

# Supplementary Material

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## Contact evolution without contact area correction

The contact area fraction calculated as  $A^h/A_0 = N_p/N^2$  without correcting the contact area near the perimeter of the contact zones is used to obtain the contact evolution as the external load is increased. The parameters in the contact evolution law corresponding to Cauchy, Dagum, and self-affine Cauchy surfaces are given in Table 1, Table 2, and Table x respectively. The graphical results corresponding to Cauchy, Dagum, and self-affine Cauchy surfaces are plotted in Figure 1, Figure 2, and Figure 3 respectively. The results qualitatively remain the same as those corresponding to contact area correction. In particular, for both the Cauchy and Dagum surfaces, the contact area evolution is independent of the Hurst parameter but is non-monotonically dependent on the fractal Dimension. The  $\kappa$  parameter for all the cases without the contact area correction is beyond the analytical predictions and of course higher than the corresponding values with the contact area correction.

Table 1: Different combinations of  $D$  and  $H$  in Cauchy random fields.  $(\kappa, \mu, \beta)$  values are the parameters in the corresponding contact evolution law when the contact area is not corrected.

\* The error bounds for these values are too large to be reported here with confidence.

$D$	$H$	$\kappa$	$\beta$	$\mu$
2.1	0.5	$(8.15 \pm 0.359)\sqrt{2\pi}$	-63*	5.63*
2.3	0.5	$(3.27 \pm 0.113)\sqrt{2\pi}$	$0.09 \pm 0.008$	$0.27 \pm 0.028$
2.5	0.5	$(1.36 \pm 0.006)\sqrt{2\pi}$	$0.65 \pm 0.048$	$0.89 \pm 0.028$
2.7	0.5	$(1.51 \pm 0.026)\sqrt{2\pi}$	$0.48 \pm 0.027$	$0.55 \pm 0.021$
2.9	0.5	$(2.7 \pm 0.062)\sqrt{2\pi}$	$0.44 \pm 0.009$	$0.35 \pm 0.007$
2.5	0.1	$(1.27 \pm 0.014)\sqrt{2\pi}$	$3.88 \pm 2.82$	$1.67 \pm 0.289$
2.5	0.3	$(1.35 \pm 0.009)\sqrt{2\pi}$	$0.63 \pm 0.063$	$0.87 \pm 0.039$
2.5	0.7	$(1.37 \pm 0.054)\sqrt{2\pi}$	$0.56 \pm 0.365$	$0.87 \pm 0.249$
2.5	0.9	$(1.38 \pm 0.009)\sqrt{2\pi}$	$0.72 \pm 0.061$	$0.93 \pm 0.035$

Table 2: Different combinations of  $D$  and  $H$  in Dagum random fields.  $(\kappa, \mu, \beta)$  values are the parameters in the corresponding contact evolution law when the contact area is not corrected.

$D$	$H$	$\kappa$	$\beta$	$\mu$
2.5	0.5	$(1.33 \pm 0.005)\sqrt{2\pi}$	$0.95 \pm 0.068$	$1.03 \pm 0.029$
2.6	0.5	$(1.29 \pm 0.004)\sqrt{2\pi}$	$0.76 \pm 0.03$	$0.87 \pm 0.015$
2.7	0.5	$(1.43 \pm 0.008)\sqrt{2\pi}$	$0.62 \pm 0.018$	$0.68 \pm 0.011$
2.8	0.5	$(1.68 \pm 0.011)\sqrt{2\pi}$	$0.59 \pm 0.011$	$0.58 \pm 0.007$
2.9	0.5	$(2.14 \pm 0.02)\sqrt{2\pi}$	$0.5 \pm 0.008$	$0.44 \pm 0.005$
2.9	0.6	$(2.15 \pm 0.021)\sqrt{2\pi}$	$0.49 \pm 0.008$	$0.43 \pm 0.006$
2.9	0.7	$(2.18 \pm 0.02)\sqrt{2\pi}$	$0.48 \pm 0.007$	$0.43 \pm 0.005$
2.9	0.8	$(2.22 \pm 0.045)\sqrt{2\pi}$	$0.48 \pm 0.014$	$0.42 \pm 0.011$
2.9	0.9	$(2.33 \pm 0.322)\sqrt{2\pi}$	$0.49 \pm 0.09$	$0.41 \pm 0.067$

