

# Taylor, Loftus and Tibshirani (2016, AoS)

Matsumura & Tachibana (forthcoming) のための勉強会

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[yasu0704xx.github.io](https://yasu0704xx.github.io)

## INFERENCE IN ADAPTIVE REGRESSION VIA THE KAC–RICE FORMULA

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We derive an exact  $p$ -value for testing a global null hypothesis in a general adaptive regression setting. Our approach uses the Kac–Rice formula [as described in *Random Fields and Geometry* (2007) Springer, New York] applied to the problem of maximizing a Gaussian process. The resulting test statistic has a known distribution in finite samples, assuming Gaussian errors. We examine this test statistic in the case of the lasso, group lasso, principal components and matrix completion problems. For the lasso problem, our test relates closely to the recently proposed covariance test of Lockhart et al. [*Ann. Statist.* (2004) **42** 413–468].

In a few specific settings, our proposed tests will be less powerful than other previously known (and well-established) tests. However, it should be noted that the real strength of our proposal here is its generality. We provide a framework for constructing valid tests across a wide class of regularized regression problems, and as far as we can tell, such a unified view was not possible before this work.

## Taylor, Loftus and Tibshirani (2016, AoS)

- Taylor, Loftus and Tibshirani (2016) derive an exact  $p$ -value for testing a global null hypothesis in a general **adaptive regression** setting and provide a framework for constructing valid tests across a wide class of **regularized regression** problems.
- Taylor, Loftus and Tibshirani (2016) use the **Kac-Rice formula**<sup>1</sup> applied to the problem of maximizing a Gaussian process.
- The resulting test statistic has a known distribution in finite samples, assuming Gaussian errors.
- Taylor, Loftus and Tibshirani (2016) examine this test statistic in the case of the lasso, group lasso, principal components and matrix completion problems.<sup>2</sup>

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<sup>1</sup>as described in *Random Fields and Geometry* (2007, Springer, New York)

<sup>2</sup>For the lasso problem, the test relates closely to the covariance test of Lockhart, Taylor, Tibshirani and Tibshirani (2014).

Introduction

General Regularized Regression Problems

Characterization of the Global Maximizer

Kac-Rice Formulae for the Global Maximizer and its Value

Practicalities and Examples

Discussion

# Introduction

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# General Regularized Regression Problems

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## Characterization of the Global Maximizer

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## Practicalities and Examples

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## Discussion

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