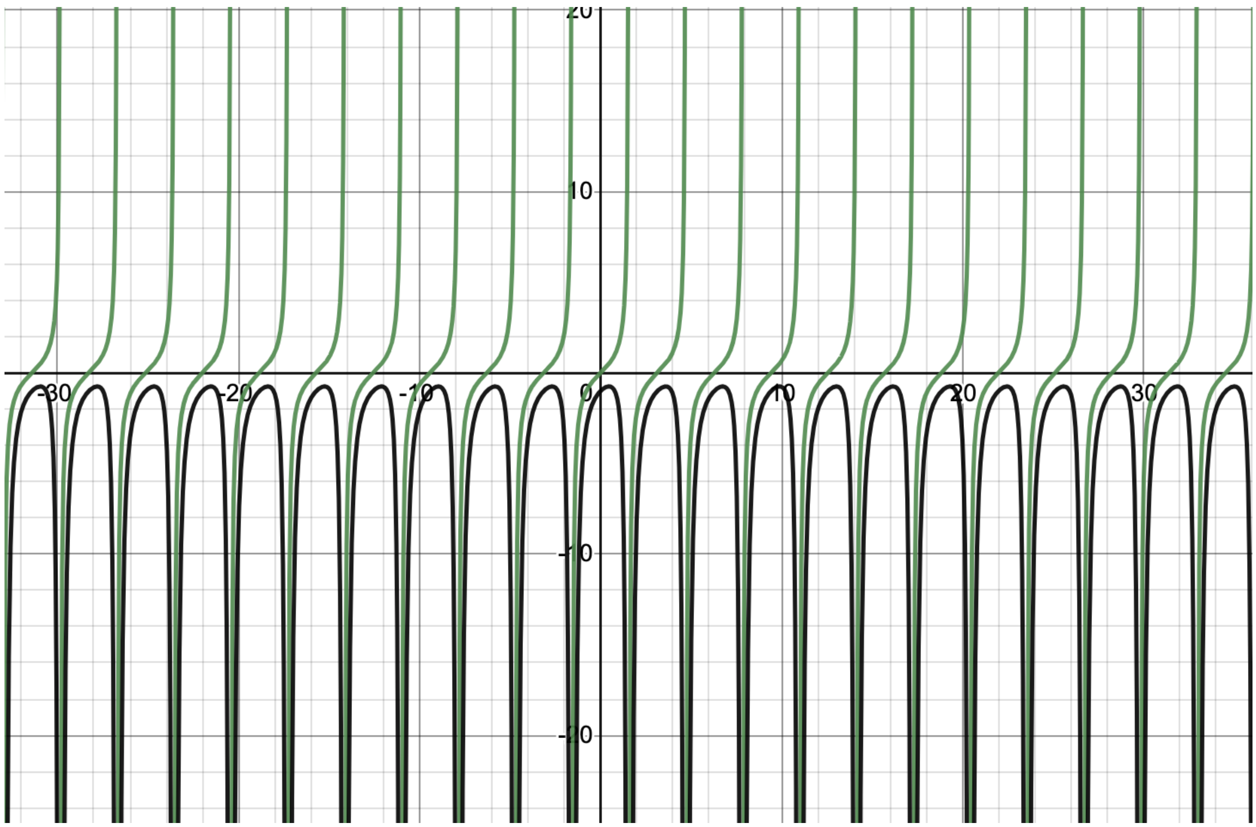

Moving Average Cross

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Table of Contents

Abstract	2
Derivations	3-4
Examples	5-7
Conclusion	8

Abstract

The moving average cross is a very simple and popular strategy in technical analysis. Aside from the curiosities that arise, it is impossible to improve the predictive power of technical analysis strategies without a solid understanding of their mathematical foundations, and this report attempts to prove the moving average strategy. Examples using simple functions that can be expressed as equations as well as real-world moving averages are given.

