FIGURE 2.5

Various relations between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

(a) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because $A \neq B$.

(b) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they are not parallel and $A \neq B$.

(c) $\vec{\mathbf{A}} \neq -\vec{\mathbf{A}}$ because they have different directions (even though $|\vec{\mathbf{A}}| = |-\vec{\mathbf{A}}| = A$).

(d) $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ because they are parallel and have identical magnitudes $A = B$.

(e) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they have different directions (are not parallel); here, their directions differ by 90°—meaning, they are orthogonal.

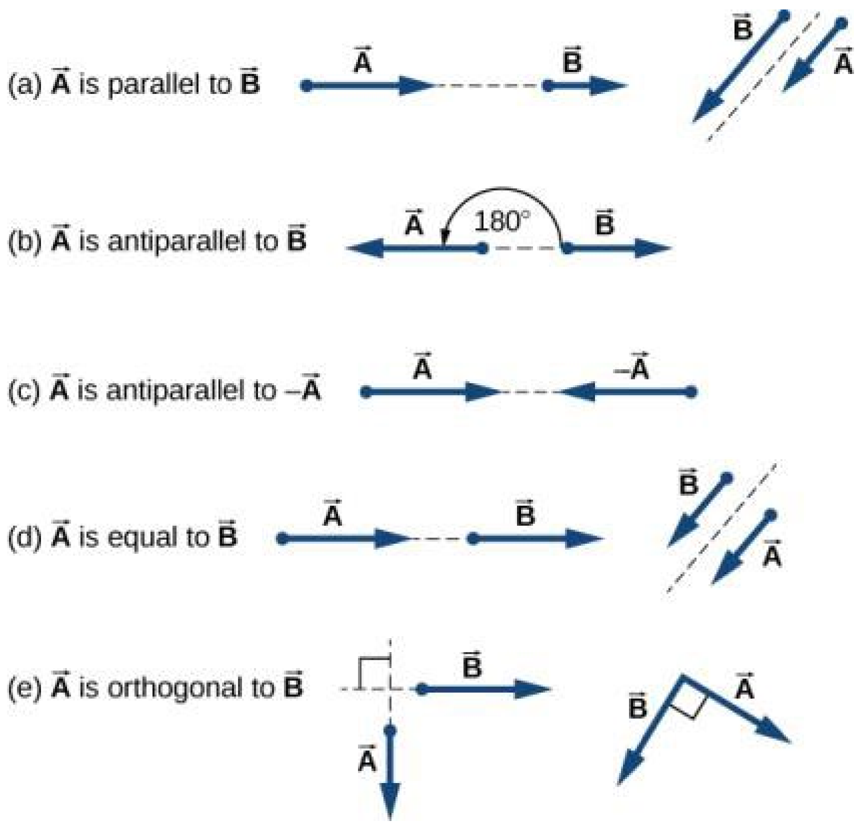
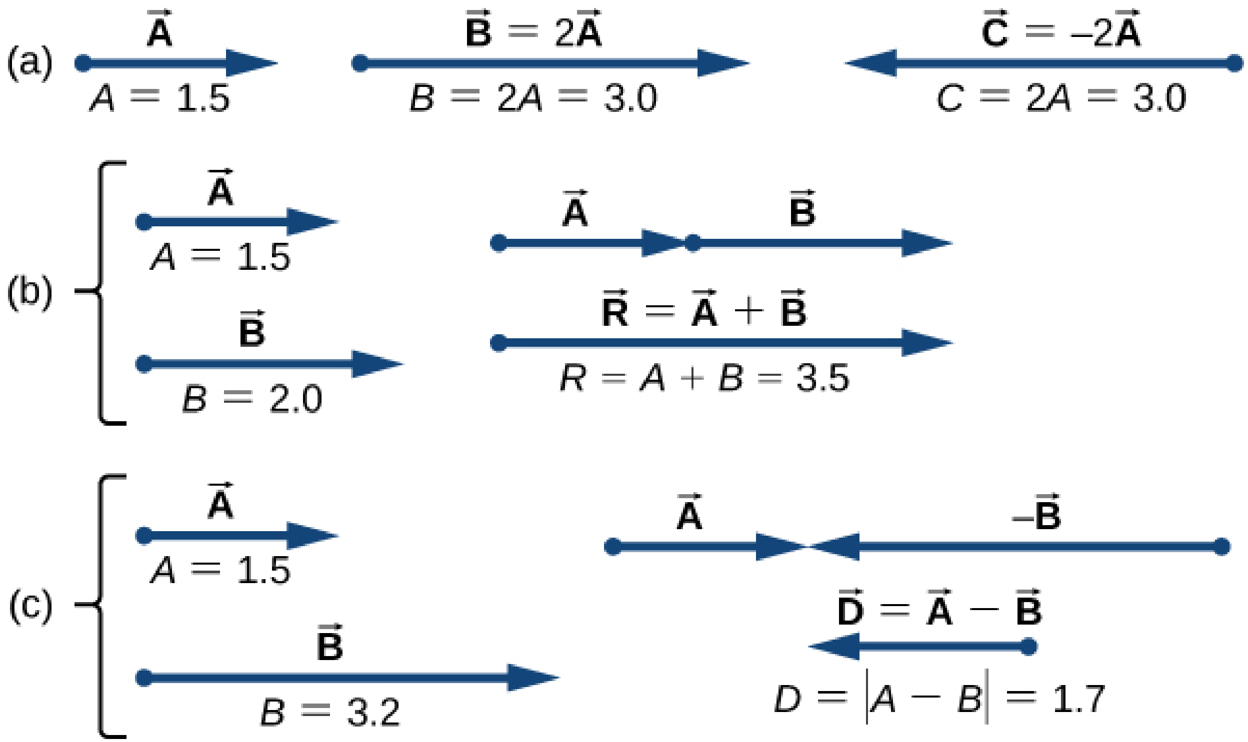


FIGURE 2.7



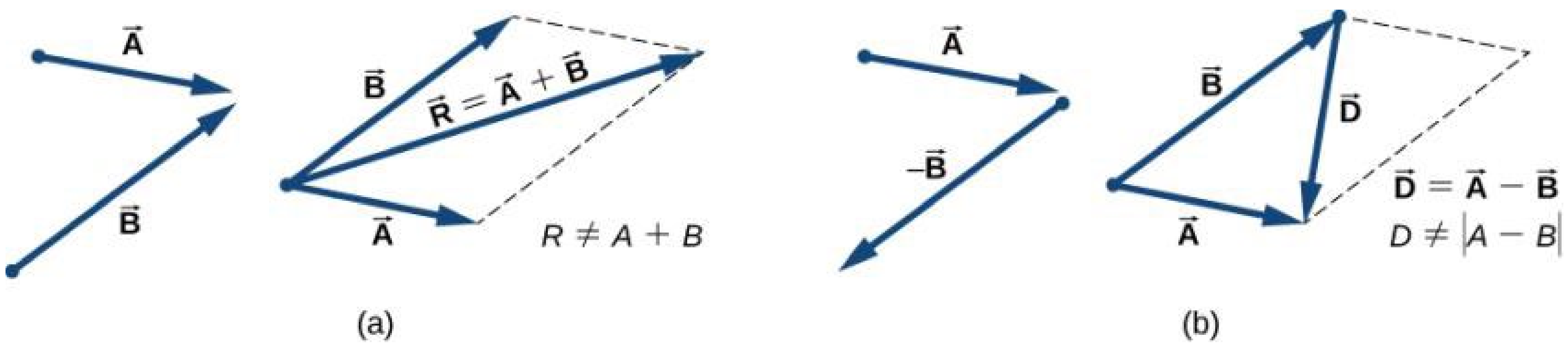
Algebra of vectors in one dimension.

