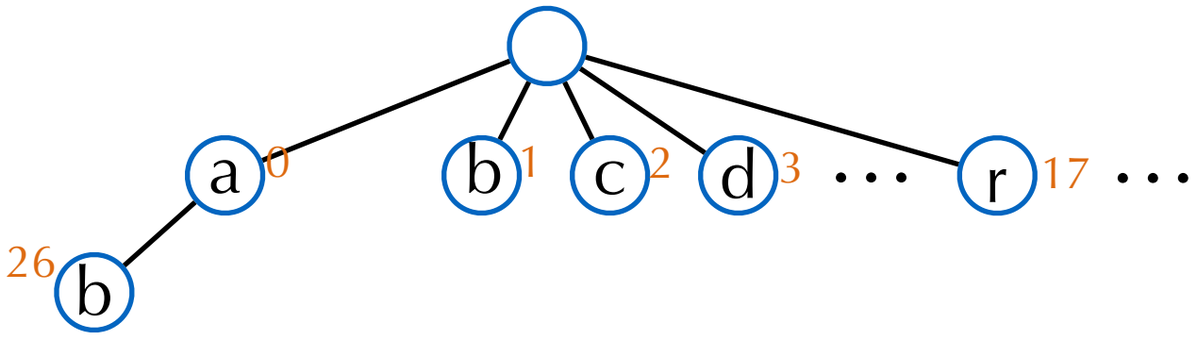

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a—z	0—25
ab	26
br	27

input 25 characters in Σ

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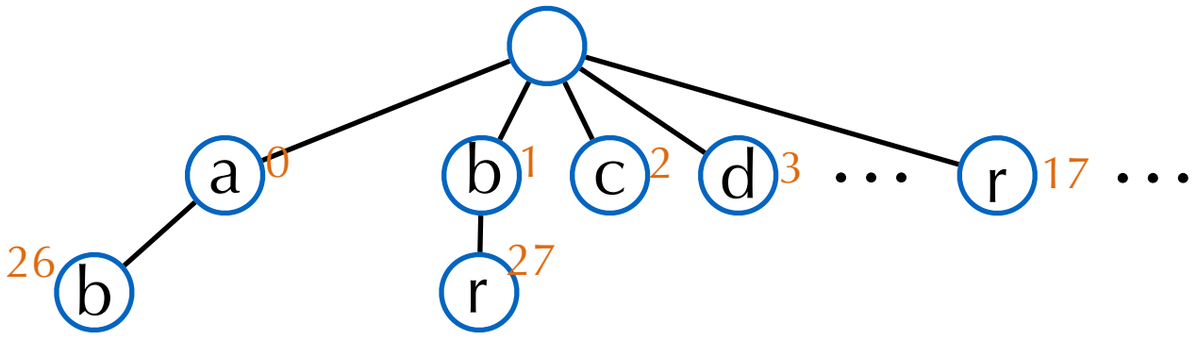

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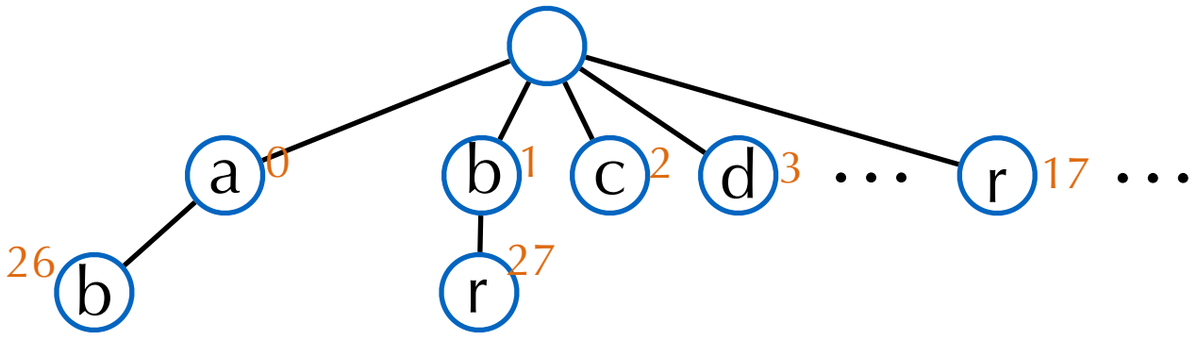

