Half Life
Lab #8
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PHYS 4C

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Introduction & Purpose

The purpose of this lab is to model radioactive decay using dice. The rate of simulated decay will then be graphed, analyzed, and compared to the statistical/theoretical model of decay.

Equipment

- 100 eight-sided dice (one side marked)
- 100 spherical “dice”
- A tray
- A stopwatch

Procedure

1. Five groups in the class will do the following:
2. Place 100 dice on the tray
3. Start the stopwatch
4. Shake the tray with the dice for 15 seconds.
5. Remove the dice with the marked side facing up.
6. Count the removed dice and record
7. Replace the removed eight-sided dice with spherical dice.
8. Repeat 2-5 within one minute until only five eight-sided dice remain in the tray
9. Sum all the individual dice counts from all groups
10. Record the data

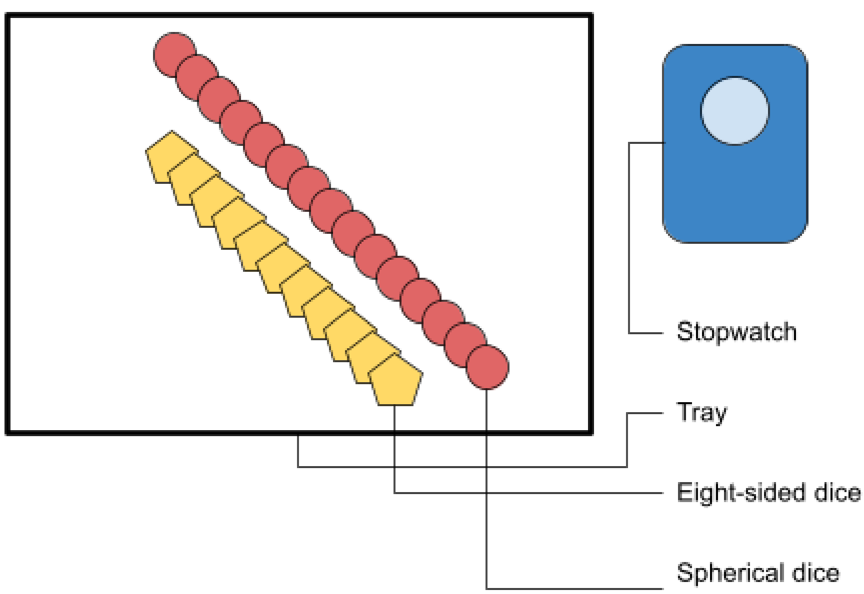

Figure 1: General lab setup

Data & Analysis

Computer Simulation of Lab

```
import pandas as pd
import random as rd

#constants
NUM_DICE = 500
TOT_TIME = 30

#integers
removed_die = 0
remaining_die = NUM_DICE

#arrays
removed_dice = [0]
remaining_dice = [NUM_DICE]

#Simulation
for i in range(TOT_TIME):
   for j in range(remaining_die):
       die_state = rd.randint(1,8)
       if die_state == 1:
           removed_die += 1
   remaining_die = NUM_DICE - removed_die
   removed_dice.append(removed_die)
   remaining_dice.append(remaining_die)

#data output
d = {'Remaining Dice':remaining_dice, 'Removed Dice':removed_dice}
lab_data = pd.DataFrame(data=d)
print(lab_data)
lab_data.to_csv('half_life.csv')
```

Theoretical	Simulation		Experiment	
Remaining Dice (T)	Remaining Dice (S)	Error (S)	Remaining Dice (E)	Error (E)
500	500	0.00%	500	0.00%
438	448	2.40%	421	3.77%
383	396	3.44%	370	3.35%
335	356	6.28%	328	2.08%
293	313	6.79%	280	4.47%
256	276	7.62%	241	6.03%
224	243	8.29%	215	4.19%
196	222	13.06%	186	5.27%
172	197	14.67%	164	4.54%
150	173	15.08%	145	3.54%
132	147	11.75%	129	1.93%
115	128	11.21%	112	2.69%
101	116	15.18%	96	4.68%
88	109	23.69%	87	1.27%
77	100	29.69%	76	1.43%
67	89	31.92%	69	2.27%
59	77	30.43%	59	0.06%
52	68	31.64%	50	3.20%
45	60	32.75%	46	1.78%
40	51	28.96%	37	6.44%
35	48	38.71%	32	7.53%
30	39	28.80%	29	4.22%
26	34	28.33%	26	1.86%
23	32	38.04%	23	0.79%
20	30	47.90%	20	1.40%
18	29	63.39%	18	1.41%
16	27	73.85%	17	9.46%

