Regression Discontinuity Designs

Hansen (2022, Chapter 21)

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https://yasu0704xx.github.io

Introduction

- Regression discontinuity designs (RDDs) are quasi-experimental designs which allow researchers to identify the causal effect of endogenous treatment on an outcome based on discontinuous policy rules.
- Local randomization is a key idea.
  - Consider a certain discontinuous rule under which treatment (e.g. college scholarship) is determined by whether a continuous covariate (e.g. admission score) is greater than a known threshold.
  - If all factors determined prior to the treatment are balanced just above and just below the threshold, the average causal effect can be estimated by comparing the mean outcome just above the threshold with that just below the threshold.

Literature

- Here we review Chapter 21 of Hansen (2022) [25].
- Excellent reviews/textbooks on regression discontinuity designs include Cattaneo and Titiunik (2022) [15], and Cattaneo, Idrobo and Titiunik (2021, 2024) [12] [13].
- The common software package is `rdrobust` by Calonico, Cattaneo, Farrell and Titiunik.
- 日本語の文献：
    - 川口・澤田（2024）[50]
    - 末石（2024）[52]
    - 高野（2025）[51]

Contents

Identification

Setup: Rubin Causal Model

- $Y_i(d) \in \mathbb{R}$, $d \in \{0,1\}$: potential outcome
- Y_i : observed outcome
- $D_i \in \{0,1\}$: treatment, which may be endogenous in that some unobserved factors may affect both D_i and Y_i.
- $\theta = Y_i(1) - Y_i(0)$: treatment effect for an individual
  $\Rightarrow$ We cannot identify θ without restrictive assumptions, because either $Y_i(1)$ or $Y_i(0)$ is unobservable.
- Instead, we are often interested in causal parameters such as ATE$= \mathbb{E}[Y_i(1) - Y_i(0)]$, ATT$= \mathbb{E}[Y_i(1) - Y_i(0)|D = 1]$, etc.

Sharp RD

- Suppose that treatment is determined by

$$D_i = 1(X_i \geq c), \tag{1}$$

  where the cut-off c is determined by policy or rule and common to all individuals.
- The covariate X_i is called the score, forcing variable, running variable, assignment variable, etc.
- In a standard RD setting, X_i is assumed to be continuously distributed on a subset of $\mathbb{R}$.

Identification in Sharp RD

Assume that, for each $d \in \{0, 1\}$,

$$\mathbb{E}[Y_i(d)|X_i = x] \text{ is continuous at } x = c. \tag{2}$$

Under Assumptions (1) and (2), the average causal effect at the cutoff point $\tau_{SRD} \equiv \mathbb{E}[Y_i(1) - Y_i(0)|X_i = c]$ *is identified by*

$$\tau_{SRD} = \lim_{x \downarrow c} \mathbb{E}[Y_i|X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i|X_i = x], \tag{3}$$

where $\lim_{x\downarrow c}$ *and* $\lim_{x\uparrow c}$ *denote the right and left limits at* $x = c$, *respectively.*

Proof By construction,

$$Y_i = Y_i(0) \cdot 1(X_i < c) + Y_i(1) \cdot 1(X_i \geq c).$$

Taking expectations conditional on $X_i = x$, we obtain

$$\begin{aligned}
&\mathbb{E}[Y_i|X_i = x] \\
&= \mathbb{E}[Y_i(0)|X_i = x]1(X_i < c) + \mathbb{E}[Y_i(1)|X_i = x]1(X_i \geq c)
\end{aligned}$$

Since $\mathbb{E}[Y_i(0)|X_i = x]$ and $\mathbb{E}[Y_i(1)|X_i = x]$ are continuous at $x = c$, they are identified by

$$\mathbb{E}[Y_i(0)|X_i = x] = \lim_{x \uparrow c} \mathbb{E}[Y_i|X_i = x],$$
$$\mathbb{E}[Y_i(1)|X_i = x] = \lim_{x \downarrow c} \mathbb{E}[Y_i|X_i = x],$$

which completes the proof.

- Assumption (2) means that the conditional expectation of the untreated and treated outcome are continuously affected by the running variable.
- It is implied that the distributions of confounding factors, including observable covariates determined prior to the treatment, are balanced near the cutoff.
- In particular, there should be no policy/legal/experimental changes at the cutoff, except for the treatment assignment.
- Counterfactual: The continuity of $\mathbb{E}[Y_i(d)|X_i = x]$ at $x = c$ cannot be directly examined, since $Y_i(0)$ and $Y_i(1)$ are unobservable under $X_i \geq c$ and $X_i < c$, respectively.

