15CSE201 : Data Structures and Algorithms

Sequences: Vectors and Iterators

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Based on the reference materials by Prof. Goodrich, OCW METU and Dr. Vidhya Balasubramanian

Sequences

- In linear data structure each object comes before another
- Represent relationship of “next” and “previous” between related objects
  - Examples
    - Packets in a network
    - Order of instructions in a program

Sequences

- Sequence:
  - collection of elements organized in a specified order
  - allows random access by rank or position
- Stack: sequence that can be accessed in LIFO fashion
- Queue: sequence that can be accessed in FIFO fashion
- Vector: sequence with random access by rank

Places in a Sequence

- Rank
  - Place specified by number of places before the place in question
  - Abstraction of the concept of array index
  - If an element is the r^{th} element then its rank is r-1
    - Previous element has rank r-1
  - Rank may change whenever the sequence is updated
    - When a new element is inserted at the beginning of the sequence, rank of all other elements increase by 1

Concept of Rank

- Rank of an element e in S
  - Number is elements that precede e in S
  - First element has rank 0, and last one has rank n-1
  - An exception is thrown if an incorrect rank is specified (e.g., a negative rank)
- Rank r does not have to mean that element e is stored at index r in the array
  - Refers to the index without referring to the actual implementation

Vector

- **Vector**
  - Linear sequence that supports access by their ranks
  - Can specify where to insert or remove an element

Vector ADT: An Overview

- Extends the array data structure by providing a sequence of objects
- Provides fast random access while maintaining ability to automatically resize when needed
- Provides accessor functions
  - Index into middle of a sequence
  - Add and remove elements by their indices
  - The index is referred to as rank

Vector ADT: Main Operations

- elemAtRank(r)
  - Returns element S at rank r
  - Input::Integer; Output:: object;
  - Error if $r<0$ or $r>n-1$
- replaceAtRank(r,e)
  - Replace with e the element at rank r.
  - Input: integer r and object e; Output: None
  - Error if $r<0$ or $r>n-1$

Vector ADT: Main Operations

- insertAtRank(r,e)
  - Insert a new element into S so that it has rank r
  - Input:: Integer r and object e; Output:none;
  - Error if r<0 or r>n
- removeAtRank(r)
  - Remove element from S at rank r.
  - Input: integer; Output: None
  - Error if r<0 or r>n-1
- Also supports size() and isEmpty() operations

Vector Example

Operation	Output	Queue Contents
Insert 7 at rank 0	-	(7)
Insert 4 at rank 0	-	(4,7)
Return the element at rank 1	7	(4,7)
Insert 2 at rank 2	-	(4,7,2)
Return the element at rank 3	“error”	(4,7,2)
Remove the element at rank 1	-	(4,2)
Insert 5 at rank 1	-	(4,5,2)
Insert 3 at rank 1	-	(4,3,5,2)
Insert 9 at rank 4	1	(4,3,5,2,9)
Return element at rank 2	5	(4,3,5,2,9)
size()	5	(4,3,5,2,9)

Applications of Vectors

- Direct application
  - Sorted collection of objects
  - Serves as a simple database
- Indirect application
  - Component of other data structures
  - Widely used for implementing many algorithms

Realization of a deque using a vector

Deque Function	Realization with Vector Functions
size()	size()
isEmpty()	isEmpty()
first()	elemAtRank(0)
last()	elemAtRank(size()-1)
insertFirst(e)	InsertAtRank(0,e)
insertLast(e)	InsertAtRank(size(),e)
removeFirst()	removeAtRank(0)
removeLast()	removeAtRank(size()-1)

Vector Interface (C++)

```
template <typename Object>
class ArrayVector {
public:
    int size() const: //returns number of objects in the vector
    bool isEmpty() //returns true if vector is empty, false otherwise
    Object& elemAtRank(int r) //access element at rank r
    void replaceAtRank(int r, const Object& obj): //replace element at given
        rank
    void removAtRank(int r): //remove element at given rank
    void insertAtRank(int r, const Object& obj): //insert element at given rank
}
```

Vector Interface (Python)

class MyVector():

def size(**self**): //returns number of objects in the vector

def isEmpty(**self**) //returns true if vector is empty, false otherwise

def elemAtRank(**self**, r) //access element at rank r

def replaceAtRank(**self,** r, value): //replace element at given rank

def removAtRank(**self,** r): //remove element at given rank

def insertAtRank(**self,** r, value): //insert element at given rank

Array based implementation of a Vector

Here A[i] stores element at rank i , and maintain a value n to keep track of size