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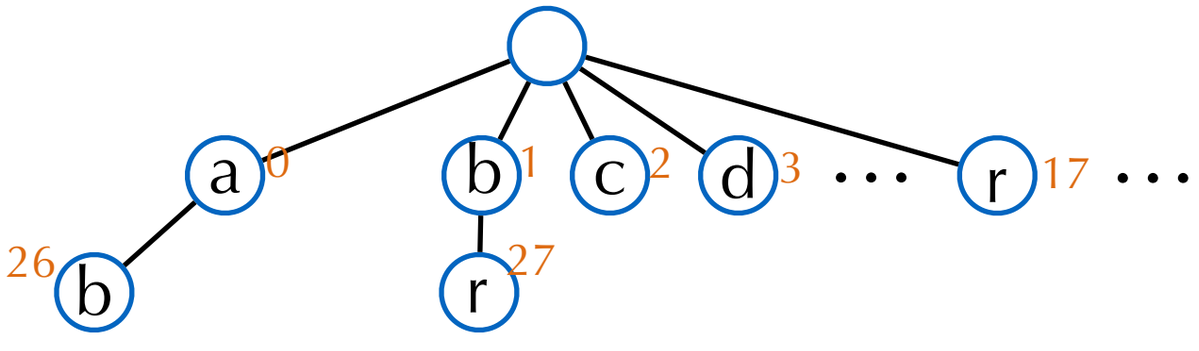

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a—z	0—25
ab	26
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ra	28
ac	29
ca	30
ad	31
da	32

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a|b|r|a|c|a|d|abracadabra

0,1,17,0,2,0,3

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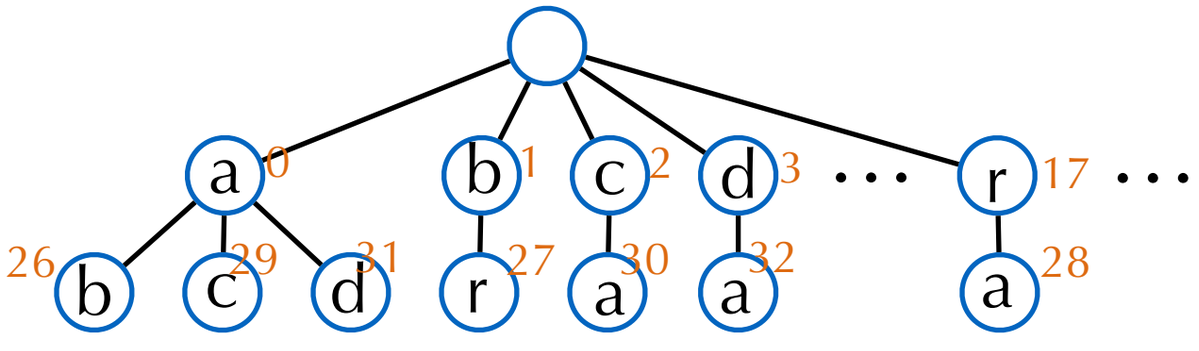

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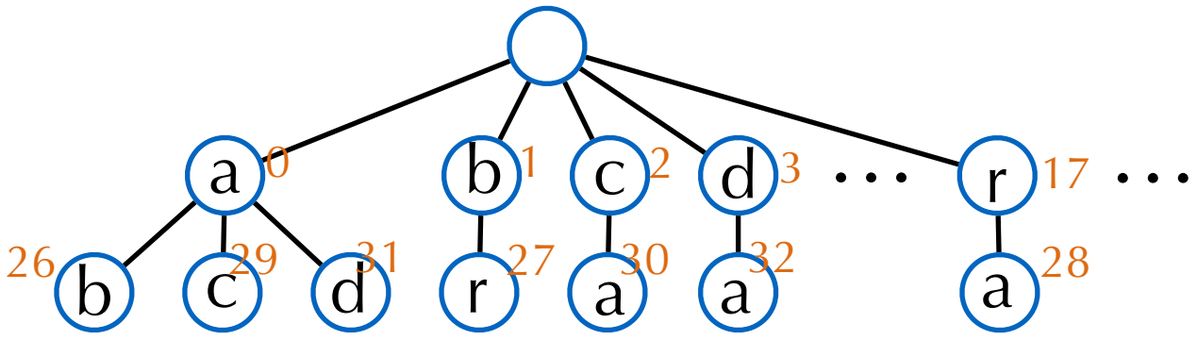

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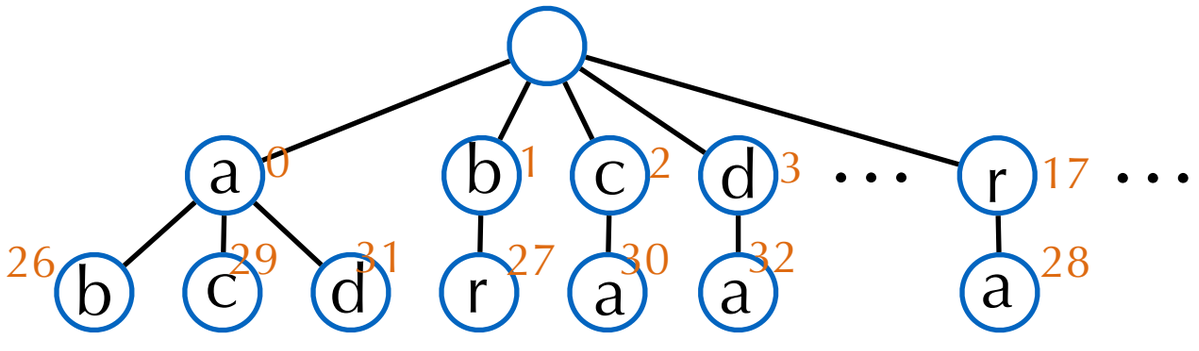

