Math 340 Homework 4

Dr. Ebrahimian

Due 9/27/2024 before the class starts

- You are expected to solve all of the following problems, but only problems under "Problems for Grading" must be submitted for grading. You will have a quiz on Friday 9/27/2024 based on these problems. **Late submission will not be accepted.**
- If you are not typing your work (which is fine) please make sure your work is legible.
- Prove **all** of your answers.

Problems for Grading

Instructions for submission: Same as before!

1. (10 pts) Determine if each of the following is a linear transformation. If it is linear, provide a proof using the definition of linear mappings. If it is not, by an example prove that it fails to satisfy one of the conditions of linear mappings.

   (a) $L : \mathbb{R}^2 \to \mathbb{R}^3$, $L(x, y) = (x + 2y, y, -x)$.

   (b) $L : \mathbb{R}^3 \to \mathbb{R}^2$, $L(x, y, z) = (x + y, z - 1)$.

2. (10 pts) Find all linear transformations $T : \mathbb{R}^3 \to \mathbb{R}^2$ satisfying all of the following:

$$T(1, 2, 0) = (0, 2), T(-1, 1, 1) = (-2, 3), \text{ and } T(1, -2, -1) = (1, -3).$$

3. (10 pts) Let $\alpha \in [0, 2\pi)$ be an angle. Consider the transformation $T_\alpha : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates every point around the z-axis with angle α. Assume we know T_α is linear. Find M_{T_α}.

4. (15 pts) True or false? If true provide a proof, and if false provide a counter-example.

   (a) If for a square matrix A we have $A^2 = 0$, then $A = 0$.

   (b) If the two products AB and BA are defined, then A and B must be square matrices.

   (c) $AB = BA$ for every two 2×2 matrices A and B

5. (10 pts) Suppose $T : V \to W$ is a linear transformation between vector spaces. Using induction, prove that for every $c_1, \ldots, c_n \in \mathbb{R}$ and every $\mathbf{v}_1, \ldots, \mathbf{v}_n \in V$, we have

$$T(c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n) = c_1T(\mathbf{v}_1) + \cdots + c_nT(\mathbf{v}_n).$$

6. (10 pts) Suppose $L : V \to W$ is a bijective linear transformation. Prove that $L^{-1} : W \to V$ is linear.

Practice Problems

The following examples and exercises are from the "Honors Linear Algebra and Multivariable Calculus" PDF file posted on ELMS under "Files".

7. Example 4.12.
8. Example 4.13.
9. Example 4.16.
10. Example 4.17.
11. Example 4.20.
12. Exercise 4.9.
13. Exercise 4.14.

Challenge Problem

Exercises 4.23, 4.24, and 4.27.