

# Nonparametric Density Estimation

Sections 17.1-17.8 of Hansen (2022)

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<https://yasu0704xx.github.io>

## About Me



I am a second-year master's student in [Graduate School of Economics, Kyoto University](#).

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This slide is available on

<https://github.com/yasu0704xx/Econometrics2025>.

# Introduction

- As a general rule, density functions can take any shape. They are inherently **nonparametric** and cannot be described by a finite set of parameters.
- That is, functional and/or distributional specifications relied on when estimating density functions may be incorrect.
- If we assume that such specifications are “true,” we might obtain incorrect empirical conclusions.
- Thus, it would be desirable if we develop estimation procedures without requiring functional and/or distributional specifications.
- **Nonparametric kernel methods** achieve such a goal.

- Here we review Sections 17.1-17.8 of Hansen (2022) [1].
- We proceed with a discussion of how to estimate the probability density function (PDF) of a real-valued random variable  $X$  for which we have  $n$  IID observations  $X_1, \dots, X_n$ .
- We assume that  $X$  has a continuous density  $f(x)$ .
- The goal is to estimate  $f(x)$  either at a single point  $x$  or a set of points in the interior of the support of  $X$ .

# References

- Excellent textbooks on nonparametric density estimation include Silverman (1986) [5] and Scott (1992) [4].
- The following textbooks are often referred:
  - Silverman (1986) [5],
  - Scott (1992) [4],
  - van der Vaart (1998, Chapter 24) [6],
  - Pagan and Ullah (1999, Chapter 2) [3], and
  - Li and Racine (2007, Chapter 1) [2].
- 日本語の文献：
  - 西山・人見 (2023, 第 1 章) [3]
  - 末石 (2015, 第 9 章) [2]
  - 清水 (2023, 第 5 章) [1]

Histogram

Kernel Density Estimator

Bias

Variance

Variance Estimation and Standard Errors

Integrated Mean Squared Error (IMSE)

Optimal Kernel

Refernces

# Histogram

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# Kernel Density Estimator

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# Bias

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# Variance

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# Variance Estimation and Standard Errors

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## Integrated Mean Squared Error (IMSE)






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## Optimal Kernel

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## Refernces




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-  Hansen, B. E. (2022). *Probability and Statistics for Economists*. Princeton.
-  Li, Q. and J. S. Racine (2007). *Nonparametric Econometrics: Theory and Practice*. Princeton.
-  Pagan, A. and A. Ullah (1999). *Nonparametric Econometrics*. Cambridge.
-  Scott, D. W. (1992). *Multivariate Density Estimation: Theory, Practice, and Visualization*. Wiley.
-  Silverman, B. W. (1986). *Density Estimation for Statistics and Data Analysis*. Chapman and Hall.



van der Vaart, A. W. (1998). *Asymptotic Statistics*.  
Cambridge.



-  清水泰隆 (2023) 『統計学への漸近論，その先は』 内田老鶴圃.
-  末石直也 (2015) 『計量経済学：マイクロデータ分析へのいざない』 日本評論社.
-  西山慶彦，人見光太郎 (2023) 『ノン・セミパラメトリック統計解析』 共立出版.