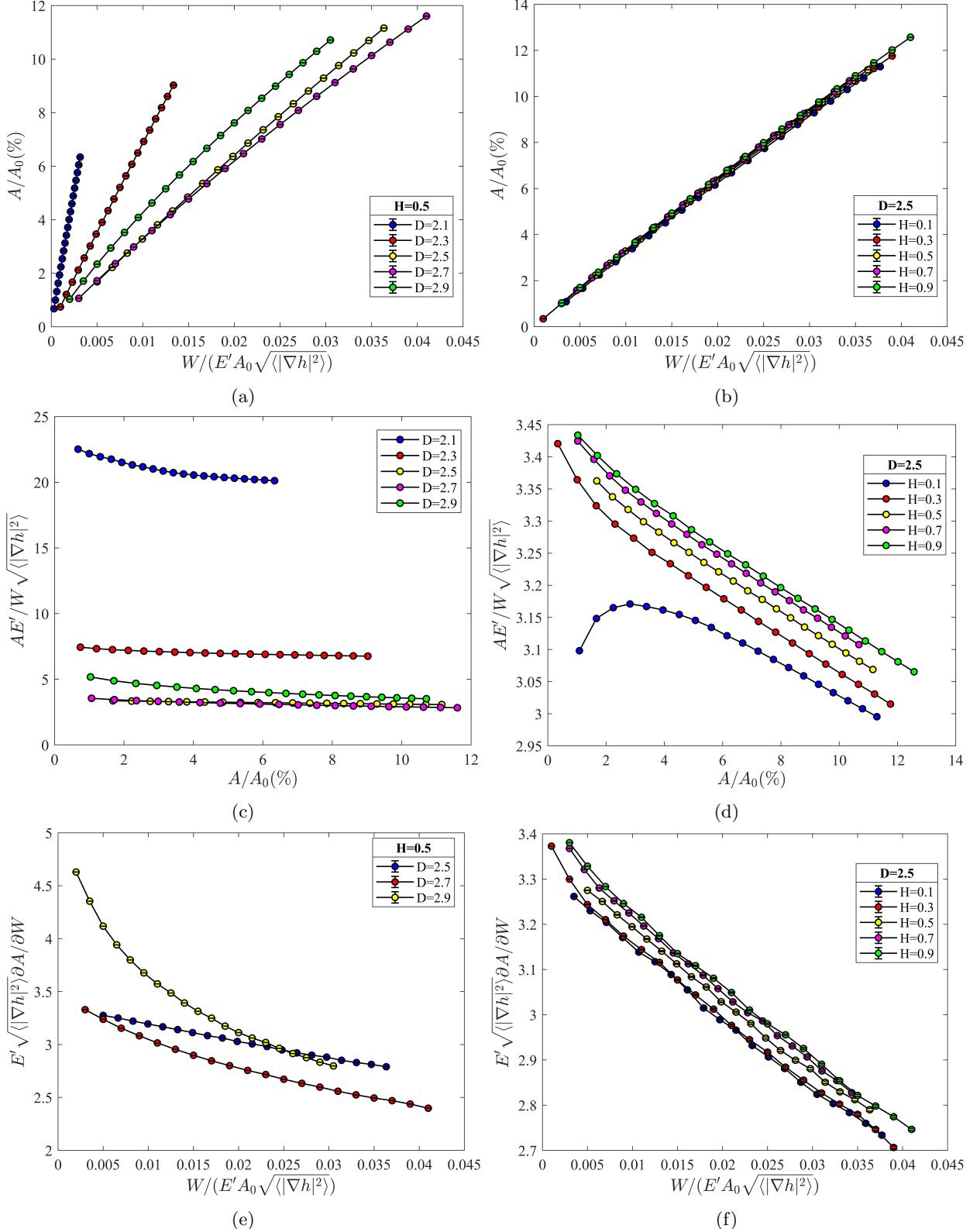


Figure 1: Contact area evolution on Cauchy random surfaces without contact area correction. The variation of true contact area fraction with normalized applied external load: (a)  $H = 0.5$  and varying  $D$  (b)  $D = 2.5$  and varying  $H$ . The secant of the contact area fraction versus the external load plots: (c)  $H = 0.5$  and varying  $D$  (d)  $D = 2.5$  and varying  $H$ . Slope of the contact area fraction versus the external load plots: (e)  $H = 0.5$  and varying  $D$  (f)  $D = 2.5$  and varying  $H$ .

