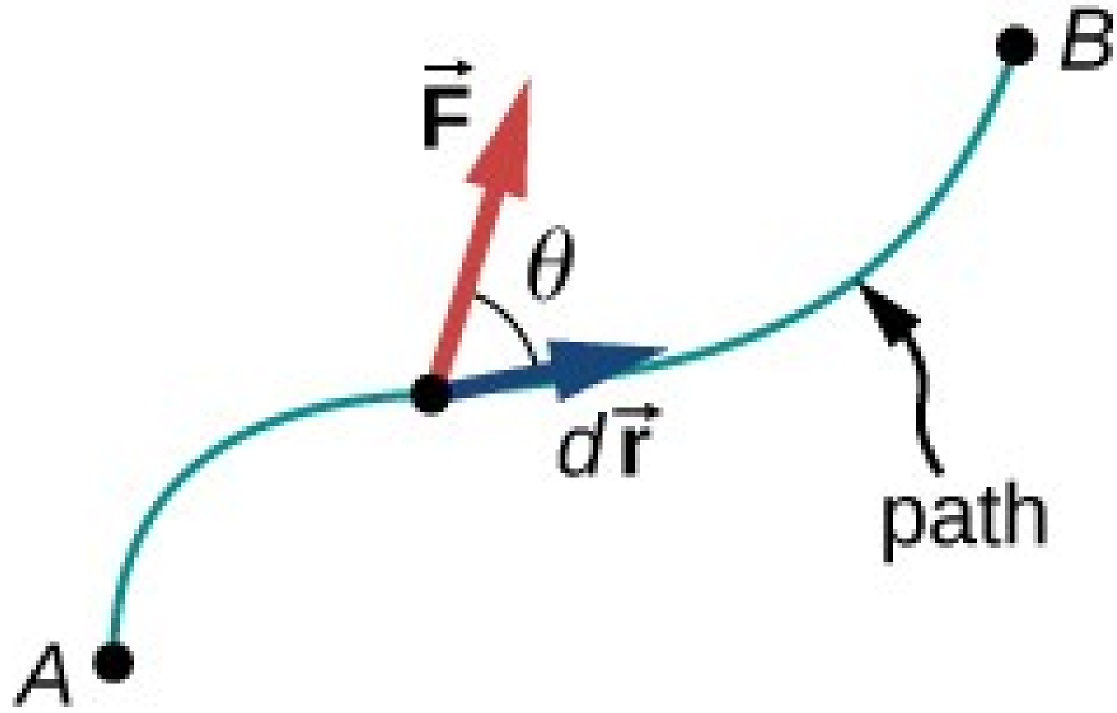
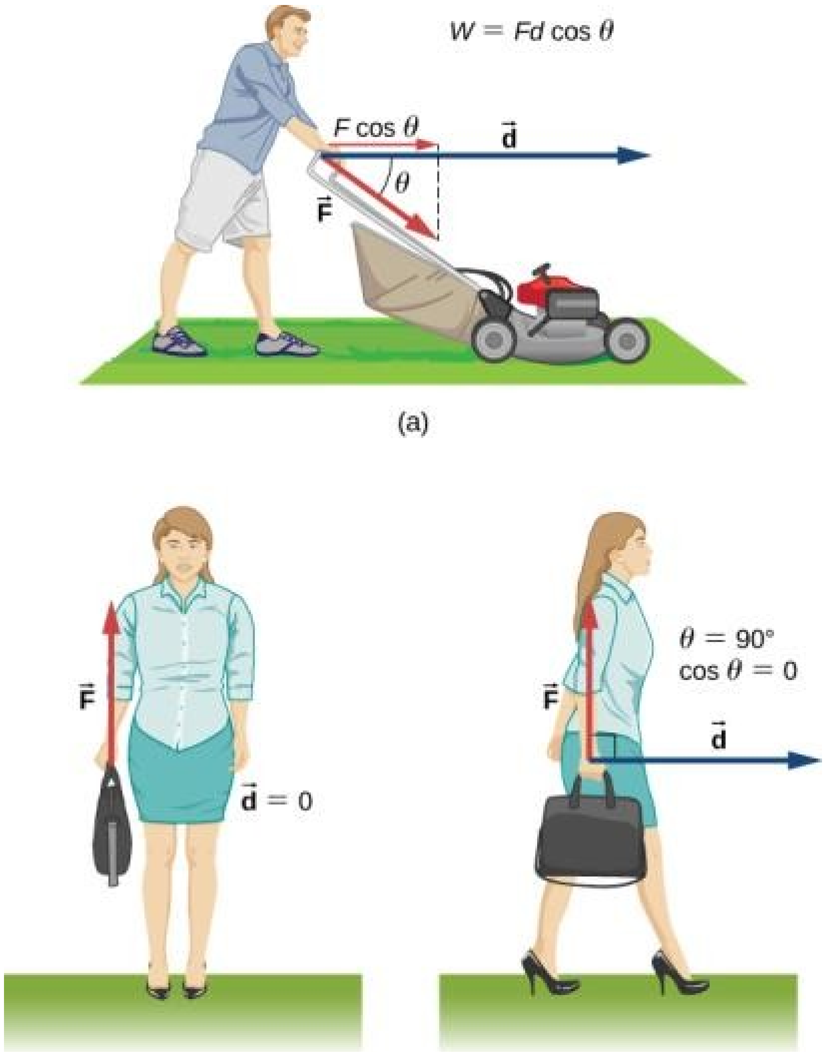
FIGURE 7.2 WORK (JOULE = NEWTON-METER)



Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between *A* and *B*. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.

FIGURE 7.3



Work done by a constant force.

(a) A person pushes a lawn mower with a constant force. The component of the force parallel to the displacement is the work done, as shown in the equation in the figure.

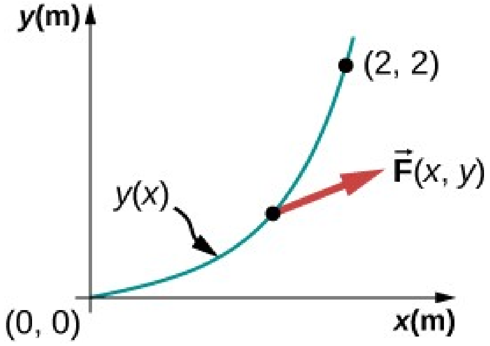
(b) A person holds a briefcase. No work is done because the displacement is zero.

