Math 340 Homework 1

Dr. Ebrahimian

Due 9/6/2024 before the class starts

- You are expected to solve all 16 of the following problems, but only the first 8 must be submitted for grading. You will have a quiz on Friday 9/6 based on these problems. **Late submission will not be accepted.**
- It is highly recommended (but not required) that you LaTeX your homework.
- If you are not typing your work (which is fine) please make sure your work is legible.
- Justify all of your solutions.
- Your are encouraged to work with your classmates, but your submission must be written by yourself in your own words.

Problems for Grading

Instructions for submission:

- To submit your homework, go to ELMS. Hit "Gradescope" on the left panel. That should allow you to upload a PDF file of your homework.
- Each problem must go on a separate page. **Make sure you assign each page to the appropriate problem number.**
- Please make sure your work is legible. You could use the (free) DocScan app to scan and upload your homework.
- Sometime in the next day or two run a test and make sure this all works out so you do not face any issues right before the deadline.
- Homework must be submitted before the class starts on the due date. GradeScope will not allow late submissions.
- You can read more about submitting homework on Gradescope here.
- **All proofs must be complete and solutions must be fully justified.**

- Read and follow the directions carefully. If a problem is asking you to use a certain method, you must use that method to solve the problem.

1. (10 pts) Let X be a nonempty set with n elements. How many one-to-one functions $f : X \to X$ are there?

2. (10 pts) Prove that

$$\bigcup_{x\in[0,1]} ([x,1] \times [0,x^2]) = \{(x,y) \mid 0 \le x \le 1 \text{ and } 0 \le y \le x^2\}$$

3. (10 pts) For n sets $A_1, A_2, \ldots, A_n$, prove that $A_1 \times A_2 \times \cdots \times A_n = \emptyset$ if and only if $A_i = \emptyset$ for some i.

   Hint: Proof by contradiction might be useful.

4. (10 pts) Prove parts (b) and (c) of Theorem 1.2: Suppose $f : A \to B$ is a function, and $T_i \subseteq B$ for $i \in I$. Then

   (a) $f^{-1}(\bigcup_{i\in I} T_i) = \bigcup_{i\in I} f^{-1}(T_i)$.

   (b) $f^{-1}(\bigcap_{i\in I} T_i) = \bigcap_{i\in I} f^{-1}(T_i)$.

5. (10 pts) The graph of a function $f : X \to Y$ is defined by $\Gamma_f = \{(x, f(x)) \mid x \in X\}$. Prove that two functions $f, g : X \to Y$ are equal if and only if $\Gamma_f = \Gamma_g$.

6. (10 pts) Let C be the unit circle $x^2 + y^2 = 1$ on the xy-plane. Describe the set $C \times [0,1]$ in $\mathbb{R}^3$.

7. (10 pts) Prove that if for a real number x, the number x^2 is irrational, then so is x.

8. (10 pts) Suppose functions $f, g : \mathbb{R} \to \mathbb{R}$ are n-times differentiable at some $x_0 \in \mathbb{R}$. Prove

$$(fg)^{(n)}(x_0) = \sum \binom{n}{k} f^{(k)}(x_0) g^{(n-k)}(x_0).$$

Practice Problems

The following examples and exercises are from the "Honors Linear Algebra and Multivariable Calculus" PDF file posted on ELMS under "Files".

9. Example 1.13.

10. Example 1.17.

11. Example 1.20.

12. Example 1.22.

13. Example 1.23.

14. Exercise 1.4.

15. Exercise 1.5.

16. Exercise 1.9.

Challenge Problems

Exercises 1.25 and 1.26.