EXAMPLE 3.3

Instantaneous Velocity Versus Average Velocity

The position of a particle is given by $x(t) = 3.0t + 0.5t^3$ m.

a. Using Equation 3.4 and Equation 3.7, find the instantaneous velocity at $t = 2.0$ s.
b. Calculate the average velocity between 1.0 s and 3.0 s.

Strategy

Equation 3.4 gives the instantaneous velocity of the particle as the derivative of the position function. Looking at the form of the position function given, we see that it is a polynomial in t. Therefore, we can use Equation 3.7, the power rule from calculus, to find the solution. We use Equation 3.6 to calculate the average velocity of the particle.

Solution

a. $v(t) = \frac{dx(t)}{dt} = 3.0 + 1.5t^2$ m/s.
   Substituting $t = 2.0$ s into this equation gives $v(2.0\text{ s}) = [3.0 + 1.5(2.0)^2]$ m/s $= 9.0$ m/s.
b. To determine the average velocity of the particle between 1.0 s and 3.0 s, we calculate the values of $x(1.0\text{ s})$ and $x(3.0\text{ s})$:

$$x(1.0\text{ s}) = \left[(3.0)(1.0) + 0.5(1.0)^3\right]\text{ m} = 3.5\text{ m}$$

$$x(3.0\text{ s}) = \left[(3.0)(3.0) + 0.5(3.0)^3\right]\text{ m} = 22.5\text{ m}.$$

Then the average velocity is

$$\bar{v} = \frac{x(3.0\text{ s}) - x(1.0\text{ s})}{t(3.0\text{ s}) - t(1.0\text{ s})} = \frac{22.5 - 3.5\text{ m}}{3.0 - 1.0\text{ s}} = 9.5\text{ m/s}.$$

Significance

In the limit that the time interval used to calculate $\bar{v}$ goes to zero, the value obtained for $\bar{v}$ converges to the value of v.

EXAMPLE 3.4

Instantaneous Velocity Versus Speed

Consider the motion of a particle in which the position is $x(t) = 3.0t - 3t^2$ m.

a. What is the instantaneous velocity at $t = 0.25$ s, $t = 0.50$ s, and $t = 1.0$ s?
b. What is the speed of the particle at these times?

Solution

a. $v(t) = \frac{dx(t)}{dt} = 3.0 - 6.0t \text{ m/s}\ v(0.25 \text{ s}) = 1.50 \text{ m/s},\ \ v(0.5 \text{ s}) = 0 \text{ m/s},\ \ v(1.0 \text{ s}) = -3.0 \text{ m/s}$
b. $\text{Speed} = |v(t)| = 1.50 \text{ m/s}, 0.0 \text{ m/s}, \text{and } 3.0 \text{ m/s}$

