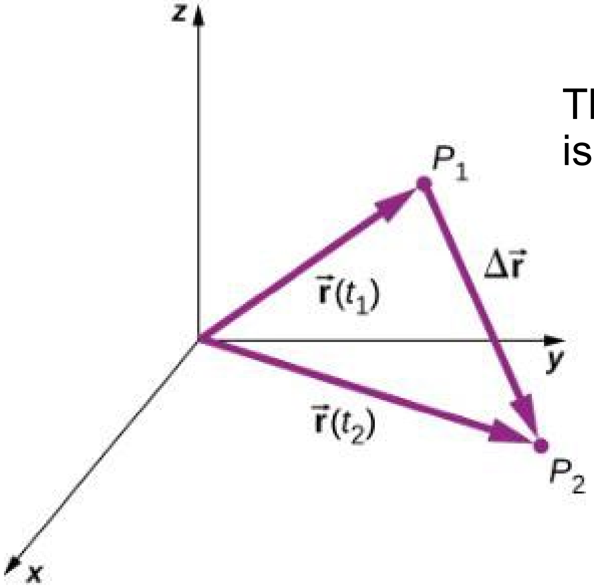
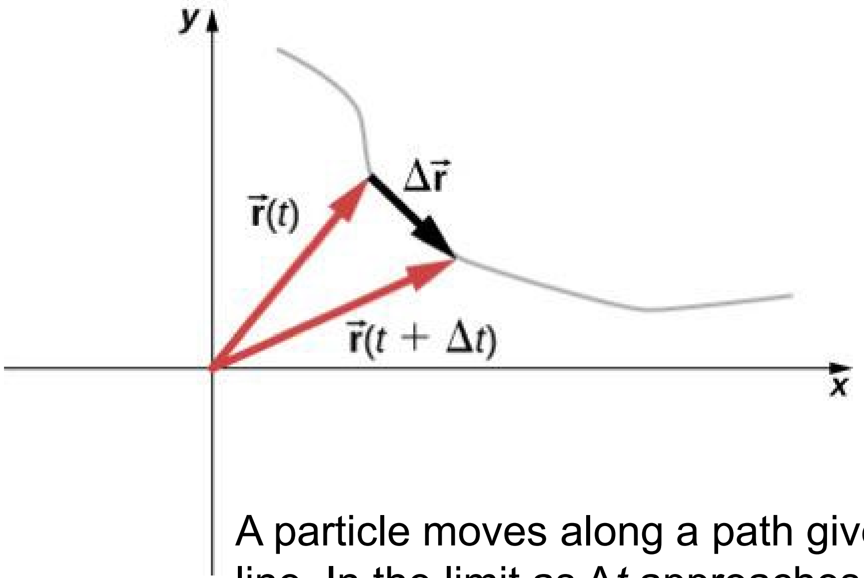
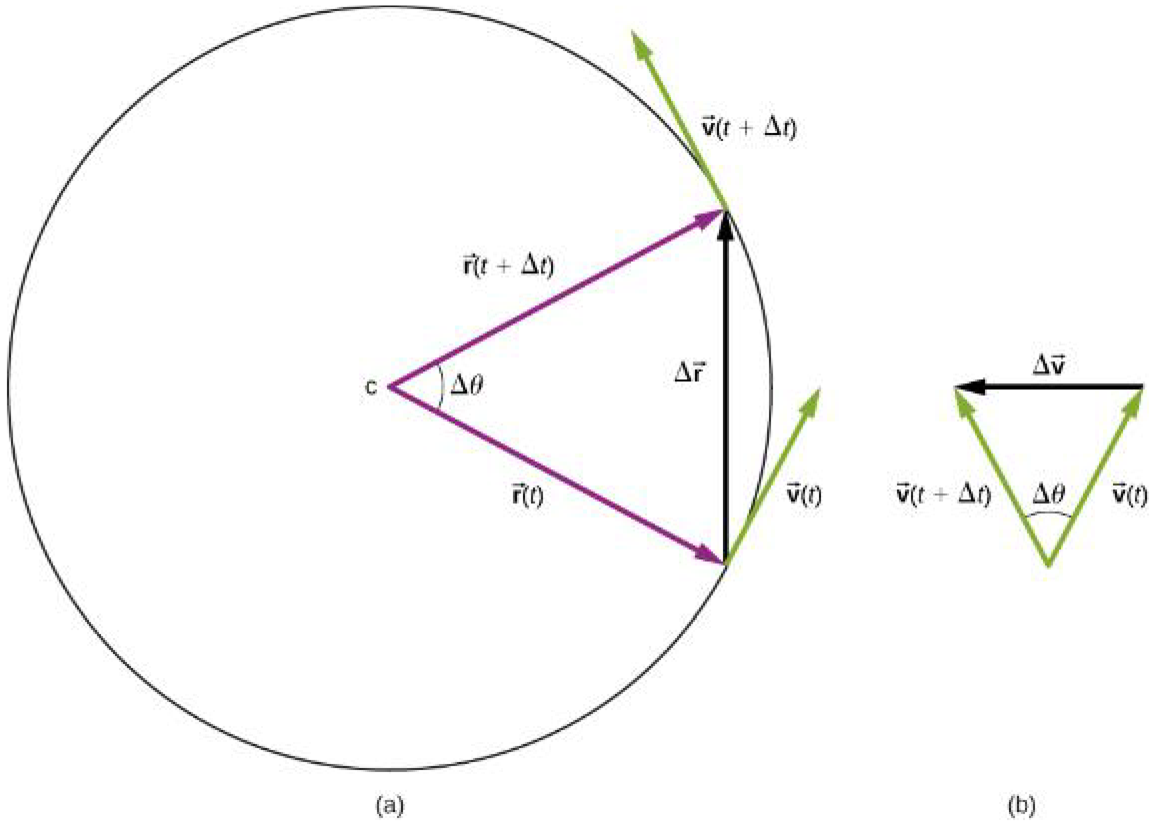


The displacement $\Delta\vec{r} = \vec{r}(t_2) - \vec{r}(t_1)$ is the vector from P_1 to P_2.



