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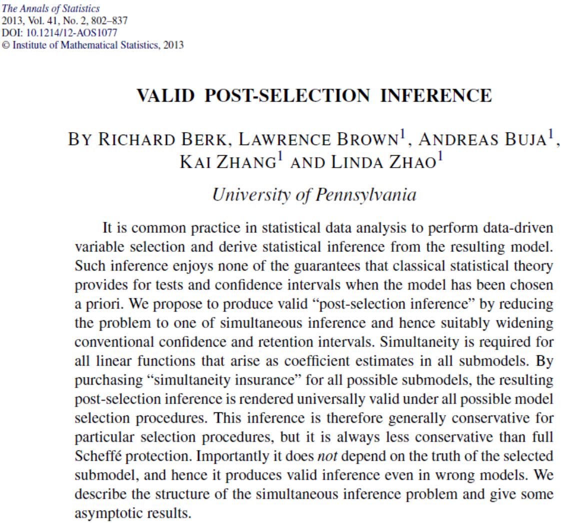

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VALID POST-SELECTION INFERENCE

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It is common practice in statistical data analysis to perform data-driven variable selection and derive statistical inference from the resulting model. Such inference enjoys none of the guarantees that classical statistical theory provides for tests and confidence intervals when the model has been chosen a priori. We propose to produce valid "post-selection inference" by reducing the problem to one of simultaneous inference and hence suitably widening conventional confidence and retention intervals. Simultaneity is required for all linear functions that arise as coefficient estimates in all submodels. By purchasing "simultaneity insurance" for all possible submodels, the resulting post-selection inference is rendered universally valid under all possible model selection procedures. This inference is therefore generally conservative for particular selection procedures, but it is always less conservative than full Scheffé protection. Importantly it does *not* depend on the truth of the selected submodel, and hence it produces valid inference even in wrong models. We describe the structure of the simultaneous inference problem and give some asymptotic results.

This slide is available on
https://github.com/yasu0704xx/ArticleReview.

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