Fuzzy RD

- In the fuzzy RD, D_i is partially determined by whether X_i is no less than a known fixed cutoff c, such that

$$\lim_{x \downarrow c} \mathbb{E}[D_i | X = x] \neq \lim_{x \uparrow c} \mathbb{E}[D_i | X = x],$$

  where $\lim_{x \downarrow c}$ and $\lim_{x \uparrow c}$ denote the right and left limits at $x = c$, respectively.
- Notice that $\mathbb{E}[D_i | X_i] = \mathbb{P}(D_i = 1 | X_i)$.

Identification in Fuzzy RD

- Define $Z_i = 1(X_i \geq c)$.
- Let $D_i(z), z \in \{0, 1\}$ be the potential treatment status when $Z_i = z$. By construction, $D = Z_i D_i(1) + (1 - Z_i) D_i(0)$.
- Consider the following causal parameter for the "compliers:"

$$\tau_{\mathsf{FRD}} \equiv \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i(1) > D_i(0), X_i \in \{c - \epsilon, c + \epsilon\}].$$

- Under several assumptions, τ_{FRD} can be identified by the local Wald estimand:

$$\tau_{\mathsf{FRD}} = \frac{\lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X = x]}. \tag{4}$$

- The arguments are quite similar to the identification of LATE parameter in the IV estimations (so skiped in the class).
- See Hahn, Todd and van der Klaauw (2001) [24], Dong (2018) [20], and Hansen (2022, Sections 21.10-11) [25] for details.

Estimation

Identification Results (re)

- Recall that the causal parameters τ_{SRD} and τ_{FRD} are identified respectively by (3) and (4):

$$\tau_{\mathsf{SRD}} = \lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x],$$
$$\tau_{\mathsf{FRD}} = \frac{\lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X = x]}.$$

- For expositional purpose, denote one of Y_i and D_i by A_i, and define the following one-sided limits:

$$\mu_A^+ \equiv \lim_{x \downarrow c} \mathbb{E}[A_i | X_i = x], \quad \mu_A^- \equiv \lim_{x \uparrow c} \mathbb{E}[A_i | X_i = x].$$

- For estimating τ_{SRD} and τ_{FRD}, it suffices to estimate $\mu_Y^+, \mu_Y^-, \mu_D^+$, and μ_D^-.

Estimation Procedures: Local Polynomial Regressions

- The quantities $\mu_Y^+, \mu_Y^-, \mu_D^+$, and μ_D^- are commonly estimated by local polynomial regressions (LPRs).
- As we studied before, the local constant (Nadaraya-Watson) regression causes the boundary bias. LPRs can circumvent such boundary bias (Hahn, Todd and van der Klaauw, 2001 [24]; Porter, 2003 [41]).
- A recommended choice of local polynomial order p is 1 or 2. [1]
- On the other hand, there are several recommendations on bandwidth selection, which we study later.

[1] Gelman and Imbens (2019) [22] argue that controlling for global high-order polynomials in regression discontinuity analysis is a flawed approach with three major problems: it leads to noisy estimates, sensitivity to the degree of the polynomial, and poor coverage of confidence intervals.

- Consider the following p-th order LPRs:

$$\hat{\beta}_A^+ \equiv \arg\min_{\beta \in \mathbb{R}^{p+1}} \sum_{i=1}^{n} 1(X_i \geq c) \left(A_i - [r_p(X_i - c)]^T \beta\right)^2 K\left(\frac{X_i - c}{h}\right),$$

$$\hat{\beta}_A^- \equiv \arg\min_{\beta \in \mathbb{R}^{p+1}} \sum_{i=1}^{n} 1(X_i < c) \left(A_i - [r_p(X_i - c)]^T \beta\right)^2 K\left(\frac{X_i - c}{h}\right),$$

where $p \geq 1$, $r_p(x) \equiv (1, x, \cdots, x^p)$ is a vector of polynomials, $h > 0$ is bandwidth, and $K(\cdot)$ is a kernel function.

- The LPR estimators of μ_A^+ and μ_A^- are the first elements of $\hat{\beta}_A^+$ and $\hat{\beta}_A^-$, respectively:

$$\hat{\mu}_A^+ \equiv e_1^T \hat{\beta}_A^+, \quad \hat{\mu}_A^- \equiv e_1^T \hat{\beta}_A^-,$$

  where $e_1 \equiv (1, 0, \cdots, 0)^T$ is the first unit vector.