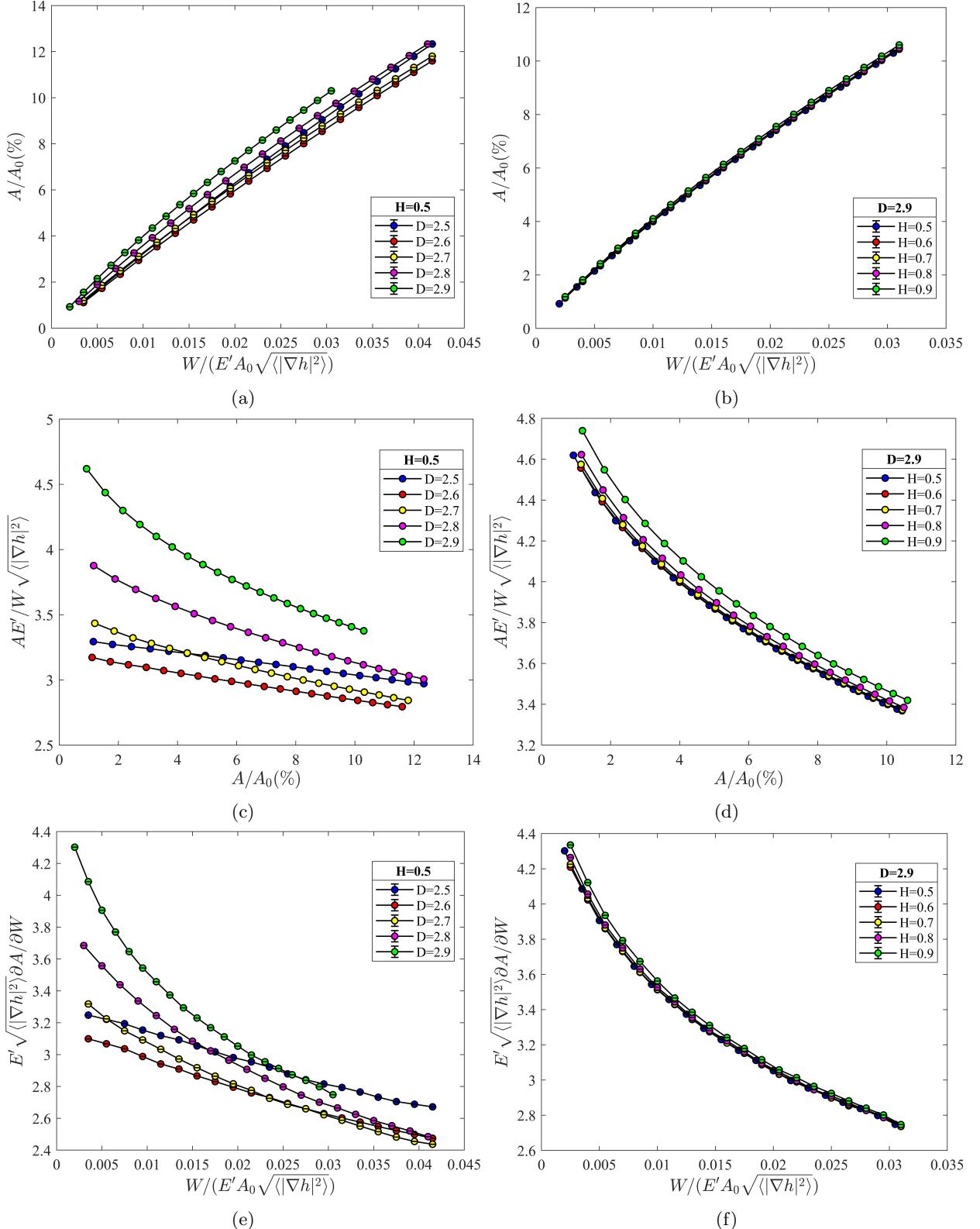


Figure 2: Contact area evolution on Dagum random surfaces without contact area correction. The variation of true contact area fraction with normalized applied external load: (a)  $H = 0.5$  and varying  $D$  (b)  $D = 2.9$  and varying  $H$ . The secant of the contact area fraction versus the external load plots: (c)  $H = 0.5$  and varying  $D$  (d)  $D = 2.9$  and varying  $H$ . Slope of the contact area fraction versus the external load plots: (e)  $H = 0.5$  and varying  $D$  (f)  $D = 2.9$  and varying  $H$ .

Table 3: Different combinations of  $D$  and  $H$  in self-affine ( $D = n + 1 - H$ ) Cauchy random fields.  $(\kappa, \mu, \beta)$  values are the parameters in the corresponding contact evolution law.  $RMSPE$  is the root mean square percentage error between the contact evolution fit and the simulation data when the contact area is not corrected.