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abr|a|c|a|d|abr|acadabra

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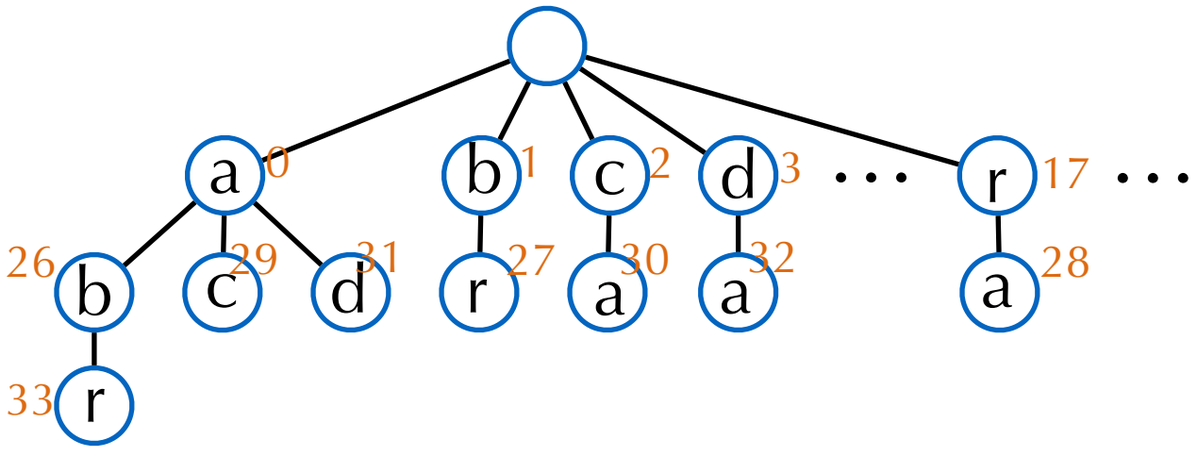

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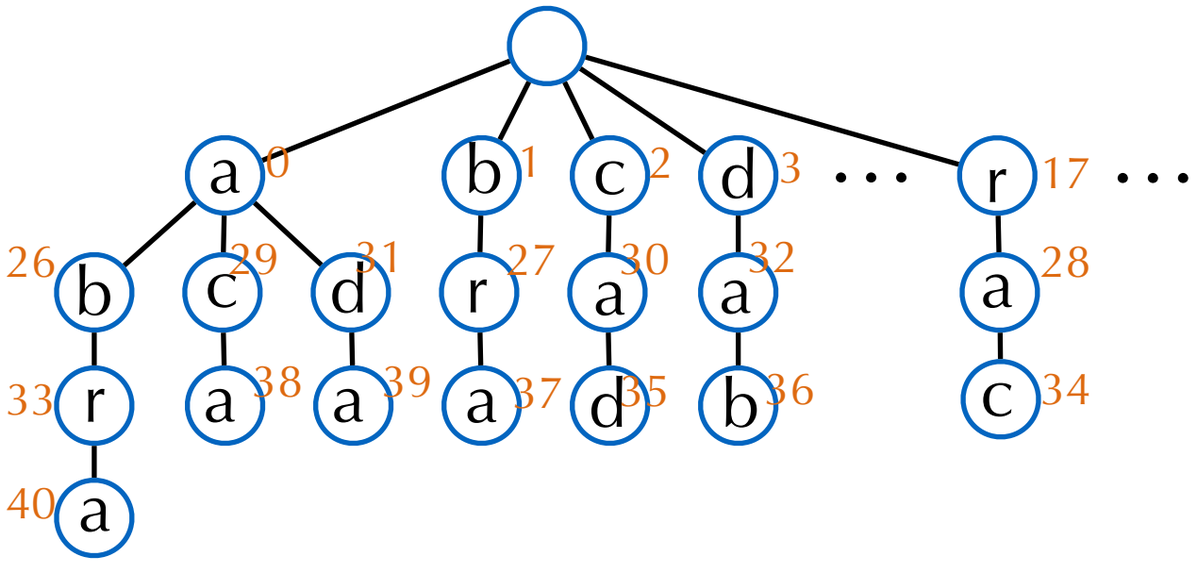

Trie growing during the encoding

$A = \{a, \ldots, z\}$
$B = \{0,\ldots,9\}$

a—z	0—25
ab	26
br	27
ra	28
ac	29
ca	30
ad	31
da	32
abr	33
rac	34
cad	35
…	…

Practical Experiment

```
_MAXASCII = 127      # 2^7 - 1
_SIZE = 2            # num bytes for the code words
_MAXCOUNT = 65535    # 2^(8*_SIZE) - 1

def compress(inname,outname):
    prefix = Trie(sizeAlpha=_MAXASCII)
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```

ASCII characters use 7 bits out of 1 byte

Here, each character is encoded in 2 bytes

```
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```

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179372	hamlet.txt

Compression ratio: 2.08

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Decoding: Reconstruct the Code on the Fly

0,1,17,0,2,0,3,26,28,30,32,27,29,31,33,0

a

b

r

a

c

a

d

ab

ra …

No trie in the decoding phase

a—z	0—25
ab	26
br	27
ra	28
ac	29
ca	30
ad	31
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…	…

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…	…

Exceptional case

a|b|ab|aba

0,1,26,28

ab	26
ba	27
aba	28

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0,1,17,0,2,0,3,26,28,30,32,27,29,31,33,0

a
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…	…

Exceptional case

abababa

0,1,26,28

ab	26
ba	27
aba	28

0,1,26,28

a
b
ab

ab	26
ba	27

?

