Math 340 Homework 2

Dr. Ebrahimian

Due 9/13/2024 before the class starts

- You are expected to solve all of the following problems. You will have a quiz on Friday 9/13 based on all of the following problems. Only the ones under "Problems for Grading" must be submitted for grading. **Late submission will not be accepted.**
- It is highly recommended (but not required) that you LaTeX your homework.
- If you are not typing your work (which is fine) please make sure your work is legible.
- Justify all of your solutions.
- Your are encouraged to work with your classmates, but your submission must be written by yourself in your own words.

Problems for Grading

Instructions for submission:

- To submit your homework, go to ELMS. Hit "Gradescope" on the left panel. That should allow you to upload a PDF file of your homework.
- Each problem must go on a separate page. **Make sure you assign each page to the appropriate problem number.**
- Please make sure your work is legible. You could use the (free) DocScan app to scan and upload your homework.
- Sometime in the next day or two run a test and make sure this all works out so you do not face any issues right before the deadline.
- Homework must be submitted before the class starts on the due date. GradeScope will not allow late submissions.
- You can read more about submitting homework on Gradescope here.
- **All proofs must be complete and solutions must be fully justified.**

- Read and follow the directions carefully. If a problem is asking you to use a certain method, you must use that method to solve the problem.

1. (10 pts) Prove the following set is a subspace of $\mathbb{R}^3$, once by showing it satisfies all vector space properties I-VII (sorry, not sorry!), and once by applying the Subspace Criterion.

$$\{(x, y, z) \in \mathbb{R}^3 \mid x + y + 2z = 0, \text{ and } z - 2y + 3x = 0\}$$

2. (10 pts) Suppose U and W are subspaces of $\mathbb{R}^n$ for which $U \cup W$ is also a subspace of $\mathbb{R}^n$. Prove that $U \subseteq W$ or $W \subseteq U$.

   Hint: Use proof by contradiction.

3. (10 pts) Determine if each of the following matrices is in echelon form, reduced echelon form or neither. If the matrix is not in reduced echelon form, turn it into reduced echelon form by appropriate elementary row operations. In each step make sure you specify which row operation is used.

   (a) $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ 1 & 2 & 3 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 2 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 5 \end{pmatrix}$

4. (10 pts) Using elementary row operations, find all solutions of each system or show the system has no solutions.

   (a) $\begin{cases} x_1 + 3x_2 + x_4 = 5 \\ x_2 - x_3 + 5x_4 = 1 \\ 2x_1 - x_3 + x_4 = 0 \end{cases}$

   (b) $\begin{cases} x_1 + x_2 + 3x_3 - x_4 = 5 \\ x_2 - x_3 + 5x_4 = -2 \\ 2x_1 + 3x_2 + 5x_3 + 3x_4 = 0 \end{cases}$

5. (10 pts) Consider the homogeneous system

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1k}x_k = 0 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2k}x_k = 0 \\ \vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nk}x_k = 0 \end{cases}$$

   Prove that the set of vectors $(x_1, x_2, \ldots, x_k) \in \mathbb{R}^k$ satisfying the system above is a subspace of $\mathbb{R}^k$.

6. (10 pts) Show that if a matrix B is obtained by applying an elementary row operation to a matrix A, then Row $(A) =$ Row (B). (Hint: Check each of the three row operations separately. You could use Example 2.14.) By an example show that Col $(A) =$ Col (B) does not always hold.

Practice Problems

7. Example 2.9 from Math 340 Notes.

8. Example 2.11 from Math 340 Notes.

9. Example 2.12 from Math 340 Notes.

10. Example 2.13 from Math 340 Notes.

11. Example 2.16 from Math 340 Notes.

12. Exercise 2.5 from Math 340 Notes.

13. Exercise 2.11 from Math 340 Notes.

14. Exercise 2.20 from Math 340 Notes.

Challenge Problem

Exercises 2.21 from Math 340 Notes.