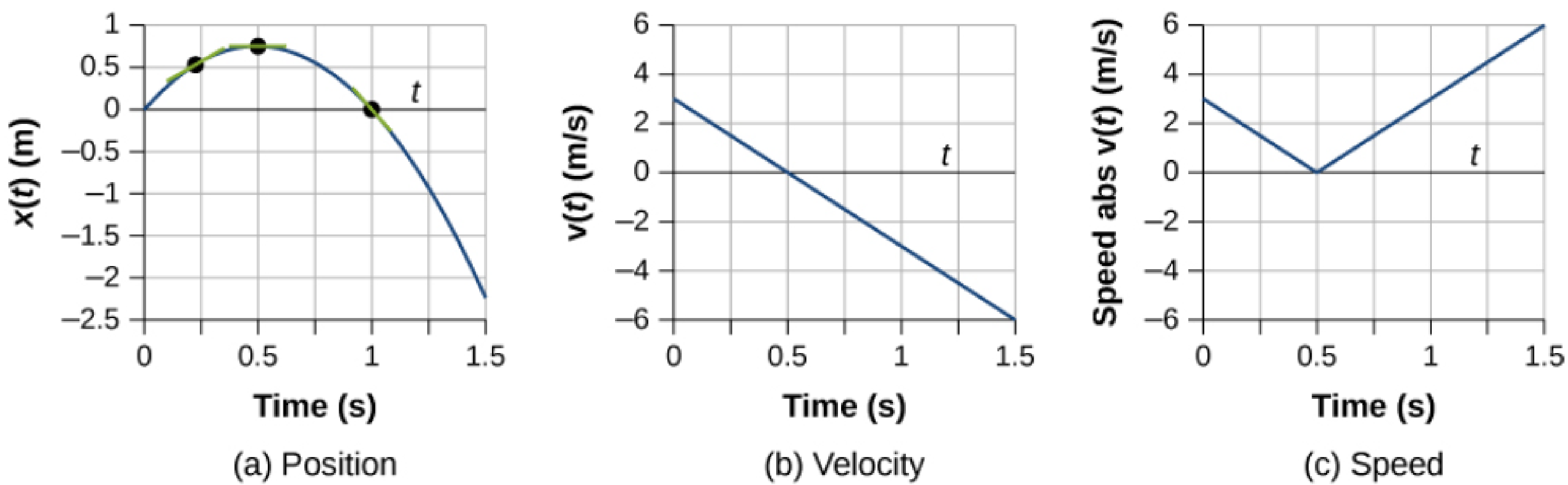


Figure 3.9 (a) Position: $x(t)$ versus time. (b) Velocity: $v(t)$ versus time. The slope of the position graph is the velocity. A rough comparison of the slopes of the tangent lines in (a) at 0.25 s, 0.5 s, and 1.0 s with the values for velocity at the corresponding times indicates they are the same values. (c) Speed: $|v(t)|$ versus time. Speed is always a positive number.

$v(t) = 20\,t - 5\,t^2$ m/s
$a(t) = dv(t)/dt = 20 - 10\,t$ m/s^2

EXAMPLE 3.6

Calculating Instantaneous Acceleration

A particle is in motion and is accelerating. The functional form of the velocity is $v(t) = 20t - 5t^2$ m/s.

a. Find the functional form of the acceleration.
b. Find the instantaneous velocity at $t = 1, 2, 3,$ and 5 s.
c. Find the instantaneous acceleration at $t = 1, 2, 3,$ and 5 s.
d. Interpret the results of (c) in terms of the directions of the acceleration and velocity vectors.

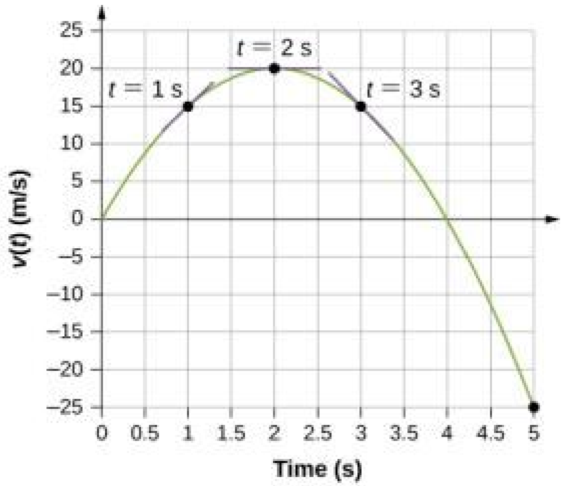
Strategy

We find the functional form of acceleration by taking the derivative of the velocity function. Then, we calculate the values of instantaneous velocity and acceleration from the given functions for each. For part (d), we need to compare the directions of velocity and acceleration at each time.