Decoding: Reconstruct the Code on the Fly

0,1,17,0,2,0,3,26,28,30,32,27,29,31,33,0

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Exceptional case

abababa

0,1,26,28

ab	26
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0,1,26,28

a
b
ab

ab	26
ba	27
?	has to be aba

Resulting Code

```
def getnext(str,index):
    res = str[index]
    for i in range(1,_SIZE): res = res*256+str[index+i]
    return res
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```
def uncompress(inname,outname):
    dic = [chr(c) for c in range(_MAXASCII)]+[None]*(_MAXCOUNT-_MAXASCII)
    count = _MAXASCII # initial size of the code
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        val = dic[getnext(text,0)]
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            if new is None: new=val+val[0] # exceptional case
            elif new==_MAXASCII: break # end of file
            if count<_MAXCOUNT:
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```
>>> uncompress("hamlet.lzw","hamlet2.txt")
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```
% diff -s hamlet.txt hamlet2.txt
Files hamlet.txt and hamlet2.txt are identical
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>>> uncompress("hamlet.lzw","hamlet2.txt")
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Resulting Code

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def getnext(str,index):
    res = str[index]
    for i in range(1,_SIZE): res = res*256+str[index+i]
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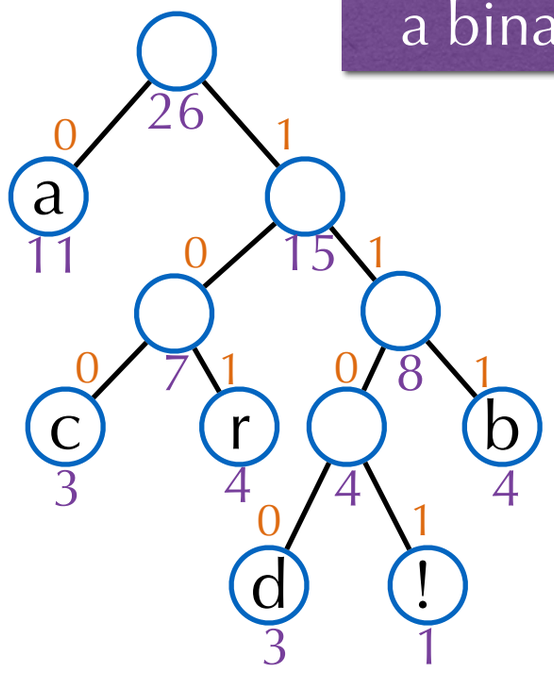
III. Huffman Encoding

Recall (CSE102)

Use fewer bits for frequent letters

`abracadabracadabra!`



Code given by a binary trie

a	0
c	100
r	101
d	1100
$!$	1101
b	111

letter	num. occurrences
a	11
b	4
c	3
d	3
r	4
!	1

Encoding/Decoding straightforward given the trie

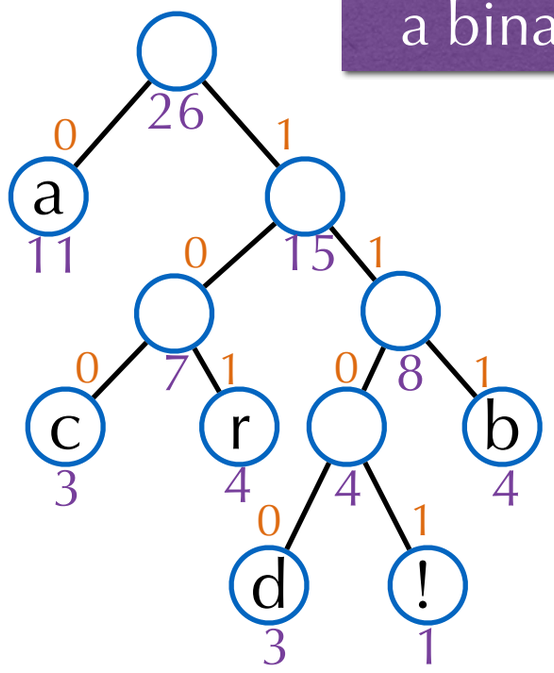
01111010100011000111101010001100011110101000110001111010 1101

26 letters —> only 60 bits (plus trie)

Recall (CSE102)

Use fewer bits for frequent letters

`abracadabracadabra!`



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Encoding/Decoding straightforward given the trie

0|11110101000110001111010100011000111101010001100011110101101

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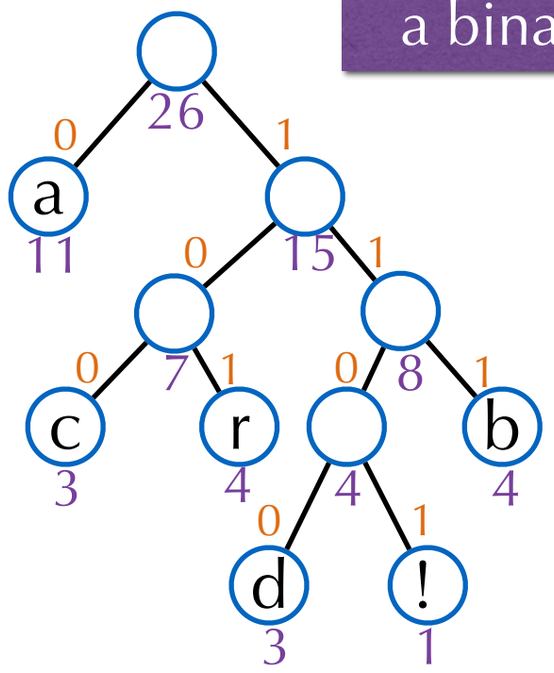

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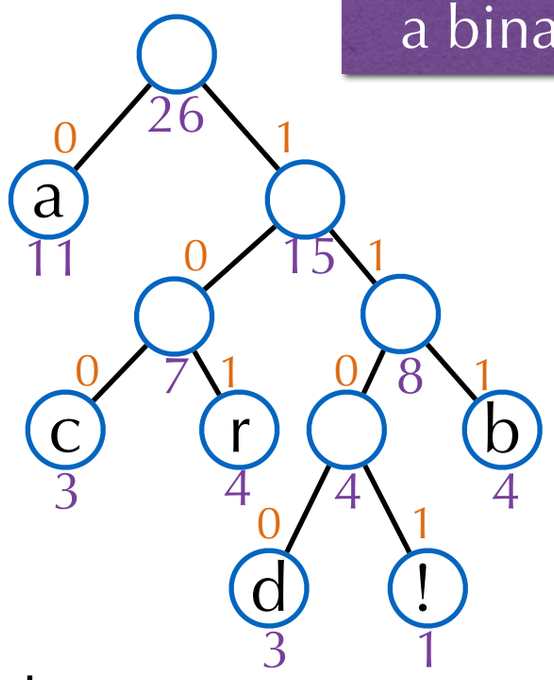

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Encoding/Decoding straightforward given the trie