- The causal parameters τ_{SRD} and τ_{FRD} can be estimated respectively by

$$\hat{\tau}_{\mathsf{SRD}} = \hat{\mu}_Y^+ - \hat{\mu}_Y^-, \quad \hat{\tau}_{\mathsf{FRD}} = \frac{\hat{\mu}_Y^+ - \hat{\mu}_Y^-}{\hat{\mu}_D^+ - \hat{\mu}_D^-}.$$

Bandwidth Selection

- Under certain regularity conditions, we can show that

$$\mathbb{E}[\hat{\tau}_{\mathsf{SRD}} - \tau_{\mathsf{SRD}} | X_1, \cdots, X_n] = h^{p+1}\mathcal{B},$$

$$\mathsf{Var}[\hat{\tau}_{\mathsf{SRD}} | X_1, \cdots, X_n] = \frac{1}{nh}\mathcal{V},$$

$$\sqrt{nh}\left(\hat{\tau}_{\mathsf{SRD}} - \tau_{\mathsf{SRD}} - h^{p+1}\mathcal{B}\right) \xrightarrow{d} \mathsf{Normal}(0, \mathcal{V}).$$

- The AMSE is given by

$$\mathsf{AMSE}(\hat{\tau}_{\mathsf{SRD}}) = h^{2(p+1)}\mathcal{B}^2 + \frac{1}{nh}\mathcal{V}.$$

- Imbens and Kalyanaraman (2012) [27] propose the (A)MSE optimal bandwidth, which satisfies that

$$h_{\mathsf{IK2012}} \propto n^{-\frac{1}{2p+3}}.$$

Inference

Asymptotic Bias

- If we are interested in the point estimations of μ_Y^+ and μ_Y^-, it is sufficient to select IK2012's MSE optimal bandwidth.
- In practice, however, we are also interested in inference: IK2012's bandwidth does not satisfy that $nh^{2p+3} \to 0$. Then, we cannot eliminate the asymptotic bias.

How to select bandwidth?

1. Undersmoothing
2. Robust bias-corrected inference (CCT2014 [8], CCF2020 [7])
3. Uniformly honest inference (KR2018 [32])

Undersmoothing

- A naive solution to mitigate asymptotic bias is to use undersmoothing.
- By using undersmoothing bandwidth such that

$$\sqrt{nh}h^{p+1} \to 0,$$

the standard t statistic satisfies

$$\frac{\sqrt{nh}(\hat{\tau}_{\mathsf{SRD}} - \tau_{\mathsf{SRD}})}{\sqrt{\mathcal{V}_{\mathsf{SRD}}}} \xrightarrow{d} \text{Normal}(0, 1).$$

- Calonico, Cattaneo and Titiunik (2014) [8], and Calonico, Cattaneo and Farrell (2020) [7] propose to correct asymptotic bias for valid statistical inference, not to eliminate the bias by undersmoothing.
- Let $\hat{\mathcal{B}}$ denote the estimator of $\mathcal{B}$ based on LPRs using bandwidth b, which can differ from h. The bias-corrected estimator of τ_{SRD} is given by

$$\hat{\tau}^{\mathsf{bc}}_{\mathsf{SRD}} \equiv \hat{\tau}_{\mathsf{SRD}} - h^{p+1}\hat{\mathcal{B}}.$$

- Under certain conditions on h and b and regularity conditions, CCT2014 [8] show that the robust bias-corrected t statistic satisfies

$$T_{\mathsf{SRD}}^{\mathsf{bc}} \equiv \frac{\sqrt{nh}\left(\hat{\tau}_{\mathsf{SRD}}^{\mathsf{bc}} - \tau_{\mathsf{SRD}}\right)}{\sqrt{\mathcal{V}_{\mathsf{SRD}}^{\mathsf{bc}}}} \xrightarrow{d} \mathsf{Normal}(0,1),$$

where $\mathcal{V}_{\mathsf{SRD}}^{\mathsf{bc}} \equiv \mathcal{V}_{\mathsf{SRD}} + \mathcal{C}_{\mathsf{SRD}}^{\mathsf{bc}}$ and $\mathcal{C}_{\mathsf{SRD}}^{\mathsf{bc}}$ is a correction term.

- Using an estimator of $\mathcal{V}_{\mathsf{SRD}}^{\mathsf{bc}}$, the $1-\alpha$ confidence interval of τ_{SRD} based on $T_{\mathsf{SRD}}^{\mathsf{bc}}$ is given by

$$\left[\hat{\tau}_{\mathsf{SRD}}^{\mathsf{bc}} - z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{\mathcal{V}}_{\mathsf{SRD}}^{\mathsf{bc}}}{nh}}, \hat{\tau}_{\mathsf{SRD}}^{\mathsf{bc}} + z_{1-\frac{\alpha}{2}}\sqrt{\frac{\hat{\mathcal{V}}_{\mathsf{SRD}}^{\mathsf{bc}}}{nh}}\right]. \tag{5}$$

- CCF2020 [7] propose to select a bandwidth which minimizes the coverage rate error of (5).

Uniformly Honest Inference (KR2018)

- Kolesar and Rothe (2018) [32] propose another bias correction.
- Assuming that the true regression function is Lipschitz continuous, and analyzing the upper bound of the bias, they construct a confidence interval uniformly robust to any DGPs.
- Some characteristics of KR2018 are that they do not require $h \to 0$, and thus they allow the runnning variable to be discretely distributed.