$D$	$H$	$\kappa$	$\beta$	$\mu$
2.4	0.6	$(1.81 \pm 0.007)\sqrt{2\pi}$	$0.62 \pm 0.048$	$0.97 \pm 0.039$
2.5	0.5	$(1.36 \pm 0.006)\sqrt{2\pi}$	$0.65 \pm 0.048$	$0.89 \pm 0.028$
2.6	0.4	$(1.3 \pm 0.003)\sqrt{2\pi}$	$0.71 \pm 0.023$	$0.81 \pm 0.013$
2.7	0.3	$(1.5 \pm 0.005)\sqrt{2\pi}$	$0.6 \pm 0.015$	$0.65 \pm 0.01$
2.8	0.2	$(1.9 \pm 0.005)\sqrt{2\pi}$	$0.5 \pm 0.0042$	$0.48 \pm 0.004$

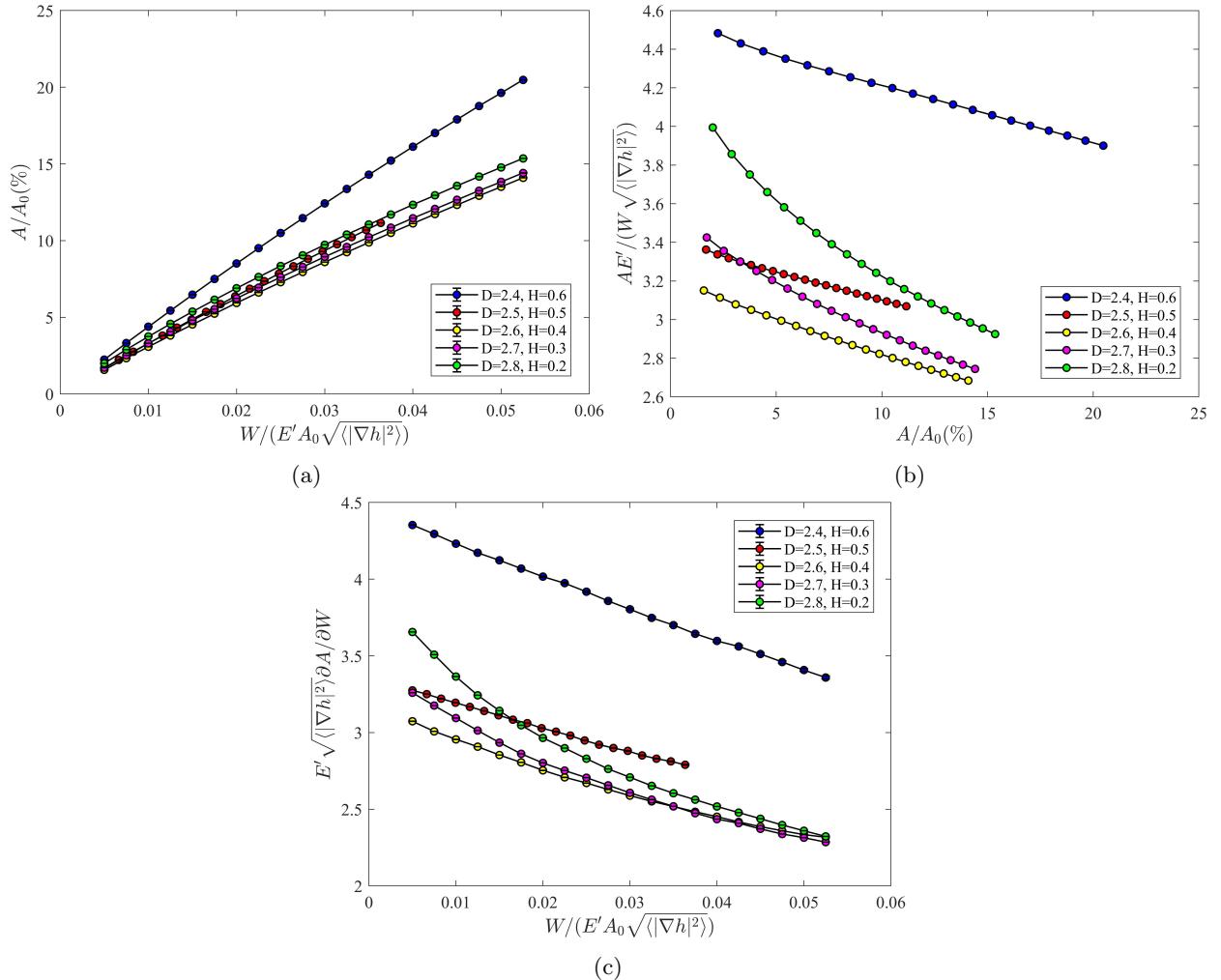


Figure 3: Contact area evolution on self-affine ( $D = n + 1 - H$ ) Cauchy random surfaces without contact area correction. (a) The variation of true contact area fraction with normalized applied external load. (b) The secant of the contact area fraction versus the external load plots. (c) Slope of the contact area fraction versus the external load plots.