(c) The person in (b) walks horizontally while holding the briefcase. No work is done because cos is zero.

EXAMPLE 7.4

Work Done by a Variable Force over a Curved Path

An object moves along a parabolic path $y = (0.5\ \text{m}^{-1})x^2$ from the origin $A = (0, 0)$ to the point $B = (2\ \text{m}, 2\ \text{m})$ under the action of a force $\vec{\mathbf{F}} = (5\ \text{N/m})y\hat{\mathbf{i}} + (10\ \text{N/m})x\hat{\mathbf{j}}$ (Figure 7.6). Calculate the work done.



The parabolic path of a particle acted on by a given force.

$$y = (0.5\ \text{m}^{-1})x^2 \text{ and } dy = 2(0.5\ \text{m}^{-1})x dx.$$

Then, the integral for the work is just a definite integral of a function of x.

Solution

The infinitesimal element of work is

$$\begin{aligned} dW &= F_x dx + F_y dy = (5\ \text{N/m})y dx + (10\ \text{N/m})x dy \\ &= (5\ \text{N/m})(0.5\ \text{m}^{-1})x^2 dx + (10\ \text{N/m})2(0.5\ \text{m}^{-1})x^2 dx = (12.5\ \text{N/m}^2)x^2 dx. \end{aligned}$$

The integral of x^2 is $x^3/3$, so

$$W = \int_0^{2\ \text{m}} (12.5\ \text{N/m}^2)x^2 dx = (12.5\ \text{N/m}^2)\frac{x^3}{3}\bigg|_0^{2\ \text{m}} = (12.5\ \text{N/m}^2)\left(\frac{8}{3}\right) = 33.3\ \text{J}.$$

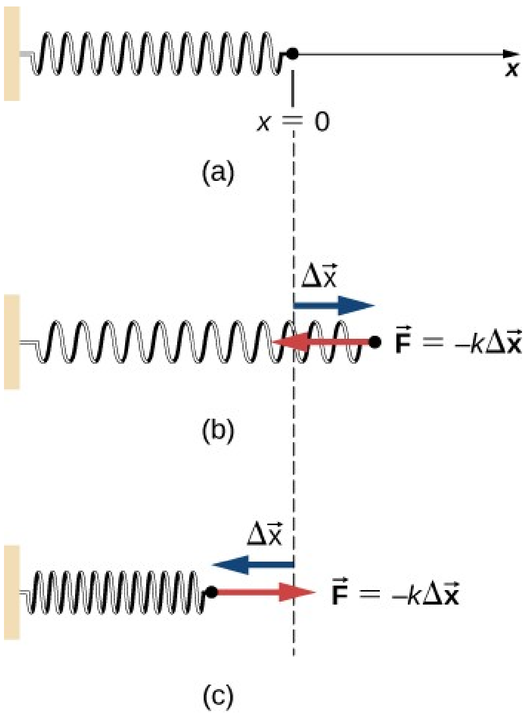
FIGURE 7.7

(a) The spring exerts no force at its equilibrium position.

The spring exerts a force in the opposite direction to either an extension or a compression. This is conveyed by the negative sign F= - kx

(b) an extension

(c) a compression.

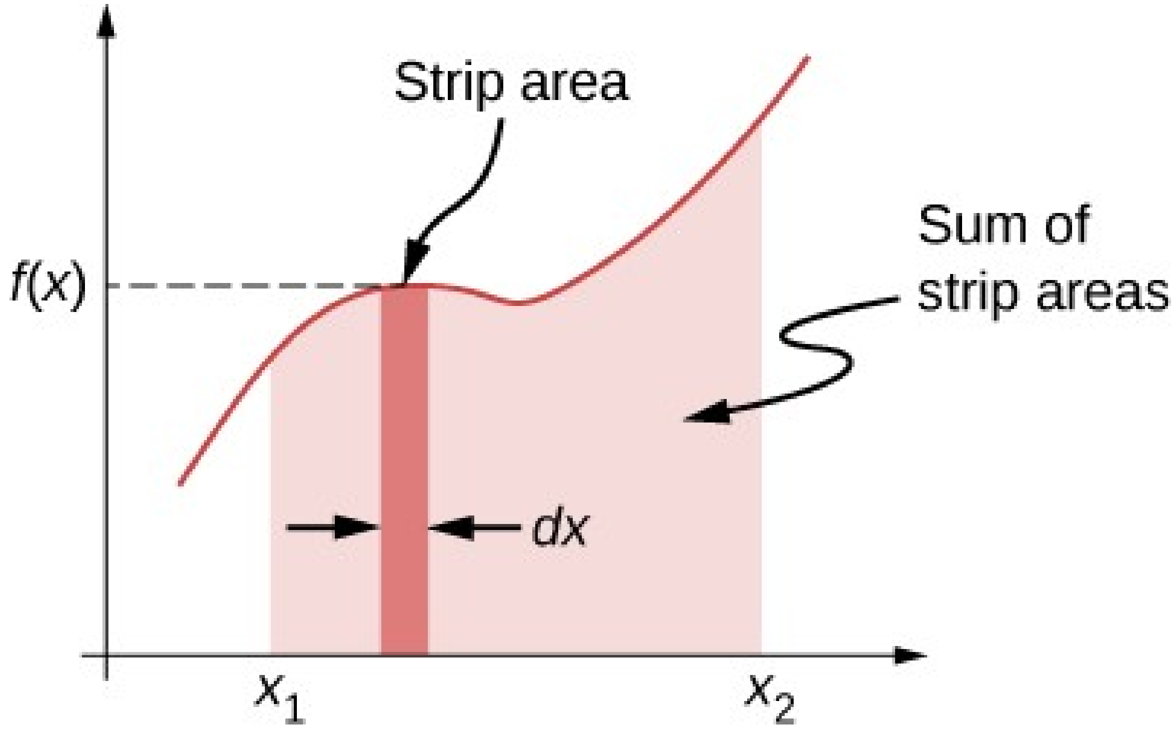


The work done in stretching the spring is the same in magnitude as in compressing it:

$$W_{\text{spring},AB} = \int_A^B F_x dx = -k \int_A^B x dx = -k \frac{x^2}{2}\Bigg|_A^B = -\frac{1}{2}k\left(x_B^2 - x_A^2\right).$$

Better write this as the absolute value of the difference between W at A and at B. Work done on the spring is positive, either by extension or compression. It increases the potential energy stored in the spring (later).

