### **TOPIC 3: DIVIDE AND CONQUER**

Write a Program to find both the maximum and minimum values in the array. Implement using any programming language of your choice.

Execute your code and provide the maximum and minimum values found. Input:

```
N=8, a[] = \{5,7,3,4,9,12,6,2\}
Output : Min = 2, Max = 12 \text{ Test}
Cases :
Input : N=9, a[] = \{1,3,5,7,9,11,13,15,17\}
Output : Min = 1, Max = 17 \text{ Test}
Cases :
Input : N=10, a[] = \{22,34,35,36,43,67,12,13,15,17\}
```

Output: Min 12, Max 67

```
def find min max(arr):
2
        minimum = arr[0]
        maximum = arr[0]
        for num in arr:
4
            if num < minimum:
5
                minimum = num
7 -
            if num > maximum:
8
                maximum = num
9
        return minimum, maximum
10
   a1 = [5, 7, 3, 4, 9, 12, 6, 2]
   min1, max1 = find_min_max(a1)
12
   print("Input:", a1)
13 print("Min =", min1, ", Max =", max1)
```

```
Input: [5, 7, 3, 4, 9, 12, 6, 2]
Min = 2 , Max = 12
=== Code Execution Successful ===
```

2. Consider an array of integers sorted in ascending order: 2,4,6,8,10,12,14,18. Write a Program to find both the maximum and minimum values in the array. Implement using any programming language of your choice. Execute your code and provide the maximum and minimum values found.

Input: N=8, 2,4,6,8,10,12,14,18.

Output : Min = 2, Max = 18 Test

Cases:

Input : N=9, a[] = {11,13,15,17,19,21,23,35,37}

Output: Min = 11, Max = 37 Test

Cases:

Input : N=10,  $a[] = \{22,34,35,36,43,67,12,13,15,17\}$ 

Output: Min 12, Max 67

```
1 def find_min_max(arr):
2    return arr[0], arr[-1]
3   a1 = [2,4,6,8,10,12,14,18]
4   min1, max1 = find_min_max(a1)
5   print("Input:", a1)
6   print("Min =", min1, ", Max =", max1)
```

```
Output

Input: [2, 4, 6, 8, 10, 12, 14, 18]

Min = 2 , Max = 18

=== Code Execution Successful ===
```

3. You are given an unsorted array 31,23,35,27,11,21,15,28. Write a program for Merge Sort and implement using any programming language of your choice.

Test Cases:

Input : N=8,  $a[] = \{31,23,35,27,11,21,15,28\}$  Output

: 11,15,21,23,27,28,31,35

Test Cases:

Input : N=10,  $a[] = \{22,34,25,36,43,67,52,13,65,17\}$ 

Output: 13,17,22,25,34,36,43,52,65,67

```
1 def merge_sort(arr):
        if len(arr) > 1:
            mid = len(arr) // 2
            left = arr[:mid]
            right = arr[mid:]
 6
            merge_sort(left)
            merge_sort(right)
8
            i = j = k = 0
            while i < len(left) and j < len(right):</pre>
10
                if left[i] < right[j]:</pre>
                    arr[k] = left[i]
12
                    i += 1
14
                    arr[k] = right[j]
16
            while i < len(left):</pre>
18
                arr[k] = left[i]
19
20
            while j < len(right):</pre>
                arr[k] = right[j]
23
                k += 1
24
25
        return arr
26 a1 = [31, 23, 35, 27, 11, 21, 15, 28]
   print("Input :", a1)
   print("Sorted Output:", merge_sort(a1))
```

```
Output

Input : [31, 23, 35, 27, 11, 21, 15, 28]

Sorted Output: [11, 15, 21, 23, 27, 28, 31, 35]

=== Code Execution Successful ===
```

4. Implement the Merge Sort algorithm in a programming language of your choice and test it on the array 12,4,78,23,45,67,89,1. Modify your implementation to count the number of comparisons made during the sorting process. Print this count along with the sorted array.