A particle moves along a path given by the gray line. In the limit as Δt approaches zero, the velocity vector becomes tangent to the path of the particle.

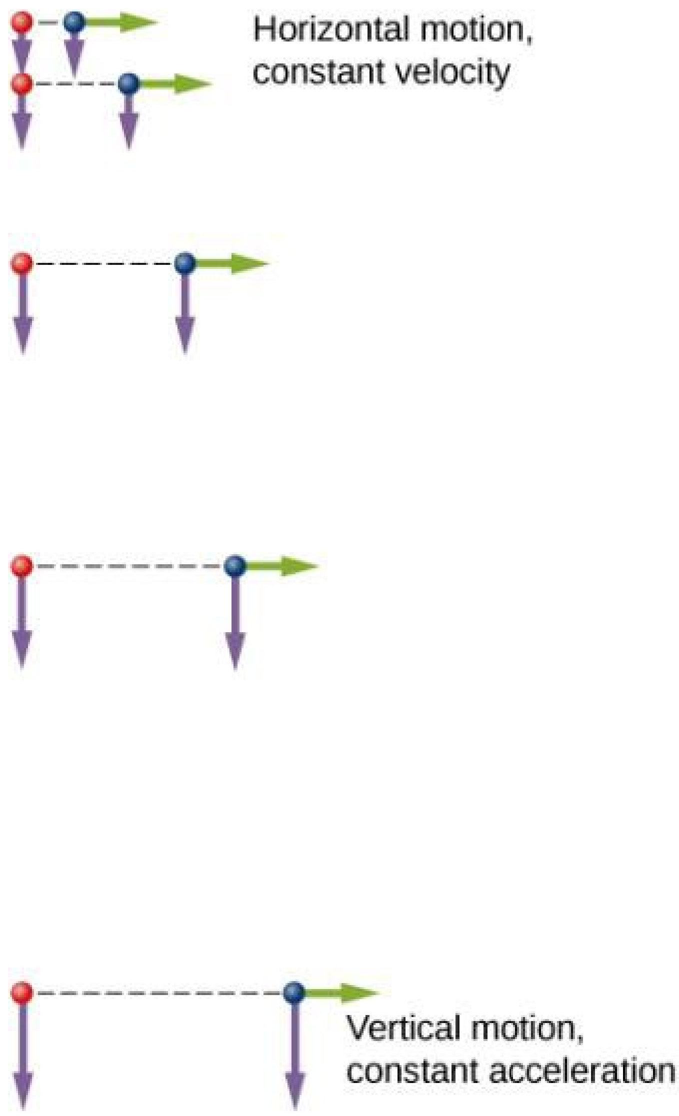
FIGURE 4.18



(a) A particle is moving in a circle at a constant speed, with position and velocity vectors at times t and $t + \Delta t$.

(b) Velocity vectors forming a triangle. The two triangles in the figure are similar. The vector $\Delta\vec{v}$ points toward the center of the circle in the limit $\Delta t \rightarrow 0$.

FIGURE 4.8



A diagram of the motions of two identical balls: one falls from rest and the other has an initial horizontal velocity. Each subsequent position is an equal time interval. Arrows represent the horizontal and vertical velocities at each position. The ball on the right has an initial horizontal velocity whereas the ball on the left has no horizontal velocity. Despite the difference in horizontal velocities, the vertical velocities and positions are identical for both balls, which shows the vertical and horizontal motions are independent.

