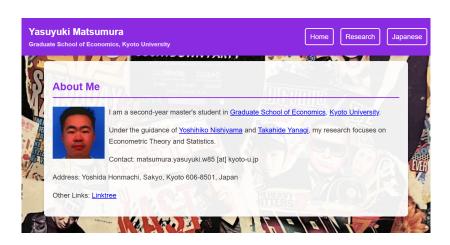
## Nonparametric Density Estimation

Sections 17.1-17.8 of Hansen (2022)

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https://yasu0704xx.github.io



### Introduction

- As a general rule, density functions can take any shape. They
  are inherently nonparametric and cannot be described by a
  finite set of parameters.
- That is, functional and/or distributional specifications relied on when estimating density functions may be incorrect.
- If we assume that such specifications are "true," we might obtain incorrect empirical conclusions.
- Thus, it would be disirable if we develop estimation procedures without requiring functional and/or distributional specifications.
- Nonparametric kernel methods achieve such a goal.

### Setup

- Here we review Sections 17.1-17.8 of Hansen (2022) [1].
- We proceed with a discussion of how to estimate the probability density function (PDF) of a real-valued random variable X for which we have n IID observations  $X_1, \dots X_n$ .
- We assume that X has a continuous density f(x).
- The goal is to estimate f(x) either at a single point x or a set of points in the interior of the support of X.

### References

- Excellent textbooks on nonparametric density estimation include Silverman (1986) [5] and Scott (1992) [4].
- The following textbooks are often refered:
  - Silverman (1986) [5],
  - Scott (1992) [4],
  - van der Vaart (1998, Chapter 24) [6],
  - Pagan and Ullah (1999, Chapter 2) [3], and
  - Li and Racine (2007, Chapter 1) [2].
- 日本語の文献:
  - 西山・人見 (2023, 第1章) [3]
  - 末石 (2015, 第9章) [2]
  - 清水 (2023, 第5章) [1]

Histogram

Kernel Density Estimator

Bias

Variance

Variance Estimation and Standard Errors

Integrated Mean Squared Error (IMSE)

Optimal Kernel

Refernces

# Histogram

# **Kernel Density Estimator**

### **Bias**

### **V**ariance

# Variance Estimation and Standard Errors

# Integrated Mean Squared Error (IMSE)

# **Optimal Kernel**

### Refernces

### References i

- Hansen, B. E. (2022). *Probability and Statistics for Economists*. Princeton.
- Li, Q. and J. S. Racine (2007). *Nonparametric Econometrics:* Theory and Practice. Princeton.
- Pagan, A. and A. Ullah (1999). *Nonparametric Econometrics*. Cambridge.
- Scott, D. W. (1992). Multivariate Density Estimation: Theory, Practice, and Visualization. Wiley.
- Silverman, B. W. (1986). Density Estimation for Statistics and Data Analysis. Chapman and Hall.

### References ii



an der Vaart, A. W. (1998). Asymptotic Statistics. Cambridge.

### 日本語の文献

- 清水泰隆 (2023)『統計学への漸近論, その先は』内田老鶴圃.
- 末石直也 (2015) 『計量経済学:ミクロデータ分析へのいざない』日本評論社.
- 西山慶彦, 人見光太郎 (2023) 『ノン・セミパラメトリック統計解析』共立出版.