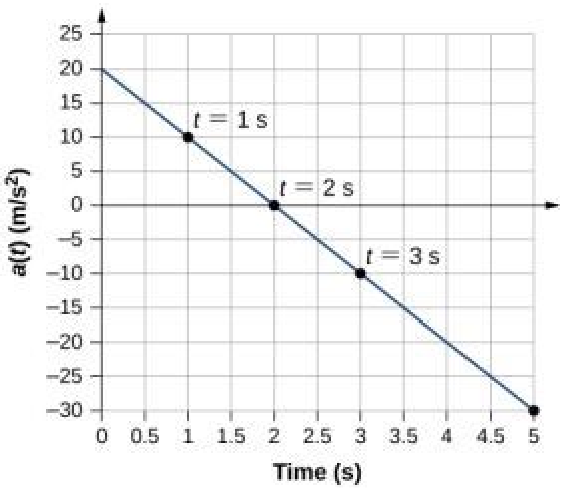
Solution

a. $a(t) = \frac{dv(t)}{dt} = 20 - 10t \text{ m/s}^2$
b. $v(1\text{ s}) = 15 \text{ m/s}, v(2\text{ s}) = 20 \text{ m/s}, v(3\text{ s}) = 15 \text{ m/s}, v(5\text{ s}) = -25 \text{ m/s}$
c. $a(1\text{ s}) = 10 \text{ m/s}^2, a(2\text{ s}) = 0 \text{ m/s}^2, a(3\text{ s}) = -10 \text{ m/s}^2, a(5\text{ s}) = -30 \text{ m/s}^2$
d. At $t = 1$ s, velocity $v(1\text{ s}) = 15$ m/s is positive and acceleration is positive, so both velocity and acceleration are in the same direction. The particle is moving faster.

FIGURE 3.16



(a) Velocity

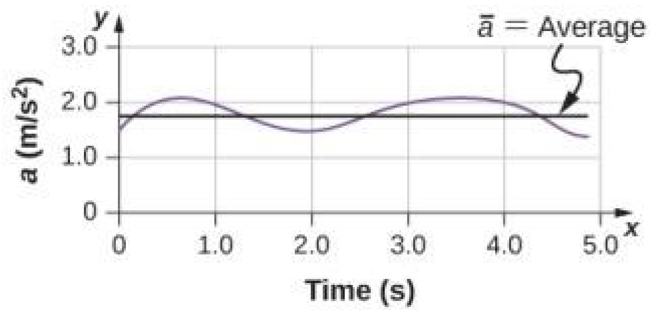


(b) Acceleration

(a) Velocity versus time. Tangent lines are indicated at times 1, 2, and 3 s. The slopes of the tangents lines are the accelerations. At *t* = 3 s, velocity is positive. At *t* = 5 s, velocity is negative, indicating the particle has reversed direction.

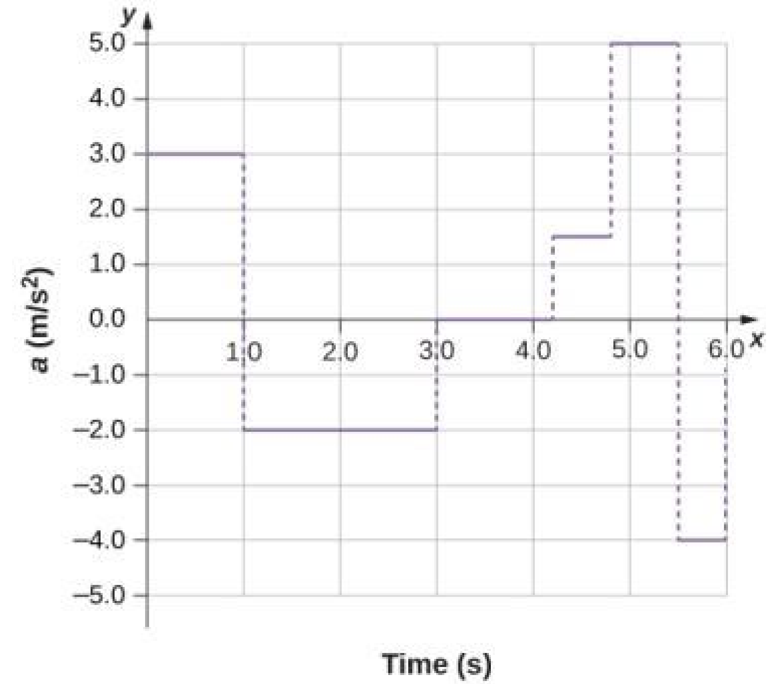
(b) Acceleration versus time. Comparing the values of accelerations given by the black dots with the corresponding slopes of the tangent lines (slopes of lines through black dots) in (a), we see they are identical.

FIGURE 3.17



Instantaneous velocity and acceleration
More basic. Can always calculate average v and a for a given time interval

(a)



(b)

Graphs of instantaneous acceleration versus time for two different one-dimensional motions.

(a) Acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time.

(b) Acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0–1.0 s) with constant or nearly constant acceleration in such a situation.

