# Semiparametric Single Index Models

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# Introduction

- A semiparametric single index model is given by

$$Y = g(X^T \beta_0) + u,$$

where

$Y \in \mathbb{R}$ : a dependent variable,
$X \in \mathbb{R}^q$ : a $q \times 1$ explanatory vector,
$\beta_0 \in \mathbb{R}^q$ : a $q \times 1$ vector of unknown parameters,
$u \in \mathbb{R}$ : an error term which satisfies $\mathbb{E}(u \mid X) = 0$,
$g(\cdot)$ : an unknown function.

# Introduction

- Even though $x$ is a $q \times 1$ vector, the term $x^T \beta_0$ is a scalar of a single linear combination, which is called a single index.
- By the form of the single index model, we obtain

$$\mathbb{E}(Y \mid X) = g(X^T \beta_0),$$

  which means that the conditional expectation of $Y$ only depends on the vector $X$ through a single index $X^T \beta_0$.
- The model is SEMIPARAMETRIC when $\beta \in \mathbb{R}^q$ is estimated with the parametric methods and $g(\cdot)$ with the nonparametric methods.
- Some of the PARAMETRIC single index models are really familiar with us.

# Examples of Parametric Single Index Model

- If $g(\cdot)$ is the identity function, then the model turns out to be a linear regression model:

$$Y = g(X^T\beta_0) + u = X^T\beta_0 + u.$$

- If $g(\cdot)$ is the CDF of Normal$(0, 1)$, then the model turns out to be a probit model.
- If $g(\cdot)$ is the CDF of logistic distribution, then the model turns out to be a logistic regression model.

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- このほかに，ECON 718 NonParametric Econometrics (Bruce Hansen, Spring 2009, University of Wisconsin-Madison) や，セミノンパラメトリック計量分析（末石直也，2014 年度後期，京都大学大学院経済学研究科）のレクチャーノートを参照した.