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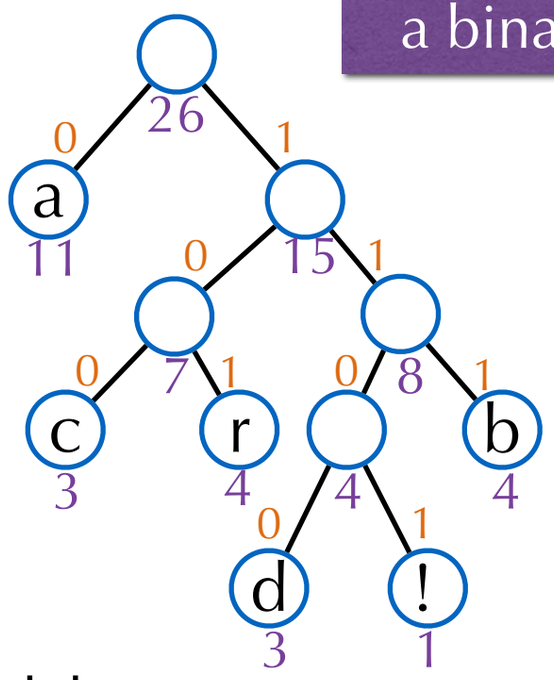

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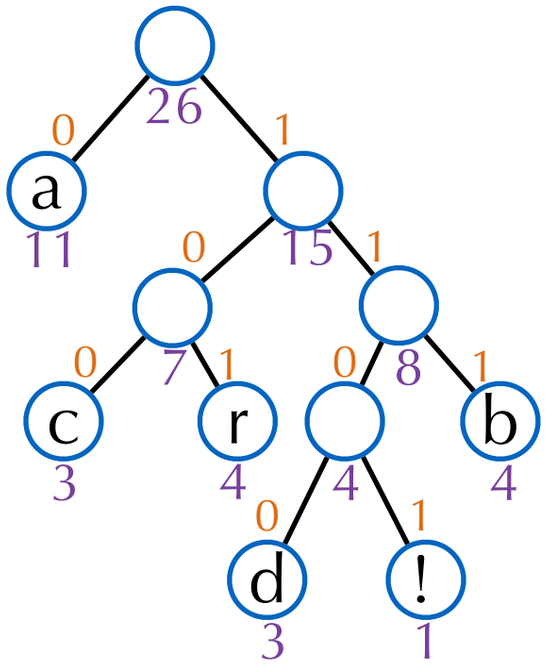
Optimal Tries



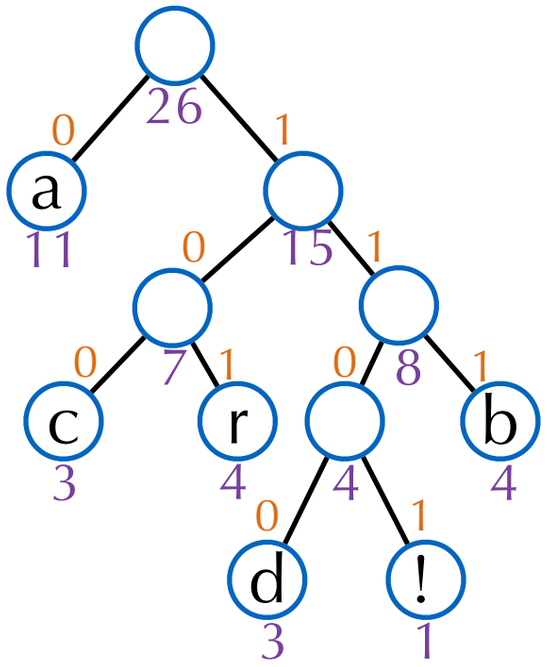
Def. Weighted external path length:

$$W(T) := \sum_{\text{leaf } \ell} \text{weight}(\ell) \times \text{depth}(\ell)\,.$$

number of occurrences

W(T) = length of the encoded string

Optimal Tries



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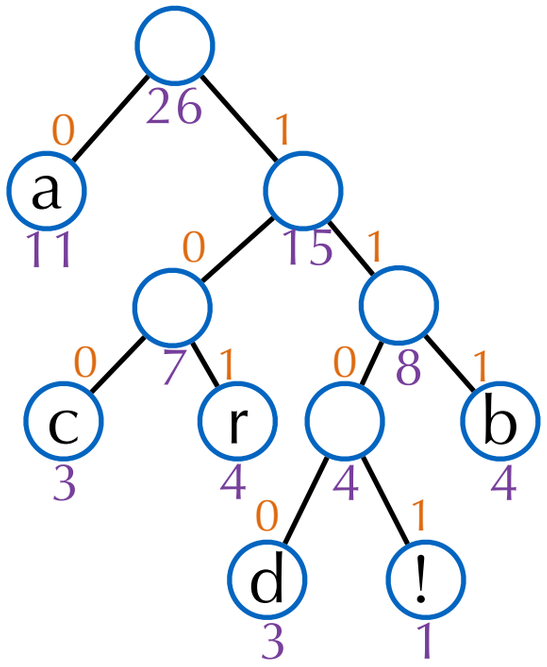

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two (sibling) leaves ℓ_1, ℓ_2 of minimal weights n_1, n_2 are at its lowest level;

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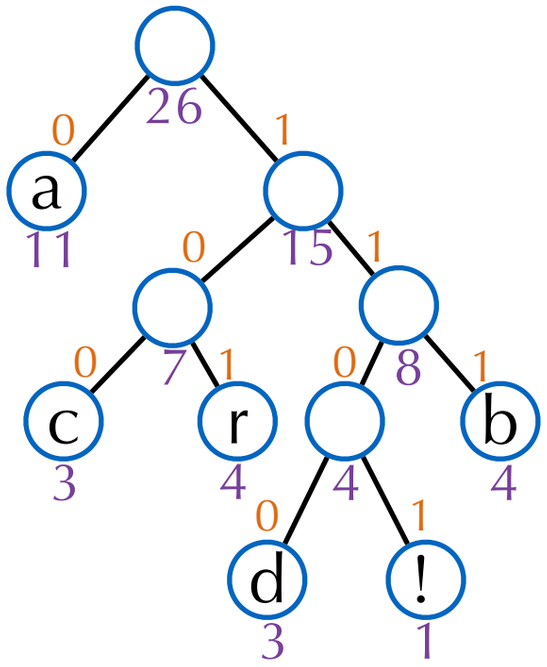

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Otherwise, exchange

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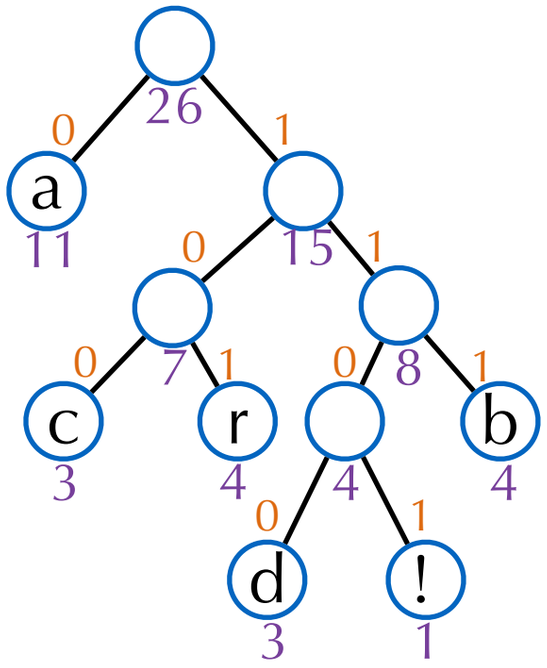

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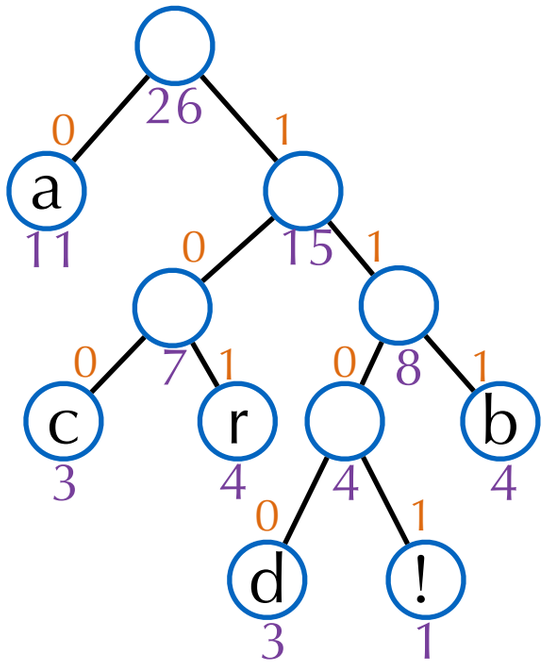

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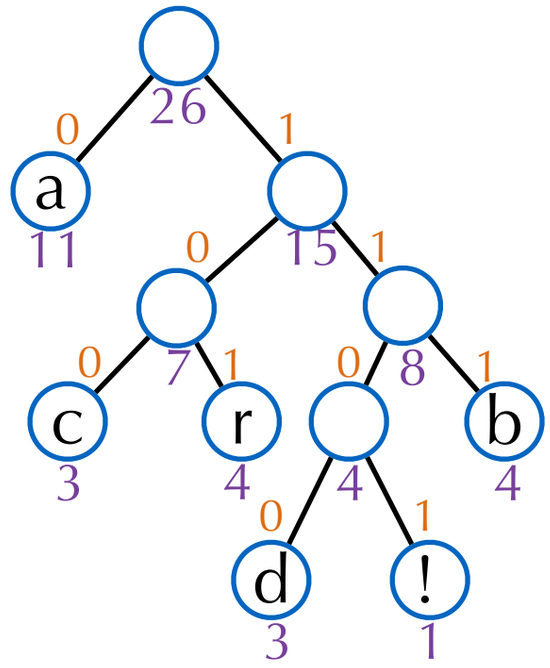
$W(T) = W(T^\star) + n_1 + n_2$.

Huffman's Algorithm