Algorithm elemAtRank(r)

Return A[r]

Algorithm replaceAtRank(r,e)

A[r] = e

Algorithm size()

Return n-1 // the number of elements inserted must be tracked

Algorithm isEmpty()

Return (n==0)

Array ADT Functions

Algorithm insertAtRank(r,e)

for i = n-1, n-2, …,r **do**

A[i+1] = A[i]   // make room for new element

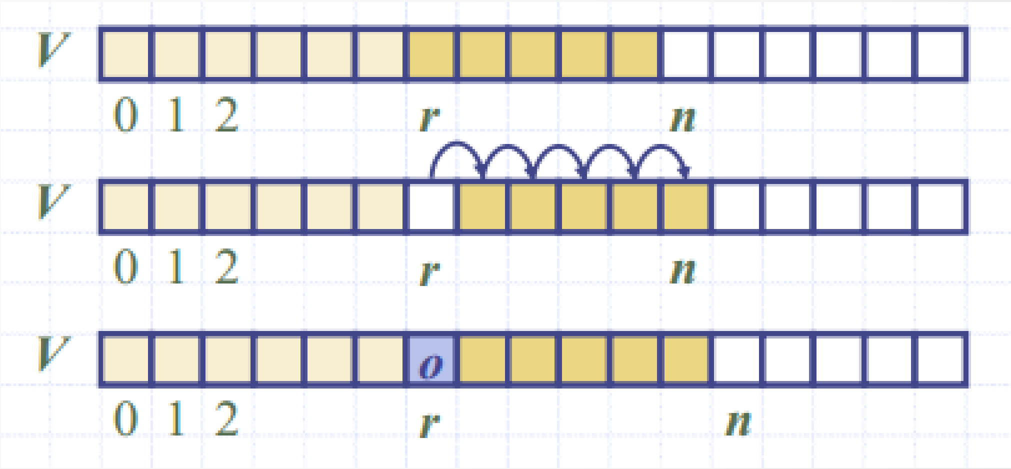
A[r] ← e

n ← n+1

Worst case running time

O(n)

When r=0



Array ADT Functions

Algorithm removeAtRank(r)

for i = r, r+1, … , n-2 **do**

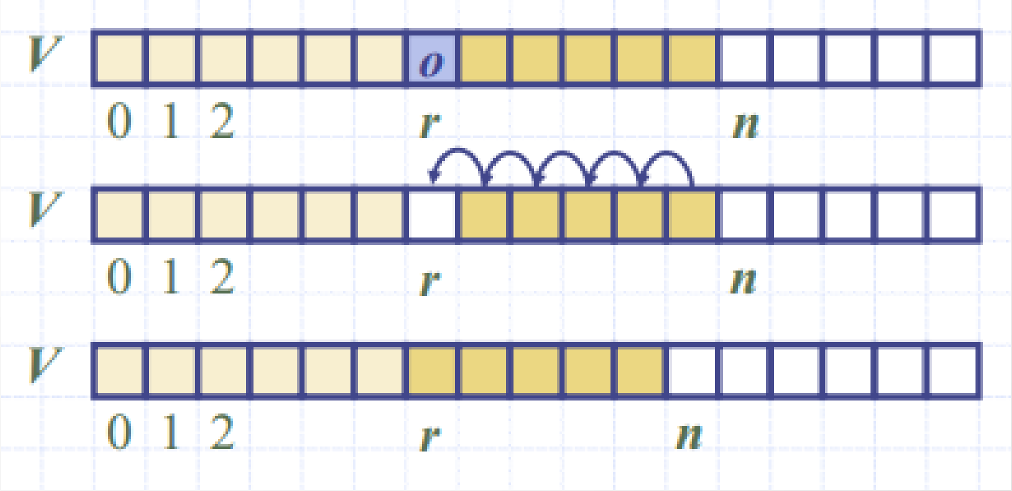
A[i] ← A[i+1]

n ← n-1

Worst case running time

O(n)

When r = 0



Exercise

- How can the functions of a stack be realized using a vector
- Consider a vector V. Consider the sequence of operations. Show the state of the vector and output an error if the operation is illegal.
  - For i= 1 to 10
    - Insert i at rank I
    - If i is even delete element at rank i-1

Complexity Analysis

Time Complexity

- size – $O(1)$
- isEmpty – $O(1)$
- elemAtRank– $O(1)$
- replaceAtRank – $O(1)$
- removeAtRank – $O(n)$
- InsertAtRank - O(n)

Space Complexity – O(n)