FIGURE 7.8



A curve of $f(x)$ versus x showing the area of an infinitesimal strip, $f(x)dx$, and the sum of such areas, which is the integral of $f(x)$ from x_1 to x_2.

KINETIC ENERGY

$$K = \frac{1}{2}mv^2.$$

EXAMPLE 7.7

Kinetic Energy Relative to Different Frames

A 75.0-kg person walks down the central aisle of a subway car at a speed of 1.50 m/s relative to the car, whereas the train is moving at 15.0 m/s relative to the tracks. (a) What is the person's kinetic energy relative to the car? (b) What is the person's kinetic energy relative to the tracks? (c) What is the person's kinetic energy relative to a frame moving with the person?

The possible motions of a person walking in a train are (a) toward the front of the car and (b) toward the back of the car.

Do the velocity addition and then derive the kinetic energy

a. $K = \frac{1}{2}(75.0\,\text{kg})(1.50\,\text{m/s})^2 = 84.4\,\text{J}.$

b. $v_{PT} = (15.0 \pm 1.50)\,\text{m/s}$. Therefore, the two possible values for kinetic energy relative to the car are

$$K = \frac{1}{2}(75.0\,\text{kg})(13.5\,\text{m/s})^2 = 6.83\,\text{kJ}$$

and

$$K = \frac{1}{2}(75.0\,\text{kg})(16.5\,\text{m/s})^2 = 10.2\,\text{kJ}.$$

Work-Energy Theorem

The net work done on a particle equals the change in the particle's kinetic energy:

$$W_{\text{net}} = K_B - K_A. \tag{7.9}$$

Let's start by looking at the net work done on a particle as it moves over an infinitesimal displacement, which is the dot product of the net force and the displacement: $dW_{\text{net}} = \vec{\mathbf{F}}_{\text{net}} \cdot d\vec{\mathbf{r}}$. Newton's second law tells us that $\vec{\mathbf{F}}_{\text{net}} = m(d\vec{\mathbf{v}}/dt)$, so $dW_{\text{net}} = m(d\vec{\mathbf{v}}/dt) \cdot d\vec{\mathbf{r}}$. For the mathematical functions describing the motion of a physical particle, we can rearrange the differentials dt, etc., as algebraic quantities in this expression, that is,

$$dW_{\text{net}} = m\left(\frac{d\vec{\mathbf{v}}}{dt}\right) \cdot d\vec{\mathbf{r}} = md\vec{\mathbf{v}} \cdot \left(\frac{d\vec{\mathbf{r}}}{dt}\right) = m\vec{\mathbf{v}} \cdot d\vec{\mathbf{v}},$$

where we substituted the velocity for the time derivative of the displacement and used the commutative property of the dot product [Equation 2.30]. Since derivatives and integrals of scalars are probably more familiar to you at this point, we express the dot product in terms of Cartesian coordinates before we integrate between any two points A and B on the particle's trajectory. This gives us the net work done on the particle:

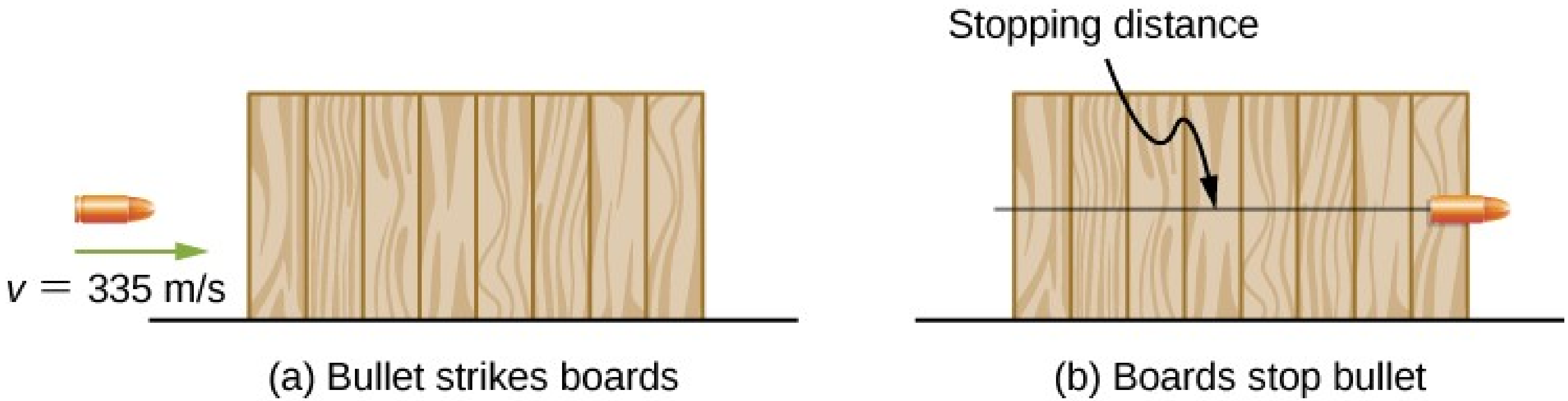
$$\begin{aligned} W_{\text{net},AB} &= \int_A^B (mv_x dv_x + mv_y dv_y + mv_z dv_z) \\ &= \tfrac{1}{2}m\left|v_x^2 + v_y^2 + v_z^2\right|_A^B = \left|\tfrac{1}{2}mv^2\right|_A^B = K_B - K_A. \end{aligned} \tag{7.8}$$

In the middle step, we used the fact that the square of the velocity is the sum of the squares of its Cartesian components, and in the last step, we used the definition of the particle's kinetic energy. This important result is called the **work-energy theorem** (Figure 7.11).