- Consider the following LLR with the uniform kernel:

$$\min_{\alpha,\tau_h,\beta,\gamma} \sum_{i=1}^{n} 1(|X_i - c| \leq h) \left[Y_i - \alpha - \tau_h D_i - \beta(X_i - c) - \gamma D_i (X_i - c)\right]^2 .$$

- Define

$$n_h = \sum_{i=1}^{n} 1(|X_i - c| \leq h),$$
$$\tilde{\tau}_h = \mathbb{E}[\hat{\tau}_h | X_1, \cdots, X_n].$$

- Let $\dfrac{\hat{\sigma}^2}{n_h}$ be the estimator of $\text{Var}(\hat{\tau}_h | X_1, \cdots, X_n)$.

- By simple calculation, we obtain

$$\frac{\sqrt{n_h}(\hat{\tau}_h - \tau_{\mathsf{SRD}})}{\hat{\sigma}} = \frac{\sqrt{n_h}(\hat{\tau}_h - \tilde{\tau}_{\mathsf{SRD}})}{\hat{\sigma}} + \frac{\sqrt{n_h}(\tilde{\tau}_h - \tau_{\mathsf{SRD}})}{\hat{\sigma}}$$

- For any h (even if h violates $h \to 0$), the first term of RHS satisfies that

$$\frac{\sqrt{n_h}(\hat{\tau}_h - \tilde{\tau}_{\mathsf{SRD}})}{\hat{\sigma}} \xrightarrow{d} \text{Normal}(0, 1).$$

- The second term is the bias term. Let us analyze its upper bound.

- Assume that, for a known constant K,[2]

$$\mu(x) = \mathbb{E}[Y_i | X_i = x] \in \mathcal{M},$$

where

$$\mathcal{M} = \{\mu : |\mu'(a) - \mu'(b)| \leq K|a-b| \text{ for all } a, b < c \text{ and all } a, b > c\}.$$

- Then, for $\hat{\sigma}$ that is estimated with certain methods, we can obtain

$$\gamma_{sup} \equiv \sup_{\mu \in \mathcal{M}} \frac{\sqrt{n_h}|\tilde{\tau}_h - \tau_{\mathsf{SRD}}|}{\hat{\sigma}}.$$

[2] We cannot choose K by data-driven procedures.

- Based on this result, we can construct the following confidence interval:

$$\mathsf{CI} = \left[\hat{\tau}_h - \mathsf{cv}_{1-\alpha}(\gamma_{sup})\frac{\hat{\sigma}}{\sqrt{n_h}}, \hat{\tau}_h + \mathsf{cv}_{1-\alpha}(\gamma_{sup})\frac{\hat{\sigma}}{\sqrt{n_h}}\right],$$

where $\mathsf{cv}_{1-\alpha}(\gamma)$ denotes the $1-\alpha$ quantile of $|\mathsf{Normal}(\gamma, 1)|$.

- The CI above is an asymptotically uniform confidence interval with respect to $\mathcal{M}$:

$$\lim_{n\to\infty} \inf_{\mu\in\mathcal{M}} \mathbb{P}_\mu(\tau_{\mathsf{SRD}} \in \mathsf{CI}) \geq 1-\alpha,$$

which KR2018 [32] call the honest confidence interval.

Covariates

Covariates

Purpose of Covariate Adjustment

- Efficiency gains in the RD effect estimator
- Not for relaxing identification assumptions (i.e., not for the unconfoundedness)

RD with Covariates

- Robinson (1988) [43]
- Calonico, Cataneo, Farrell and Titiunik (2019) [9]

RD with High-Dimensional Covariates

- Kreiss and Rothe (2023) [33]
- Arai, Otsu and Seo (2024) [4]
- Noack, Olma and Rothe (2025) [38]
- Review: Chernozhukov et al. (2025, Chapter 17) [16]

Covariate Adjustment Relying on Robinson (1988)

- Relying on Robinson's (1988) [43] estimation of semiparametric partially linear model, we can adjust covariates to RD estimation.
- The estimator reaches the semiparametric efficiency bound. However, the finite-sample property is not so good.
- See Section 21.7 of Hansen (2022) [25] for details.

Covariate-Adjusted RD Estimation (CCFT2019)

- Calonico, Cattaneo, Farrell and Titiunik (2019) [9] recommend to implement the WLS based on

$$\tilde{Y}_i = \tilde{\alpha} + D_i\tilde{\tau} + X_i\tilde{\beta}_- + D_iX_i\tilde{\beta}_+ + Z_i^T\tilde{\gamma},$$

  where

  - $c = 0$ is a normalized cutoff point,
  - Z_i is a vector of auxiliary covariates of units satisfying $X_i \in \{-h, h\}$,
  - the weight function is assumed to be the unifrom or triangular kernel.