Circular Array Implementation

- The restriction that element at rank r is stored at index r causes higher cost for insertions and deletions
  - Can be rectified using circular array like the one used for a queue
  - If we use the array in a circular fashion, insertAtRank(0) and removeAtRank(0) run in O(1) time
  - In an insertAtRank operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one

Comparison of the Strategies

- We compare the incremental strategy and the doubling strategy by analyzing the total time ***T(n)*** needed to perform a series of n push operations
- Assume that we start with an empty stack represented by an array of size 1
- We call amortized time of a push operation the average time taken by a push over the series of operations, i.e., ***T(n)/n***

Incremental Strategy Analysis

- We replace the array $k = n/c$ times
- The total time $T(n)$ of a series of n push operations is proportional to

$n + c + 2c + 3c + 4c + \ldots + kc =$

$n + c(1 + 2 + 3 + \ldots + k) =$

$n + ck(k + 1)/2$

- Since c is a constant, $T(n)$ is $O(n+k^2)$, i.e., $O(n^2)$
- The amortized time of a push operation is $O(n)$

Doubling Strategy Analysis

- We replace the array $k = \log_2 n$ times
- The total time T(n) of a series of n push operations is proportional to

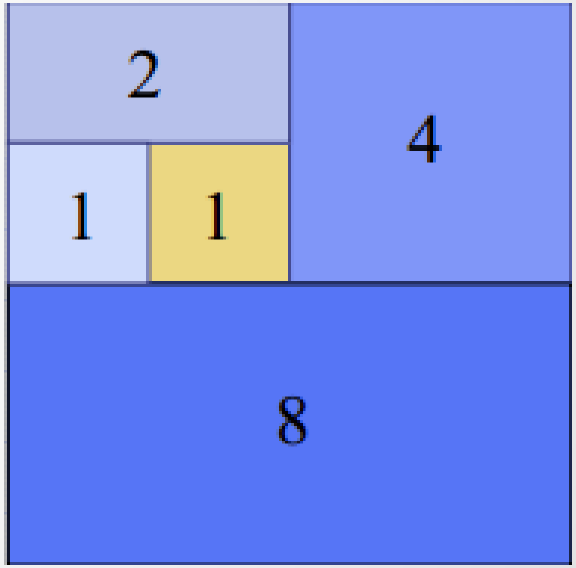
$n + 1 + 2 + 4 + 8 + \ldots + 2^k =$

$n + 2^k + 1 - 1 = 2n - 1$

- T(n) is O(n)
- The amortized time of a push operation

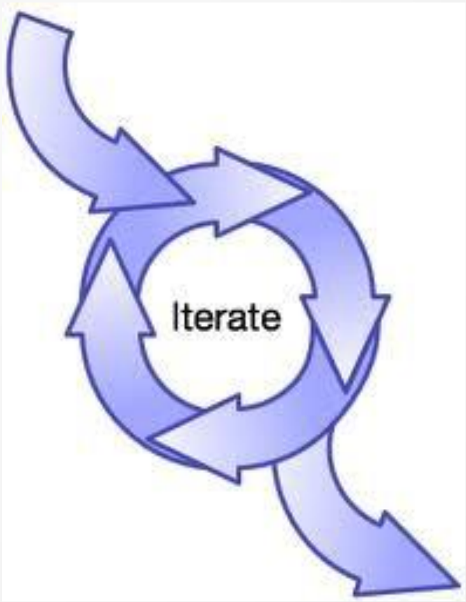
  is O(1)

Geometric Series



Iterators

- abstracts the process of scanning through a collection of elements one element at a time
- Consists of
  - a sequence S
  - a current position in S
  - a way of stepping to the next position in S making it the current position



ObjectIterator ADT

hasNext():

- Test whether there are elements left in the iterator
- Input:None; Output: Boolean

next():

- Return the next element in the iterator and step to the next position
- Input: None; Output: Object
- Can be implemented by using a pointer to the current element

Iterators

- Provides a unified scheme to access all the elements of a container
  - Independent from the specific organization of the collection
  - Can be augmented with can augment the Stack, Queue, Vector, List and Sequence ADTS
    - May be implemented with an array or singly linked list
    - e.g next() in a stack can be implemented using pop()
- Extends the concept of position by adding a traversal capability
  - Returns objects according to their linear ordering

Notions and Types

- Snapshot
  - freezes the contents of the data structure at a given time
- Dynamic
  - follows changes to the data structure
- Forward and Backward Iterators
  - Allows movement in both directions on a certain ordering of elements
  - Can also support repeated access to current element

End of Lecture 8