### Test Cases:

Input : N= 8,  $a[] = \{12,4,78,23,45,67,89,1\}$ 

Output: 1,4,12,23,45,67,78,89

### Test Cases:

Input: N=7,  $a[] = \{38,27,43,3,9,82,10\}$  Output: 3,9,10,27,38,43,82,

```
comparisons = 0
2 def merge_sort(arr):
        global comparisons
4
        if len(arr) > 1:
 5
            mid = len(arr) // 2
6
            left = arr[:mid]
7
            right = arr[mid:]
8
            merge_sort(left)
            merge_sort(right)
9
            i = j = k = 0
10
11
            while i < len(left) and j < len(right):</pre>
12
                 comparisons += 1
13
                 if left[i] < right[j]:</pre>
14
                     arr[k] = left[i]
15
                     i += 1
16 -
                 else:
17
                     arr[k] = right[j]
                     i += 1
18
19
                 k += 1
            while i < len(left):
20 -
21
                 arr[k] = left[i]
22
                 i += 1
                 k += 1
23
24
            while j < len(right):
                 arr[k] = right[j]
25
```

```
26
                i += 1
                k += 1
27
28
        return arr
29
    arr1 = [12, 4, 78, 23, 45, 67, 89, 1]
    comparisons = 0
30
   sorted_arr1 = merge_sort(arr1[:]) |
31
   print("Input:", arr1)
32
   print("Sorted Output:", sorted_arr1)
33
   print("Comparisons:", comparisons)
34
```

```
Output
Input: [12, 4, 78, 23, 45, 67, 89, 1]
Sorted Output: [1, 4, 12, 23, 45, 67, 78, 89]
Comparisons: 16

=== Code Execution Successful ===
```

5. Given an unsorted array 10,16,8,12,15,6,3,9,5 Write a program to perform Quick Sort. Choose the first element as the pivot and partition the array accordingly. Show the array after this partition. Recursively apply Quick Sort on the sub-arrays formed. Display the array after each recursive call until the entire array is sorted.

```
Input: N=9, a[]= {10,16,8,12,15,6,3,9,5}
```

Output: 3,5,6,8,9,10,12,15,16

Test Cases:

Input: N=8,  $a[] = \{12,4,78,23,45,67,89,1\}$  Output:

1,4,12,23,45,67,78,89

Test Cases:

Input: N=7,  $a[] = \{38,27,43,3,9,82,10\}$  Output:

3,9,10,27,38,43,82,

```
1 def partition(arr, low, high):
        pivot = arr[low]
        i = low + 1
3
        j = high
4
5
        while True:
6 -
            while i <= j and arr[i] <= pivot:</pre>
8 -
            while i <= j and arr[j] > pivot:
               j -= 1
9
10
            if i <= j:
11
                arr[i], arr[j] = arr[j], arr[i]
12
            else:
13
14
        arr[low], arr[j] = arr[j], arr[low]
        return j
15
16 def quick_sort(arr, low, high):
17
        if low < high:
            pivot_index = partition(arr, low, high)
18
            print("Array after partitioning with pivot", arr[pivot_index], ":", arr)
19
20
            quick_sort(arr, low, pivot_index - 1)
21
            quick_sort(arr, pivot_index + 1, high)
22 arr1 = [10, 16, 8, 12, 15, 6, 3, 9, 5]
23 print("Initial array:", arr1)
24 quick_sort(arr1, 0, len(arr1) - 1)
25 print("Final sorted array:", arr1)
```

```
Output

Initial array: [10, 16, 8, 12, 15, 6, 3, 9, 5]
Array after partitioning with pivot 10 : [6, 5, 8, 9, 3, 10, 15, 12, 16]
Array after partitioning with pivot 6 : [3, 5, 6, 9, 8, 10, 15, 12, 16]
Array after partitioning with pivot 3 : [3, 5, 6, 9, 8, 10, 15, 12, 16]
Array after partitioning with pivot 9 : [3, 5, 6, 8, 9, 10, 15, 12, 16]
Array after partitioning with pivot 15 : [3, 5, 6, 8, 9, 10, 12, 15, 16]
Final sorted array: [3, 5, 6, 8, 9, 10, 12, 15, 16]
=== Code Execution Successful ===
```

6. Implement the Quick Sort algorithm in a programming language of your choice and test it on the array 19,72,35,46,58,91,22,31. Choose the middle element as the pivot and partition the array accordingly. Show the array after this partition. Recursively apply Quick Sort on the

sub-arrays formed. Display the array after each recursive call until the entire array is sorted. Execute your code and show the sorted array. Input: N=

8, 
$$a[] = \{19,72,35,46,58,91,22,31\}$$

Output: 19,22,31,35,46,58,72,91

Test Cases:

Input: N=8,  $a[] = \{31,23,35,27,11,21,15,28\}$  Output

: 11,15,21,23,27,28,31,35

Test Cases:

Input: N=10, a[] = {22,34,25,36,43,67, 52,13,65,17}

Output: 13,17,22,25,34,36,43,52,65,67

```
1 def partition(arr, low, high):
2
        mid = (low + high) // 2
3
        pivot = arr[mid]
        i = low
4
 5
        j = high
        while i <= j:
6
 7 -
            while arr[i] < pivot:</pre>
8
                i += 1
            while arr[j] > pivot:
9 -
                 j -= 1
10
11
            if i \leftarrow j:
12
                arr[i], arr[j] = arr[j], arr[i]
13
                 i += 1
                j -= 1
14
15
        return i, j
16 def quick_sort(arr, low, high):
17 -
        if low < high:
18
            i, j = partition(arr, low, high)
            print("Array after partition with current pivot:", arr)
19
20
            quick_sort(arr, low, j)
21
            quick_sort(arr, i, high)
22
    arr1 = [19, 72, 35, 46, 58, 91, 22, 31]
   print("Initial array:", arr1)
23
   quick_sort(arr1, 0, len(arr1) - 1)
   print("Final sorted array:", arr1)
```

```
Output

Initial array: [19, 72, 35, 46, 58, 91, 22, 31]

Array after partition with current pivot: [19, 31, 35, 22, 58, 91, 46, 72]

Array after partition with current pivot: [19, 22, 35, 31, 58, 91, 46, 72]

Array after partition with current pivot: [19, 22, 35, 31, 58, 91, 46, 72]

Array after partition with current pivot: [19, 22, 31, 35, 58, 91, 46, 72]

Array after partition with current pivot: [19, 22, 31, 35, 58, 72, 46, 91]

Array after partition with current pivot: [19, 22, 31, 35, 58, 46, 72, 91]

Array after partition with current pivot: [19, 22, 31, 35, 46, 58, 72, 91]

Final sorted array: [19, 22, 31, 35, 46, 58, 72, 91]

=== Code Execution Successful ===
```

7. Implement the Binary Search algorithm in a programming language of your choice and test it on the array

5,10,15,20,25,30,35,40,45 to find the position of the element 20.

Execute your code and provide the index of the element 20. Modify your implementation to count the number of comparisons made during the search process. Print this count along with the result.

```
\begin{split} &\text{Input}: N=9, \ a[]=\{5,10,15,20,25,30,35,40,45\}, \ search \ key=20\\ &\text{Output}: 4 \ Test\\ &\text{cases}\\ &\text{Input}: N=6, \ a[]=\{10,20,30,40,50,60\}, \ search \ key=50 \ Output: 5\\ &\text{Input}: N=7, \ a[]=\{21,32,40,54,65,76,87\}, \ search \ key=32 \end{split}
```

Output: 2

```
def binary_search(arr, key):
 2
        low = 0
 3
        high = len(arr) - 1
 4
        comparisons = 0
        while low <= high:
 5
            mid = (low + high) // 2
 6
            comparisons += 1
            if arr[mid] == key:
 8
                print(f"Element {key} found at position {mid + 1}")
 9
                print(f"Number of comparisons: {comparisons}\n")
10
11
                return mid + 1
12 -
            elif arr[mid] < key:</pre>
                low = mid + 1
13
14 -
            else:
15
                high = mid - 1
16
        print(f"Element {key} not found")
17
        print(f"Number of comparisons: {comparisons}\n")
18
        return -1
19
    arr1 = [5, 10, 15, 20, 25, 30, 35, 40, 45]
    print("Test Case 1:")
20
    binary_search(arr1, 20)
```

Test Case 1:

Element 20 found at position 4

Number of comparisons: 4

=== Code Execution Successful ===

8. You are given a sorted array 3,9,14,19,25,31,42,47,53 and asked to find the position of the element 31 using Binary Search. Show the mid-point calculations and the steps involved in finding the element. Display, what would happen if the array was not sorted, how would this impact the performance and correctness of the Binary Search algorithm?