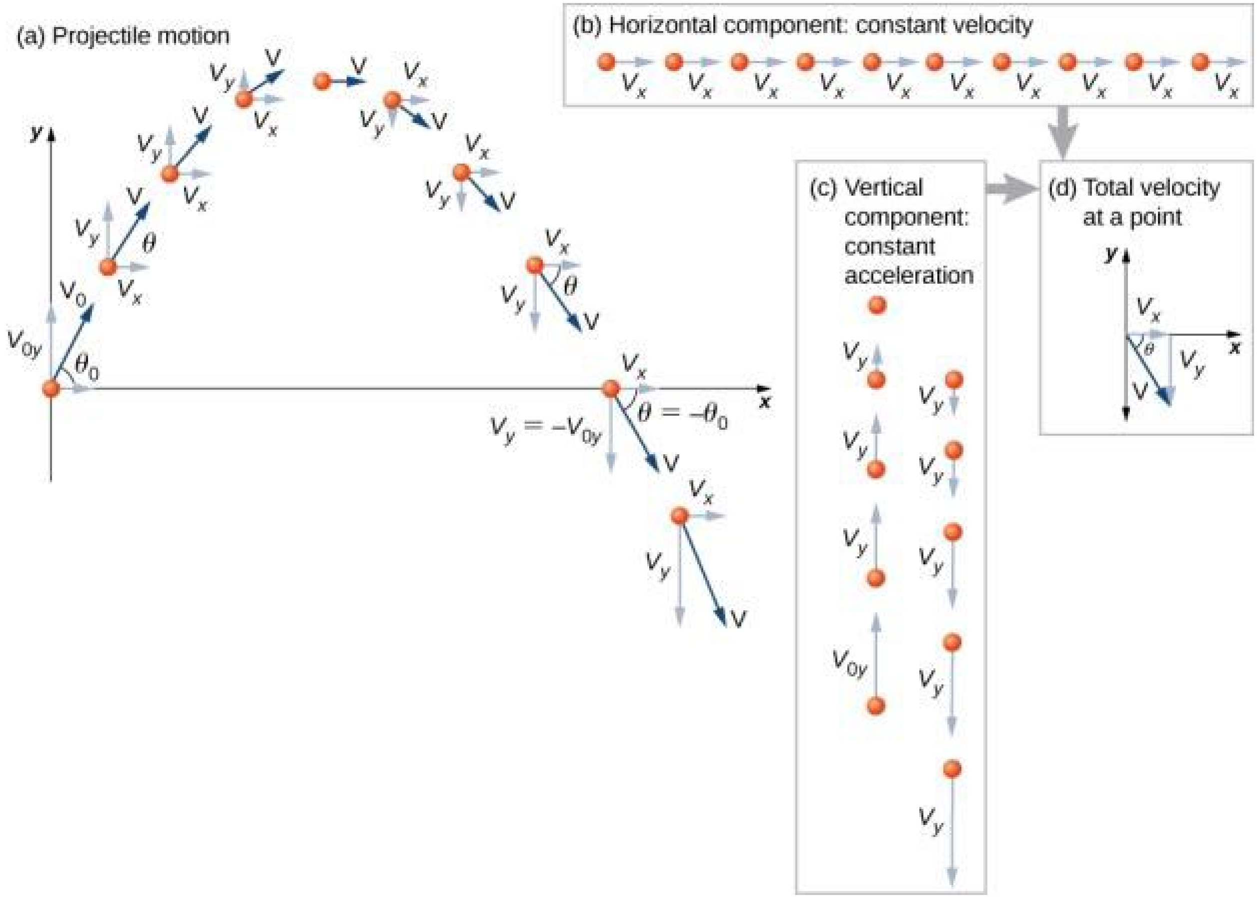
Demo:

C2-21 (Projectiles dropped & shot),

C2-22 (Monkey) next lecture

C2-25 (Funnel Cart)

FIGURE 4.12



(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes.

(b) The horizontal motion is simple, because $a_x = 0$ and v_x is a constant.

(c) The velocity in the vertical direction begins to decrease as the object rises. At its highest point, the vertical velocity is zero. As the object falls toward Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity.

(d) The x and y motions are recombined to give the total velocity at any given point on the trajectory.

EXAMPLE 4.6

A Skier

Figure 4.10 shows a skier moving with an acceleration of $2.1\ \text{m/s}^2$ down a slope of $15°$ at $t = 0$. With the origin of the coordinate system at the front of the lodge, her initial position and velocity are

$$\vec{\mathbf{r}}(0) = (75.0\hat{\mathbf{i}} - 50.0\hat{\mathbf{j}})\ \text{m}$$

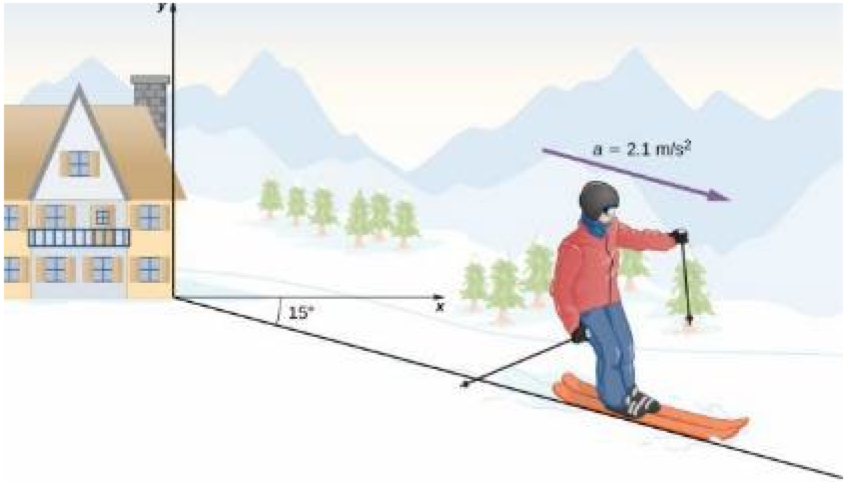
and

$$\vec{\mathbf{v}}(0) = (4.1\hat{\mathbf{i}} - 1.1\hat{\mathbf{j}})\ \text{m/s}.$$

(a) What are the x- and y-components of the skier's position and velocity as functions of time? (b) What are her position and velocity at $t = 10.0$ s?