Derivations

Let $f(x)$ be an arbitrary function that is differentiable on the interval (a, b). The average value of the function on this interval is:

$$\frac{1}{b-a}\int_a^b f(x)dx$$

For the moving average, let the point *b* be *x* and let the point a be *x* - *b*. By the fundamental theorem of calculus, the resulting expression is:

$$\frac{1}{b}\int_0^x f(x)dx - \frac{1}{b}\int_0^{x-b} f(x)dx$$

If $f(x)$ is at a local extrema when it is equivalent to its moving average, the following equation can be used to express this relation:

$$\frac{\mathrm{df}}{\mathrm{d}x} \approx f(x) - \frac{1}{b}[\int_0^x f(x)dx - \int_0^{x-b} f(x)dx]$$

Differentiating the equation results in the following:

$$\frac{\mathrm{df}}{\mathrm{d}x} - \frac{\mathrm{d^2f}}{\mathrm{d}x^2} \approx \frac{1}{b}[f(x) - f(x-b)]$$

Let $f(x)$ be modeled by a linear function or be approximately linear on the given interval. Since the moving average is intended to closely resemble the original function, let *b* approach 0. The resulting equation is:

$$\frac{df}{dx} \approx \lim_{b\to 0}\left(\frac{f(x) - f(x-b)}{b}\right)$$

The two terms of this equation are equivalent, since this is the definition of a derivative. Thus, the assumption that a moving average over a small interval subtracted from a function is zero when the function is at an extrema is true if the second derivative of a function is zero (the function is linear). In practice, this means that a trendline with a small standard deviation can be drawn to model the function. If this is not possible, multiple trendlines can be drawn.

The equation for a tangent line of a function at a point *p* for a function $f(x)$ is:

$$f'(p)(x-p) \, + \, f(p)$$

The moving average of which is:

$$f'(p)(x-p-1) + f(p)$$

Let the function $h(x)$ be the previous repeated for all points *p* = *x*, satisfying the following:

$$h(x) \; = f(x) \; - \; f'(x)$$

$$f'(x) \approx f(x) - h(x)$$

This means that the extrema of any differentiable function can be found using *h*, since the equation above is always true. If the equation of the function is unknown, *h* can be written as the following:

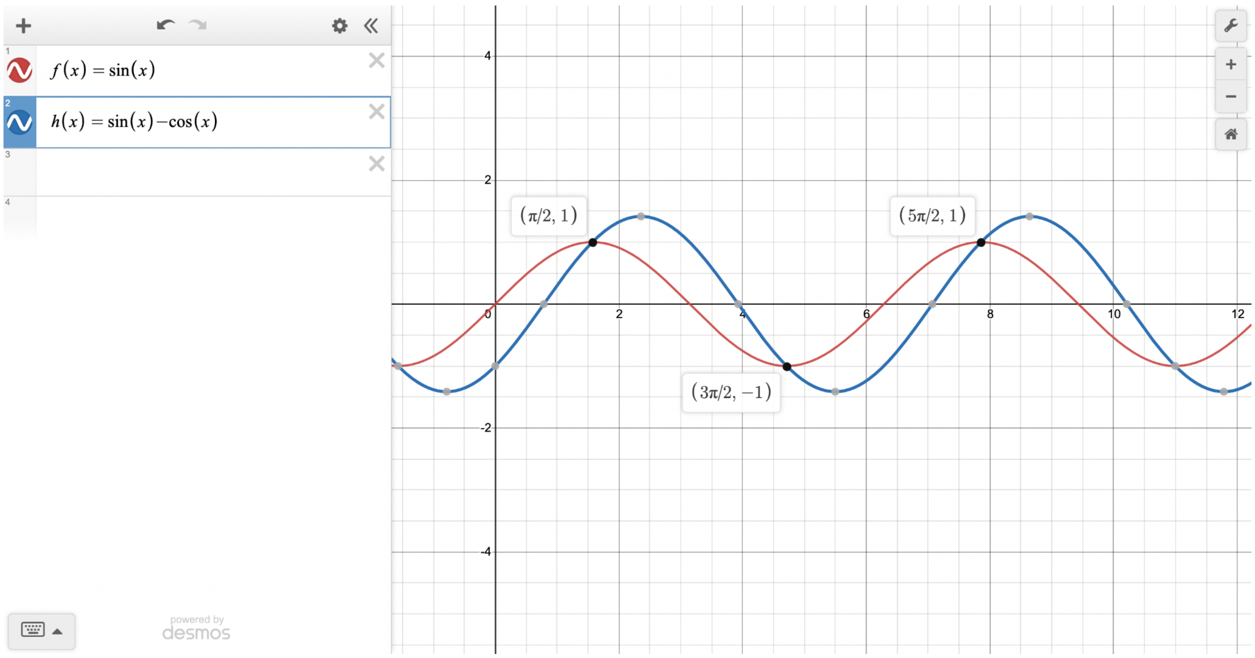
$$h(x) \; \approx \; f(x) \; - \; \lim_{b \to 0}\left(\frac{f(x) \; - \; f(x-b)}{b} \right)$$

