Regression Discontinuity Designs

Hansen (2022, Chapter 21)

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Last Updated: June 3, 2025

https://yasu0704xx.github.io

Introduction

- Regression discontinuity designs (RDDs) are quasi-experimental designs which allow researchers to identify the causal effect of endogenous treatment on an outcome based on discontinuous policy rules.
- Local randomization is a key idea.
  - Consider a certain discontinuous rule under which treatment (e.g. college scholarship) is determined by whether a continuous covariate (e.g. admission score) is greater than a known threshold.
  - If all factors determined prior to the treatment are balanced just above and just below the threshold, the average causal effect can be estimated by comparing the mean outcome just above the threshold with that just below the threshold.

Literature

- Here we review Chapter 21 of Hansen (2022) [25].
- Excellent reviews/textbooks on regression discontinuity designs include Abadie and Cattaneo (2018) [1], and Cattaneo, Idrobo and Titiunik (2021, 2024) [13] [14].
- The common software package is `rdrobust` by Calonico, Cattaneo, Farrell and Titiunik.
- 日本語の文献：
  - 川口・澤田（2024）[47]
  - 末石（2024）[48]
  - 高野（2025）[49]

Contents

Identification

Setup: Rubin Causal Model

- $Y_i(d) \in \mathbb{R}$, $d \in \{0, 1\}$: potential outcome
- Y_i : observed outcome
- $D_i \in \{0, 1\}$: treatment, which may be endogenous in that some unobserved factors may affect both D_i and Y_i.
- $\theta = Y_i(1) - Y_i(0)$: treatment effect for an individual
  $\Rightarrow$ We cannot identify θ without restrictive assumptions, because either $Y_i(1)$ or $Y_i(0)$ is unobservable.
- Instead, we are often interested in causal parameters such as ATE$= \mathbb{E}[Y_i(1) - Y_i(0)]$, ATT$= \mathbb{E}[Y_i(1) - Y_i(0)|D = 1]$, etc.

Sharp RD

- Suppose that treatment is determined by

$$D_i = 1(X_i \geq c), \tag{1}$$

  where the cut-off c is determined by policy or rule and common to all individuals.
- The covariate X_i is called the score, forcing variable, running variable, assignment variable, etc.
- In a standard RD setting, X_i is assumed to be continuously distributed on a subset of $\mathbb{R}$.

Identification in Sharp RD

Assume that, for each $d \in \{0,1\}$,

$$\mathbb{E}[Y_i(d)|X_i = x] \text{ is continuous at } x = c. \tag{2}$$

Under Assumptions (1) and (2), the average causal effect at the cutoff point $\tau_{SRD} \equiv \mathbb{E}[Y_i(1) - Y_i(0)|X_i = c]$ *is identified by*

$$\tau_{SRD} = \lim_{x \downarrow c} \mathbb{E}[Y_i|X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i|X_i = x], \tag{3}$$

where $\lim_{x \downarrow c}$ *and* $\lim_{x \uparrow c}$ *denote the right and left limits at* $x = c$*, respectively.*

Proof By construction,

$$Y_i = Y_i(0) \cdot 1(X_i < c) + Y_i(1) \cdot 1(X_i \geq c).$$

Taking expectations conditional on $X_i = x$, we obtain

$$\begin{aligned}
&\mathbb{E}[Y_i|X_i = x] \\
&= \mathbb{E}[Y_i(0)|X_i = x]1(X_i < c) + \mathbb{E}[Y_i(1)|X_i = x]1(X_i \geq c)
\end{aligned}$$

Since $\mathbb{E}[Y_i(0)|X_i = x]$ and $\mathbb{E}[Y_i(1)|X_i = x]$ are continuous at $x = c$, they are identified by

$$\mathbb{E}[Y_i(0)|X_i = x] = \lim_{x \uparrow c} \mathbb{E}[Y_i|X_i = x],$$
$$\mathbb{E}[Y_i(1)|X_i = x] = \lim_{x \downarrow c} \mathbb{E}[Y_i|X_i = x],$$

which completes the proof.