Input: N=9, a[] = {3,9,14,19,25,31,42,47,53}, search key = 31

Output: 6 Test

cases

Input : N=7, a[] = {13,19,24,29,35,41,42}, search key = 42

Output: 7 Test

cases

Input : N=6, a[] = {20,40,60,80,100,120}, search key = 60

Output: 3

```
1 def binary_search(arr, key):
2
        low = 0
3
        high = len(arr) - 1
4
        step = 1
        print(f"\nSearching for {key} in {arr}")
        while low <= high:
            mid = (low + high) // 2
8
            print(f"Step {step}: low={low}, high={high}, mid={mid}, a[mid]
                ={arr[mid]}")
9
            step += 1
10
            if arr[mid] == key:
11
                print(f"Element {key} found at position {mid + 1}\n")
12
                return mid + 1
13
            elif arr[mid] < key:</pre>
14
                print(f"{arr[mid]} < {key} → Move right half\n")</pre>
15
                low = mid + 1
16 -
            else:
17
                print(f"{arr[mid]} > {key} → Move left half\n")
                high = mid - 1
18
19
        print(f"Element {key} not found in the array.\n")
20
        return -1
21
   a1 = [3, 9, 14, 19, 25, 31, 42, 47, 53]
    binary_search(a1, 31)
```

```
Searching for 31 in [3, 9, 14, 19, 25, 31, 42, 47, 53]

Step 1: low=0, high=8, mid=4, a[mid]=25

25 < 31 → Move right half

Step 2: low=5, high=8, mid=6, a[mid]=42

42 > 31 → Move left half

Step 3: low=5, high=5, mid=5, a[mid]=31

Element 31 found at position 6

=== Code Execution Successful ===
```

9. Given an array of points where points[i] = [xi, yi] represents a point on the X-Y plane and an integer k, return the k closest points to the origin (0, 0).

```
Input : points = [[1,3],[-2,2],[5,8],[0,1]],k=2
Output:[[-2, 2], [0, 1]]
```

- (ii) Input: points = [[1, 3], [-2, 2]], k = 1
  Output: [[-2, 2]]
- (iii) Input: points = [[3, 3], [5, -1], [-2, 4]], k = 2

**Output**: [[3, 3], [-2, 4]]

```
1 import math
2 def k_closest(points, k):
3    points.sort(key=lambda p: p[0]**2 + p[1]**2)
4    return points[:k]
5 points1 = [[1, 3], [-2, 2], [5, 8], [0, 1]]
6 k1 = 2
7 print("Input:", points1, "k =", k1)
8 print("Output:", k_closest(points1, k1))
9
```

```
Output

Input: [[1, 3], [-2, 2], [5, 8], [0, 1]] k = 2

Output: [[0, 1], [-2, 2]]

=== Code Execution Successful ===
```

10. Given four lists A, B, C, D of integer values, Write a program to compute how many tuples n(i, j, k, l) there are such that A[i] + B[j] + C[k] + D[l] is zero.

Input: 
$$A = [1, 2], B = [-2, -1], C = [-1, 2], D = [0, 2]$$
Output: 2

(ii) Input: A = [0], B = [0], C = [0], D = [0]

Output: 1

```
def four_sum_count(A, B, C, D):
        ab_sum = {}
 3
        count = 0
        for a in A:
            for b in B:
 6
                s = a + b
                ab\_sum[s] = ab\_sum.get(s, 0) + 1
 8
        for c in C:
 9
            for d in D:
                target = -(c + d)
10
11 -
                if target in ab_sum:
12
                    count += ab_sum[target]
13
        return count
14 A1 = [1, 2]
15 B1 = [-2, -1]
16 \quad C1 = [-1, 2]
17 D1 = [0, 2]
18 print("Input:")
19 print("A =", A1, "B =", B1, "C =", C1, "D =", D1)
20 print("Output:", four_sum_count(A1, B1, C1, D1))
```

Input:

$$A = [1, 2] B = [-2, -1] C = [-1, 2] D = [0, 2]$$

Output: 2

=== Code Execution Successful ===

11. To Implement the Median of Medians algorithm ensures that you handle the worst-case time complexity efficiently while finding the k-th smallest element in an unsorted array.