Calculate displacement from velocity, **Integration!**

$v(t) = dx/dt$ => $x(t) = \int v(t)dt$ (integrate from initial time to final time) + x_0

= ave v in the time interval + x_0 $x = x_0 + \bar{v}t,$

velocity from acceleration:

$a(t) = dv/dt$ => $v(t) = \int a(t)dt$ (integrate from initial time to final time) + v_0

= ave a in the time interval + v_0

Assume constant acceleration: [blackboard derivation]

[See Sec. 3.6 Finding Velocity and Displacement from Acceleration:]

$v(t) = v_0 + a\,t$ Eq (1)

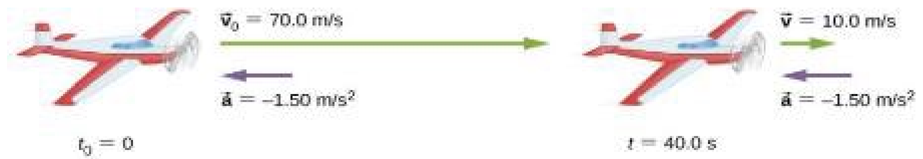
$x(t) = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2$ Eq (2)

Eliminating t we get an equation for x, v and a:

$v^2 = v_0^2 + 2a(x - x_0)$ (constant a). Eq (3)

The airplane lands with an initial velocity of 70.0 m/s and then decelerates with 1.50 m/s^2 for 40.0 s

Calculate its final velocity



EXAMPLE 3.7

Calculating Final Velocity

An airplane lands with an initial velocity of 70.0 m/s and then accelerates opposite to the motion at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

First, we identify the knowns: $v_0 = 70$ m/s, $a = -1.50$ m/s^2, $t = 40$ s.

Second, we identify the unknown; in this case, it is final velocity v_f.

Last, we determine which equation to use. To do this we figure out which kinematic equation gives the unknown in terms of the knowns. We calculate the final velocity using Equation 3.12, $v = v_0 + at$.

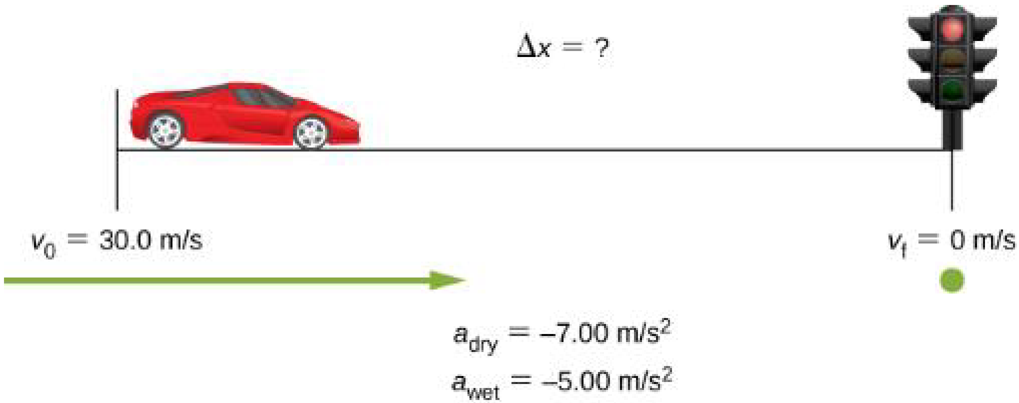
Solution

Substitute the known values and solve:

$$v = v_0 + at = 70.0 \text{ m/s} + \left(-1.50 \text{ m/s}^2\right)(40.0 \text{ s}) = 10.0 \text{ m/s}.$$

Figure 3.19 is a sketch that shows the acceleration and velocity vectors.

FIGURE 3.22: BRAKING DISTANCE



$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

$$x - 0 = \frac{0^2 - (30.0\ \text{m/s})^2}{2(-7.00\text{m/s}^2)}.$$

This problem does not involve time
=> use Eq. (3)

$x = 64.3$ m on dry concrete.