EXAMPLE 7.10

Determining a Stopping Force

A bullet has a mass of 40 grains (2.60 g) and a muzzle velocity of 1100 ft./s (335 m/s). It can penetrate eight 1-inch pine boards, each with thickness 0.75 inches. What is the average stopping force exerted by the wood, as shown in Figure 7.13?



The boards exert a force to stop the bullet. They do work against the bullet and the bullet loses kinetic energy by an amount:

$$W_{\text{net}} = -F_{\text{ave}}\Delta s_{\text{stop}} = -K_{\text{initial}},$$

$$F_{\text{ave}} = \frac{\frac{1}{2}mv^2}{\Delta s_{\text{stop}}} = \frac{\frac{1}{2}(2.6 \times 10^{-3}\,\text{kg})(335\,\text{m/s})^2}{0.152\,\text{m}} = 960\,\text{N}.$$

POWER (WATTS)

$$P = \frac{dW}{dt} = \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \left(\frac{d\vec{\mathbf{r}}}{dt}\right) = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}},$$

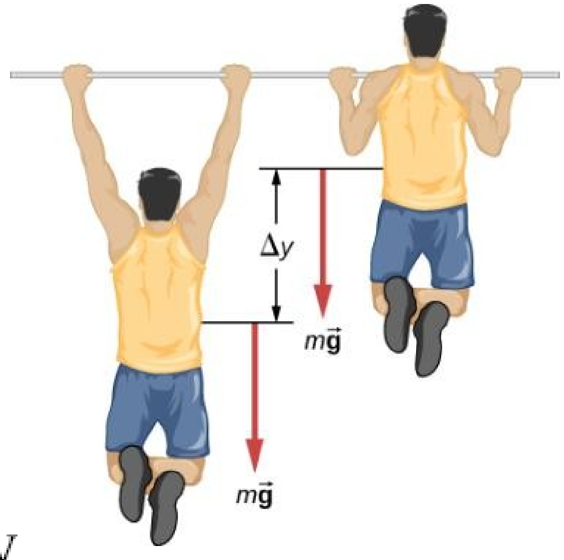
EXAMPLE 7.11

Pull-Up Power

An 80-kg army trainee does pull-ups on a horizontal bar (Figure 7.14). It takes the trainee 0.8 seconds to raise the body from a lower position to where the chin is above the bar. How much power do the trainee's muscles supply moving his body from the lower position to where the chin is above the bar? (*Hint:* Make reasonable estimates for any quantities needed.)

Let's assume that $\Delta y = 2\text{ft} \approx 60\text{ cm}$.

Also assume that the arms comprise 10% of the body mass and are not included in the moving mass

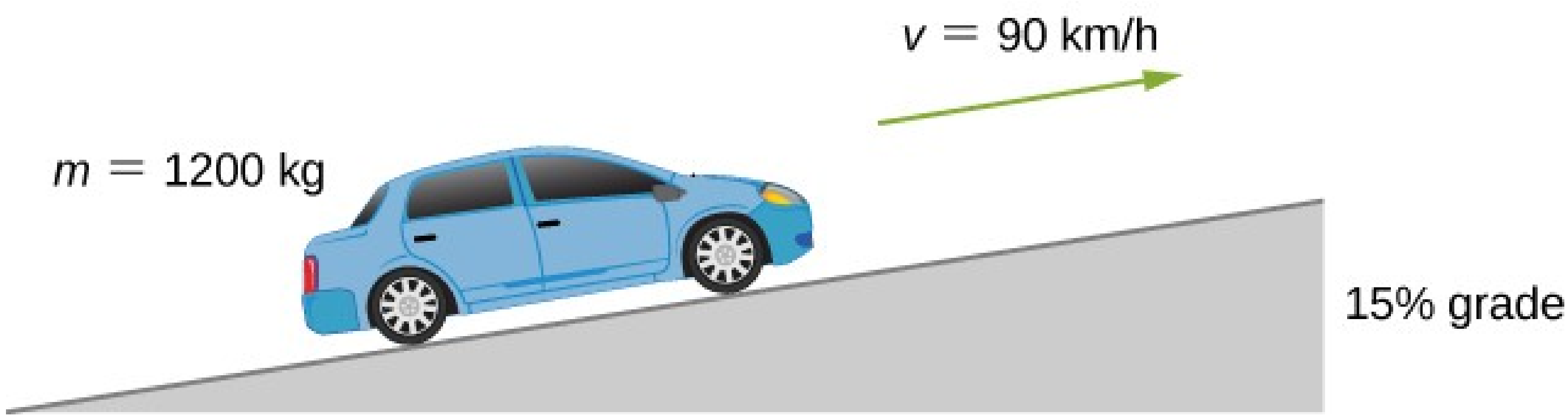


$$P = \frac{mg(\Delta y)}{t} = \frac{0.9\,(80\text{ kg})(9.8\text{ m/s}^2)(0.60\text{ m})}{0.8\text{ s}} = 529\text{ W}.$$

EXAMPLE 7.12

Automotive Power Driving Uphill

How much power must an automobile engine expend to move a 1200-kg car up a 15% grade at 90 km/h (Figure 7.15)? Assume that 25% of this power is dissipated overcoming air resistance and friction.



Since the weight is in the –y direction => no work done for x component of motion. The power supplied by the engine to move the car is the power expended against gravity.

$$m\vec{\mathbf{g}} \cdot \vec{\mathbf{v}} = mgv \sin\theta,$$

tan θ = 15 º

$$0.75\,P = mgv \sin(\tan^{-1} 0.15),$$

Given 75% of the power is sup

$$P = \frac{(1200 \times 9.8\text{ N})(90\text{ m}/3.6\text{ s})\sin(8.53°)}{0.75} = 58\text{ kW},$$

EXAMPLE 7.8

Special Names for Kinetic Energy

(a) A player lobs a mid-court pass with a 624-g basketball, which covers 15 m in 2 s. What is the basketball's horizontal translational kinetic energy while in flight? (b) An average molecule of air, in the basketball in part (a), has a mass of 29 u, and an average speed of 500 m/s, relative to the basketball. There are about 3×10^{23} molecules inside it, moving in random directions, when the ball is properly inflated. What is the average translational kinetic energy of the random motion of all the molecules inside, relative to the basketball? (c) How fast would the basketball have to travel relative to the court, as in part (a), so as to have a kinetic energy equal to the amount in part (b)?