(a) Multiplication by a scalar.

(b) Addition of two vectors ($\vec{\mathbf{R}}$ is called the *resultant* of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$).

(c) Subtraction of two vectors ($\vec{\mathbf{D}}$ is the difference of vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$).

FIGURE 2.10

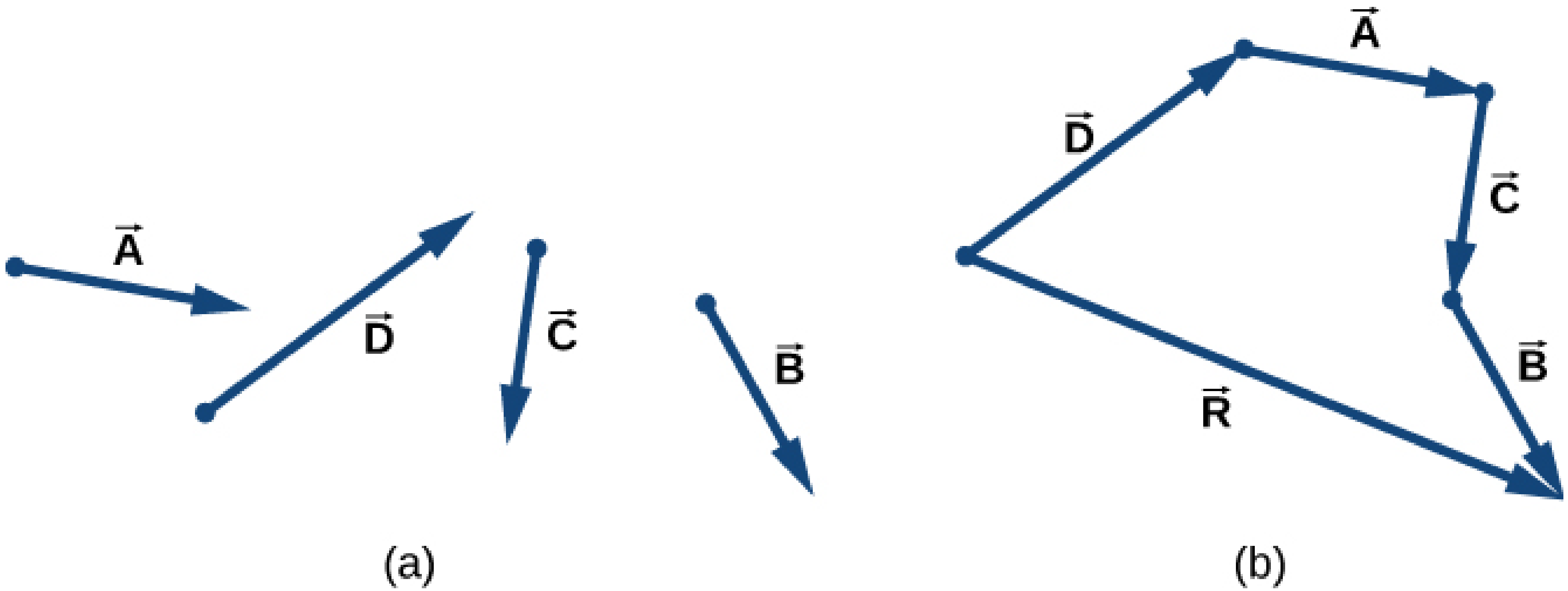


The parallelogram rule for the addition of two vectors. Make the parallel translation of each vector to a point where their origins (marked by the dot) coincide and construct a parallelogram with two sides on the vectors and the other two sides (indicated by dashed lines) parallel to the vectors.

(a) Draw the resultant vector $\vec{\mathbf{R}}$ along the diagonal of the parallelogram from the common point to the opposite corner. Length $\vec{\mathbf{R}}$ of the resultant vector is not equal to the sum of the magnitudes of the two vectors.

(b) Draw the difference vector $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ along the diagonal connecting the ends of the vectors. Place the origin of vector $\vec{\mathbf{D}}$ at the end of vector $\vec{\mathbf{B}}$ and the end (arrowhead) of vector $\vec{\mathbf{D}}$ at the end of vector $\vec{\mathbf{A}}$. Length D of the difference vector is not equal to the difference of magnitudes of the two vectors.

FIGURE 2.12

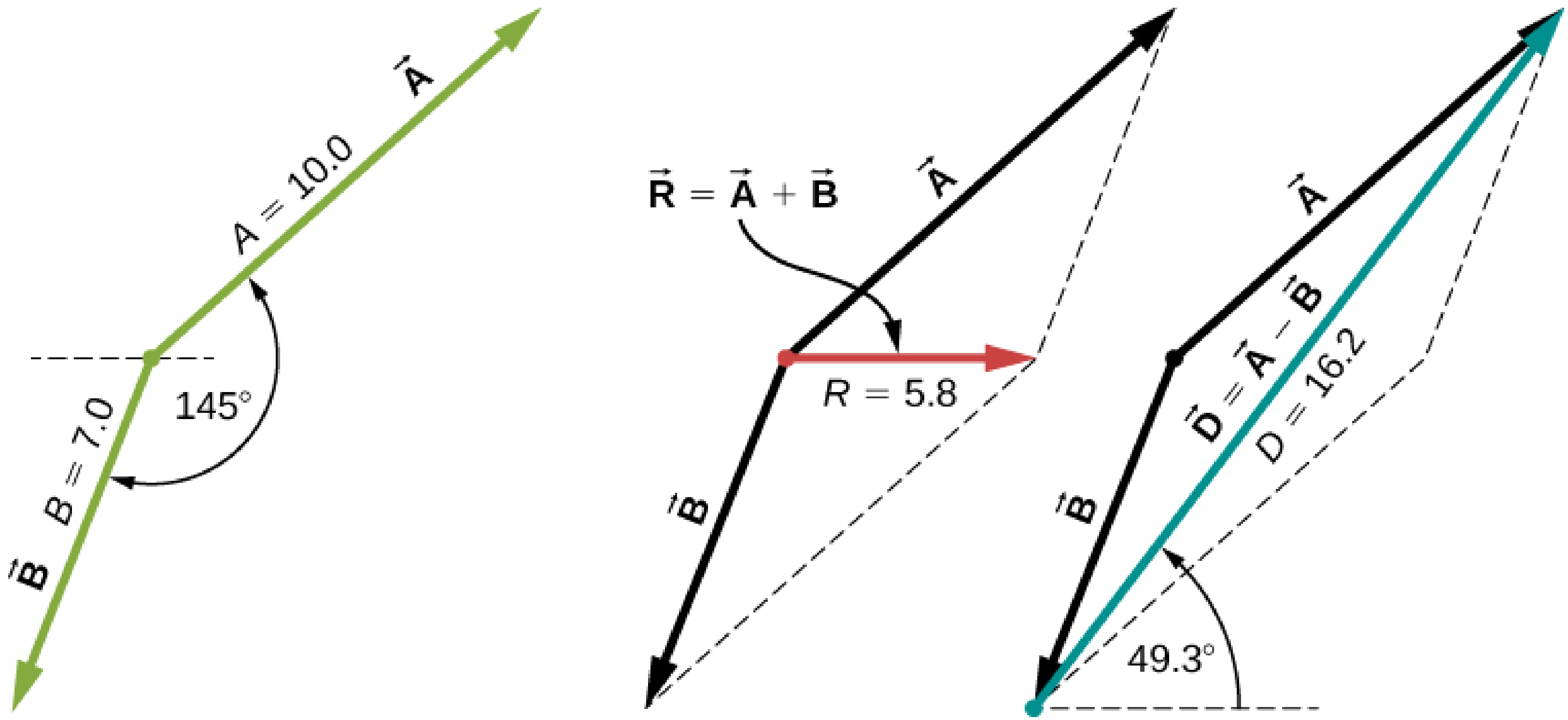


Tail-to-head method for drawing the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$.