```
class NodeHuffman:

    def __init__(self, val, child=[None,None]):
        self.val = val
        self.child = child
```

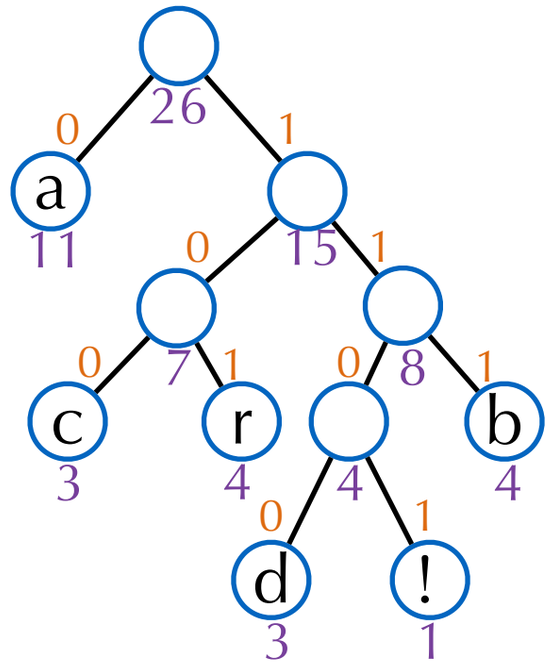
```
def maketrie(text,wordsize):
    numocc = getnumberoccurrences(text,wordsize)
    minpq = PQ() # Priority queue of trees by weights
    for c in range(2**(8*wordsize)):
        if numocc[c]>0:
            minpq.insert(-numocc[c],NodeHuffman(c))
    while minpq.size>1:
        n1,l1 = minpq.deletemax()
        n2,l2 = minpq.deletemax()
        minpq.insert(n1+n2,NodeHuffman(None,child=[l1,l2]))
    return minpq.deletemax()[1]
```

Thm. Huffman's algorithm constructs a prefix-free code with minimal $W(T)$.

Proof by induction on the number of nodes

Huffman (1952)

Huffman's Algorithm

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Start from a forest with one tree per letter

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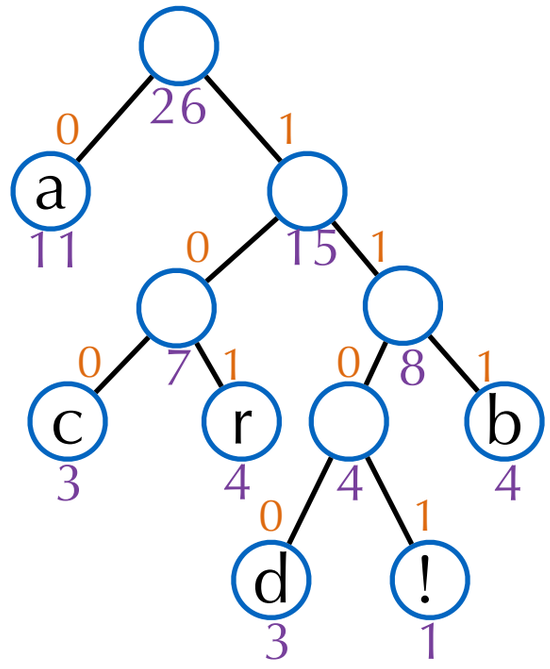


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Start from a forest with one tree per letter

Combine two trees of minimal weight

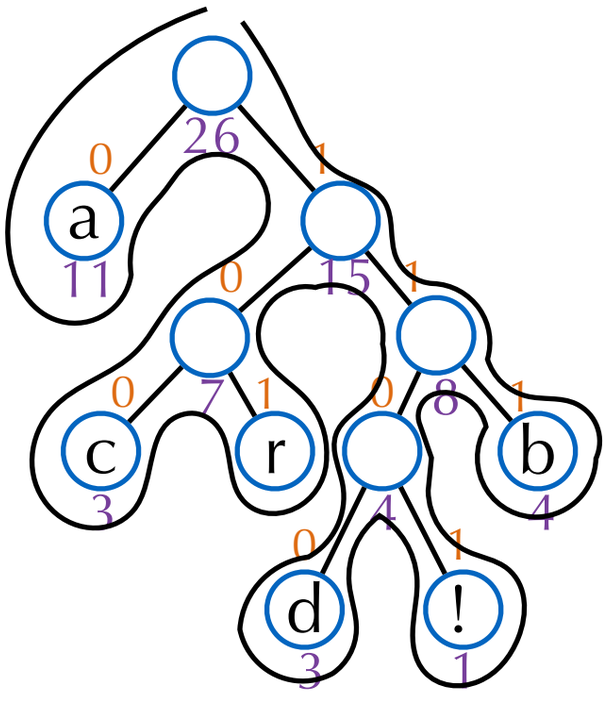
Thm. Huffman's algorithm constructs a prefix-free code with minimal $W(T)$.

Proof by induction on the number of nodes

Huffman (1952)

Communicating the Trie

Use a preorder traversal,
with 0 for nodes and 1+letter for leaves



01a001c1r001d1!1b