- Assumption (2) means that the conditional expectation of the untreated and treated outcome are continuously affected by the running variable.
- It is implied that the distributions of confounding factors, including observable covariates determined prior to the treatment, are balanced near the cutoff.
- In particular, there should be no policy/legal/experimental changes at the cutoff, except for the treatment assignment.
- Counterfactual: The continuity of $\mathbb{E}[Y_i(d)|X_i = x]$ at $x = c$ cannot be directly examined, since $Y_i(0)$ and $Y_i(1)$ are unobservable under $X_i \geq c$ and $X_i < c$, respectively.

Fuzzy RD

- In the fuzzy RD, D_i is partially determined by whether X_i is no less than a known fixed cutoff c, such that

$$\lim_{x \downarrow c} \mathbb{E}[D_i|X = x] \neq \lim_{x \uparrow c} \mathbb{E}[D_i|X = x],$$

  where $\lim_{x \downarrow c}$ and $\lim_{x \uparrow c}$ denote the right and left limits at $x = c$, respectively.
- Notice that $\mathbb{E}[D_i|X_i] = \mathbb{P}(D_i = 1|X_i)$.

Identification in Fuzzy RD

- Define $Z_i = 1(X_i \geq c)$.
- Let $D_i(z), z \in \{0, 1\}$ be the potential treatment status when $Z_i = z$. By construction, $D = Z_i D_i(1) + (1 - Z_i) D_i(0)$.
- Consider the following causal parameter for the "compliers:"

$$\tau_{\text{FRD}} \equiv \mathbb{E}[Y_i(1) - Y_i(0) \mid D_i(1) > D_i(0), X_i \in \{c - \epsilon, c + \epsilon\}].$$

- Under several assumptions, τ_{FRD} can be identified by the local Wald estimand:

$$\tau_{\text{FRD}} = \frac{\lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X = x]}. \tag{4}$$

- The arguments are quite similar to the identification of LATE parameter in the IV estimations (so skiped in the class).
- See Hahn, Todd and van der Klaauw (2001) [24], Dong (2018) [20], and Hansen (2022, Sections 21.10-11) [25] for details.

Estimation

Identification Results

- Recall that the causal parameters τ_{SRD} and τ_{FRD} are identified respectively by (3) and (4):

$$\tau_{\mathsf{SRD}} = \lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x],$$
$$\tau_{\mathsf{FRD}} = \frac{\lim_{x \downarrow c} \mathbb{E}[Y_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[Y_i | X_i = x]}{\lim_{x \downarrow c} \mathbb{E}[D_i | X_i = x] - \lim_{x \uparrow c} \mathbb{E}[D_i | X = x]}.$$

- For expositional purpose, denote one of Y_i and D_i by A_i, and define the following one-sided limits:

$$\mu_A^+ \equiv \lim_{x \downarrow c} \mathbb{E}[A_i | X_i = x], \quad \mu_A^- \equiv \lim_{x \uparrow c} \mathbb{E}[A_i | X_i = x].$$

- For estimating τ_{SRD} and τ_{FRD}, it suffices to estimate $\mu_Y^+, \mu_Y^-, \mu_D^+$, and μ_D^-.

Estimation Procedures: Local Polynomial Regressions

- The quantities $\mu_Y^+, \mu_Y^-, \mu_D^+$, and μ_D^- are commonly estimated by local polynomial regressions (LPRs).
- As we study before, the local constant (Nadaraya-Watson) regression causes the boundary bias. LPRs can circumvent such boundary bias (Hahn, Todd and van der Klaauw, 2001 [24]; Porter, 2003 [41]).
- A recommended choice of local polynomial order p is 1 or 2. [1]
- On the other hand, there are several recommendations on bandwidth selection, which we study later.

[1] Gelman and Imbens (2019) [22] argue that controlling for global high-order polynomials in regression discontinuity analysis is a flawed approach with three major problems: it leads to noisy estimates, sensitivity to the degree of the polynomial, and poor coverage of confidence intervals.