Table 1: Raw data with error calculation

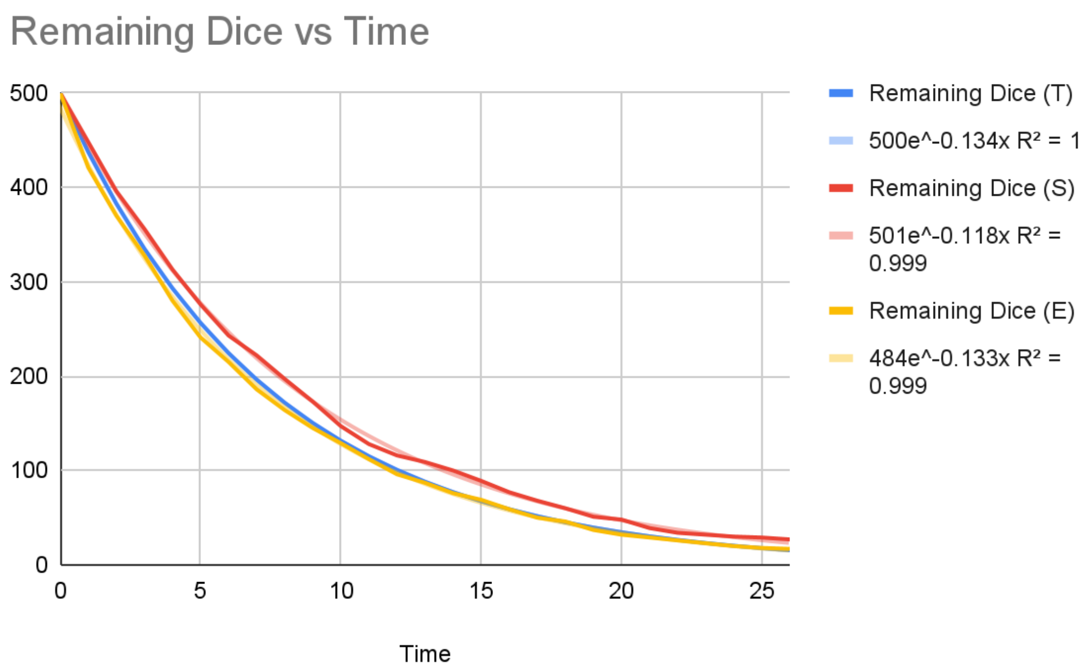

Figure 2: Plot of the theoretical decay (blue), computer simulated decay (red), and experimental decay (yellow)

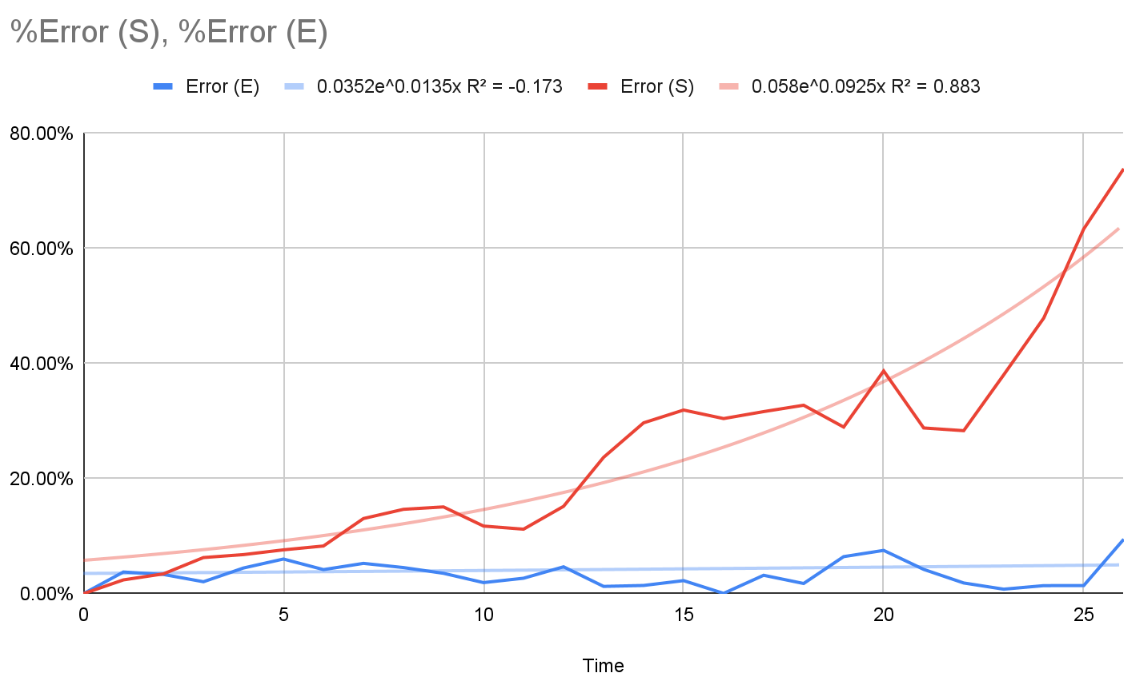

Figure 3: Plot of the percent error of the experimental decay (blue) and computer simulated decay (red) relative to the theoretical decay.