  That is, they consider a LLR estimator based on a kernel with bounded support.
- Under the assumption that there are no treatment effects on the covariates, $\tilde{\tau}$ is a consistent estimator of τ_{SRD}.
- Inference procedures based on the estimator $\tilde{\tau}$ are similar to the ones discussed in CCT2014 [8] and CCF2020 [7].

High-Dimensional Covariates

- Kreiss and Rothe (2023) [33] and Arai, Otsu and Seo (2024) [4] propose to implement certain Lasso-type selections to chosse active covariates and address the bias caused by these regularizations.
- Kreiss and Rothe (2023) : a "localized" version of Lasso regression
  - They inherit the framework from CCFT2019 [9].
- Arai, Otsu and Seo (2024) : the debiased Lasso (Zhang and Zhang, 2014 [49])

Falsification Test

Identification Assumptions

Identification Assumption (2) in Sharp RD

For each $d \in \{0, 1\}$,

$$\mathbb{E}[Y_i(d)|X_i = x] \text{ is continuous at } x = c.$$

- Counterfactual: The continuity of $\mathbb{E}[Y_i(d)|X_i = x]$ at $x = c$ cannot be directly examined, since $Y_i(0)$ and $Y_i(1)$ are unobservable under $X_i \geq c$ and $X_i < c$, respectively.
- Instead, researchers often examine certain necessary conditions.
- No manipulation: Under the above assumptions, it is necessary for the density of running variable X_i to be continuous at the cutoff point.

Manipulation Test

- Testing the continuity of the density of the assignment variable:
  - McCrary (2008) [36]
  - Otsu, Xu and Matsushita (2013) [40]
  - Cattaneo, Jansson and Ma (2020) [14]
  - Software packages by CJM2020: `lpdensity`, `rddensity`
  - Imai and Okamoto (2024) [26]
- Testing the continuity of the conditional distribution of covariates:
  - Lee (2008) [34]
  - Canay and Kamat (2018) [10]
  - Fusejima, Ishihara and Sawada (2025) [21]

Placebo Test

- Take several placebo cutoff points.
- At a placebo cutoff point $c' < c$, researchers can observe whether the density of untreated potential outcome $Y_i(0)$ is continuous or not.
- Similarly, at $c' > c$, they can observe whether the density of treated potential outcome $Y_i(1)$ is continuous or not.
- If the continuity is observed, then there arises some plausibility of the identification assumptions in Theorem 21.1.
- However, it is just a plausibility. Note that the continuity at placebo cutoff points is neither necessary nor sufficient for the identification assumptions.

Caveats on Pretesting

- Recent works argue that, in general, researchers should not implement such pretesting.
  - Roth (2022) [44] : Pre-trend test
  - Sueishi (2023) [47] : Hausman test
- Pretesting analysis in the RD setting can be found in Section 5.2 of Fusejima, Ishihara and Sawada (2025) [21].

Practical Recommendation

Practical Recommendation in Sharp RD

- The polynomial order p of LPRs should be 1 or 2.
- For point estimation, use IK2012's (A)MSE optimal bandwidth.
- For inference, use CCT2014 & CCF2020's bandwidth.
- The common package `rdrobust` is equipped with point estimation relying on IK2012 and robust bias-corrected inference by CCT2014 & CCF2020.
- Observable covariates can be utilized to gain efficiency of the sharp RD estimator.
- Note that pretesting for Assumption (2) may distort the estimation and inference results.

Empirical Application

"Waiting for Life"

- The first study relying on RD: Thistlehwaite and Cambell (1960) [48]
- Angrist and Lavy (1999) [3], Black (1999) [6]
    - RD was "waiting for life" (Cook, 2008 [17])
- Hahn, Todd, and Klaauw (2001) [24] formalize general RDDs and establish identification results for treatment effects.

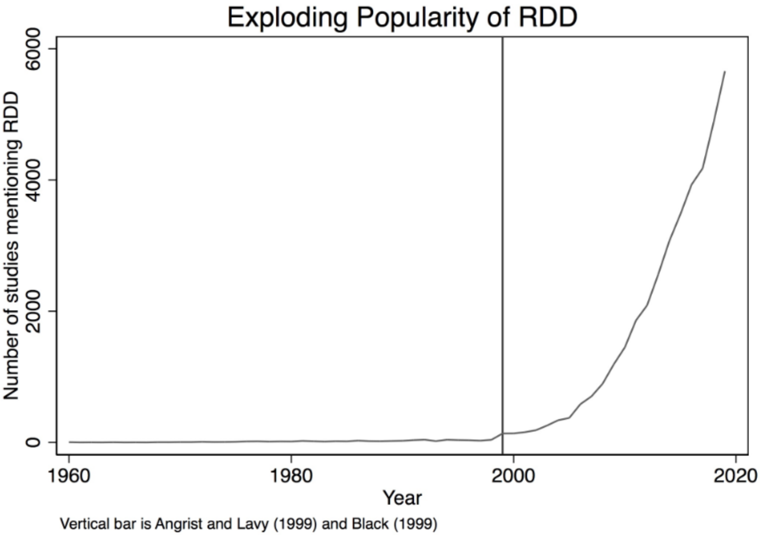


Figure 6.1 of Cunningham (2021) [18]

