EE1390 - Matrix Project

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Question

Q.15 from JEE Problems in Linear Algebra 2D

Find the eccentricity of an ellipse having centre at the origin, axes along the coordinate axes and passing through the points

$$P = \begin{pmatrix} 4 \\ -1 \end{pmatrix}, Q = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

Solution in matrix form

Given points P, Q which passes through the ellipse

$$P = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\text{Q}= \begin{pmatrix} -2 \\ 2 \end{pmatrix}$$

The equation of the ellipse in matrix form with origin as the centre.

$$X^T V X = 1$$

where

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$V = \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix}$$

Solution contd...

$$\begin{pmatrix} 4 & -1 \end{pmatrix} \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \begin{pmatrix} 4 \\ -1 \end{pmatrix} = 1$$

$$\begin{pmatrix} -2 & 2 \end{pmatrix} \begin{pmatrix} \frac{1}{a^2} & 0 \\ 0 & \frac{1}{b^2} \end{pmatrix} \begin{pmatrix} -2 \\ 2 \end{pmatrix} = 1$$

Solution contd..

By solving the matrix equation, we get

$$a^2 = 20$$

$$b^2 = 5$$

$$\text{Eccentricity of the ellipse } e = \sqrt{1 - \frac{b^2}{a^2}}$$

$$= \frac{\sqrt{3}}{2}$$

Plot