Estimation of τ_{SRD} and τ_{FRD}

- Consider the following p-th order LPRs:

$$\hat{\beta}_A^+ \equiv \arg\min_{\beta\in\mathbb{R}^{p+1}} \sum_{i=1}^{n} 1(X_i \geq c)\left(A_i - [r_p(X_i - c)]^T\beta\right)^2 K\left(\frac{X_i - c}{h}\right),$$

$$\hat{\beta}_A^- \equiv \arg\min_{\beta\in\mathbb{R}^{p+1}} \sum_{i=1}^{n} 1(X_i < c)\left(A_i - [r_p(X_i - c)]^T\beta\right)^2 K\left(\frac{X_i - c}{h}\right),$$

where $p \geq 1$, $r_p(x) \equiv (1, x, \cdots, x^p)$ is a vector of polynomials, $h > 0$ is bandwidth, and $K(\cdot)$ is a kernel function.

- The LPR estimators of μ_A^+ and μ_A^- are the first elements of $\hat{\beta}_A^+$ and $\hat{\beta}_A^-$. respectively:

$$\hat{\mu}_A^+ \equiv e_1^T \hat{\beta}_A^+, \quad \hat{\mu}_A^- \equiv e_1^T \hat{\beta}_A^-,$$

where $e_1 \equiv (1, 0, \cdots, 0)^T$ is the first unit vector.

- The causal parameters τ_{SRD} and τ_{FRD} can be estimated respectively by

$$\hat{\tau}_{\mathsf{SRD}} = \hat{\mu}_Y^+ - \hat{\mu}_Y^-, \quad \hat{\tau}_{\mathsf{FRD}} = \frac{\hat{\mu}_Y^+ - \hat{\mu}_Y^-}{\hat{\mu}_D^+ - \hat{\mu}_D^-}.$$

Bandwidth Selection

- Imbens and Kalyanaraman (2012) [27] propose the (A)MSE optimal bandwidth, which satisfies that

$$h_{\text{IK2012}} \propto n^{-\frac{1}{2p+3}}.$$

- If we are intersted in the point estimations of μ_Y^+ and μ_Y^-, it is sufficient to select IK2012's bandwidth.
- In practice, however, we are also interested in inference: IK2012's MSE optimal bandwidth does not satisfy that $nh^5 \to 0$. Then, we cannot eliminate the asymptotic bias.
- How to select bandwidth?
    1. Undersmoothing
    2. Robust bias-corrected inference (CCT2014 [9], CCF2020 [8])
    3. Uniformly honest inference (KR2018 [32])

Inference

Undersmoothing

- A naive solution to address asymptotic bias is to use undersmoothing.
- By using undersmoothing bandwidth such that $\sqrt{nh}h^{p+1} \to 0$, the standard t statistic satisfies

$$\frac{\hat{\tau}_{\mathsf{SRD}} - \tau_{\mathsf{SRD}}}{\sqrt{\frac{\mathcal{V}_{\mathsf{SRD}}}{nh}}} \xrightarrow{d} \text{Normal}(0,1).$$

Robust Bias-Corrected Inference

Robust bias-corrected:

Calonico, Cattaneo and Titiunik (2014) [9]

Calonico, Cattaneo and Farrell (2020) [8]

Uniformly Honest Inference

Unifromly honest CI:
Kolesar and Rothe (2018) [32]

Covariates

Notes

Covariate-adjust

Calonico, Cataneo, Farrell and Titiunik (2019) [10]

Noack, Olma and Rothe (2025) [38]

High-dimensional covariates

Kreiss and Rothe (2023) [33]

Arai, Otsu and Seo (2024) [5]

Review: Chernozhukov et al. (2025) [16]

Falsification Test

Identification Assumptions

Theorem 21.1 (re)

Assume that treatent is assigned as $D = 1(X \geq c)$. Suppose that $m_0(x)$ and $m_1(x)$ are continuous at $x = c$. Then,

$$\bar{\theta} = m(c+) - m(c-).$$

- Counterfactual: The continuity of $m_0(x)$ and $m_1(x)$ cannot be directly examined, since Y_0 and Y_1 are unobservable under $X \geq c$ and $X < c$, respectively.
- Instead, researchers often examine certain necessary conditions.
- No manipulation: Under the above assumptions, it is necessary for the density of running variable X to be continuous at the cutoff point.