$$arr = [12, 3, 5, 7, 19] k = 2$$

Expected Output:5

$$arr = [12, 3, 5, 7, 4, 19, 26] k = 3$$

Expected Output:5

$$arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] k = 6$$

Expected Output:6

```
1 def partition(arr, low, high, pivot):
 2
        pivot_value = arr[pivot]
        arr[pivot], arr[high] = arr[high], arr[pivot]
4
        store_index = low
 5
        for i in range(low, high):
 6
            if arr[i] < pivot_value:</pre>
                arr[i], arr[store_index] = arr[store_index], arr[i]
 8
                store_index += 1
9
        arr[store_index], arr[high] = arr[high], arr[store_index]
10
        return store_index
11 def find_median(arr):
12
        arr.sort()
13
        mid = len(arr) // 2
14
        return arr[mid]
15 def select(arr, low, high, k):
16
        if low == high:
17
            return arr[low]
18
        medians = []
19
        i = low
20 -
        while i <= high:
21
            group = arr[i:i + 5]
22
            medians.append(find_median(group))
            i += 5
23
24
        if len(medians) == 1:
            median_of_medians = medians[0]
25
        else:
26
```

```
27
            median_of_medians = select(medians, 0, len(medians) - 1, len(medians)
                [] [2]
28
        pivot_index = arr.index(median_of_medians)
29
        pivot_index = partition(arr, low, high, pivot_index)
30
        rank = pivot_index - low + 1
31
32
        if rank == k:
33
            return arr[pivot_index]
34
        elif k < rank:
35
            return select(arr, low, pivot_index - 1, k)
36
37
            return select(arr, pivot_index + 1, high, k - rank)
38 def kth_smallest(arr, k):
39
        if k < 1 or k > len(arr):
40
            return None
41
        return select(arr, 0, len(arr) - 1, k)
42
    arr1 = [12, 3, 5, 7, 19]
43
   print("Input:", arr1, "k = 2")
44 print("Output:", kth_smallest(arr1[:], 2))
```

Input: [12, 3, 5, 7, 19] k = 2

Output: 7

=== Code Execution Successful ===

12. To Implement a function median\_of\_medians(arr, k) that takes an unsorted array arr and an integer k, and returns the k-th smallest element in the array.

$$arr = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] k = 6$$

$$arr = [23, 17, 31, 44, 55, 21, 20, 18, 19, 27] k = 5$$

Output: An integer representing the k-th smallest element in the array.

```
1 def median_of_medians(arr, k):
        def partition(arr, pivot):
            low = [x for x in arr if x < pivot]</pre>
            high = [x for x in arr if x > pivot]
4
            equal = [x for x in arr if x == pivot]
            return low, equal, high
 8
        def select(arr, k):
9 -
            if len(arr) <= 5:
10
                arr.sort()
11
                return arr[k - 1]
            medians = [sorted(arr[i:i + 5])[len(arr[i:i + 5]) // 2] for i in range(0
12
                , len(arr), 5)]
            pivot = select(medians, len(medians) // 2 + 1)
13
14
            low, equal, high = partition(arr, pivot)
15 -
            if k <= len(low):</pre>
16
                return select(low, k)
17
            elif k <= len(low) + len(equal):</pre>
18
                return pivot
19 -
            else:
20
                return select(high, k - len(low) - len(equal))
21
        return select(arr, k)
22 arr1 = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10]
23 k1 = 6
24 print("Input:", arr1, "k =", k1)
25 print("Output:", median_of_medians(arr1, k1))
```

```
Output

Input: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10] k = 6
Output: 6

=== Code Execution Successful ===
```

13. Write a program to implement Meet in the Middle Technique. Given an array of integers and a target sum, find the subset whose sum is closest to the target. You will use the Meet in the Middle technique to efficiently find this subset.

```
a) Set[] = \{45, 34, 4, 12, 5, 2\} Target Sum : 42
```

```
b) Set[]= \{1, 3, 2, 7, 4, 6\}
```

### Target sum = 10:

```
from itertools import combinations
    import bisect
3 def meet_in_middle(arr, target):
        n = len(arr)
        half = n // 2
6
        left = arr[:half]
7
        right = arr[half:]
8 -
        def subset_sums(nums):
9
            sums = []
            for r in range(len(nums) + 1):
10 -
11 -
                for comb in combinations(nums, r):
12
                     sums.append(sum(comb))
13
            return sums
        left_sums = subset_sums(left)
14
15
        right_sums = subset_sums(right)
16
        right_sums.sort()
17
        closest_sum = None
18
        min_diff = float('inf')
        for s in left sums:
19 -
20
            remaining = target - s
21
            idx = bisect.bisect_left(right_sums, remaining)
22 -
            for j in [idx, idx - 1]:
23
                if 0 <= j < len(right_sums):</pre>
24
                     total = s + right_sums[j]
25
                     diff = abs(target - total)
```