(a) Four vectors of different magnitudes and directions.

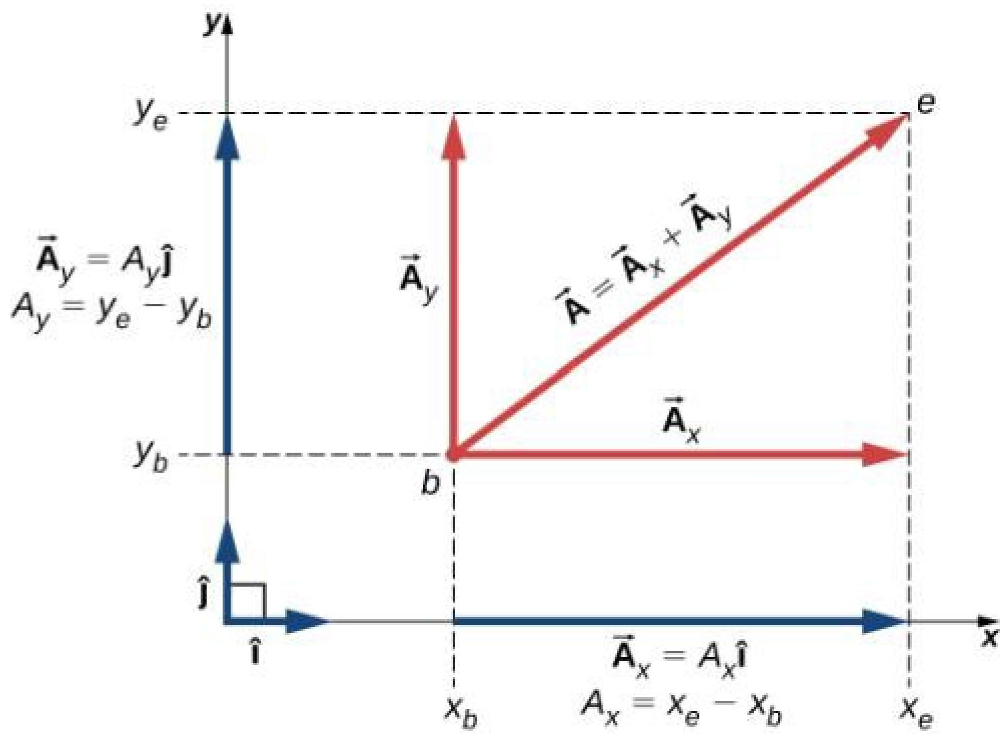
(b) Vectors in (a) are translated to new positions where the origin ("tail") of one vector is at the end ("head") of another vector. The resultant vector is drawn from the origin ("tail") of the first vector to the end ("head") of the last vector in this arrangement.

FIGURE 2.14



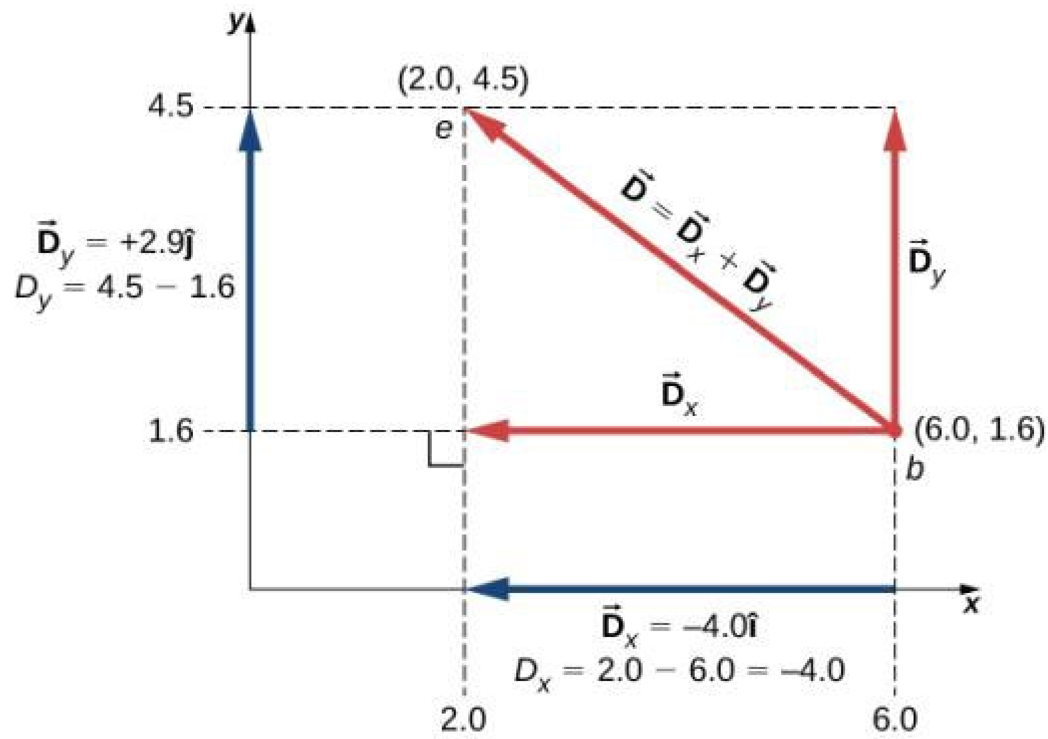
Using the parallelogram rule to solve (a) (finding the resultant, red) and (b) (finding the difference, blue).

FIGURE 2.16



Vector $\vec{\mathbf{A}}$ in a plane in the Cartesian coordinate system is the vector sum of its vector *x*- and *y*-components. The *x*-vector component $\vec{\mathbf{A}}_x$ is the orthogonal projection of vector $\vec{\mathbf{A}}$ onto the *x*-axis. The *y*-vector component $\vec{\mathbf{A}}_y$ is the orthogonal projection of vector onto the *y*-axis. The numbers A_x and A_y that multiply the unit vectors are the scalar components of the vector.

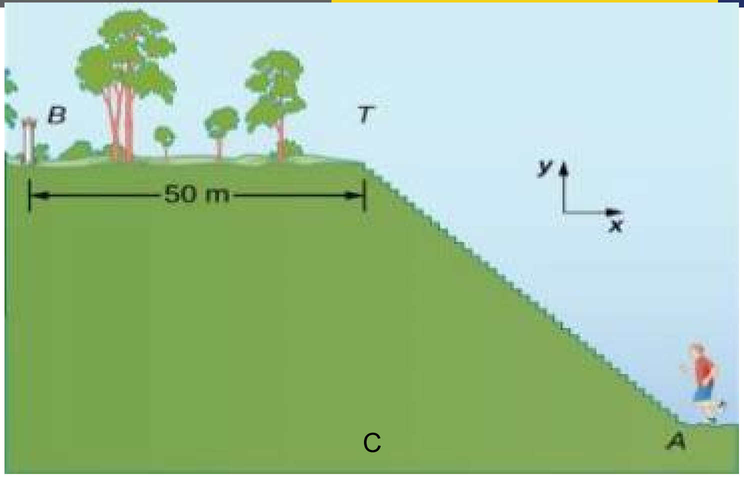
FIGURE 2.17



The graph of the displacement vector. The vector points from the origin point at *b* to the end point at *e*.

