

# Kaneko and Toyama (2025)

Student Presentation in Empirical Industrial Organization

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<https://yasu0704xx.github.io>

DEMAND ESTIMATION WITH FLEXIBLE INCOME  
EFFECT: AN APPLICATION TO PASS-THROUGH  
AND MERGER ANALYSIS\*

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This article proposes a semiparametric discrete choice model that incorporates a nonparametric specification for income effects. The model allows for the flexible estimation of demand curvature, which has significant implications for pricing and policy analysis in oligopolistic markets. Our estimation algorithm adopts a method of sieve approximation with shape restrictions in a nested fixed-point algorithm. Applying this framework to the Japanese automobile market, we conduct a pass-through analysis of feebates and merger simulations. Our model predicts a higher pass-through rate and more significant merger effects than parametric demand models, highlighting the importance of flexibly estimating demand curvature.

- Kaneko & Toyama (2025) propose a **semiparametric discrete choice model**.
  - Their model overcome certain problems arising with parametric specifications for the income effect, resulting in **accurate estimation of demand curvature, price elasticity, and welfare changes in the presense of income effects**.
- Applying the proposed framework to data from the Japanese automobile industry, they conduct merger simulations and pass-through analysis for a feebate policy (subsidy for eco-friendlly cars).<sup>1</sup>
  - Pass-through analysis of feebates  $\Rightarrow$  High pass-through rate
  - Merger simulations (Toyota & Honda)  $\Rightarrow$  More significant merger effects

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<sup>1</sup>To be skipped in the class.

Introduction

Demand Model

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# Introduction

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# Consumer Demand Model

- Price elasticity and the substitution patterns implied by the consumer demand model are key factors firms must consider when making pricing decisions in oligopolistic markets.
- Consumer demand is also essential for evaluating the welfare consequences of a firm's strategic behavior and policy changes.
- Therefore, accurate measurement of consumer demand is critical for various applications.
  - Merger analysis
  - Pass-through analysis of cost shocks and taxes
  - Introduction of new products

# Specification for Income Effects

- The majority of existing econometric methods for estimating consumer demand for differentiated products rely on parametric specifications, which could be problematic because such parametrizing often imposes strict restrictions on the shape of demand curve.
- To address this concern, Kaneko & Toyama (2025) propose a **semiparametric discrete choice model**.
  - A nonparametric specification for income effects
  - Adopting a method of sieve approximation with shape restrictions in a nested fixed-point algorithm
  - Parametric component: utility derived from product characteristics
- This allows for the flexible estimation of demand curvature and price elasticity patterns, which has significant implications for pricing and policy analysis in oligopolistic markets.

## Demand Estimation with Flexible Income Effect

- To estimate their model, Kaneko & Toyama (2025) combine a method of **sieve approximation** (Chen, 2007) and **nested fixed point algorithm** (BLP):
- First, they approximate the nonparametric function of the income effect using a sieve (a linear combination of known basis functions). They select Bernstein polynomials as the basis function due to their shape-preserving properties. After implementing the sieve approximation, their model is closely aligns with the standard parametric framework of BLP.
- Second, they use a nested fixed point algorithm to implement sieve GMM estimation.



# Demand Model

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# Utility Maximization Problem

- Let  $U(m, j)$  denote the direct utility function.
  - $m$  is a  $d_m$  dimensional vector representing the consumption of continuous choice goods.
  - $j \in \mathbb{J} = \{0, 1, \dots, J\}$  corresponds to an alternative in the discrete choice decision, with  $J$  products available in the market. The index  $j = 0$  indicates the outside goods.
- The utility maximization problem is given by

$$\begin{aligned} \max_{(m,j) \in \mathbb{R}_+^{d_m} \times \mathbb{J}} \quad & U(m, j) \\ \text{s.t.} \quad & P_m^T m + p_j \leq y_i, \end{aligned} \tag{1}$$

where  $P_m$  is a  $d_m$  dimensional vector of prices of continuous choice goods,  $p_j$  is the price of alternative  $j$ , and  $y_i$  is income.

## Conditional Indirect Utility Function

- Conditional on choice  $j$  in the discrete choice, the conditional indirect utility function is defined as

$$V(P_m, y - p_j, j) \equiv \max_{m \in \mathbb{R}_+^{d_m}} U(m, j) \text{ s.t. } P_m^T m \leq y_i - p_j. \quad (2)$$

Note that we define  $p_0 = 0$  as choosing the outside good incurs no costs.

- Assume that the direct utility function satisfies

$$U(m, j) = v(j) + u(m), \quad (3)$$

which suggests that the utility derived from differentiated goods is independent from that of all other goods.