Set: [45, 34, 4, 12, 5, 2] Target Sum: 42

Closest Subset Sum: 41

=== Code Execution Successful ===

14. Write a program to implement Meet in the Middle Technique. Given a large array of integers and an exact sum E, determine if there is any subset that sums exactly to E. Utilize the Meet in the Middle technique to handle the potentially large size of the array. Return true if there is a subset that sums exactly to E, otherwise return false.

a) 
$$E = \{1, 3, 9, 2, 7, 12\}$$
 exact Sum = 15

b) 
$$E = \{3, 34, 4, 12, 5, 2\}$$
 exact Sum = 15

```
from itertools import combinations
   import bisect
3 def meet_in_middle_exact(arr, target):
        n = len(arr)
5
        half = n // 2
        left = arr[:half]
6
        right = arr[half:]
        def subset_sums(nums):
8
9
            sums = []
            for r in range(len(nums) + 1):
10 -
11
                for comb in combinations(nums, r):
                    sums.append(sum(comb))
12
13
            return sums
        left_sums = subset_sums(left)
14
15
        right_sums = subset_sums(right)
16
        right_sums.sort()
17
        for s in left_sums:
18
            remaining = target - s
19
            idx = bisect.bisect_left(right_sums, remaining)
20 -
            if idx < len(right_sums) and right_sums[idx] == remaining:</pre>
21
                return True
22
        return False
23 arr1 = [1, 3, 9, 2, 7, 12]
24 target1 = 15
25 print("Set:", arr1, "Target Sum:", target1)
26 print("Subset exists:", meet_in_middle_exact(arr1, target1))
```

```
Set: [1, 3, 9, 2, 7, 12] Target Sum: 15
Subset exists: True

=== Code Execution Successful ===
```

15 Given two 2×2 Matrices A and B

Use Strassen's matrix multiplication algorithm to compute the product matrix C such that C=A×B.

# **Test Cases:**

Consider the following matrices for testing your implementation:

# **Test Case 1:**

Expected Output: C=(18

14

62 66)

```
1 def strassen_2x2(A, B):
       a, b, c, d = A[0][0], A[0][1], A[1][0], A[1][1]
3
       e, f, g, h = B[0][0], B[0][1], B[1][0], B[1][1]
       P1 = a * (f - h)
       P2 = (a + b) * h
5
6
       P3 = (c + d) * e
       P4 = d * (g - e)
8
       P5 = (a + d) * (e + h)
9
       P6 = (b - d) * (g + h)
10
       P7 = (a - c) * (e + f)
11
       C11 = P5 + P4 - P2 + P6
12
       C12 = P1 + P2
13
       C21 = P3 + P4
14
       C22 = P1 + P5 - P3 - P7
15
       return [[C11, C12], [C21, C22]]
16 A1 = [[1, 7], [3, 5]]
17 B1 = [[1, 3], [7, 5]]
18 C1 = strassen_2x2(A1, B1)
19 print("Matrix A:", A1)
20 print("Matrix B:", B1)
21 print("Product Matrix C:", C1)
```

```
Output

Matrix A: [[1, 7], [3, 5]]

Matrix B: [[1, 3], [7, 5]]

Product Matrix C: [[50, 38], [38, 34]]

=== Code Execution Successful ===
```

16 Given two integers X=1234 and Y=5678: Use the Karatsuba algorithm to compute the product  $Z=X \times Y$ 

### **Test Case 1:**

Input: x=1234,y=5678

Expected Output: z=1234×5678=7016652

```
1 def karatsuba(x, y):
        if x < 10 or y < 10:
 3
            return x * y
        n = max(len(str(x)), len(str(y)))
 4
        m = n // 2
       high1, low1 = divmod(x, 10**m)
 6
       high2, low2 = divmod(y, 10**m)
 8
       z0 = karatsuba(low1, low2)
 9
        z1 = karatsuba((low1 + high1), (low2 + high2))
10
        z2 = karatsuba(high1, high2)
11
        return z2 * 10**(2*m) + (z1 - z2 - z0) * 10**m + z0
12 x = 1234
13 y = 5678
14 z = karatsuba(x, y)
15 print(f"Input: x = \{x\}, y = \{y\}")
16 print(f"Output: z = \{x\} \times \{y\} = \{z\}")
```

```
Input: x = 1234, y = 5678
Output: z = 1234 x 5678 = 7006652
=== Code Execution Successful ===
```