$$a_x = (2.1\ \text{m/s}^2)\cos(15°) = 2.0\ \text{m/s}^2$$

$$a_y = (-2.1\ \text{m/s}^2)\sin 15° = -0.54\ \text{m/s}^2$$



$$x(t) = 75.0\ \text{m} + (4.1\ \text{m/s})t + \frac{1}{2}(2.0\ \text{m/s}^2)t^2$$

$$v_x(t) = 4.1\ \text{m/s} + (2.0\ \text{m/s}^2)t.$$

$$y(t) = -50.0\ \text{m} + (-1.1\ \text{m/s})t + \frac{1}{2}(-0.54\ \text{m/s}^2)t^2$$

$$v_y(t) = -1.1\ \text{m/s} + (-0.54\ \text{m/s}^2)t.$$

The trajectory of a fireworks shell. The fuse is set to explode the shell at the highest point in its trajectory, find a) the height y; b) the time t for it to reach this height c) how far is its horizontal distance; d) total displacement to firework

a) $$v_y^2 = v_{0y}^2 - 2g(y - y_0).$$

ion simplifies to

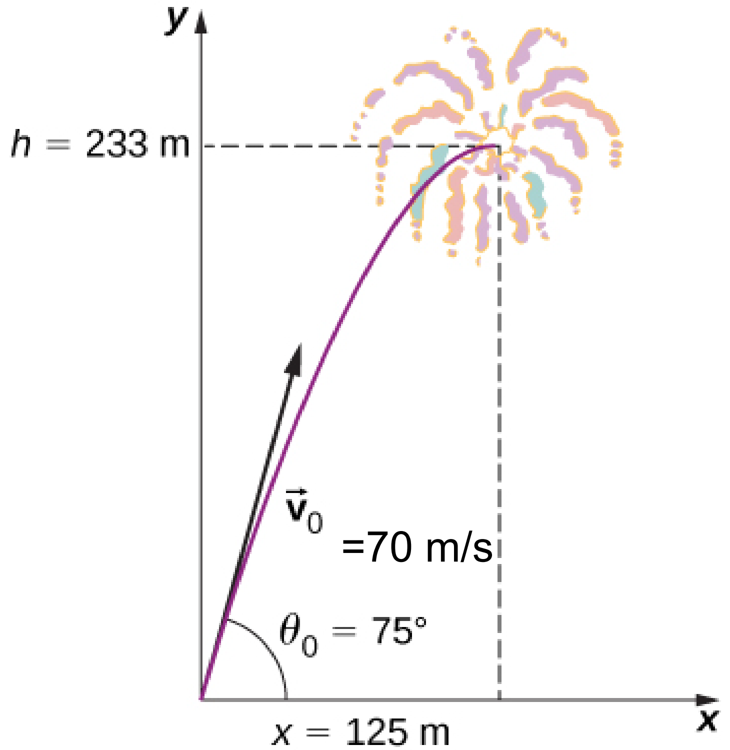
$$0 = v_{0y}^2 - 2gy.$$

$$y = \frac{v_{0y}^2}{2g}.$$

b)

to use $v_y = v_{0y} - gt.$

$$0 = v_{0y} - gt$$



c) $$x = v_x t,$$

of the velocity, which is given by

$$v_x = v_0 \cos\theta = (70.0 \text{ m/s})\cos 75° = 18.1 \text{ m/s}.$$

d) $$\vec{s} = 125\hat{\mathbf{i}} + 233\hat{\mathbf{j}}$$

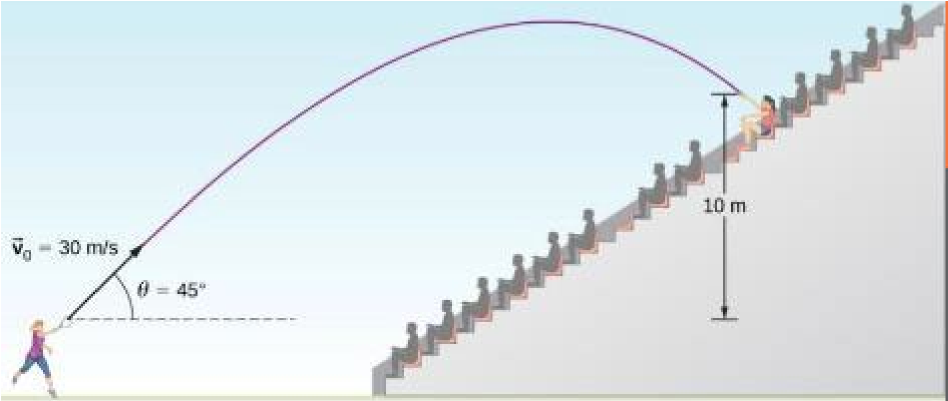
$$|\vec{s}| = \sqrt{125^2 + 233^2} = 264 \text{ m}$$

$$\Phi = \tan^{-1}\left(\frac{233}{125}\right) = 61.8°.$$

EXAMPLE 4.8

Calculating Projectile Motion: Tennis Player

A tennis player wins a match at Arthur Ashe stadium and hits a ball into the stands at 30 m/s and at an angle 45° above the horizontal (Figure 4.14). On its way down, the ball is caught by a spectator 10 m above the point where the ball was hit. (a) Calculate the time it takes the tennis ball to reach the spectator. (b) What are the magnitude and direction of the ball's velocity at impact?



a)

$$y = y_0 + v_{0y}t - \frac{1}{2}gt^2.$$

to be zero, then the final position is $y = 10$ m. Th

l velocity:

$$v_{0y} = v_0 \sin\theta_0 = (30.0\text{ m/s})\sin 45° = 21.2\text{ m/s}.$$

for y gives us

$$10.0\text{ m} = (21.2\text{ m/s})t - (4.90\text{ m/s}^2)t^2.$$

ratic equation in t:

$$(4.90\text{ m/s}^2)t^2 - (21.2\text{ m/s})t + 10.0\text{ m} = 0.$$

t = 3.79 s (descent – this is what we want) and t or 0.54 s (first pass ascent).