Motion of molecules => kinetic energy of molecules => Thermal Energy

a. The horizontal speed is (15 m)/(2 s), so the horizontal kinetic energy of the basketball is

$$\frac{1}{2}(0.624\text{ kg})(7.5\text{ m/s})^2 = 17.6\text{ J}.$$

b. The average translational kinetic energy of a molecule is

$$\frac{1}{2}(29\text{ u})(1.66 \times 10^{-27}\text{ kg/u})(500\text{ m/s})^2 = 6.02 \times 10^{-21}\text{ J},$$

and the total kinetic energy of all the molecules is

$$(3 \times 10^{23})(6.02 \times 10^{-21}\text{ J}) = 1.80\text{ kJ}.$$

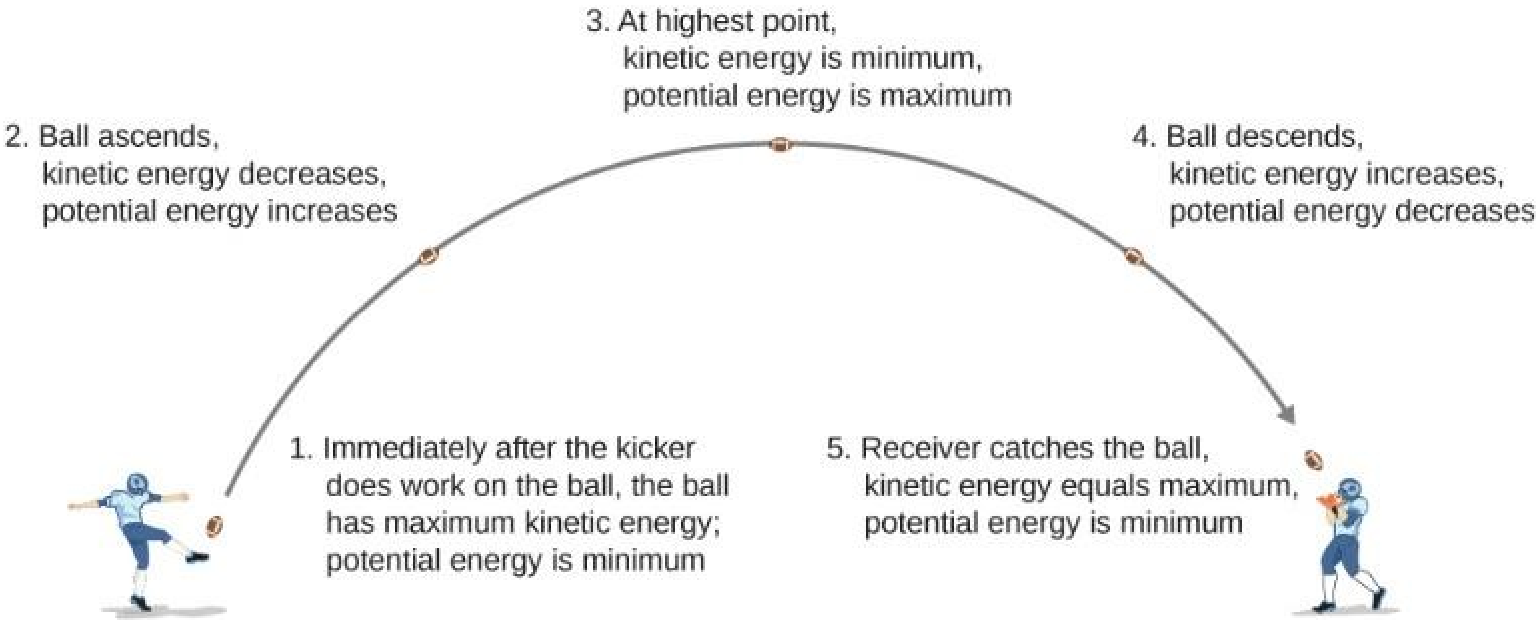
c. $v = \sqrt{2(1.8\text{ kJ})/(0.624\text{ kg})} = 76.0\text{ m/s}.$

POTENTIAL ENERGY: GRAVITATIONAL

$$\Delta U_{\text{grav}} = -W_{\text{grav},AB} = mg\left(y_B - y_A\right).$$

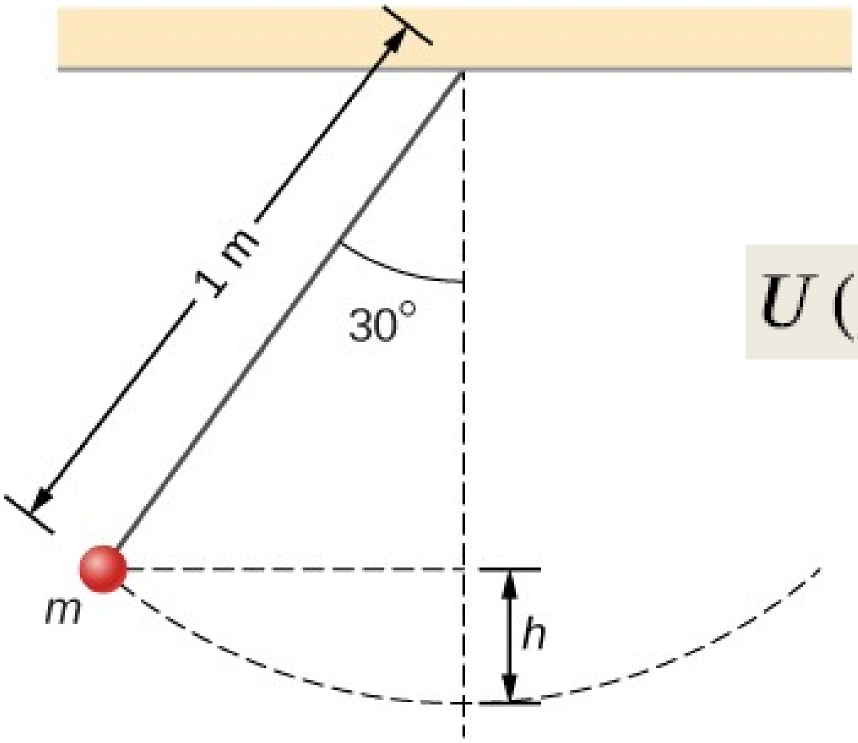
the gravitational potential energy function, near Earth's surface,

$$U(y) = mgy + \text{const.}$$



As a football starts its descent toward the wide receiver, gravitational potential energy is converted back into kinetic energy.