Working backwards from the formulas derived earlier, *h* can be written in terms of the moving average *a* as such:

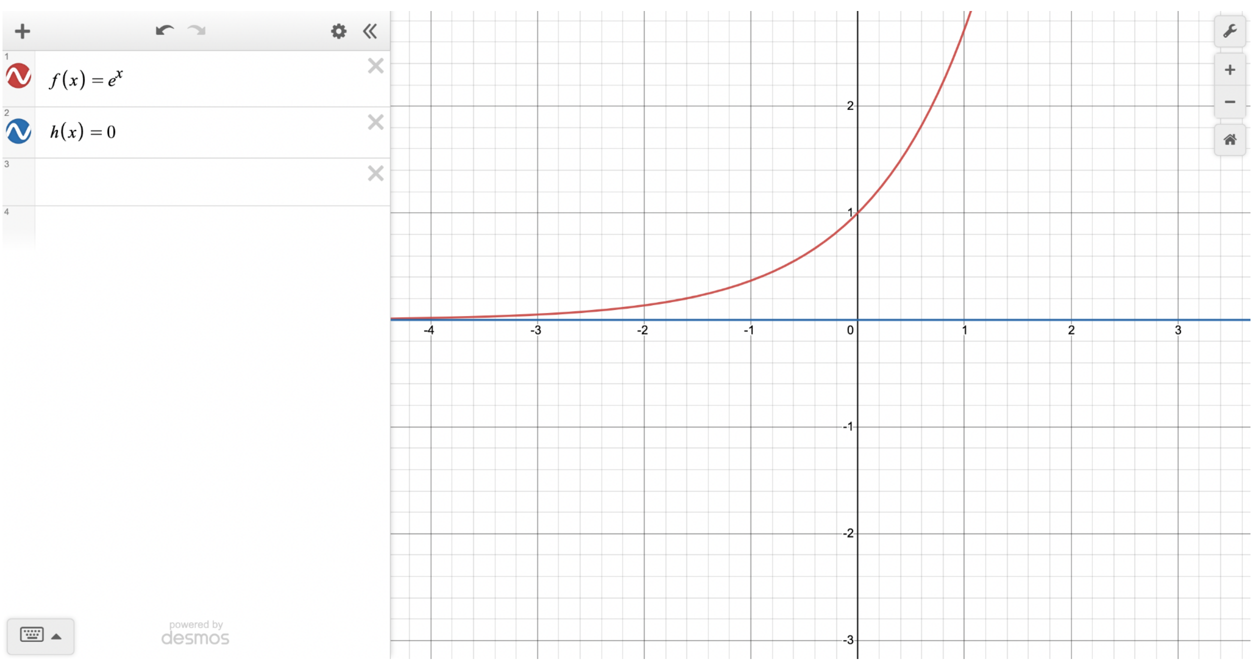
$$h(x) \; \approx \; f(x) \; - \; a'(x)$$