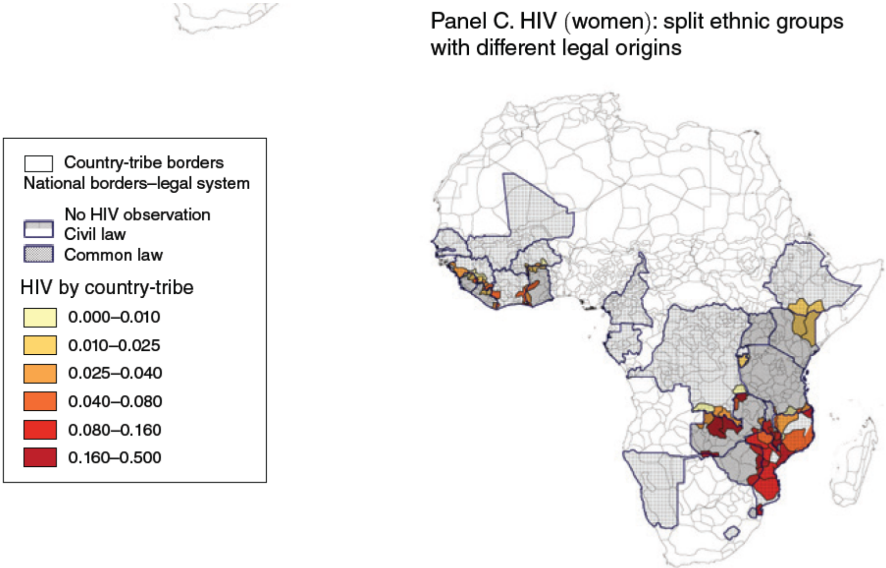
Recent Empirical RD Studies

- Here we review Anderson (2018) [2] and Kawai et al. (2023) [30].
  - For more details than below, see the OG papers and auxiliary review slides by me.
  - Slides for Anderson (2018)
  - Slides for Kawai et al. (2023)
- Other empirical RD studies:
  - Ludwig and Miller (2007) [35], Lee (2008) [34], Matsudaira (2008) [37], Battistin et al. (2009) [5], Carpenter, Christopher and Carlos Dobkin (2009) [11], Greenstone, Hornbeck and Moretti (2010) [23], Abdulkadiroglu, Angrist and Pathak (2014) [1], Ito (2014) [28], Kleven et al. (2014) [31], Shigeoka (2014) [45], Shigeoka (2016) [46], Ito and Sallee (2018) [29], Oizer (2018) [39], etc.

Anderson (2018, AER)

- Anderson (2018) [2] examines causal relationship between legal systems and female HIV infection rates in sub-Saharan Africa.
- RD, motivated by Dell (2010) [19]:
  - As-if random borders can mitigate an endogeneity that emerges within ethnicities.
- Result 1 (HIV positive rates)
  - Female: common law countries $>$ civil law countries
  - Male: no significant difference
- Result 2 (Contraception use)
  - Female: common law countries $<$ civil law countries
  - Male: common law countries $<$ civil law countries
- Common Law $\Rightarrow$ Female bargaining power $\downarrow$
  $\Rightarrow$ Negotiation for safe sex practices $\times$ $\Rightarrow$ HIV prevalence $\uparrow$

Split Ethnic Groups with Different Legal Origins



Specification

- Model:

$$Y_{rcepi} = \alpha_0 + \alpha_1 L_{rc} + \alpha_2 X_{rc} + \alpha_3 X_{rcep} + \alpha_4 X_{rcepi} + f(BD_{rcep}) + \delta_e + \gamma_r + \lambda_t + \epsilon_{rcepi}$$

- Subscripts: region, country, ethnic homeland, pixel
- Y_{rcepi} : an outcome of interest
- L_{rc} : common law legal system indicator
- $X_{rc}, X_{rcep}, X_{rcepi}$: vectors of controls
- $f(BD_{rcep})$: a second-order RD polynomial of the distance from the centroid of pixel to the nearest national border with different legal origins
- δ_e, γ_r : fixed effects w.r.t. ethnicity and region, respectively
- ϵ_{rcepi} : clustered at the ethnicity and country level
- λ_t : years of survey