FIGURE 2.26

A jogger runs up a flight of steps.



|AC|= 40m, |TC|=30m

EXAMPLE 2.13

Displacement of a Jogger

A jogger runs up a flight of 200 identical steps to the top of a hill and then runs along the top of the hill 50.0 m before he stops at a drinking fountain (Figure 2.26). His displacement vector from point A at the bottom of the steps to point B at the fountain is $\vec{\mathbf{D}}_{AB} = (-90.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}})\text{m}$. What is the height and width of each step in the flight? What is the actual distance the jogger covers? If he makes a loop and returns to point A, what is his net displacement vector?

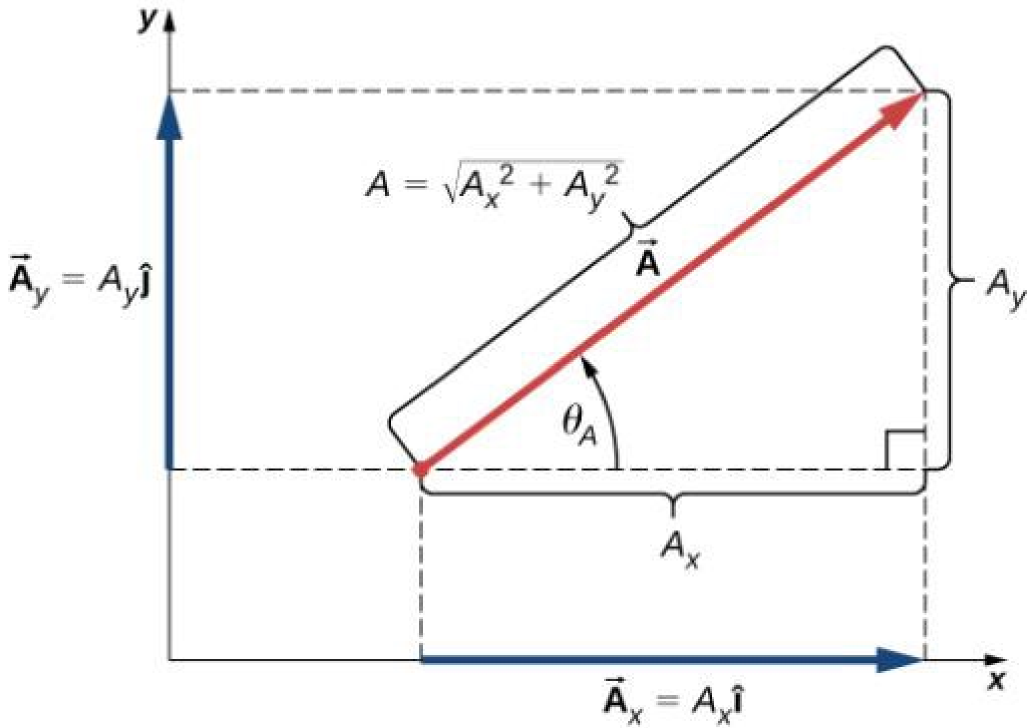
Its scalar components are $D_{ATx} = -40.0\text{ m}$ and $D_{ATy} = 30.0\text{ m}$. Therefore, we must have

$$200w = |-40.0|\text{m and } 200h = 30.0\text{ m}.$$

Hence, the step width is $w = 40.0\text{ m}/200 = 0.2\text{ m} = 20\text{ cm}$, and the step height is $h = 30.0\text{ m}/200 = 0.15\text{ m} = 15\text{ cm}$. The distance that the jogger covers along the stairs is

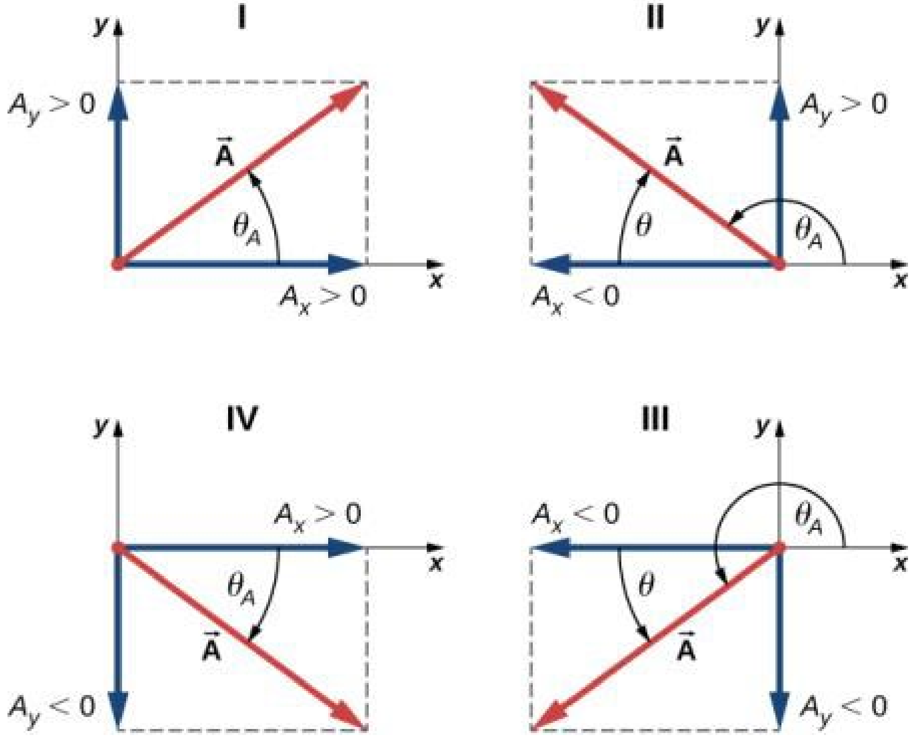
$$D_{AT} = \sqrt{D_{ATx}^2 + D_{ATy}^2} = \sqrt{(-40.0)^2 + (30.0)^2}\text{ m} = 50.0\text{ m}.$$

FIGURE 2.18



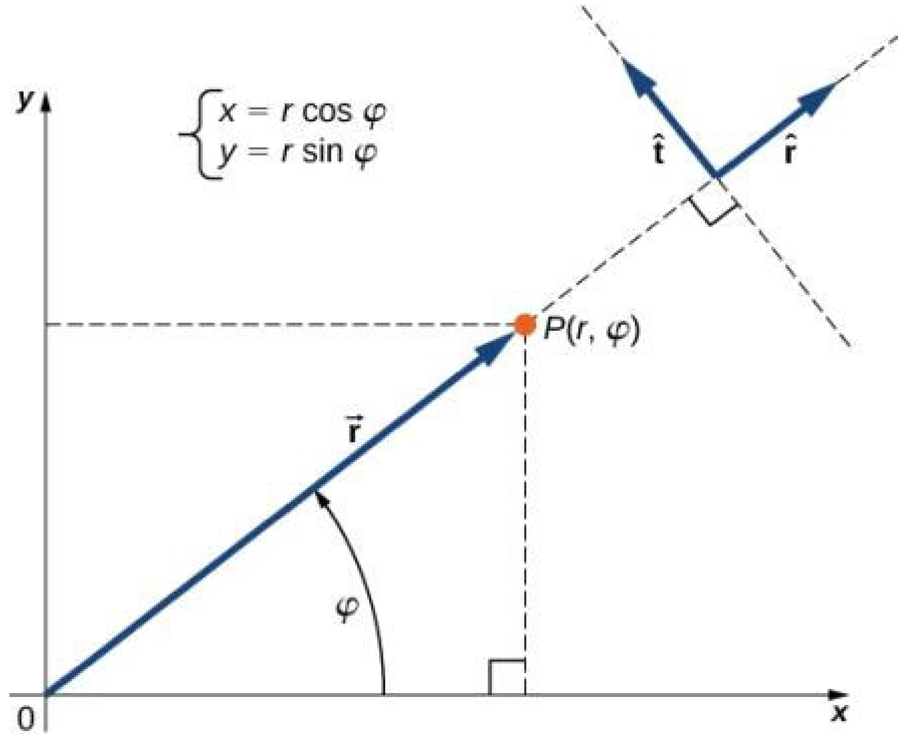
For vector $\vec{\mathbf{A}}$, its magnitude A and its direction angle θ_A are related to the magnitudes of its scalar components because A, A_x, and A_y form a right triangle.