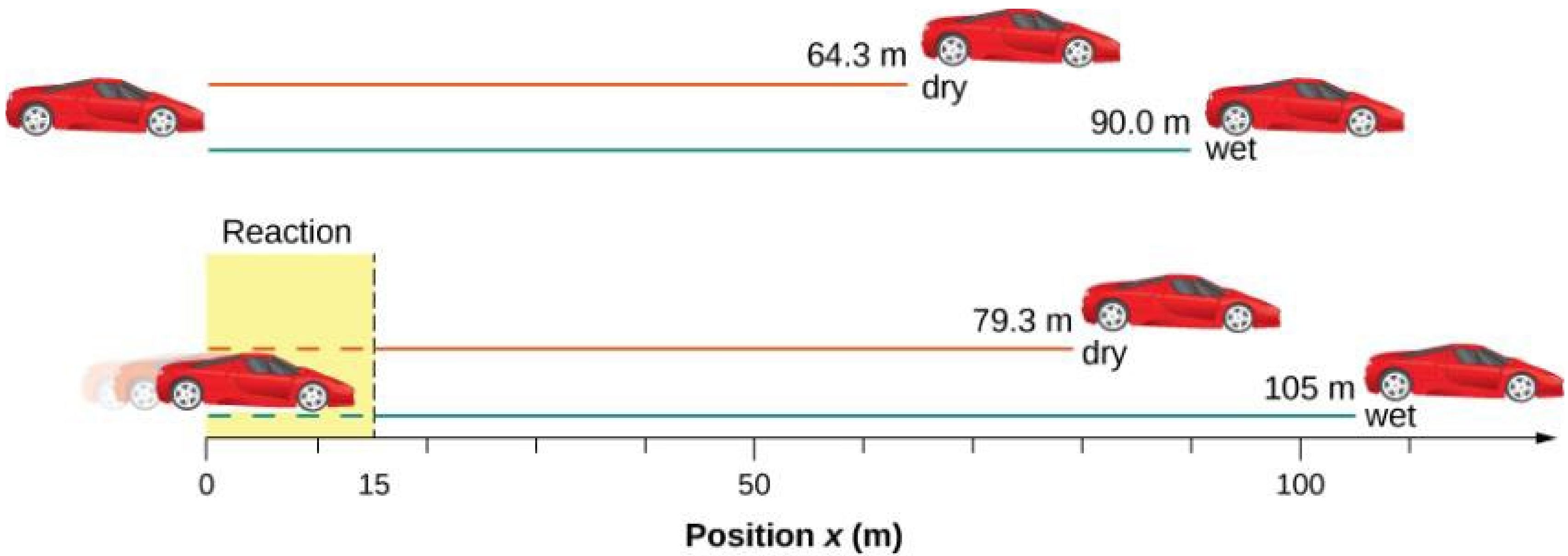
same manner as (a). The only d

$x_{\text{wet}} = 90.0$ m on wet concrete

(c) Repeat both calculations and find the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

During that reaction time, the car has travelled 30m/s x 0.5 s = 15m. Thus should add this distance to the above to get the actual braking distance.

FIGURE 3.23



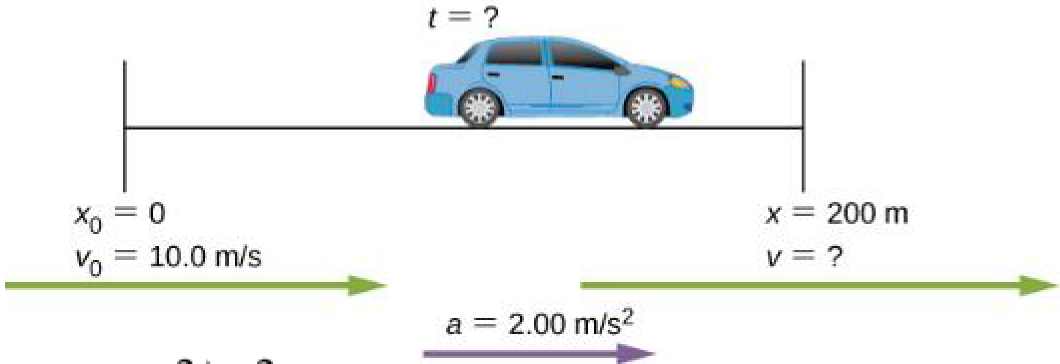
The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car traveling initially at 30.0 m/s. Also shown are the total distances traveled from the point when the driver first sees a light turn red, assuming a 0.500-s reaction time.

FIGURE 3.24 Car accelerating on a freeway ramp.