b)

$$v_x = v_0 \cos\theta_0 = (30\text{ m/s})\cos 45° = 21.2\text{ m/s}.$$

$$v_y = v_{0y} - gt.$$

n part (a) to be 21.2 m/s, we have

$$v_y = 21.2\text{ m/s} - 9.8\text{ m/s}^2(3.79\text{ s}) = -15.9\text{ m/s}.$$

final velocity $\vec{v}$ is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(21.2\text{ m/s})^2 + (-15.9\text{ m/s})^2} = 26.5\text{ m/s}.$$

ınd using the inverse tangent:

$$\theta_v = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15.9}{21.2}\right) = 36.9° \text{ below the horizon.}$$

1. Time of Flight: y (impact) = y_0

$$y - y_0 = v_{0y}t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 = 0.$$

$$t\left(v_0 \sin\theta_0 - \frac{gt}{2}\right) = 0.$$

$$T_{\text{tof}} = \frac{2(v_0 \sin\theta_0)}{g}.$$

2. Trajectory: eliminate time. Projectile launched at the origin x_0=y_0=0

$$x = v_{0x}t \Rightarrow t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta_0}.$$

Substitute this expression for t into $y = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$

Get

$$y = (\tan\theta_0)x - \left[\frac{g}{2(v_0 \cos\theta_0)^2}\right]x^2$$

3. Range: Set y=0 (impact) solve for x that's the range of the projectile.

Note: a) Maximum range at 45 degree angle

b) θ and 90- θ have the same range.

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

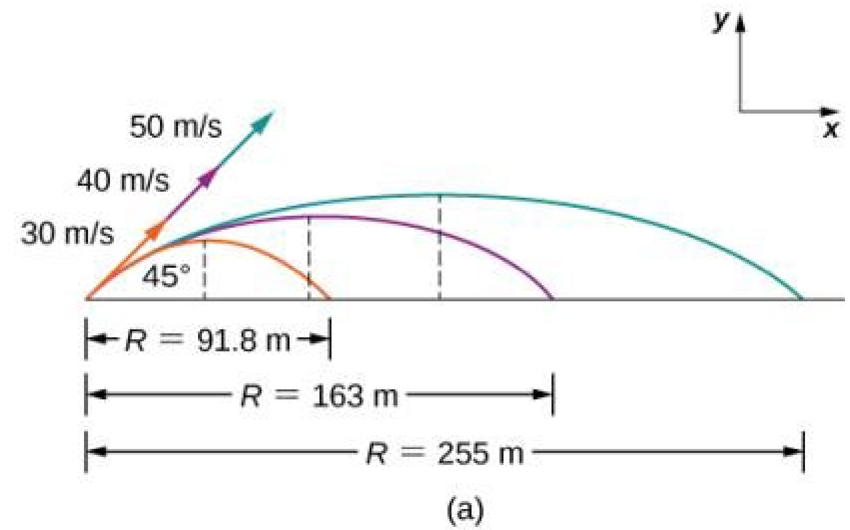
FIGURE 4.15

Trajectories of projectiles on level ground.

(a) The greater the initial speed v_0, the greater the range for a given initial angle.

(b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed.

(c) Note that the range is the same for initial angles of 15° and 75°, although the maximum heights of those paths are different.



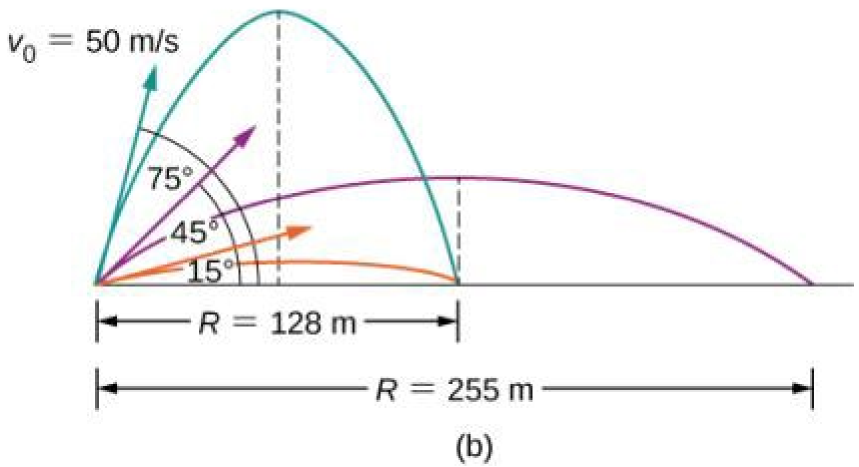
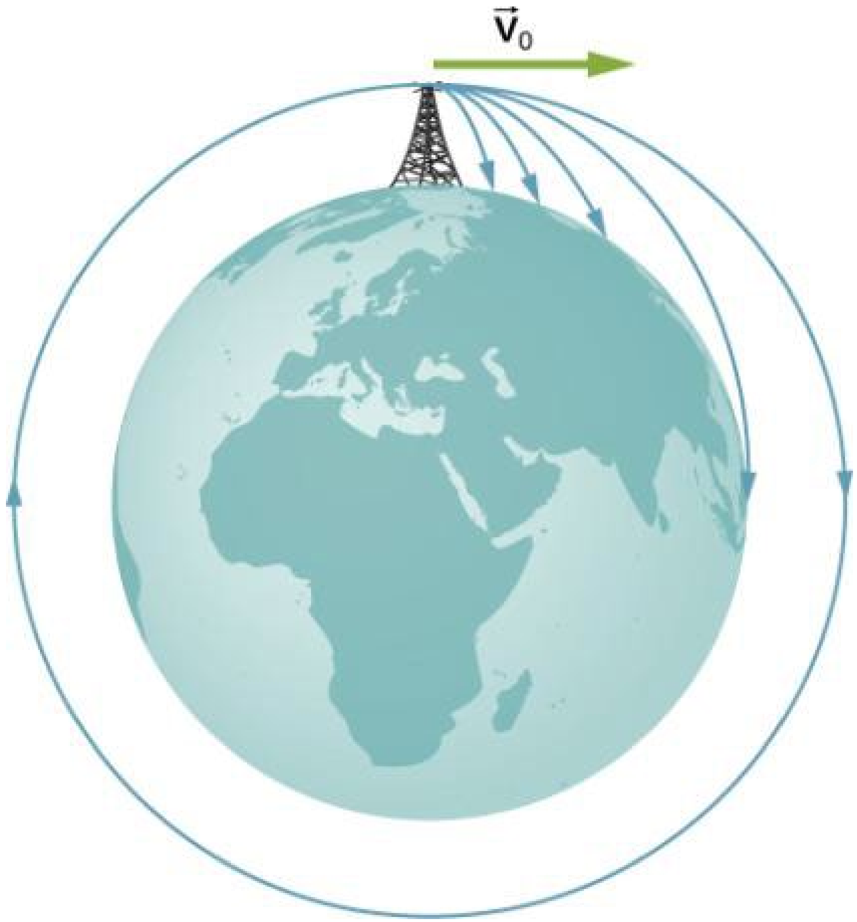