FIGURE 2.19



Scalar components of a vector may be positive or negative. Vectors in the first quadrant (I) have both scalar components positive and vectors in the third quadrant have both scalar components negative. For vectors in quadrants II and III, the direction angle of a vector is $\theta_A = \theta + 180°$.

FIGURE 2.20



Using polar coordinates, the unit vector $\hat{\boldsymbol{r}}$ defines the positive direction along the radius *r* (radial direction) and, orthogonal to it, the unit vector $\hat{\boldsymbol{t}}$ defines the positive direction of rotation by the angle φ.

EXAMPLE 2.9

Analytical Computation of a Resultant

Three displacement vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ in a plane (Figure 2.13) are specified by their magnitudes $A = 10.0$, $B = 7.0$, and $C = 8.0$, respectively, and by their respective direction angles with the horizontal direction $\alpha = 35°$, $\beta = -110°$, and $\gamma = 30°$. The physical units of the magnitudes are centimeters. Resolve the vectors to their scalar components and find the following vector sums: (a) $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, (b) $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, and (c) $\vec{\mathbf{S}} = \vec{\mathbf{A}} - 3\vec{\mathbf{B}} + \vec{\mathbf{C}}$.

Solution

We resolve the given vectors to their scalar components:

$$\begin{cases} A_x = A\cos\alpha = (10.0\text{ cm})\cos 35° = 8.19\text{ cm} \\ A_y = A\sin\alpha = (10.0\text{ cm})\sin 35° = 5.73\text{ cm} \end{cases}$$

$$\begin{cases} B_x = B\cos\beta = (7.0\text{ cm})\cos(-110°) = -2.39\text{ cm} \\ B_y = B\sin\beta = (7.0\text{ cm})\sin(-110°) = -6.58\text{ cm} \end{cases}.$$

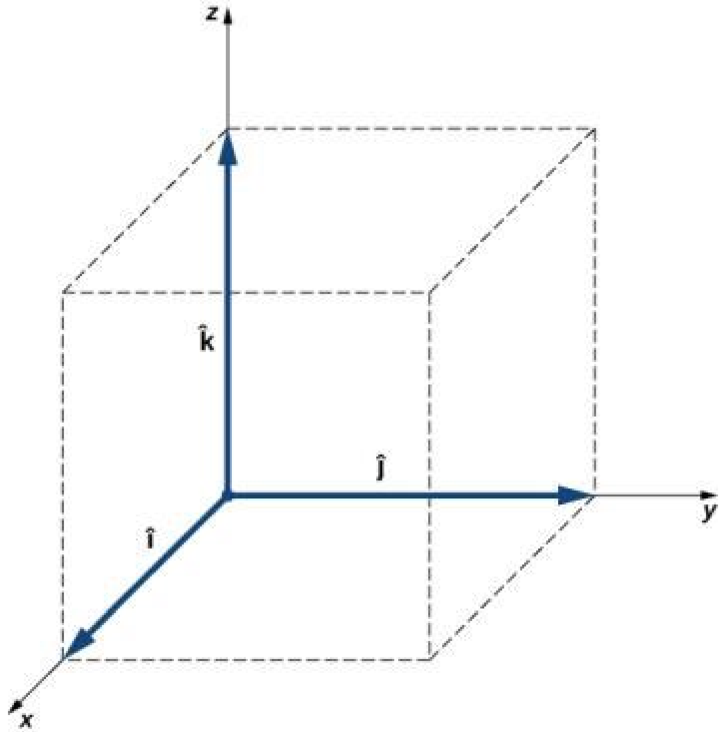
$$\begin{cases} C_x = C\cos\gamma = (8.0\text{ cm})\cos 30° = 6.93\text{ cm} \\ C_y = C\sin\gamma = (8.0\text{ cm})\sin 30° = 4.00\text{ cm} \end{cases}$$

For (a) we may substitute directly into Equation 2.25 to find the scalar components of the resultant:

$$\begin{cases} R_x = A_x + B_x + C_x = 8.19\text{ cm} - 2.39\text{ cm} + 6.93\text{ cm} = 12.73\text{ cm} \\ R_y = A_y + B_y + C_y = 5.73\text{ cm} - 6.58\text{ cm} + 4.00\text{ cm} = 3.15\text{ cm} \end{cases}.$$

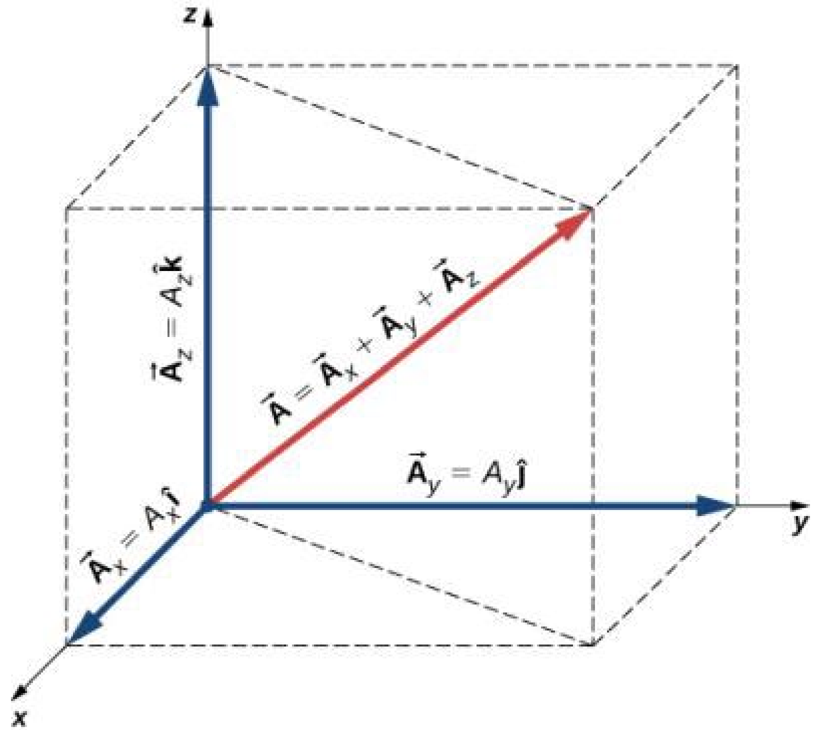
Therefore, the resultant vector is $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} = (12.7\hat{\mathbf{i}} + 3.1\hat{\mathbf{j}})\text{cm}$.

FIGURE 2.21



Three unit vectors define a Cartesian system in three-dimensional space. The order in which these unit vectors appear defines the orientation of the coordinate system. The order shown here defines the right-handed orientation.