Examples

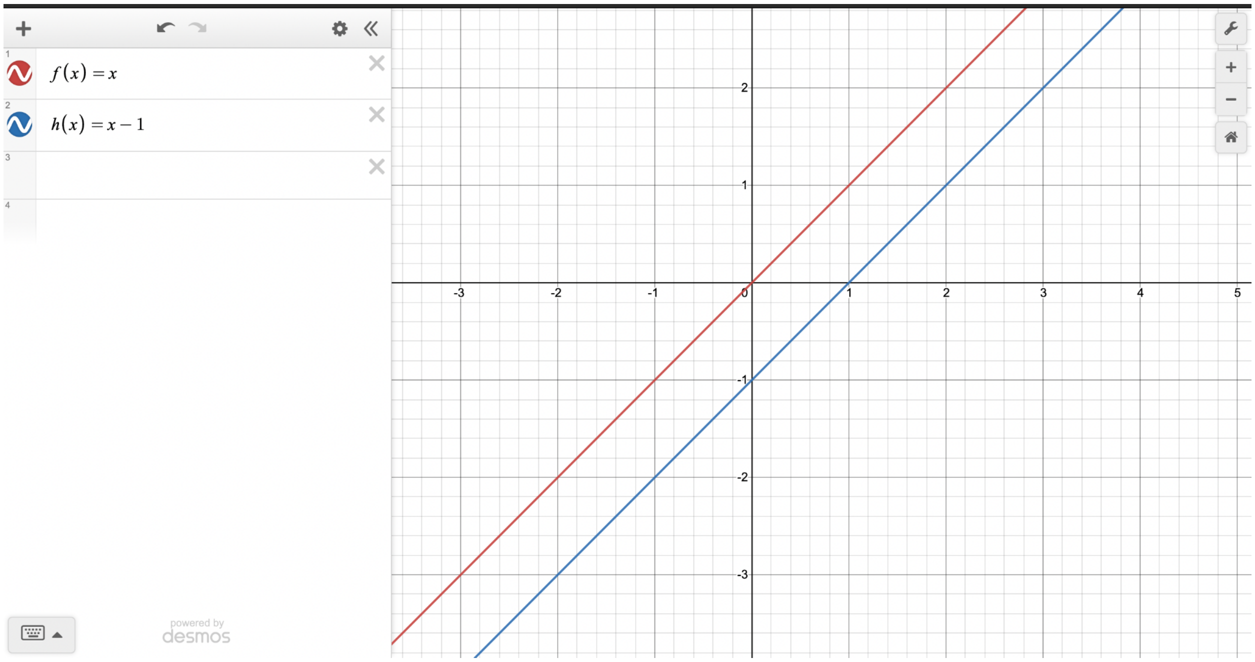
Functions with local extrema are intersected by their *h* at that point.



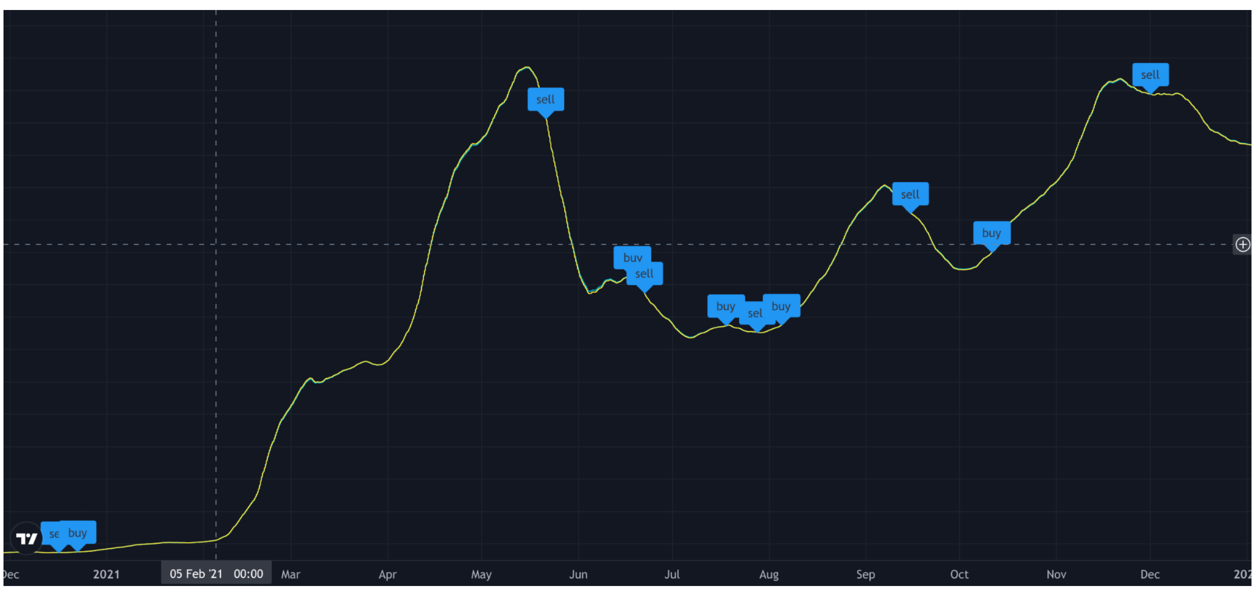
Functions that converge to a certain value also get infinitely close to intersecting their *h.* In this case, the left hand limit of the derivative of *f* is zero, hence the left hand limit of *f* is *h.*