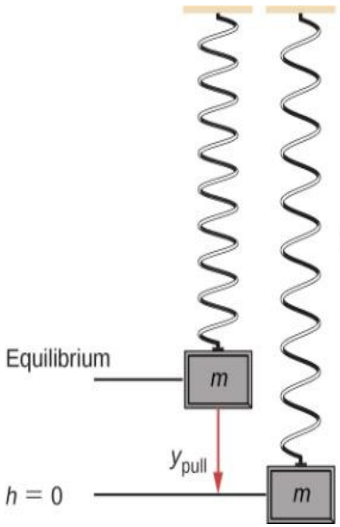
KINETIC AND POTENTIAL ENERGY OF A PENDULUM

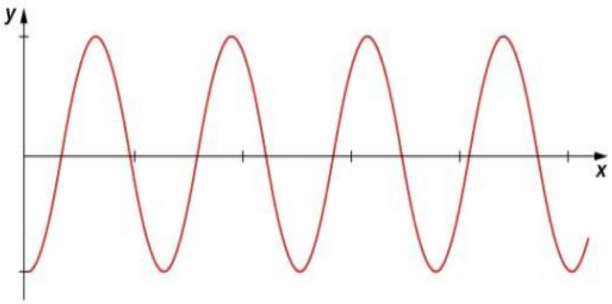


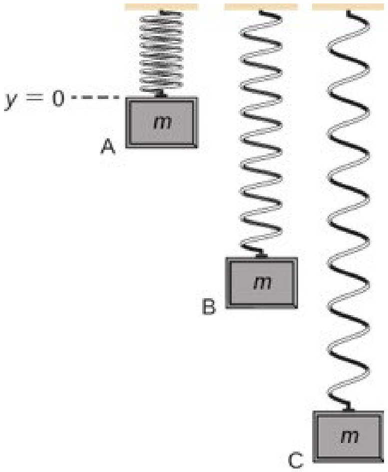
$$U(y) = mgy + \text{const.}$$

A particle hung from a string constitutes a simple pendulum. It is shown when released from rest, along with some distances used in analyzing the motion.

POTENTIAL ENERGY OF STRETCHED OR COMPRESSED SPRING







A vertical mass-spring system (*y*-axis pointing up). The mass is initially at an equilibrium position and pulled downward to y_{pull}. An oscillation begins, centered at the equilibrium position.

For bungee jump, the equilibrium point y=0 is where the elastic rope just begins its downward stretch.

Two potential energies are present:

Gravitational U_g= mgy and

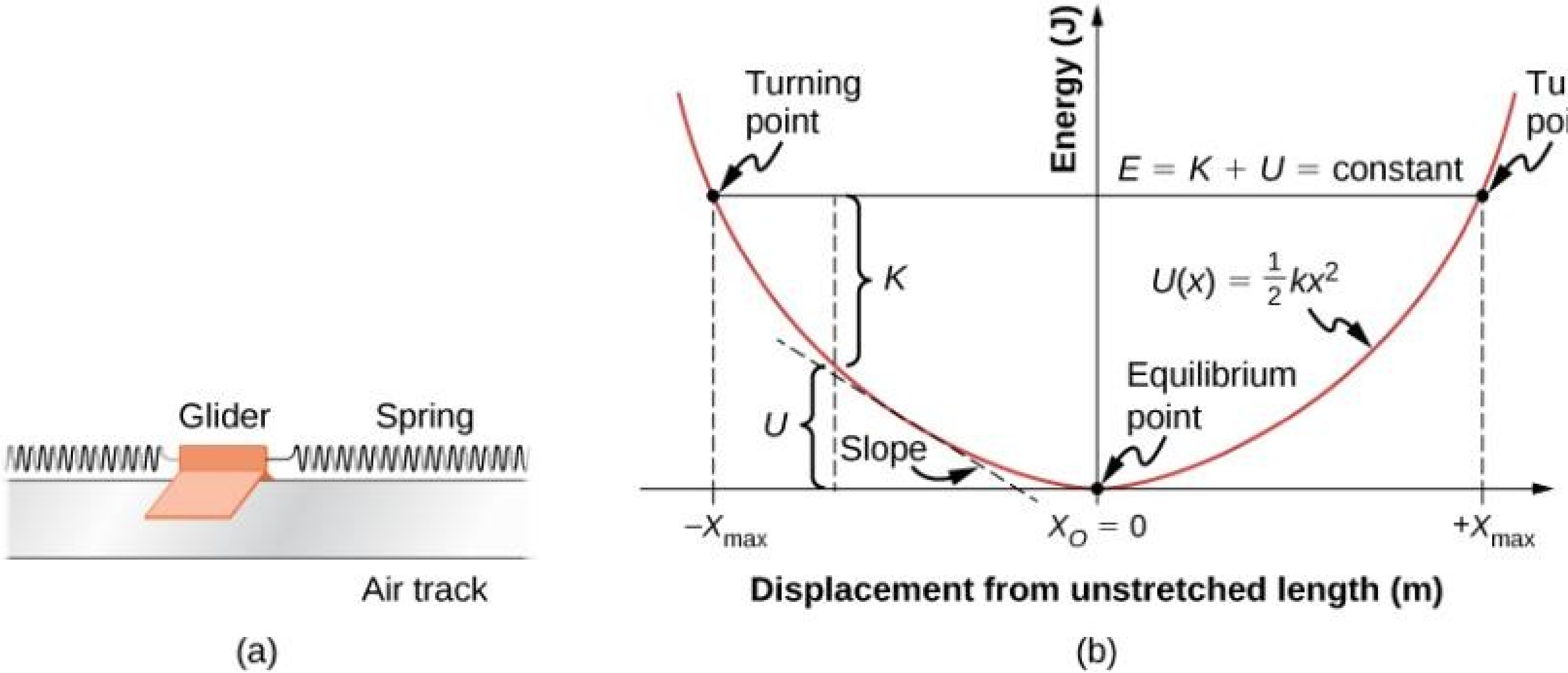
Spring's stored energy U_s = k y^2/2

$$K_A + U_A = K_C + U_C$$

$$0 = 0 + mgy_C + \frac{1}{2}k(y_C)^2$$

$$y_C = \frac{-2mg}{k}$$

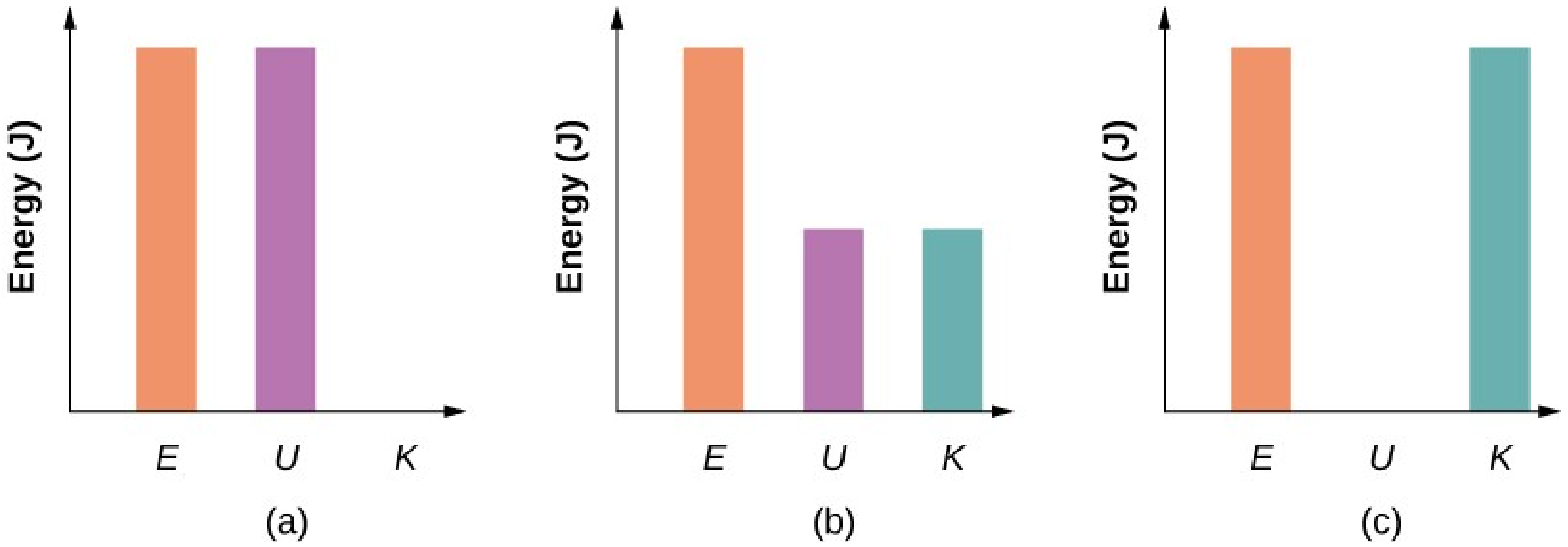
FIGURE 8.11



(a) A glider between springs on an air track is an example of a horizontal mass-spring system.

(b) The potential energy diagram for this system, with various quantities indicated.

FIGURE 8.8



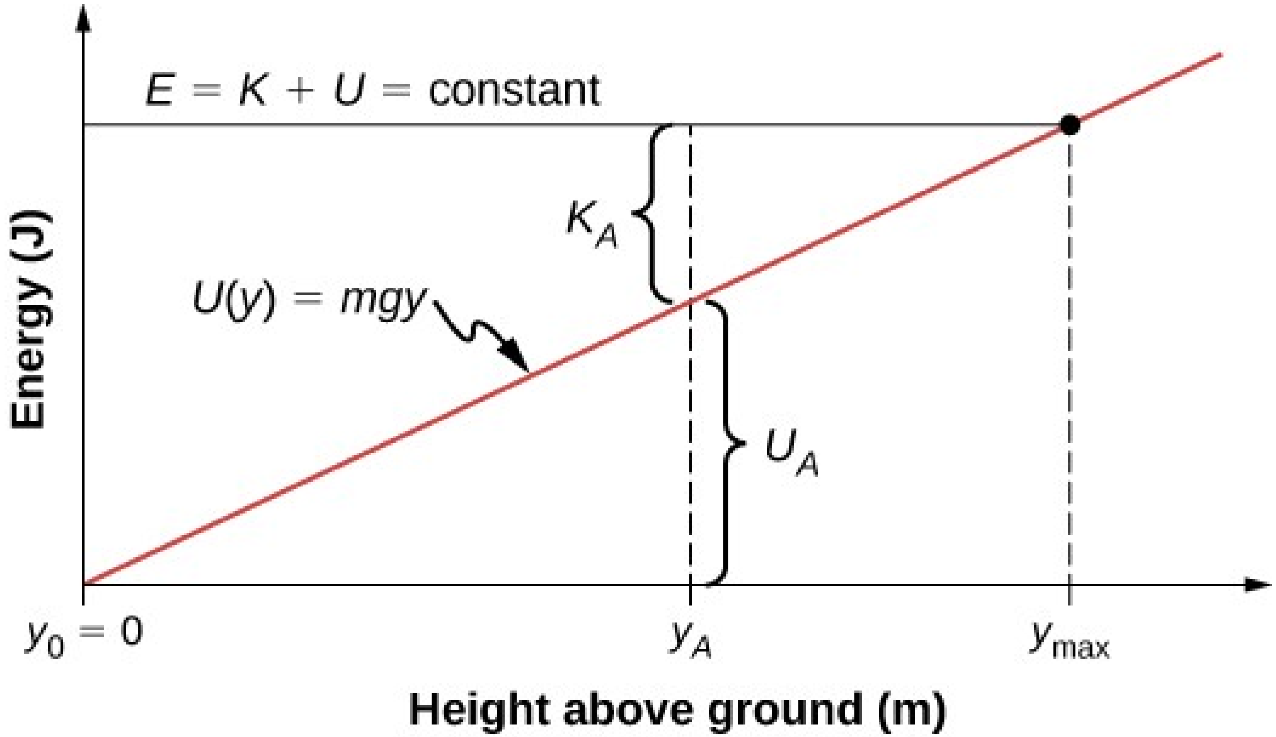
Bar graphs representing the total energy (*E*), potential energy (*U*), and kinetic energy (*K*) of the particle in different positions.