EXAMPLE 3.11

Calculating Time

Suppose a car merges into freeway traffic on a 200-m-long ramp. If its initial velocity is 10.0 m/s and it accelerates at 2.00 m/s^2, how long does it take the car to travel the 200 m up the ramp? (Such information might be useful to a traffic engineer.)



Since time is asked, and both velocity and acceleration are involved, we should use Eq (2)

$x(t) = x_0 + v_0\, t + \frac{1}{2}\, a\, t^2$

$$200\ \text{m} = 0\ \text{m} + (10.0\ \text{m/s})t + \frac{1}{2}\left(2.00\ \text{m/s}^2\right) t^2.$$

t = 10.0 sec

EXAMPLE 3.12

Acceleration of a Spaceship

A spaceship has left Earth's orbit and is on its way to the Moon. It accelerates at 20 m/s^2 for 2 min and covers a distance of 1000 km. What are the initial and final velocities of the spaceship?

Since time t is involved so we need to use Eq (1) and/or (2)

$v(t) = v_0 + a\,t$ Eq (1)

$x(t) = x_0 + v_0\,t + ½\,a\,t^2$ Eq (2)

Our targets are v_0 an

So use (2) to get v_0

$$x - x_0 = v_0 t + \frac{1}{2}at^2$$

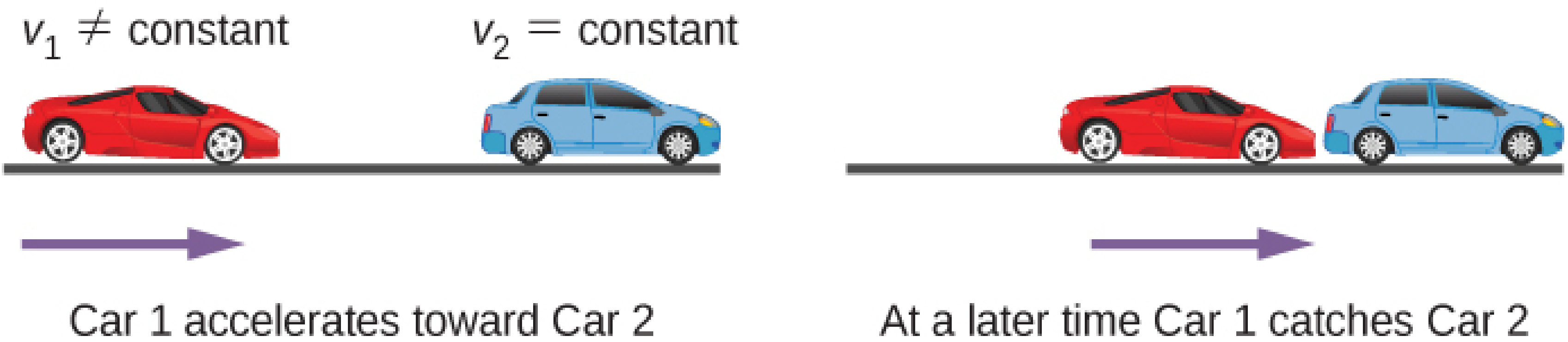
$$1.0 \times 10^6\ \mathrm{m} = v_0(120.0\ \mathrm{s}) + \frac{1}{2}(20.0\ \mathrm{m/s^2})(120.0\ \mathrm{s})^2$$

$$v_0 = 7133.3\ \mathrm{m/s}.$$

a

$$v = v_0 + at = 7133.3\ \mathrm{m/s} + (20.0\ \mathrm{m/s^2})(120.0\ \mathrm{s}) = 9533.3\ \mathrm{m/s}.$$

FIGURE 3.25 A two-body pursuit scenario where car 2 has a constant velocity and car 1 is behind with a constant acceleration.



EXAMPLE 3.13

Cheetah Catching a Gazelle

A cheetah waits in hiding behind a bush. The cheetah spots a gazelle running past at 10 m/s. At the instant the gazelle passes the cheetah, the cheetah accelerates from rest at 4 m/s^2 to catch the gazelle. (a) How long does it take the cheetah to catch the gazelle? (b) What is the displacement of the gazelle and cheetah?