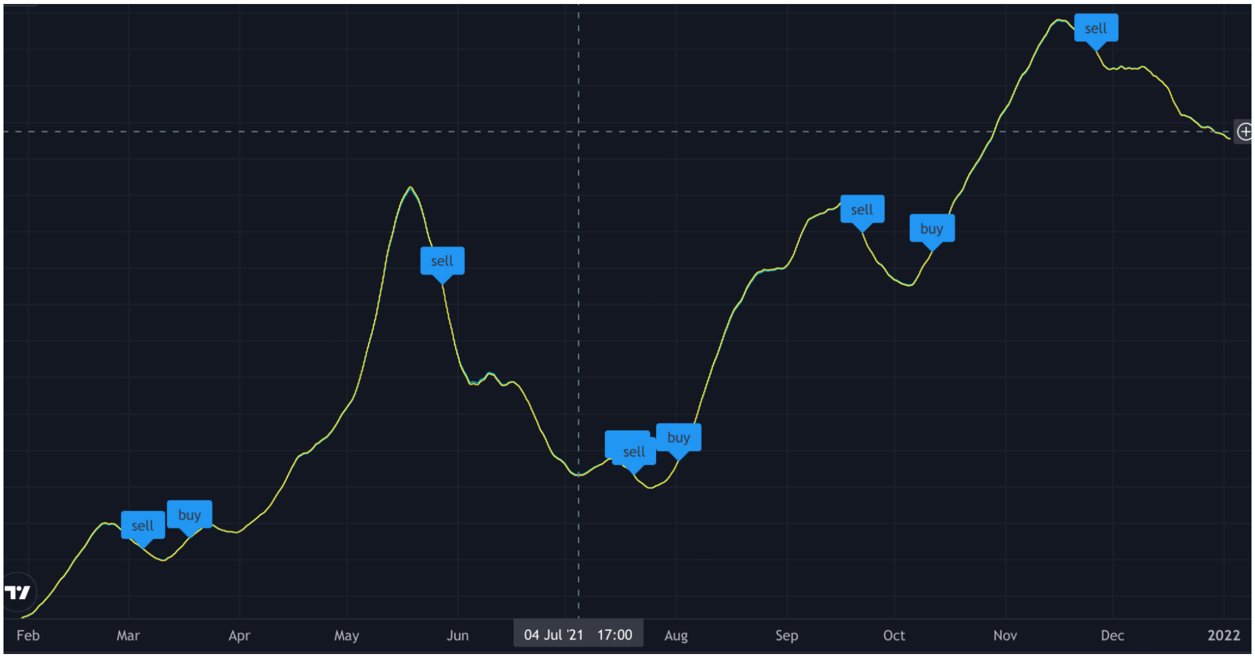


Functions with first derivatives that are never zero never intersect their *h*, as is seen in the case of a simple linear function.



In real-world cases, functions may not be differentiable within the interval (b, x), so moving averages with an appropriate *b* can be used to approximate the end behavior of the function. The following is a case where *b* = 100. *h* is so close to the moving average that it is not visible, but the labels below show the points of intersection.

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Conclusions

Moving averages can be used to provide satisfactory approximations of differentiable functions, and the first formula for h can be used to find the exact location of the zero value of the first derivative of such functions. For non-differentiable functions, a differentiable function that approximates them can be used as a substitute, but the efficacy of the strategy becomes poorer with increasing length b. In those cases, further analysis is required to confirm minimums and maximums.