FIGURE 2.22



A vector in three-dimensional space is the vector sum of its three vector components.

FIGURE 2.23

During a takeoff of IAI Heron (Figure 2.23), its position with respect to a control tower is 100 m above the ground, 300 m to the east, and 200 m to the north. One minute later, its position is 250 m above the ground, 1200 m to the east, and 2100 m to the north. What is the drone's displacement vector with respect to the control tower? What is the magnitude of its displacement vector?

Begin at coordinates (300.0 m, 200.0 m, 100.0 m)

End at coordinates (1200 m, 2100 m, 250 m),

$$\begin{cases} D_x = x_e - x_b = 1200.0\,\text{m} - 300.0\,\text{m} = 900.0\,\text{m}, \\ D_y = y_e - y_b = 2100.0\,\text{m} - 200.0\,\text{m} = 1900.0\,\text{m}, \\ D_z = z_e - z_b = 250.0\,\text{m} - 100.0\,\text{m} = 150.0\,\text{m}. \end{cases}$$

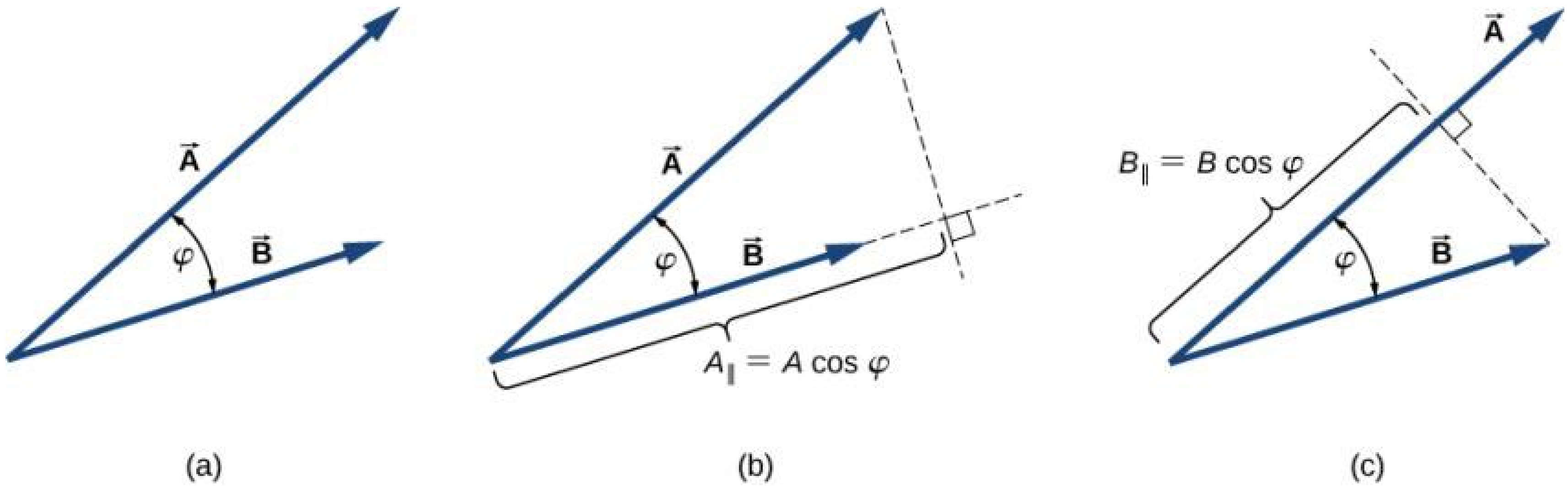
We substitute these components into Equation 2.19 to find the displacement vector:

$$\vec{\mathbf{D}} = D_x\hat{\mathbf{i}} + D_y\hat{\mathbf{j}} + D_z\hat{\mathbf{k}} = 900.0\,\text{m}\hat{\mathbf{i}} + 1900.0\,\text{m}\hat{\mathbf{j}} + 150.0\,\text{m}\hat{\mathbf{k}} = (0.90\hat{\mathbf{i}} + 1.90\hat{\mathbf{j}} + 0.15\hat{\mathbf{k}})\,\text{km}.$$

We substitute into Equation 2.21 to find the magnitude of the displacement:

$$D = \sqrt{D_x^2 + D_y^2 + D_z^2} = \sqrt{(0.90\,\text{km})^2 + (1.90\,\text{km})^2 + (0.15\,\text{km})^2} = 2.11\,\text{km}.$$

FIGURE 2.27



The scalar product of two vectors.

(a) The angle between the two vectors.

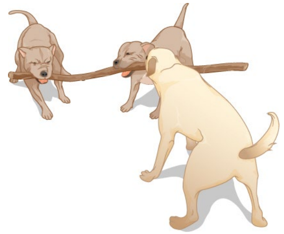
(b) The orthogonal projection $A_{||}$ of vector $\vec{\mathbf{A}}$ onto the direction of vector $\vec{\mathbf{B}}$.

(c) The orthogonal projection $B_{||}$ of vector $\vec{\mathbf{B}}$ onto the direction of vector $\vec{\mathbf{A}}$.

EXAMPLE 2.16

Angle between Two Forces

Three dogs are pulling on a stick in different directions, as shown in Figure 2.28. The first dog pulls with force $\vec{\mathbf{F}}_1 = (10.0\hat{\mathbf{i}} - 20.4\hat{\mathbf{j}} + 2.0\hat{\mathbf{k}})\text{N}$, the second dog pulls with force $\vec{\mathbf{F}}_2 = (-15.0\hat{\mathbf{i}} - 6.2\hat{\mathbf{k}})\text{N}$, and the third dog pulls with force $\vec{\mathbf{F}}_3 = (5.0\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})\text{N}$. What is the angle between forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$?



Solution

The magnitudes of forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$ are

$$F_1 = \sqrt{F_{1x}^2 + F_{1y}^2 + F_{1z}^2} = \sqrt{10.0^2 + 20.4^2 + 2.0^2}\,\text{N} = 22.8\,\text{N}$$

and

$$F_2 = \sqrt{F_{2x}^2 + F_{2y}^2 + F_{2z}^2} = \sqrt{15.0^2 + 6.2^2}\,\text{N} = 16.2\,\text{N}.$$

Substituting the scalar components into Equation 2.33 yields the scalar product

$$\begin{aligned} \vec{\mathbf{F}}_1 \cdot \vec{\mathbf{F}}_2 &= F_{1x}F_{2x} + F_{1y}F_{2y} + F_{1z}F_{2z} \\ &= (10.0\,\text{N})(-15.0\,\text{N}) + (-20.4\,\text{N})(0.0\,\text{N}) + (2.0\,\text{N})(-6.2\,\text{N}) \\ &= -162.4\,\text{N}^2. \end{aligned}$$

Finally, substituting everything into Equation 2.34 gives the angle

$$\cos\varphi = \frac{\vec{\mathbf{F}}_1 \cdot \vec{\mathbf{F}}_2}{F_1 F_2} = \frac{-162.4\,\text{N}^2}{(22.8\,\text{N})(16.2\,\text{N})} = -0.439 \Rightarrow \quad \varphi = \cos^{-1}(-0.439) = 116.0°.$$

WORK = FORCE * DISPLACEMENT

EXAMPLE 2.17

The Work of a Force

When force $\vec{\mathbf{F}}$ pulls on an object and when it causes its displacement $\vec{\mathbf{D}}$, we say the force performs work. The amount of work the force does is the scalar product $\vec{\mathbf{F}} \cdot \vec{\mathbf{D}}$. If the stick in Example 2.16 moves momentarily and gets displaced by vector $\vec{\mathbf{D}} = (-7.9\hat{\mathbf{j}} - 4.2\hat{\mathbf{k}})$ cm, how much work is done by the third dog in Example 2.16?