G moves with constant speed: Eq.(0)

C moves with constant acceleration: Eq (2)

$$x = x_0 + \bar{v}t,$$

$$x(t) = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$x_0=0$ and $v_0=0$, we are given v and a.
Need to calculate the time t
when these two displacements are equal

$$x = \bar{v}t = \frac{1}{2}at^2$$

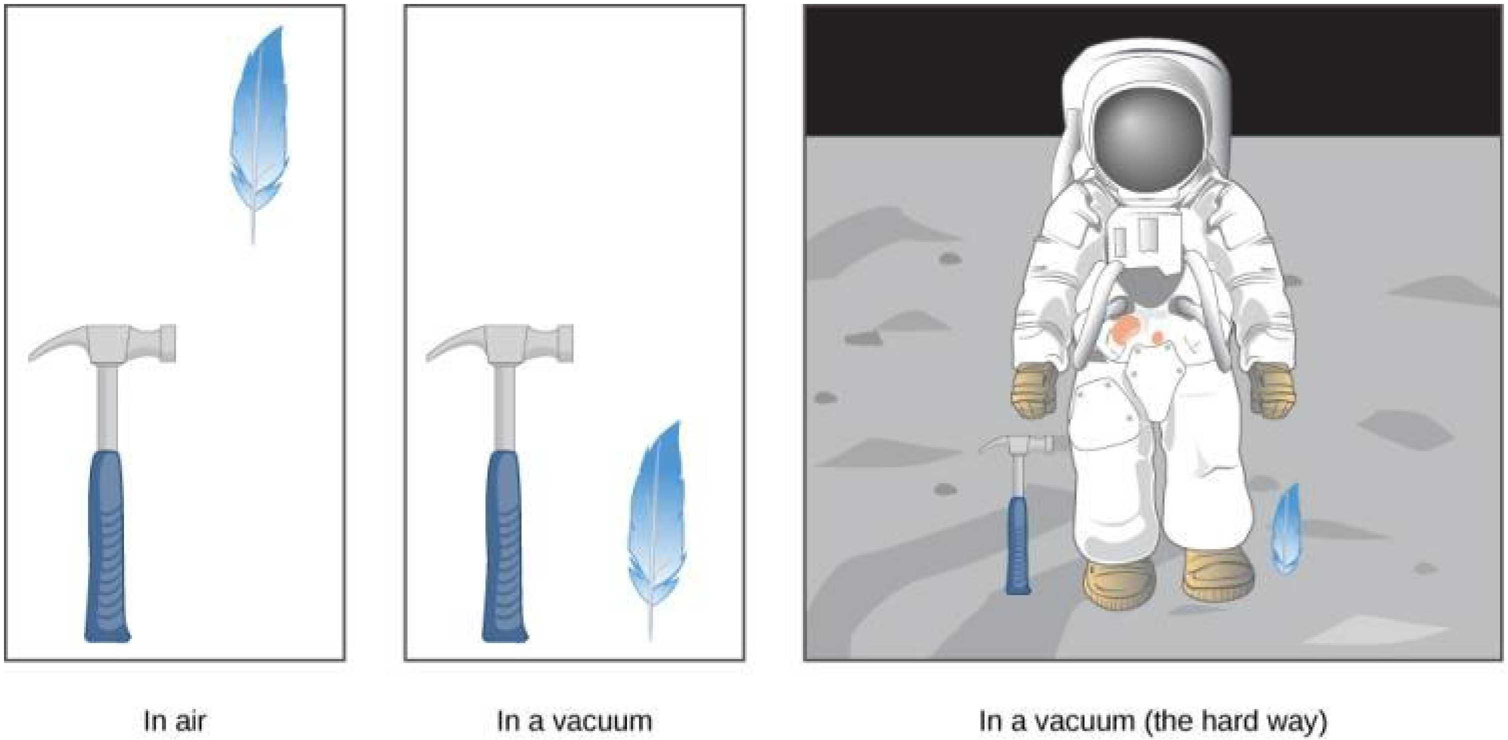
$$t = \frac{2\bar{v}}{a}. \qquad x = \frac{1}{2}at^2 = \frac{1}{2}(4\text{m/s}^2)(5)^2 = 50\text{ m}.$$

$$x = \bar{v}t = 10\text{ m/s}(5) = 50\text{ m}.$$

GRAVITATIONAL ACCELERATION

FIGURE 3.26

DEMO C4-32, 33



A hammer and a feather fall with the same constant acceleration if air resistance is negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated in 1971 on the Moon, where the acceleration from gravity is only 1.67 m/s^2 and there is no atmosphere.

