Computer Problem set 5.

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The objective of this CPS is to explore different techniques of Monte-Carlo estimation, and to compare the efficiency of the usual naive Monte-Carlo method with the *Importance sampling* method which is based om probability change via Girsanov's theorem.

Question 3

In this question we are required to run many estimations of the probability : $\mathbb{P}(S_T \geq K) = \mathbb{E}_{\mathbb{P}}(1_{S_T \geq K})$ and to compare the errors of the estimations. The plot is the following (Figure 1):

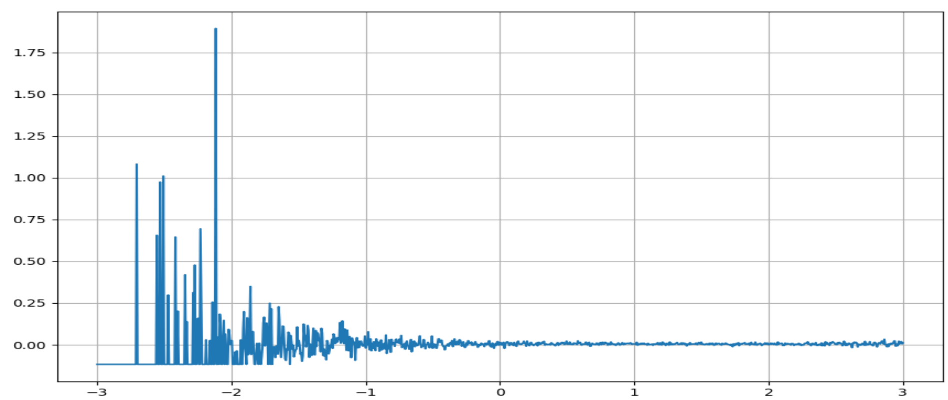

Figure 1: Estimation errors for different values of θ

This graph in only a plot of estimations errors, and these estimations are random as well. The graph produced is only then one trajectory of estimation errors for different values of θ which should be thought of as some kind of random process in θ. Ploting many trajectories of this process produces a graph similar to the following:

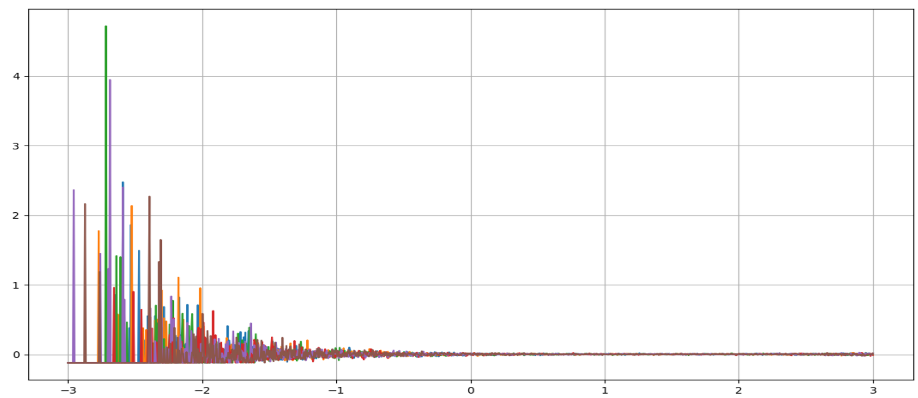

Figure 2: Estimation errors for different values of θ

Theses results give us an idea on the variance of the estimators that correspond to different values of θ. And we observe that this variance is pretty high for values of that arround -2 and tends decreases when θ increasing.