Strategy

We compute the scalar product of displacement vector $\vec{\mathbf{D}}$ with force vector $\vec{\mathbf{F}}_3 = (5.0\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})$N, which is the pull from the third dog. Let's use W_3 to denote the work done by force $\vec{\mathbf{F}}_3$ on displacement $\vec{\mathbf{D}}$.

Solution

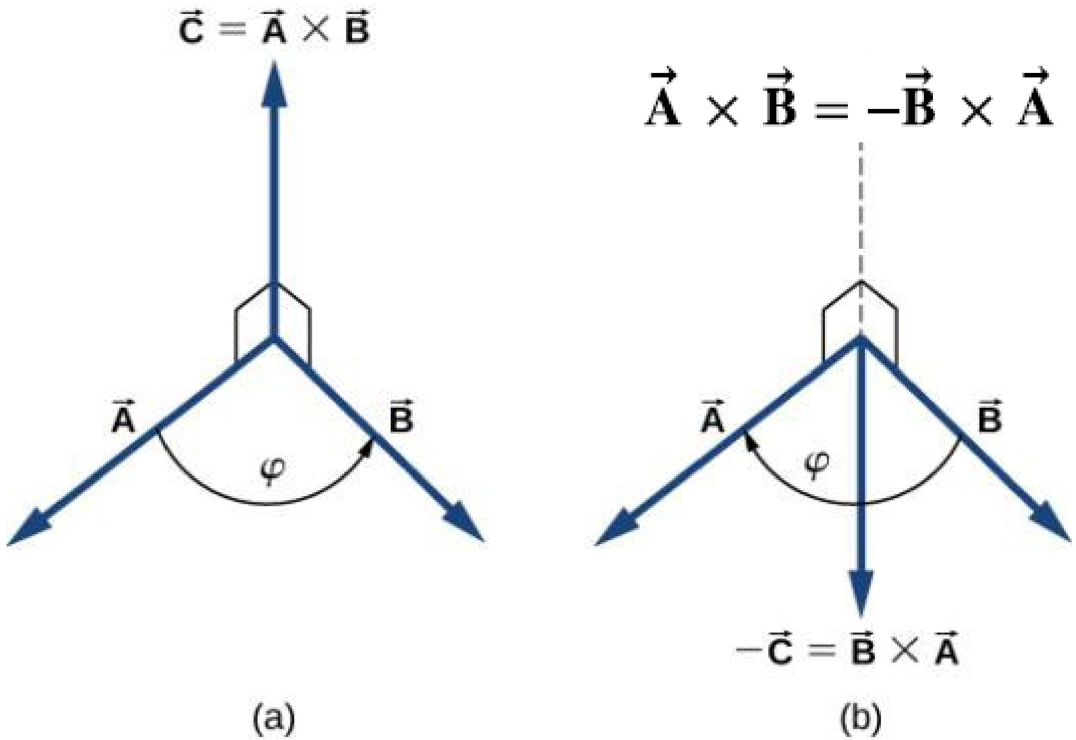
Calculating the work is a straightforward application of the dot product:

$$\begin{aligned} W_3 &= \vec{\mathbf{F}}_3 \cdot \vec{\mathbf{D}} = F_{3x} D_x + F_{3y} D_y + F_{3z} D_z \\ &= (5.0\text{ N})(0.0\text{ cm}) + (12.5\text{ N})(-7.9\text{ cm}) + (0.0\text{ N})(-4.2\text{ cm}) \\ &= -98.7\text{ N} \cdot \text{cm}. \end{aligned}$$

Significance

The SI unit of work is called the joule (J), where 1 J = 1 N · m. The unit cm · N can be written as $10^{-2}\text{m} \cdot \text{N} = 10^{-2}\text{J}$, so the answer can be expressed as $W_3 = -0.9875\text{ J} \approx -1.0\text{ J}$.

FIGURE 2.29: CROSS PRODUCT

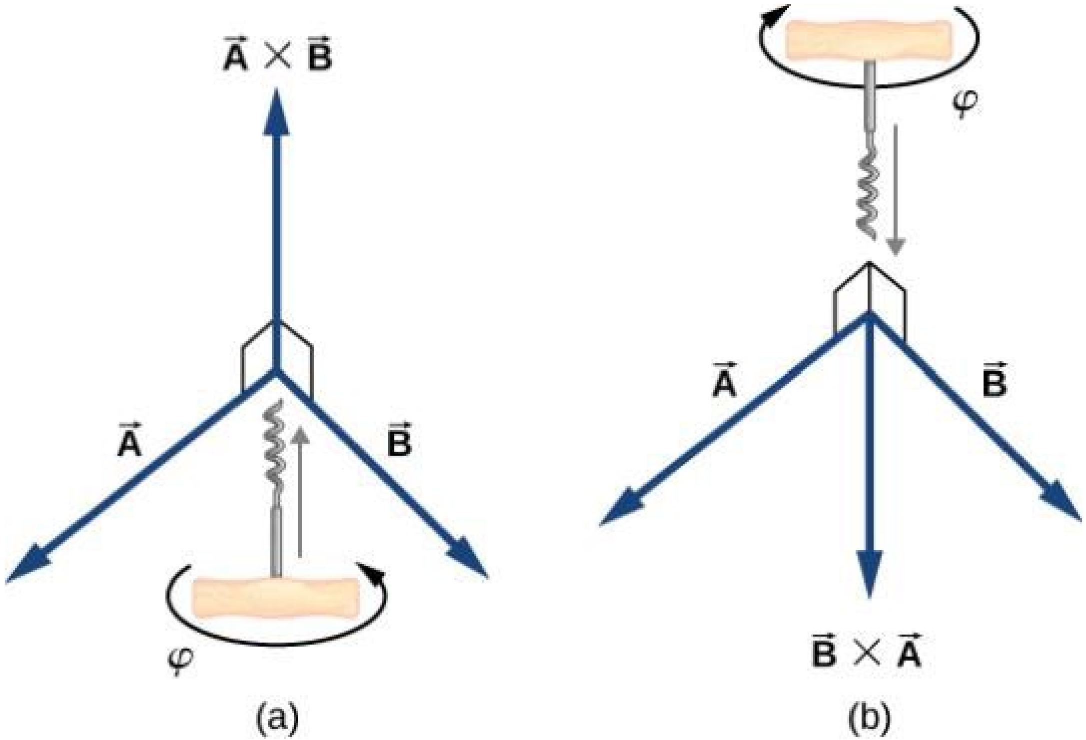


The vector product of two vectors is drawn in three-dimensional space.

(a) The vector product $\vec{A} \times \vec{B}$ is a vector perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$. Small squares drawn in perspective mark right angles between $\vec{A}$ and $\vec{C}$, and between $\vec{B}$ and $\vec{C}$ so that if $\vec{A}$ and $\vec{B}$ lie on the floor, vector $\vec{C}$ points vertically upward to the ceiling.

(b) The vector product $\vec{B} \times \vec{A}$ is a vector antiparallel to vector $\vec{A} \times \vec{B}$.

FIGURE 2.30



The corkscrew right-hand rule can be used to determine the direction of the cross product $\vec{A} \times \vec{B}$. Place a corkscrew in the direction perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, and turn it in the direction from the first to the second vector in the product. The direction of the cross product is given by the progression of the corkscrew.

(a) Upward movement means the cross-product vector points up.