Use the same set of equations, but with a= -g

gravitational acceleration   g = GM/R^2 = 9.81 m/s^2

G Newton Constant, M, R = Mass and Radius of the Earth

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

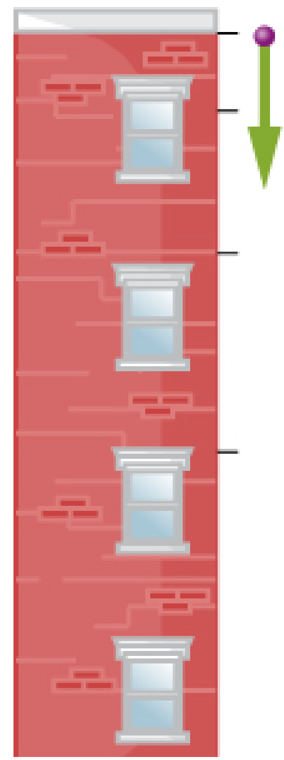
$$-98.0\ \text{m} = 0 - (4.9\ \text{m/s})t - \frac{1}{2}(9.8\ \text{m/s}^2)t^2.$$

$$t^2 + t - 20 = 0.$$

This is a quadratic equation with roots t =4.0s and -5.0s. The positive root is the one we are interested in, since time t=0 is the time when the ball is released at the top of the building.

(The time t =-5.0s represents the fact that a ball thrown upward from the ground would have been in the air for 5.0s when it passed by the top of the building moving downward at 4.9 m/s.)

Figure 3.27 **A** ball is thrown downward from the top of a 98-m-high building with initial speed v_0 = 4.9 m/s. Shown are the positions and velocities at 1-s intervals.



t (s)	x (m)	v (m/s)
0	0	–4.9
1	–9.8	–14.7
2	–29.4	–24.5
3	–58.8	–34.3
4	–98.0	–44.1

$$v = v_0 - gt = -4.9\ \text{m/s} - (9.8\ \text{m/s}^2)(4.0\ \text{s}) = -44.1\ \text{m/s}.$$

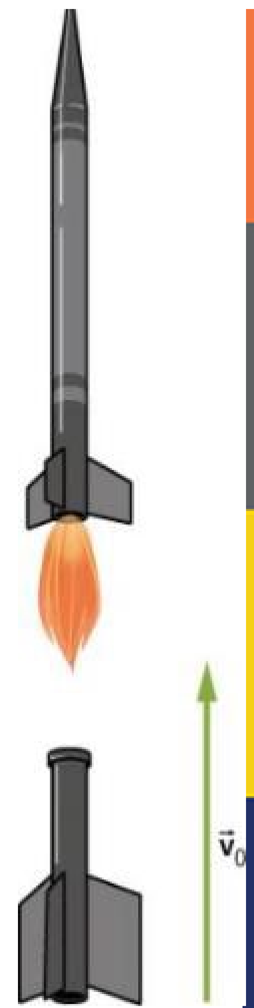
EXAMPLE 3.16

Rocket Booster

A small rocket with a booster blasts off and heads straight upward. When at a height of 5.0 km and velocity of 200.0 m/s, it releases its booster. (a) What is the maximum height the booster attains? (b) What is the velocity of the booster at a height of 6.0 km? Neglect air resistance.

We set the origin of our coordinate y_0 =0 at the point of release.

Fig 3.29



At max height v=0. Notice the problem didn't ask for time, thus Eq (3) would come in handy: a)

$v^2 = v_0^2 - 2g(y - y_0)$. With $v = 0$ and $y_0 = 0$, we can solve for y:

$$y = \frac{v_0^2}{2g} = \frac{(2.0 \times 10^2 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 2040.8 \text{ m}.$$

Add to this 5.0km, the height where the booster was released, gives max height.

b) What is the velocity of the booster at a height of 6.0 km?

This corresponds to 1km in our coordinate system.

With y_0 = 0 and v_0 = 200.0 m/s, we have

$$v^2 = (200.0 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(1.0 \times 10^3 \text{ m}) \Rightarrow v = \pm 142.8 \text{ m/s}.$$

Two more problems worked out on board.