FIGURE 4.17



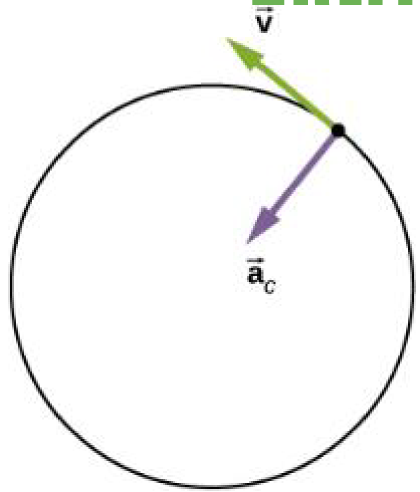
Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because Earth curves away beneath its path. With a speed of 8000 m/s, orbit is achieved.

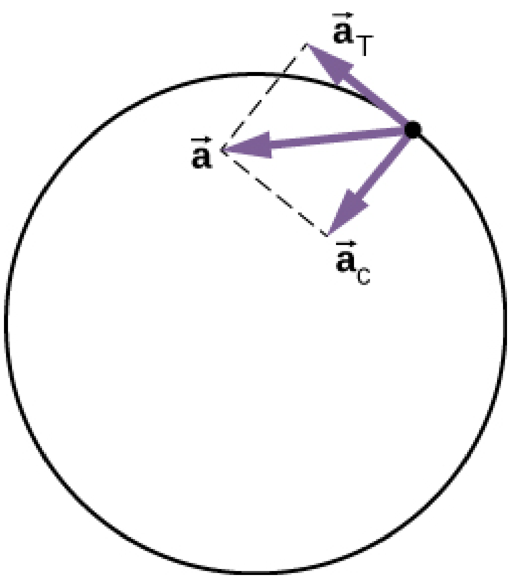
Note that all these trajectories are free falls, where gravity → 0 (later, equivalence principle).

Of special interest are the orbital trajectories. That's why you experience weightlessness.

FIGURE 4.19

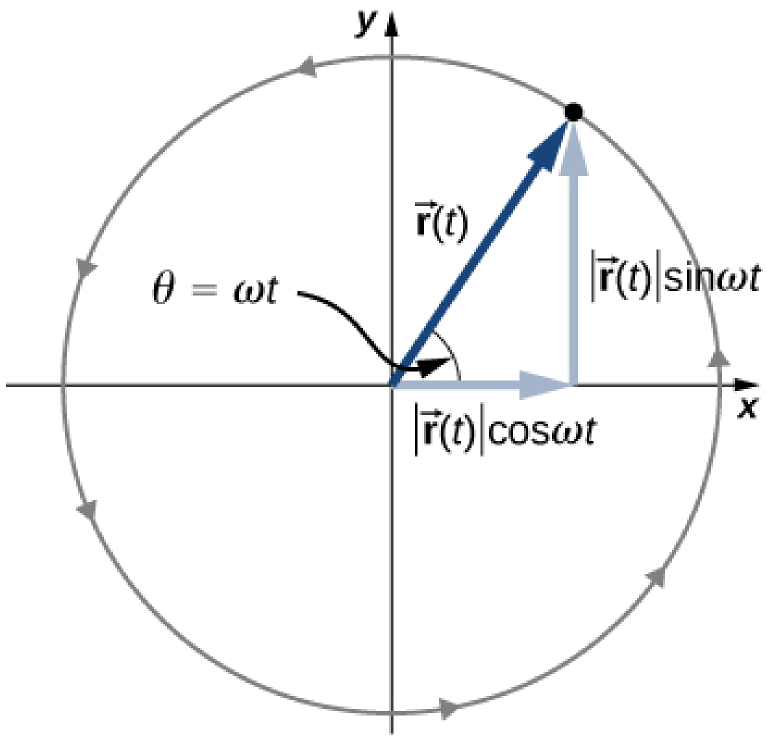
The centripetal acceleration vector points toward the center of the circular path of motion and is an acceleration in the radial direction. The velocity vector is also shown and is tangent to the circle.





The centripetal acceleration points toward the center of the circle. The tangential acceleration is tangential to the circle at the particle's position. The total acceleration is the vector sum of the tangential and centripetal accelerations, which are perpendicular.