(b) Downward movement means the cross-product vector points downward.

$$\begin{cases} \hat{\mathbf{i}} \times \hat{\mathbf{j}} = +\hat{\mathbf{k}}, \\ \hat{\mathbf{j}} \times \hat{\mathbf{k}} = +\hat{\mathbf{i}}, \\ \hat{\mathbf{k}} \times \hat{\mathbf{i}} = +\hat{\mathbf{j}}. \end{cases}$$

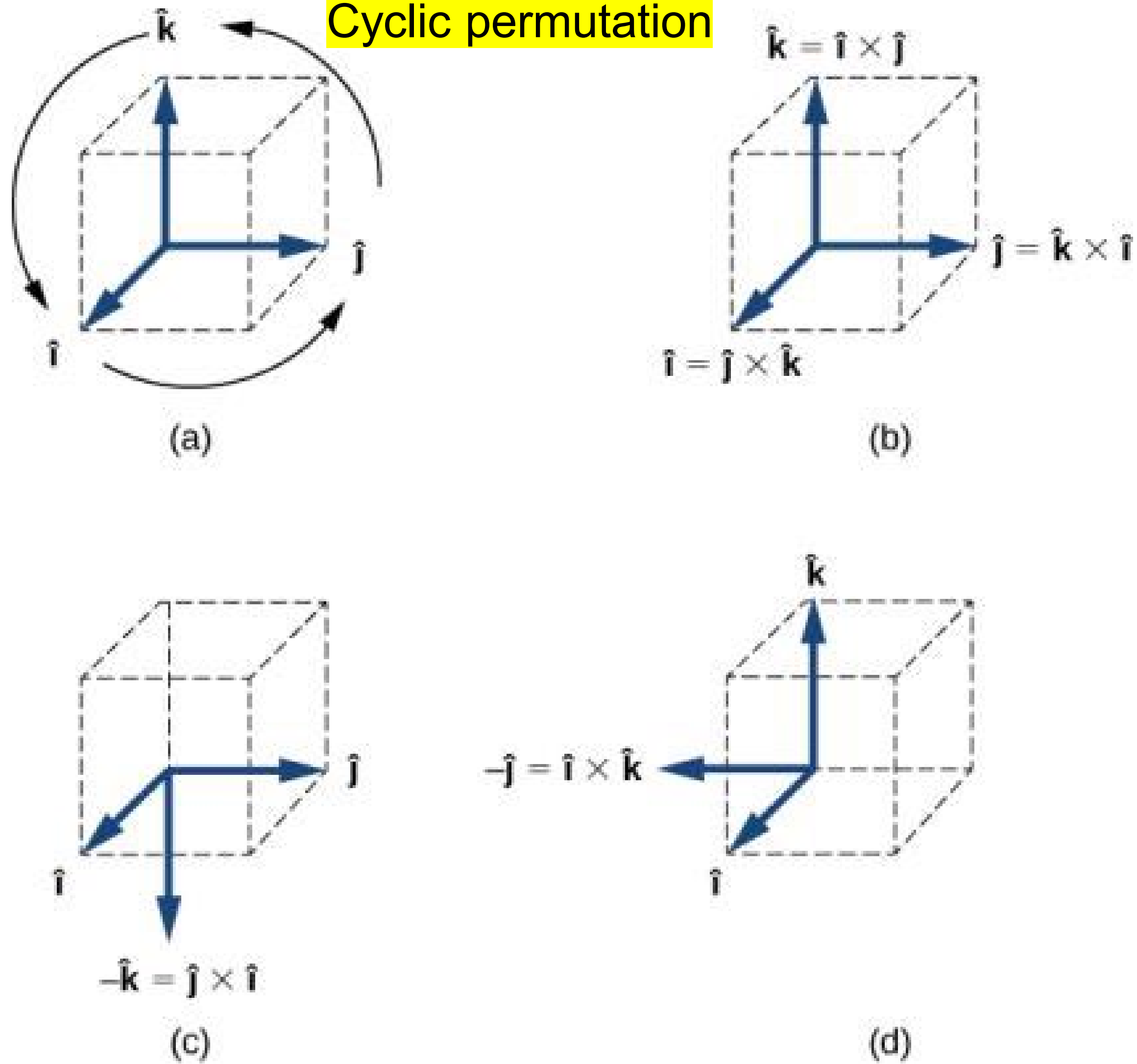
IGURE 2.32

Cyclic permutation

(a) The diagram of the cyclic order of the unit vectors of the axes.

(b) The only cross products where the unit vectors appear in the cyclic order. These products have the positive sign.

(c) (d) Two examples of cross products where the unit vectors do not appear in the cyclic order. These products have the negative sign.



$$
\begin{aligned}
\vec{\mathbf{A}} \times \vec{\mathbf{B}} &= (A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}) \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}) \\
&= A_x\hat{\mathbf{i}} \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}) + A_y\hat{\mathbf{j}} \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}) + A_z\hat{\mathbf{k}} \times (B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}) \\
&= \quad A_xB_x\hat{\mathbf{i}} \times \hat{\mathbf{i}} + A_xB_y\hat{\mathbf{i}} \times \hat{\mathbf{j}} + A_xB_z\hat{\mathbf{i}} \times \hat{\mathbf{k}} \\
&\quad + A_yB_x\hat{\mathbf{j}} \times \hat{\mathbf{i}} + A_yB_y\hat{\mathbf{j}} \times \hat{\mathbf{j}} + A_yB_z\hat{\mathbf{j}} \times \hat{\mathbf{k}} \\
&\quad + A_zB_x\hat{\mathbf{k}} \times \hat{\mathbf{i}} + A_zB_y\hat{\mathbf{k}} \times \hat{\mathbf{j}} + A_zB_z\hat{\mathbf{k}} \times \hat{\mathbf{k}} \\
&= \quad A_xB_x(0) + A_xB_y(+\hat{\mathbf{k}}) + A_xB_z(-\hat{\mathbf{j}}) \\
&\quad + A_yB_x(-\hat{\mathbf{k}}) + A_yB_y(0) + A_yB_z(+\hat{\mathbf{i}}) \\
&\quad + A_zB_x(+\hat{\mathbf{j}}) + A_zB_y(-\hat{\mathbf{i}}) + A_zB_z(0).
\end{aligned}
$$

$$
\begin{cases}
\hat{\mathbf{i}} \times \hat{\mathbf{j}} = +\hat{\mathbf{k}}, \\
\hat{\mathbf{j}} \times \hat{\mathbf{k}} = +\hat{\mathbf{i}}, \\
\hat{\mathbf{k}} \times \hat{\mathbf{i}} = +\hat{\mathbf{j}}.
\end{cases}
$$

When performing algebraic operations involving the cross product, be very careful about keeping the correct order of multiplication because the cross product is anticommutative. The last two steps that we still have to do to complete our task are, first, grouping the terms that contain a common unit vector and, second, factoring. In this way we obtain the following very useful expression for the computation of the cross product:

$$
\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}} = (A_yB_z - A_zB_y)\hat{\mathbf{i}} + (A_zB_x - A_xB_z)\hat{\mathbf{j}} + (A_xB_y - A_yB_x)\hat{\mathbf{k}}. \tag{2.40}
$$

In this expression, the scalar components of the cross-product vector are

$$
\begin{cases}
C_x = A_yB_z - A_zB_y, \\
C_y = A_zB_x - A_xB_z, \\
C_z = A_xB_y - A_yB_x.
\end{cases} \tag{2.41}
$$

Simpler using determinants (later)

A x **B** = A B sin ϕ = area //gram