HIV Prevalence Rates

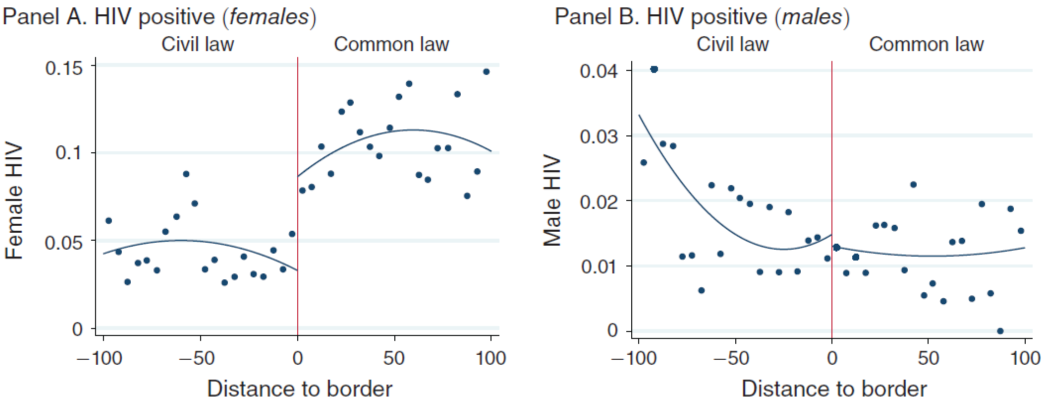


FIGURE 2. HIV POSITIVE

HIV Prevalence Rates

TABLE 1—HIV POSITIVE: FEMALES AGED 15–49

Variable	Whole sample $\leq$ 200 km	$\leq$ 150 km	$\leq$ 100 km	Non-Muslim Non-Polygynous $\leq$ 100 km	Muslim Polygynous $\leq$ 100 km
Common law	0.016	0.018	0.019	0.016	0.007
	(0.006)	(0.006)	(0.007)	(0.006)	(0.013)
Observations	118,903	99,511	77,336	55,507	21,829

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include: country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

TABLE 2—HIV POSITIVE: MALES AGED 15–49

Variable	Whole sample $\leq$ 200 km	$\leq$ 150 km	$\leq$ 100 km	Non-Muslim Non-Polygynous $\leq$ 100 km	Muslim Polygynous $\leq$ 100 km
Common law	0.001	0.001	–0.001	–0.003	0.002
	(0.006)	(0.005)	(0.005)	(0.005)	(0.01)
Observations	50,754	40,780	31,189	24,261	6,928

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include: country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

Protective Contraception

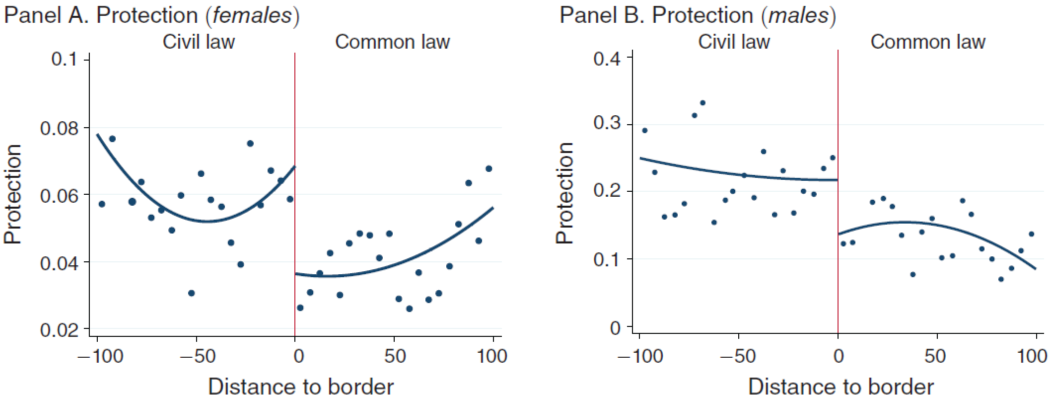


FIGURE 3. PROTECTION

Protective Contraception

TABLE 3—PROTECTIVE CONTRACEPTION: FEMALES AGED 15–49

	Whole sample			Non-Muslim Non-Polygynous	Muslim Polygynous
Variable	$\leq$ 200 km	$\leq$ 150 km	$\leq$ 100 km	$\leq$ 100 km	$\leq$ 100 km
Common law	−0.018	−0.019	−0.019	−0.024	−0.008
	(0.006)	(0.006)	(0.007)	(0.01)	(0.007)
Observations	117,263	97,285	76,698	55,261	21,437

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

TABLE 4—PROTECTIVE CONTRACEPTION: MALES AGED 15–49

	Whole sample			Non-Muslim Non-Polygynous	Muslim Polygynous
Variable	$\leq$ 200 km	$\leq$ 150 km	$\leq$ 100 km	$\leq$ 100 km	$\leq$ 100 km
Common law	−0.07	−0.07	−0.07	−0.08	−0.003
	(0.02)	(0.02)	(0.02)	(0.02)	(0.02)
Observations	81,873	67,887	52,902	46,016	6,886

Notes: Standard errors are clustered at the ethnic and country level using the approach of Cameron, Gelbach, and Miller (2011). All estimations include country, individual, and pixel controls; region fixed effects; ethnic fixed effects; second-order RD polynomial of distance to national border; and the year of the survey. Refer to the online Appendix for details on the data.