Manipulation Test

- Testing the continuity of the density of the assignment variable:
  - McCrary (2008) [36]
  - Otsu, Xu and Matsushita (2013) [40]
  - Cattaneo, Jansson and Ma (2020) [15]
  - Software packages by CJM2020: `lpdensity`, `rddensity`
  - Imai and Okamoto (2024) [26]
- Testing the continuity of the conditional distribution of covariates:
  - Lee (2008) [34]
  - Canay and Kamat (2018) [11]
  - Fusejima, Ishihara and Sawada (2025) [21]

Placebo Test

- Take several placebo cutoff points.
- At a placebo cutoff point $c' < c$, researchers can observe whether the dendity of untreated potential outcome Y_0 is continuous or not.
- Similarly, at $c' > c$, they can observe whether the density of treated potential outcome Y_1 is continuous or not.
- If the continuity is observed, then there arises some plausibility of the identification assumptions in Theorem 21.1.
- Howevere, it is just a plausibility. Note that the continuity at placebo cutoff points is neither necessary nor sufficient for the identification assumptions.

Caveats on Pretesting

- Recent works argue that, in general, researchers should not implement such pretesting.
  - Roth (2022) [42] : Pre-trend test
  - Sueishi (2023) [45] : Hausman test
- Pretesting analysis in the RD setting can be found in Section 5.2 of Fusejima, Ishihara and Sawada (2025) [21].

Practical Recommendation

Practical Recommendation in Sharp RD

- The polynomial order p of LPRs should be 1 or 2.
- For point estimation, use IK2012's (A)MSE optimal bandwidth.
- For inference, use CCT2014 & CCT2020's bandwidth or KR2018's bandwidth.
- The common package `rdrobust` is equipped with point estimation relying on IK2012 and robust bias-corrected inference by CCT2014 & CCF2020.
- `RDHonest` provides KR2018's uniformly honest inference procedures.
- Observable covariates can be utilized to gain efficiency of the sharp RD estimator.
- Note that pretesting for Assumption (2) may distort the estimation and inference results.

Empirical Application

"Waiting for Life"

- The first study relying on RD can be found in Thistlehwaite and Cambell (1960) [46].
- As Cook (2008) [17] says that RD was "waiting for life," RD was not popular until 1999, the year when Angrist and Lavy (1999) [4] and Black (1999) [7] were published.
- Hahn, Todd, and Klaauw (2001) [24] formalize general RDDs and establish identification results for treatment effects.

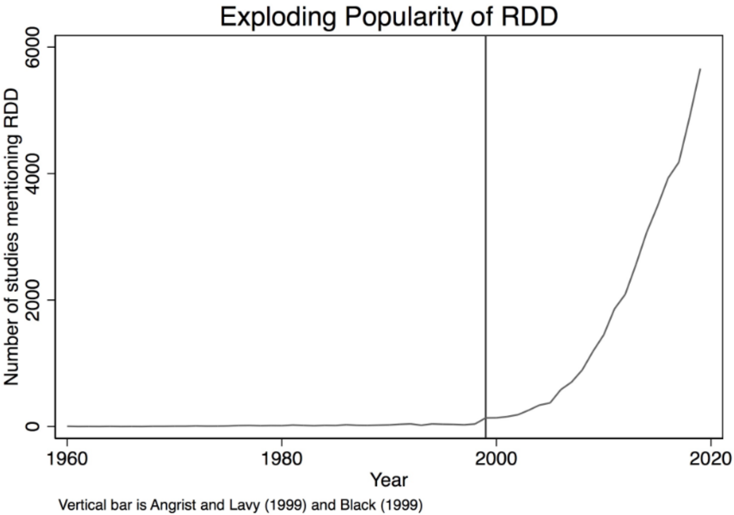

Figure 6.1 of Cunningham (2021) [18]