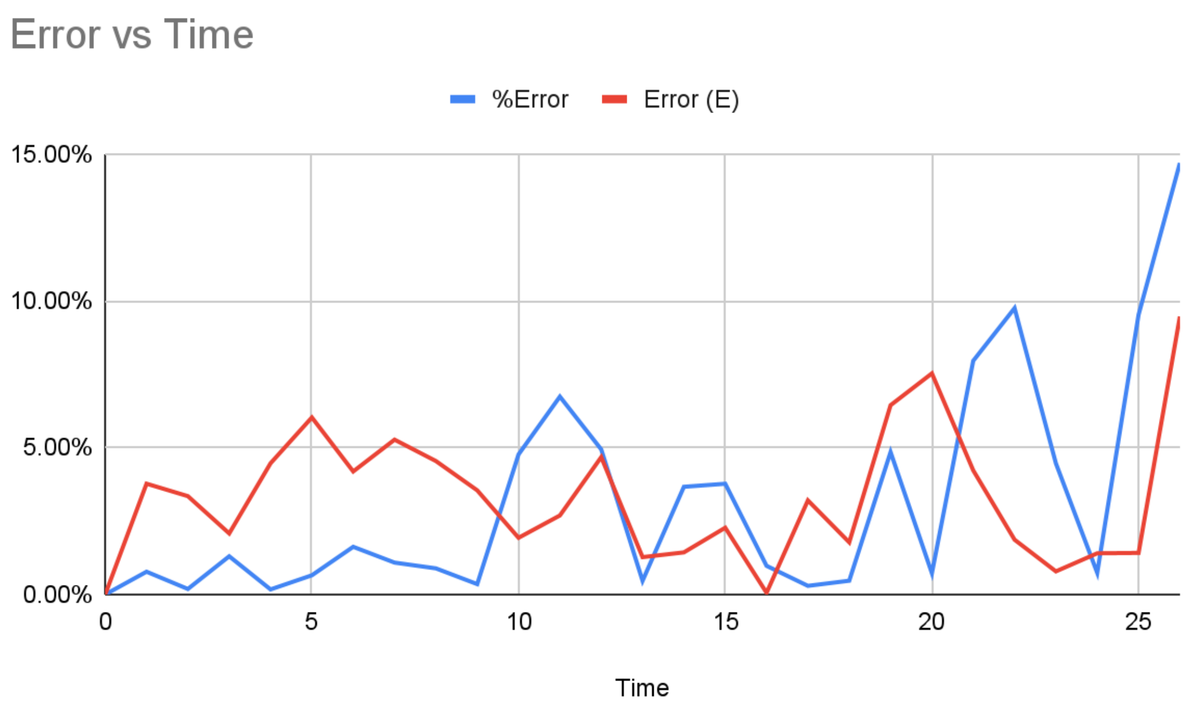

Figure 4: Error after correction (blue line) and experimental error (red line).

The probability of the dice landing on the marked side is ⅛ , since it has eight sides. Adding the spherical dice to replace the removed dice has no theoretical effect on the probability. So, the dice remaining (equation 1) can be modeled using a modified compound interest formula (equation 3).

$$1)\quad \text{remaining} = \text{initial} - \text{initial}(1 - \text{probability})^{time}$$

$$2)\quad S = P(1+r)^t$$

$$3)\quad S = 500\left(1 - \frac{1}{8}\right)^t$$

Equation 3 was used to calculate the error in the experimental and simulated data. The error function used as a regression for the percent error is exponential; the main source of error in this experiment are factors that inhibit “true randomness”, as that is the main premise of equation 3. Variations in randomness, and by extension probabilities, affect the terms of the exponential function-hence the exponential error.

As can be seen in figure 2, percent error for the experimental data appears to be relatively consistent throughout the experiment, signifying nearly random outcomes (expected probabilities) from the dice rolling. Truly random experimental outcomes means that the error function does not diverge; the small exponent of the curve fit means a slow/non-existent divergence. However, the simulation error function has a visible divergence and significantly larger exponent in the curve fit. This means that the random number generator used does not output truly random values (probabilities are skewed).

Let us assume that the “decay” observed in the lab is continuous so that it can be modeled as an exponential function of base e:

$$4) \quad S = Pe^{rt}$$

Ignoring the coefficients of all functions-since they have a negligible contribution to error-the probability deviation of the simulation can be approximated simply by comparing exponents. Thus, the difference of the simulation and experiment exponents from the theoretical exponent of the regression is the deviance from the expected probability.

Since the experimental error has no trend (see the small R values in figure 3), there is no probability deviance in the experiment that can be measured in the data; it can be approximated to be zero. The probability deviance for the simulation however can be approximated to be non-zero given a satisfactory R value in figure 3; it is 0.016 or 1.6%. In other words, since the simulation over-approximates the remaining dice, it is 1.6% less likely for a simulated dice to roll on the marked side than in theory. It can be seen in figure 4 that after accounting for this probability shift, the corrected simulation function provides a similar error to the experiment (no trend).

Conclusion

The experiment is effective at modeling radioactive decay-the dice rolls had satisfactorily random outcomes that produced a curve very similar to the theoretical exponential decay function with zero measurable variance in probability. The computer simulation however had a variance from the expected probability by 11.3% (it lagged by 1.6%), which is due to the mersenne twister algorithm used in the python random library not being statistically random.

Technically, the exponential regression model shows a probability discrepancy of 0.1% between the experimental and theoretical decay functions, but this is likely a negligible artifact from the regression algorithm. Another possibility may be physical characteristics of the dice setup, e.g. dice not turning due to friction etc.