Kawai, Nakabayashi, Ortner and Chassang (2023, REStud)

- Cartels participating in procurement auctions frequently use bid rotation or prioritize incumbents to allocate contracts.
- However, establishing a link between observed allocation patterns and firm conduct has been difficult: Cartel? Cost-based competition?
- Data-driven screens to flag suspicious firm conduct:
  - Under competition, differences in backlog or incumbency status between close winners and close losers should vanish.
  - Under cartels, bids are generated by collusive bidding. Then, the differences in these variables between close winners and close losers need not disappear.
  - Their tests of non-competitive behaviour seek to detect discontinuities in the distribution of economically relevant covariates around close winners and close losers.
- Empirical examples:
  - Ohio milk auctons (Porter and Zona, 1999 [42])
  - Auctions for construction projects let by municipalities in Tohuku, Japan

Collusion? Competition?

- Bidders with low levels of backlog (firms that have not won many auctions in the recent past) are more likely to win than bidders with high levels of backlog.
- Difficulty in discriminating between competitive and non-competitive bid rotation and incumbency patterns:
  - Suppose that firms' procurement costs are increasing with backlog.
  - Even if firms are competitive, on average, firms with lower backlog will have lower costs and be more likely to win an auction than firms with higher backlog.
  - In this environment, a test seeking to detect collusive bid rotation by comparing the unconditional backlog of winners and losers would yield false positives.

How to Detect?

- Kawai et al. (2023) [30] propose to compare the backlog of a selected group of firms: bidders that win or lose by a small margin.
- Under competition, no firm can consistently be a marginal winner or a marginal loser. Winning or losing should be as-if-random conditional on close bids. As a result, close winners and losers should be statistically similar.
- Under collusion, close winners have consistently lower levels of backlog than close losers, which is evidence of collusive bid rotation.

Regression Discontinuity Approach (based on CCT2014)

- Let $\Delta_{i,t} \equiv b_{i,t} - \wedge b_{-i,t}$ denote the difference between the bid of firm i, and the most competitive alternative bid at time t.
- If $\Delta_{i,t} < 0$, bidder i wins the auction; if $\Delta_{i,t} > 0$, bidder i loses.
- Let $x_{i,t}$ be a measure (observed by the econometrician) of firm i's backlog before bidding at time t (alternatively it could be incumbency, or another relevant covariate).
- Define β the difference in average backlog between close losers and close winners:

$$\beta = \lim_{\epsilon \downarrow 0^+} \mathbb{E}[x_{i,t}|\Delta_{i,t} = \epsilon] - \lim_{\epsilon \uparrow 0^-} \mathbb{E}[x_{i,t}|\Delta_{i,t} = \epsilon] \tag{6}$$

- Test the null: $\mathbb{H}_0 : \beta = 0$.
  - When x denotes backlog, we expect β to be strictly positive under bid rotation.
  - When x denotes incumbency status, we expect β to be strictly negative if the cartel allocates market shares according to incumbency.
- Reject $\mathbb{H}_0 \implies$ Reject "competition" (some evidence of collusion)

Japanese Procurement Auction (Tohoku Region)

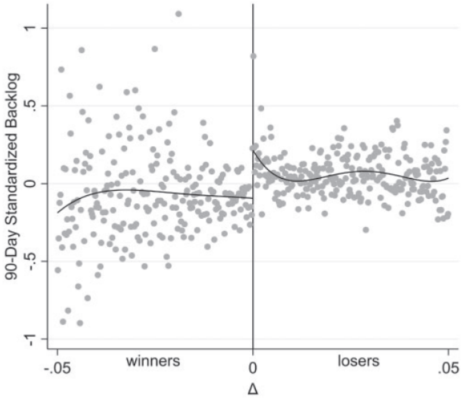


FIGURE 1

Binned scatter plot of standardized backlog, Japanese municipal auctions.

Notes: For each firm i and auction t, the standardized backlog of firm i at t is the Yen denominated amount of work it won in the 90 days prior to auction t, re-expressed in units of standard deviation from the firm's time-series average. The figure is a binned scatter plot of this measure against $\Delta_{i,t}$. See Section 5 for details.

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