10/28 (月)

① $E(\hat{m}(y.x))$ の評価.

○ $E(\hat{f}(y.x)) = f(y.x)$
$$+ \frac{1}{2} K_2 \sum_{s=1}^{q} h_s^2 f_{ss}(y.x)$$
$$+ \sum_{s=1}^{r} \lambda_s \left(\frac{1}{c_s - 1} \sum_{u^d \in D} \mathbb{1}_s(u^d, z^d) f(z^c, u^d) \right)$$
$$+ o\left(\sum_{s=1}^{q} h_s^2 + \sum_{s=1}^{r} \lambda_s \right)$$

(5.6), (5.7), (5.8) と同様に計算する.

○ $E(\hat{\mu}(x)) = \mu(x)$
$$+ \frac{1}{2} K_2 \sum_{s=q_y+1}^{q} h_s^2 \mu_{ss}(x) \times \frac{\mu(x)}{\mu(x)}$$
$$+ \sum_{s=r_y+1}^{r} \lambda_s \left(\frac{1}{c_s - 1} \sum_{u^d \in D} \mathbb{1}_s(u^d, x^d) \mu(x^c, u^d) \right) \times \frac{\mu(x)}{\mu(x)}$$
$$+ o\left(\sum_{s=q_y+1}^{q} h_s^2 + \sum_{s=r_y+1}^{r} \lambda_s \right)$$

$\Longrightarrow E(\hat{\mu}(x) g(y|x))$
$$= f(y.x)$$
$$+ \frac{1}{2} K_2 \sum_{s=q_y+1}^{q} h_s^2 \frac{\mu_{ss}(x)}{\mu(x)} f(y.x)$$
$$+ \sum_{s=r_y+1}^{r} \lambda_s \frac{1}{c_s - 1} \sum_{u^d \in D} \mathbb{1}_s(u^d, x^d) \frac{\mu(x^c, u^d)}{\mu(x)} f(y.x)$$
$$+ o\left(\sum_{s=q_y+1}^{q} h_s^2 + \sum_{s=r_y+1}^{r} \lambda_s \right)$$

○ $E(\hat{m}(y.x)) = E(\hat{f}(y.x)) - E(\hat{\mu}(x) g(y|x))$
$$= \sum_{s=1}^{q} h_s^2 B_{1s}(z) + \sum_{s=1}^{r} \lambda_s B_{2s}(z) + o\left(\sum_{s=1}^{q} h_s^2 + \sum_{s=1}^{r} \lambda_s \right)$$

② $Var(\hat{m}(y.x))$ の評価

○ $Var(\hat{m}(y.x)) = \dfrac{1}{n h_1 \cdots h_q} \times K^q \times f(y.x) + (\text{small order})$

③ $\hat{m}(y.x)$ の漸近分布

$h_s \sim n^{-\frac{1}{q+4}}$, $\lambda_s \sim n^{-\frac{2}{q+4}}$ とする。Liapunov CLT より、

$$\sqrt{n h_1 \cdots h_q}\left(\hat{m}(y.x) - \left[\sum_{s=1}^{q} h_s^2 B_{1s}(z) + \sum_{s=1}^{r} \lambda_s B_{2s}(z)\right]\right)$$

$$\xrightarrow{d} N\left(0,\ K^q \times f(y.x)\right)$$

④ $\hat{\mu}(x) \xrightarrow{p} \mu(x)$ の収束レート

$$\hat{\mu}(x) - \mu(x) = O_p\left(\sum_{s=q_y+1}^{q} h_s^2 + \frac{1}{\sqrt{n h_{q_y+1} \cdots h_q}}\right)$$

⑤ $\hat{g}(y|x)$ の漸近分布

$$\sqrt{n h_1 \cdots h_q}\left(\hat{g}(y|x) - g(y|x) - \left[\sum_{s=1}^{q} h_s^2 B_{1s}(z) + \sum_{s=1}^{r} \lambda_s B_{2s}(z)\right]\right)$$

$$= \frac{\sqrt{n h_1 \cdots h_q}\left\{\left(\hat{g}(y|x) - g(y|x)\right)\hat{\mu}(x) - \hat{\mu}(x)\left[\sum_{s=1}^{q} h_s^2 B_{1s}(z) + \sum_{s=1}^{r} \lambda_s B_{2s}(z)\right]\right\}}{\hat{\mu}(x)}$$

$$= \frac{\sqrt{n h_1 \cdots h_q}\left\{\hat{m}(y.x) - \mu(x)\left[\sum_{s=1}^{q} h_s^2 B_{1s}(z) + \sum_{s=1}^{r} \lambda_s B_{2s}(z)\right]\right\}}{\mu(x)} + op(1)$$

True $\mu(x)$

$$\xrightarrow{d} \frac{1}{\mu(x)} \times N\left(0,\ K^q \times f(y.x)\right)$$

$$= N\left(0,\ \frac{K^q f(y.x)}{\mu^2(x)}\right)$$

$$= N\left(0,\ \frac{K^q g(y|x)}{\mu(x)}\right)$$