(a) The total energy of the system equals the potential energy and the kinetic energy is zero, which is found at the highest point the particle reaches.

(b) The particle is midway between the highest and lowest point, so the kinetic energy plus potential energy bar graphs equal the total energy.

(c) The particle is at the lowest point of the swing, so the kinetic energy bar graph is the highest and equal to the total energy of the system.

FIGURE 8.10



The potential energy graph for an object in vertical free fall, with various quantities indicated.

Force F derived from potential energy difference dU over displacement dl

$$F_l = -\frac{dU}{dl}.$$

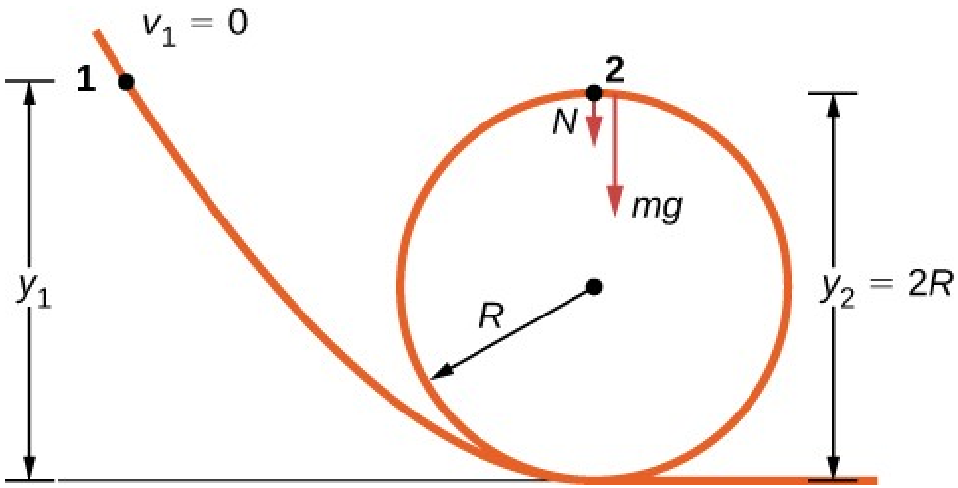
EXAMPLE 7.9

Loop-the-Loop

The frictionless track for a toy car includes a loop-the-loop of radius R. How high, measured from the bottom of the loop, must the car be placed to start from rest on the approaching section of track and go all the way around the loop?

the starting height determines the speed of the car at the top of the loop

$$-mg(y_2 - y_1) = \frac{1}{2}mv_2{}^2,$$



At the top of the loop, the normal force and gravity are both down and the acceleration is centripetal

$$a_{\text{top}} = \frac{F}{m} = \frac{N + mg}{m} = \frac{v_2^2}{R}.$$

Demo D1-53

$$N = -mg + \frac{mv_2^2}{R} = \frac{-mgR + 2mg(y_1 - R)}{R} > 0 \quad \text{or} \quad y_1 > \frac{5R}{2}.$$

D1-53: LOOP-THE-LOOP

https://lecdem.physics.umd.edu/d/d1/d1-53

Demonstrates centripetal force and conservation of energy in a rotating object

- **Description:**

This track can be described as three segments: the long upright segment, the loop, and the shorter upright segment. If you begin by placing the ball on the long upright segment at a height equal to the height of the loop (2R), the ball will roll down the track, begin to climb the loop, and then fall off and roll away. You can then repeat this at increasingly higher positions until the ball makes it all the way around the loop and begins to climb the shorter upright segment. In either case, be ready to catch it as it falls off afterwards! This is a good demonstration to encourage students to make predictions about its behaviour. Invite students to make arguments about what starting height will allow the ball to complete the full loop. A meter stick can be additionally provided upon request to aid in measuring the height.

- **Background**

- Motion of the ball down the track and around the loop-the-loop can be described in terms of gravitational potential energy, rotational and translational kinetic energy, and centripetal force. A ball of mass m and radius r must be released at some minimum height h above the bottom point of the track so that it will not leave the track while passing around the loop-the-loop. In order to stay on the track at the top of the loop the centrifugal reaction of the ball on the track must be equal to or greater than the gravitational force on the ball: mv^2/R = mg, or v^2 = gR, where v is its linear velocity at the top of the loop, R is the radius of the loop, and g is the acceleration of gravity. Conservation of energy dictates that at the top of the loop Iw^2/2 + mv^2/2 +2mgR = mgh, where the moment of inertia of the ball I = 2mr^2/5 and w is the angular velocity of the ball at the top of the loop.
- From these considerations we obtain the minimum starting height for the ball above the bottom of the loop-the-loop in order that the ball remain in contact with the track at all times: h = 2.7 R. In the case of an object sliding along a frictionless loop-the-loop, the height would be h = 2.5 R. Marks have been made at the points 2.5 R and 2.7 R. The ball remains in contact with the track at the top of the loop only when the height 2.7 R is reached, demonstrating the effect of the rotation of the rolling ball.