FIGURE 4.20



The position vector for a particle in circular motion with its components along the *x*- and *y*-axes. The particle moves counterclockwise. Angle θ is the angular frequency ω in radians per second multiplied by *t*.

EXAMPLE 4.12

Total Acceleration during Circular Motion

A particle moves in a circle of radius $r = 2.0$ m. During the time interval from $t = 1.5$ s to $t = 4.0$ s its speed varies with time according to

$$v(t) = c_1 - \frac{c_2}{t^2}, \quad c_1 = 4.0\,\text{m/s}, \quad c_2 = 6.0\,\text{m}\cdot\text{s}.$$

What is the total acceleration of the particle at $t = 2.0$ s?

$$v(2.0\text{s}) = \left(4.0 - \frac{6.0}{(2.0)^2}\right)\text{m/s} = 2.5\,\text{m/s}$$

$$a_c = \frac{v^2}{r} = \frac{(2.5\,\text{m/s})^2}{2.0\,\text{m}} = 3.1\,\text{m/s}^2$$

circle. Tangential acceleration is

$$a_T = \left|\frac{d\vec{\mathbf{v}}}{dt}\right| = \frac{2c_2}{t^3} = \frac{12.0}{(2.0)^3}\text{m/s}^2 = 1.5\,\text{m/s}^2.$$

and $\theta = \tan^{-1}\frac{3.1}{1.5} = 64°$ from the tangent to the circle.

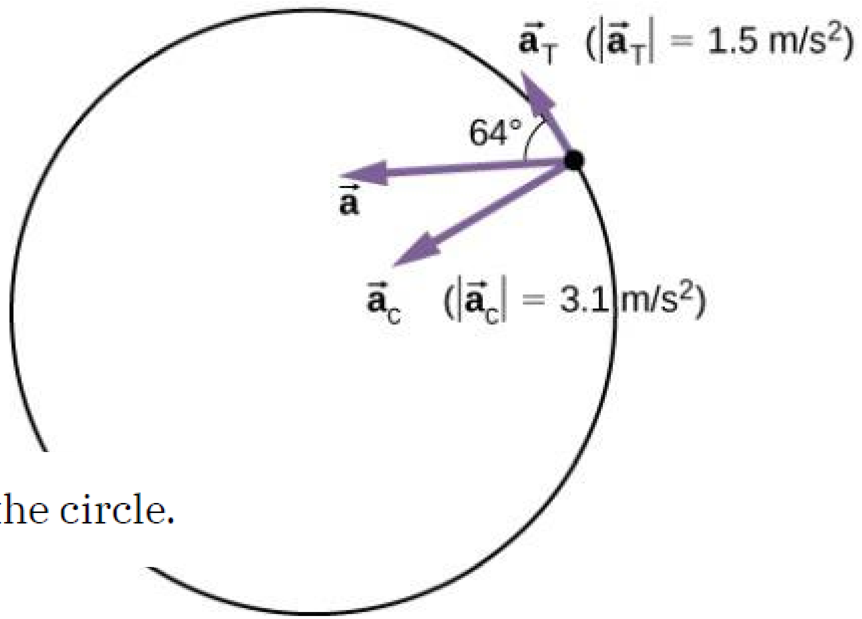
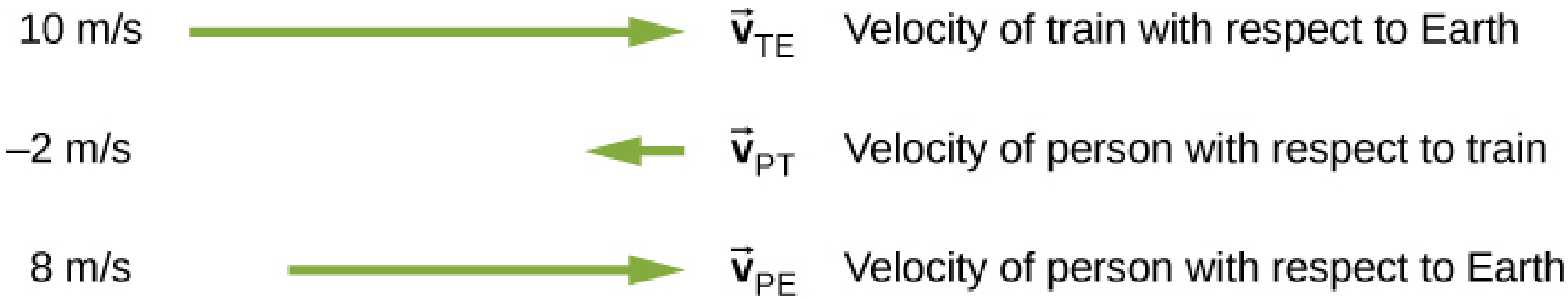


