Computer Problem set 4.

Yassine EL MAAZOUZ

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Using a forward simulation of $(W_{t_i^n})$ we can simulate $(S_{t_i^n})$ thanks to the expression:
$S_{t_i^n} = S_0 exp((\mu - \frac{\sigma^2}{2})t + \sigma W_{t_i^n})$
Let's compute the mean of S_t for a given $t \geq 0$.

$$\begin{aligned}\mathbb{E}(S_t) &= \mathbb{E}(S_0.exp((\mu - \frac{\sigma^2}{2})t + \sigma W_t)) \\ &= S_0.exp((\mu - \frac{\sigma^2}{2})t).\mathbb{E}(exp(\sigma.W_t))\end{aligned}$$

Since $\sigma.W_t$ has a $\mathcal{N}(0, \sigma^2 t)$ distribution we can conclude by using Laplace's transofmation that:

$$\mathbb{E}(S_t) = S_0.exp(\mu t)$$

It is also interesting to computer the variance of S_t since the quality of our Monte-Carlo estimations of the mean of S_t depends on the size of the variance.

A similar calculation of the mean gives us the result: $Var(S_t) = S_0^2.e^{2\mu t}(e^{\sigma^2 t} - 1)$

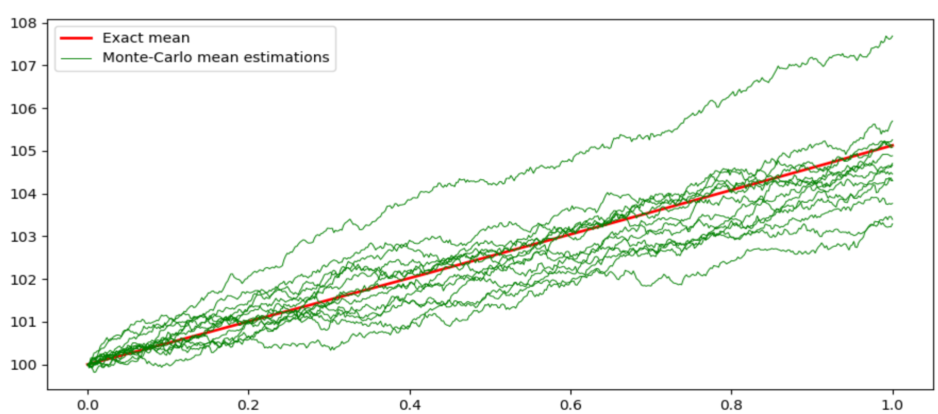

Figure 1: Estimations of mean of S_t

As we can predicted from the result $Var(S_t) = S_0^2.e^{2\mu t}(e^{\sigma^2 t} - 1)$ the quality of the estimation tend to get lower as time t becomes bigger.

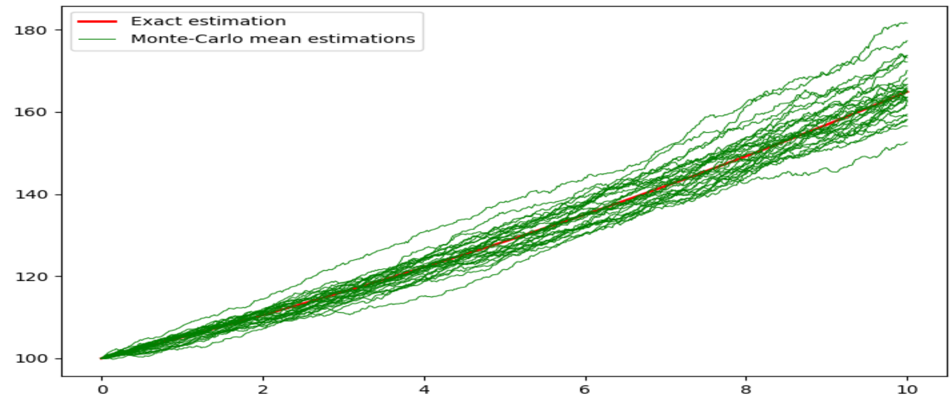

Figure 2: Estimations of mean of S_t

The value of the drift μ has a big impact on the mean of S_t since it's expression suggests an exponential variation in μt