```
def writetrie(trie,out):
    if trie.val is None:
        out.extend('0')
        writetrie(trie.child[0],out)
        writetrie(trie.child[1],out)
    else:
        out.extend('1')
        out.extend(format(trie.val,'08b'))

def readtrie(barray,index)
    if barray[index]:
        return NodeHuffman(int(barray[index+1:index+9])),index+9
    else:
        left,index = readtrie(barray,index+1)
        right,index = readtrie(barray,index)
        return NodeHuffman(None,child=[left,right]),index
```

Exercise:
modify for wordsize > 1

Summary

Lempel-Ziv-Welsh	captures repetitions, regularity, easy to decode, works in one pass
Huffman	exploit differences in frequencies, requires two passes, the code must be transmitted (possibly compressed) as well

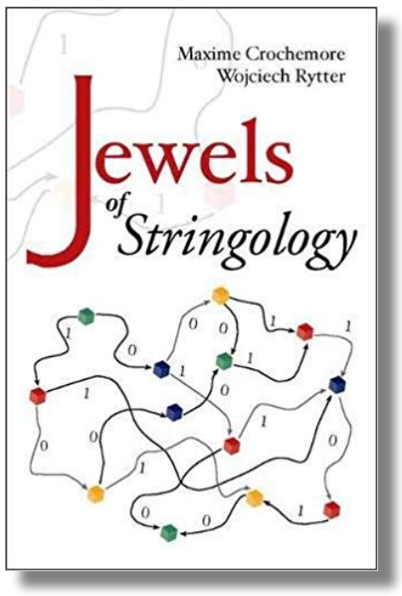
Compression routines (e.g., gzip) use (variants of) both!

text —> LZ —> Huffman

References for this lecture

The slides are designed to be self-contained.

They were prepared using the following books that I recommend if you want to learn more:



Next

NO Assignment

Next tutorial: precomputing indexes with suffix arrays

Next lecture: P vs NP

Feedback

Moodle

Questions: constantin.enea@polytechnique.edu