- The conditional indirect utility function can be rewritten as

$$V(P_m, y - p_j, j) = v(j) + \tilde{V}(P_m, y - p_j). \quad (4)$$

- Assume that the continuous good is a numeraire, with its price represented by  $P^m$ . Then, we obtain

$$\tilde{V}(P^m, y - p_j) = u\left(\frac{y - p_j}{P^m}\right),$$

implying that the utility from numeraire depends on the disposal income  $y - p_j$  after choosing alternative  $j$ . Both income  $y$  and the price of discrete choice goods  $p_j$  are deflated by the price index  $P^m$ .

- Define the income effect term by

$$f(y - p_j) \equiv \tilde{V}(P^m, y - p_j),$$

which plays a pivotal role in their empirical framework. Note that  $f(y - p_j)$  is weakly-increasing function, imposed in the estimation.

# Conditional Indirect Utility Function

- Letting  $v_{ij}$  denote consumer  $i$ 's utility from a discrete choice good  $j$ , we specify that

$$v_{ij} = \beta^T X_j + \xi_j + \epsilon_{ij} \text{ for } j = 1, \dots, J, \quad (5)$$

$$v_{i0} = \epsilon_{i0}. \quad (6)$$

where  $X_j$  is a vector of observable characteristics of product  $j$ ,  $\xi_j$  represents its unobservable characteristics, and  $\epsilon_{ij}$  is an IID idiosyncratic shock that follows the type I extreme-value distribution.

- Hence, the conditional indirect utility function of consumer  $i$  when choosing  $j$  is given by

$$V_{ij} = \begin{cases} f(y_i - p_j) + \beta^T X_j + \xi_j + \epsilon_{ij} & \text{for } j = 1, \dots, J, \\ f(y_i) + \epsilon_{i0} & \text{for } j = 0. \end{cases} \quad (7)$$

# Individual Choice Probability

- Define the choice set of consumer  $i$  as

$$\mathbb{J}_{it} = \{0\} \cup \{j \in \{1, \dots, J_t\} : y_{it} - p_{jt} \geq 0\}, \quad (8)$$

where  $J_t$  is the total number of products available in market  $t$ . Note that  $\mathbb{J}_{it}$  is decided by the budget constraint given in (1).

- Given the conditional indirect utility  $V_{ijt}$  (7), the discrete choice problem is described as

$$\max_{j \in \mathbb{J}_{it}} V_{ijt}. \quad (9)$$

and the choice probability for consumer  $i$  selecting alternative  $j$  is derived as

$$s_{ijt}(y_{it}) = \frac{1(y_{it} \geq p_{jt}) \cdot \exp(f(y_{it} - p_{jt}) + \beta^T X_{jt} + \xi_{it})}{\exp(f(y_{it})) + \sum_{k=1}^{J_t} 1(y_{it} \geq p_{kt}) \cdot \exp(f(y_{it} - p_{kt}) + \beta^T X_{kt} + \xi_{it})}. \quad (10)$$

- Letting  $y_{it}$  follow the distribution of income  $G_t(y_{it})$  (that is, capture the consumer heterogeneity), the market share is given by

$$s_{jt} = \int s_{ijt} dG_t(y_{it}). \quad (11)$$

- Market demand  $q_{jt}$  is given by

$$g_{jt} = N_t \times s_{jt}$$

where  $N_t$  denote the market size.

## Practical Importance of the Flexible Income Effect

- Price Elasticity: The curvature of the demand curve, as indicated by its second-order derivative, is determined by the shape of income effect  $f(y - p)$ . Therefore, the flexible specification of income effect avoids imposing any predetermined restrictions on how own-price elasticity varies with price.
- Pass-Through Analysis: Imposing a functional form on the income effect can inherently restrict the demand curvature, and consequently, the pass-through-rate.
- Merger Analysis: In the simulation approach proposed by Croole et al. (1999), different curvatures of the demand function lead different simulated merger outcomes even under the same consumer demand with identical elasticities.



# Estimation Method

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- Kaneko & Toyama's (2025) semiparametric model includes the nonparametric function  $f(y - p)$  and the linear parameter  $\beta$  in the utility function.
- To estimate these model components, they employ a sieve approximation for the nonparametric function and incorporate it into the nested fixed-point (NFP) algorithm as proposed by BLP.
- Additionally, they apply a shape restriction to the nonparametric component with the aim of enhancing the precision of the parameter estimation.

# Sieve Approximation

- Sieve Approximation: Chen (2007), Blundell et al. (2007)
- Approximate  $f(\cdot)$  by the  $K$ -th order Bernstein polynomial, i.e., by a linear function of the basis function  $\Psi^K(x) = (b_0^K(x), b_1^K(x), \dots, b_K^K(x))^T$  and coefficients  $\Pi = (\pi_0, \pi_1, \dots, \pi_K)^T$ :

$$f(x) \simeq B_K(x) = \sum_{k=0}^K \pi_k b_k^K(x) \equiv \Psi^K(x)^T \Pi \quad (12)$$

where

$$b_k^K(x) = \binom{K}{k} x^k (1-x)^{K-k}, \quad (13)$$

and letting  $x$  be normalized to  $[0, 1]$ .