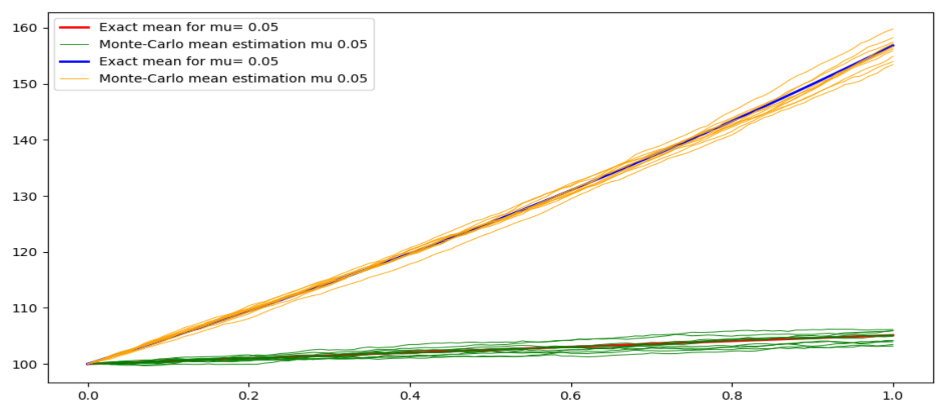

Figure 3: Estimations of mean of S_t

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In this question we simulate N copies of *Profit-loss* matrix $(PL_T^n(K))_{K \in K-list, n \in n-list}$ in order to compute the the mean and variance of $PL_T^n(K)$ for different values of n and K. This will allow us to plot the mean and volatility surface of PL.

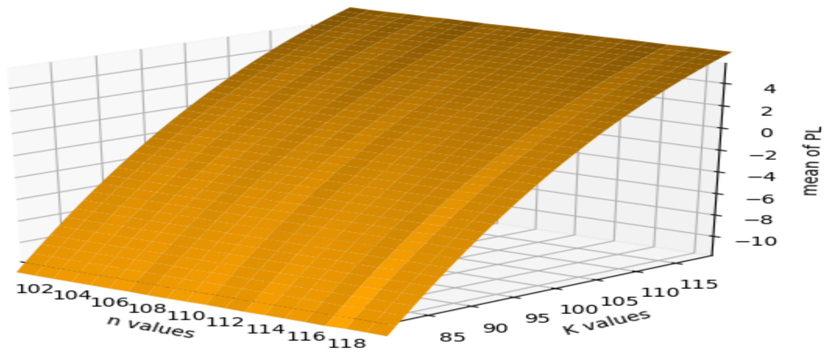

Figure 4: Estimations of volatilities of PL for different n and K values

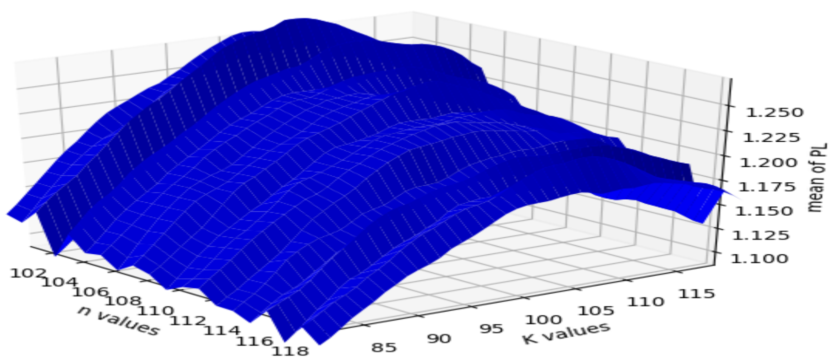

Figure 5: Estimations of mean of PL for different n and K values