Recent Empirical RD Studies

Ludwig and Miller (2007) [35],
Lee (2008) [34],
Matsudaira (2008) [37],
Battistin et al. (2009) [6],
Carpenter, Christopher and Carlos Dobkin (2009) [12],
Greenstone, Hornbeck and Moretti (2010) [23],
Abdulkadiroglu, Angrist and Pathak (2014) [2],
Ito (2014) [28],
Kleven et al. (2014) [31],
Shigeoka (2014) [43],
Shigeoka (2016) [44],
Ito and Sallee (2018) [29],
Oizer (2018) [39],
Kawai et al. (2023) [30], etc.

Anderson (2018, AER)

- Anderson (2018) [3] examines causal relationship between legal systems and female HIV infection rates in sub-Saharan Africa.
- RD, motiveted by Dell (2010) [19]:
  - As-if random borders can mitigate an endogeneity emerged within ethnicity level.
- Result 1 (HIV positive rates)
  - Female: common law countries $>$ civil law countries
  - Male: no significant difference
- Result 2 (Contraception use)
  - Female: common law countries $<$ civil law countries
  - Male: common law countries $<$ civil law countries
- Common Law $\Rightarrow$ Female bargaining power $\downarrow$
  $\Rightarrow$ Negotiation for safe sex practices $\times$ $\Rightarrow$ HIV prevalence $\uparrow$

Split Ethnic Groups with Different Legal Origins

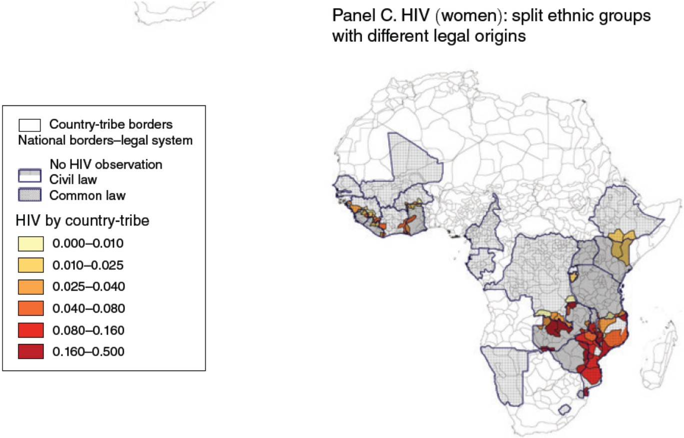

FIGURE 1. FEMALE HIV BY ETHNIC GROUP

Empirical Strategy & Main Result

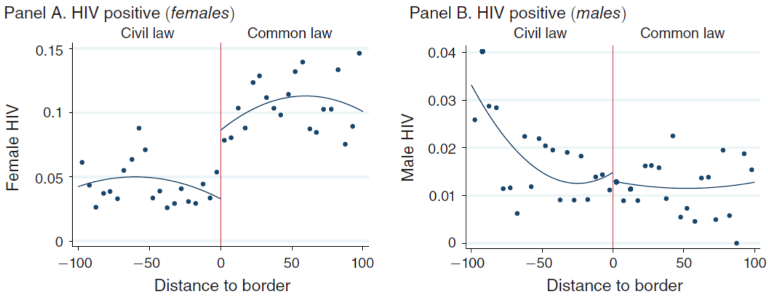

FIGURE 2. HIV POSITIVE

- Y_i = HIV Positive Rate (Proxy for Females Bargaining Power)
- D_i = Legal Origins (Civil Law vs Common Law)
- $X_i - c$ = Distance to Border
- As-if random borders of Scramble for Africa (1884)
  $\Longrightarrow$ Same cultures but different legal origins near a border!

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