FIGURE 4.25 RELATIVE VELOCITY

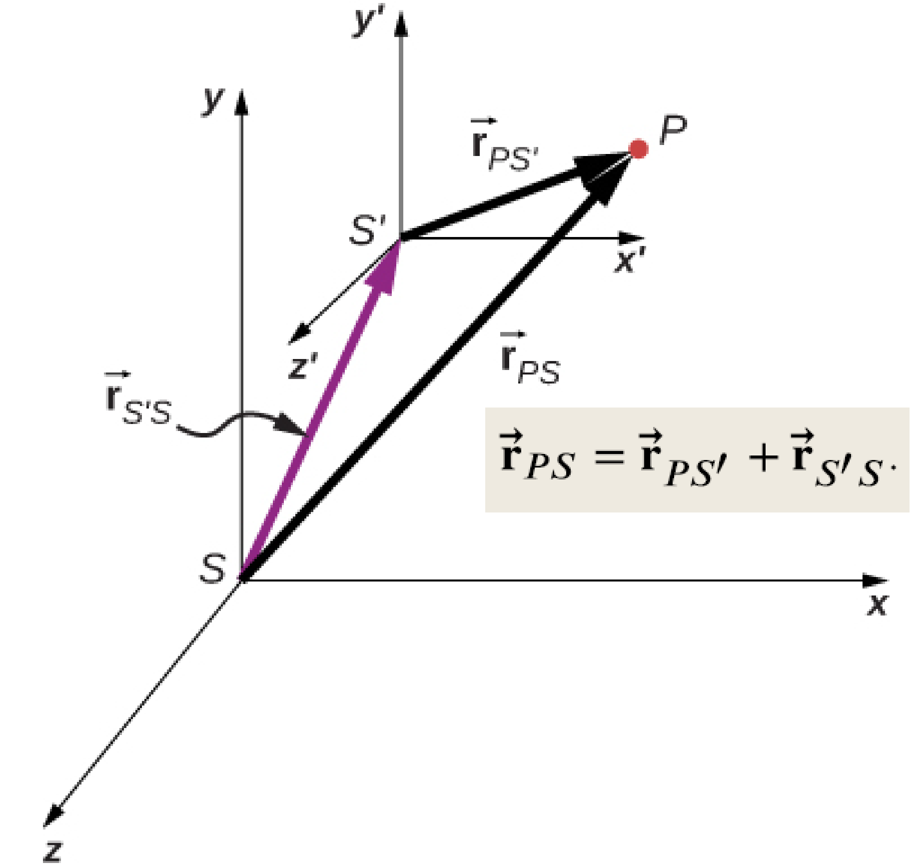


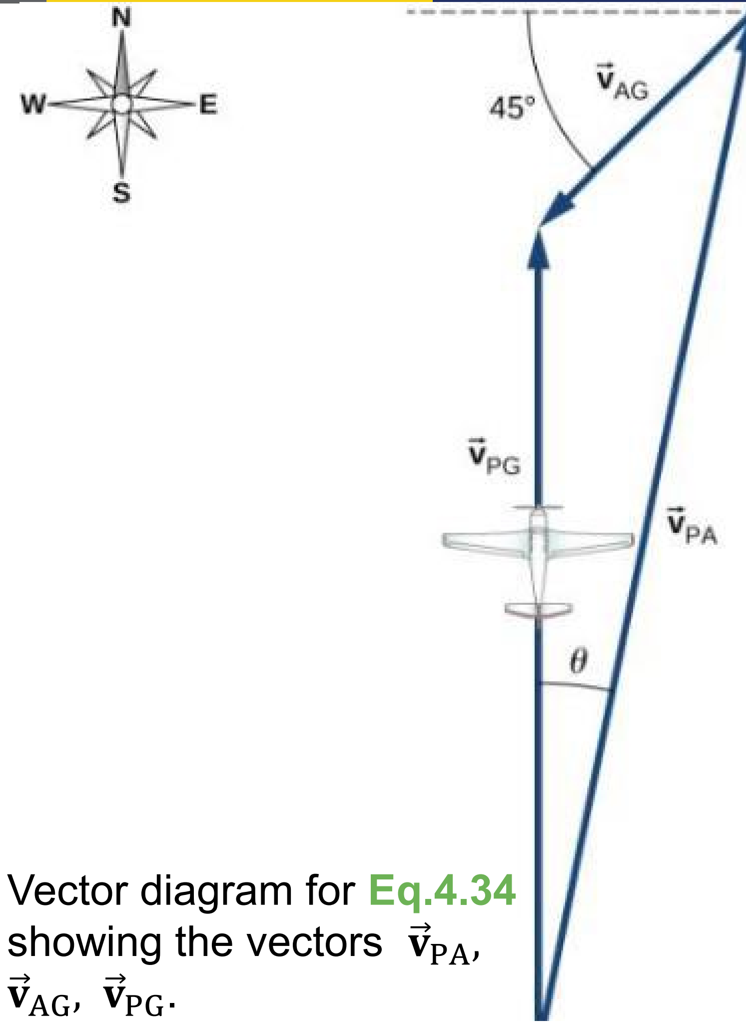
Velocity vectors of the train with respect to Earth, person with respect to the train, and person with respect to Earth.

$$\vec{\mathbf{v}}_{PE} = \vec{\mathbf{v}}_{PT} + \vec{\mathbf{v}}_{TE}$$

When constructing the vector equation, the subscripts for the coupling reference frame appear consecutively on the inside. The subscripts on the left-hand side of the equation are the same as the two outside subscripts on the right-hand side of the equation.

The positions of particle *P* relative to frames *S* and *S′* are $\vec{\mathbf{r}}_{PS}$ and $\vec{\mathbf{r}}_{PS'}$, respectively.





Vector diagram for Eq.4.34 showing the vectors $\vec{\mathbf{v}}_{\mathrm{PA}}$, $\vec{\mathbf{v}}_{\mathrm{AG}}$, $\vec{\mathbf{v}}_{\mathrm{PG}}$.

EXAMPLE 4.14

Flying a Plane in a Wind

A pilot must fly his plane due north to reach his destination. The plane can fly at 300 km/h in still air. A wind is blowing out of the northeast at 90 km/h. (a) What is the speed of the plane relative to the ground? (b) In what direction must the pilot head her plane to fly due north?