## Shape Restrictions & Normalization

- Selecting the Bernstein polynomial as a basis function can easily lead to incorporating shape restrictions into the nonparametric function.
- Recall that the nonparametric income-effect term  $f(y - p)$  is weakly increasing (monotonicity). To incorporate this shape restriction within Kaneko & Toyama's (2025) estimation, we impose constraints on the coefficients  $\Pi$ .
- Under  $\pi_k \leq \pi_{k+1}$  for all  $k$ , the derivative of the Bernstein polynomial approximation function (12) satisfies that  $B'_K(x) = K \sum_{k=0}^{K-1} (\pi_{k+1} - \pi_k) b_k^{K-1}(x) \geq 0$  for all  $k$ , which is the desired monotonicity.
- The level of the income effect cannot be identified. Thus, letting  $\pi_0 = 0$ , we normalize the approximation function as  $f(0) = 0$ .

# Approximated Model

- Under the sieve approximation above, the market share defined by (10) and (11) can be rewritten as

$$s_{jt} = \int \frac{1(y_{it} \geq p_{jt}) \cdot \exp(\Psi^K(y_{it} - p_{jt})^T \Pi + \beta^T X_{jt} + \xi_{jt})}{\text{denom.}} dG_t(y_{it}), \quad (14)$$

where the denominator is given by

$$\exp(\Psi^K(y_{it})^T \Pi) + \sum_{k=1}^{J_t} 1(y_{it} \geq p_{kt}) \cdot \exp(\Psi^K(y_{it} - p_{kt})^T \Pi + \beta^T X_{kt} + \xi_{kt})$$

- Note that there emerges **an endogeneity** between the product price  $p_{jt}$  and the unobserved product characteristics  $\xi_{jt}$ . Then, we introduce **IVs**, for example, proposed by BLP, Konishi & Zhao (2017), and Kitano (2022), among others.

- Moment Conditions: for  $b = 1, \dots, B$ ,

$$\mathbb{E} [\xi_{jt}(\theta) p_b(X_{jt}, W_{jt})] = 0, \quad (15)$$

where  $X_{jt}$  is a vector of exogenous variables,  $W_{jt}$  is a vector of IVs,  $\theta = (\beta, \Pi)$ ,  $\{p_b(X_{jt}, W_{jt})\}_{b=1, \dots, B}$  is a sequence of known functions that can approximate any real-valued square-integrable functions of  $X_{jt}$  and  $W_{jt}$  as  $B \rightarrow \infty$ .

- GMM Criterion:

$$\xi(\theta)^T \tilde{P} \left( \tilde{P}^T \tilde{P} \right)^{-} \tilde{P}^T \xi(\theta)^T, \quad (16)$$

where  $\xi(\theta)^T$  is a vector that stacks  $\xi_{jt}$ 's. The matrix  $\tilde{P} = [P, P \otimes X]$  denotes a matrix of instruments, for the choice of which we follow Chetverikov et al. (2018).

- Calculation of the objective function & numerical optimization procedures are as follows:<sup>2</sup>
  - 1. Calculate the vector of mean utility  $\delta$  by applying a contraction-mapping algorithm.
  - 2. Run a linear regression of  $\delta$  on  $X$  and obtain  $\hat{\beta}$  and the residual  $\hat{\xi}_{jt}$ .
  - 3. Calculate the value of the objective function (16).
  - 4. Run a nonlinear optimization routine over  $\Pi$ .<sup>3</sup>
- Inference: Generalized residual bootstrap (Chen & Pouzo, 2015).

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<sup>2</sup>See BLP (1995) for details.

<sup>3</sup>Note that  $\beta$  appearing in the mean utility function can be obtained by employing a linear GMM (concentration out: Nevo, 2001).

See Sections IV, V, VI, and VII of Kaneko & Toyama (2025).



## Conclusion

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# Conclusion

- Kaneko & Toyama (2025) propose a new empirical framework for a differentiated product demand model with a nonparametric income effect.
- The proposed model is a semiparametric model with endogeneity. They estimate the model by combining the nested fixed point algorithm proposed by BLP and a sieve approximation with shape restriction.
- Monte Carlo simulations suggest significant gains in estimating the nonparametric term of the income effect by incorporating the shape restriction (Skipped in the class).
- Applying their framework to Japanese automobile data, they demonstrate the importance of a flexible income effect specification (Skipped in the class).