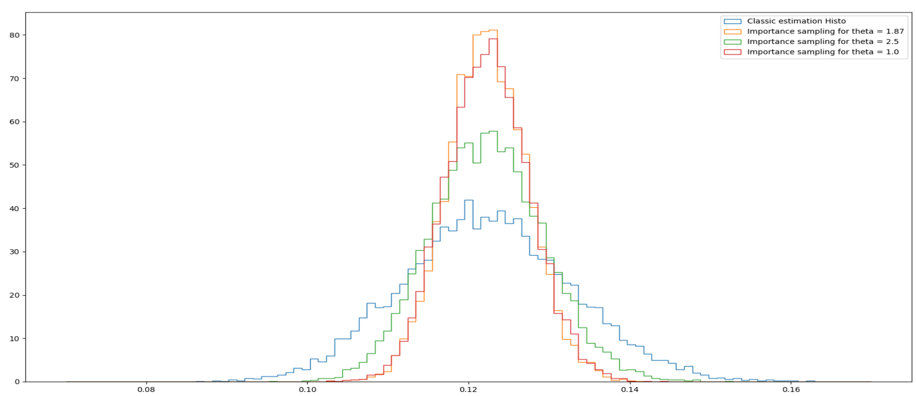

Figure 3: histograms of estimations for different θ values

Question 5

In order to choose a value of θ that is optimal in the sens that the estimator of Δ_0 associated to θ is more precise we need to study the variance of that estimator and it's behaviour when θ changes.

The optimal value for θ will then be the value that minimizes that variance. We estimate the variance using the provided formula:

$$V^\theta = \mathbb{E}^{\mathbb{Q}^\theta}[(Z^\theta)^{-2}.1_{S_T \geq K}] - \Delta_0$$

The resulting graph is the following:

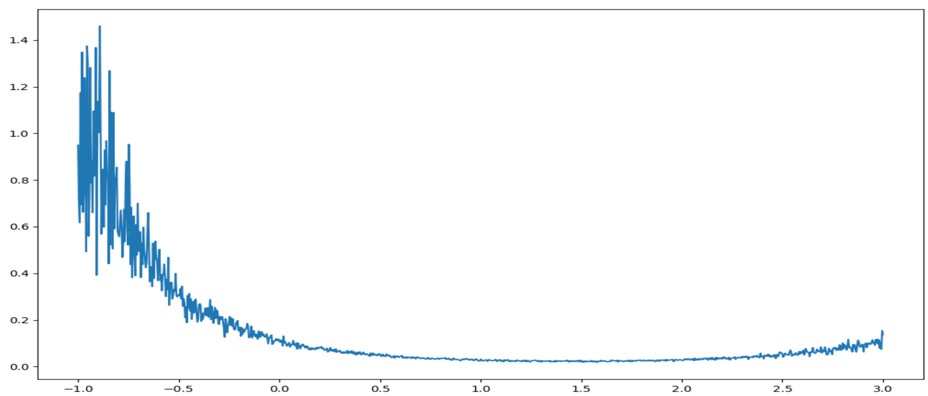

Figure 4: Estimated Variance for M = 1000

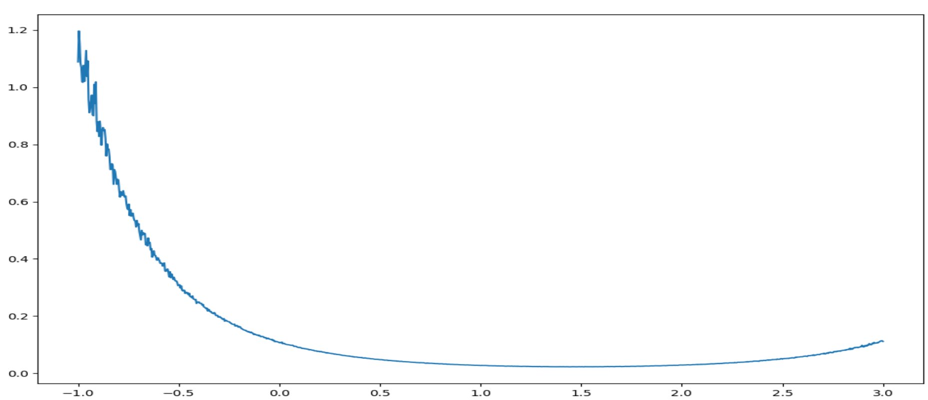

Figure 5: Estimated Variance for M = 100000

This shoes that the result is stable when M changes, and when the simulation effort M increases the estimation of the variance is more precise. we also notice that the variance of the estimator dramatically increases when θ is less that -1 which explains the results of the figures 1 and 2. And the optimal value of θ is around 1.5 because the minimum of the variance is between 1.0 and 2.0.