

Array Inversion

Inversion count

Inversion count of an array

Given an array, find the number of inversions of it. If $(i \text{ and } (A[i] > A[j]))$ then the pair (i, j) is called an inversion of an array A . We need to count all such pairs in the array.

For example,

Input: $A[] = [1, 9, 6, 4, 5]$

Output: Inversion count is 5

There are 5 inversions in the array – $(9, 6), (9, 4), (9, 5), (6, 4), (6, 5)$

Approach - 1

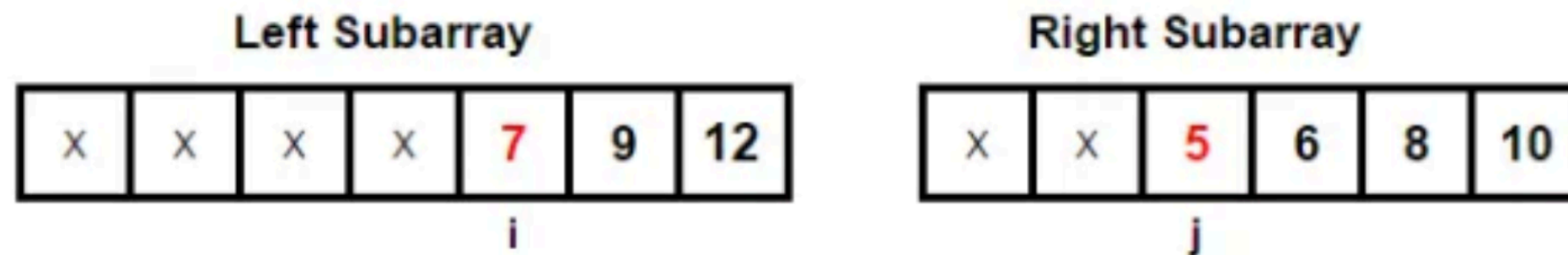
- Simple solution would be for each element of the array, count all elements less than it to its right and add the count to output.
- Complexity will be $O(n^2)$

```
for (i = 0 to n - 1 ) {  
    for ( j = i + 1 to n ) {  
        if (A[ i ] > A[ j ] )  
            inversionCount++  
    }  
}
```

Approach 2

- Using Merge Sort
- Basically for each element of the array, we count all elements more than it to its left and add the count to output.
- This happens inside merge function of merge sort
- Let us consider two subarrays involved in merge process

Merge process



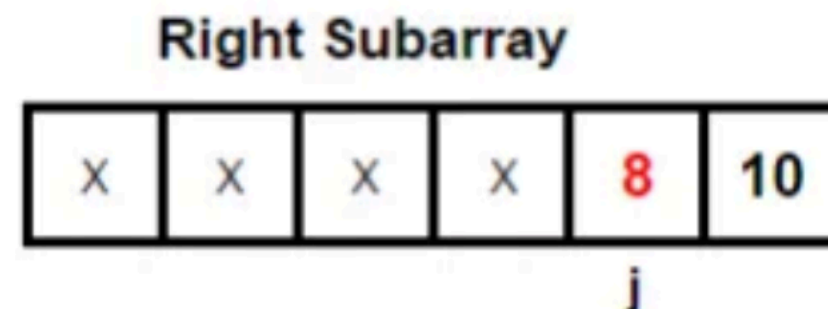
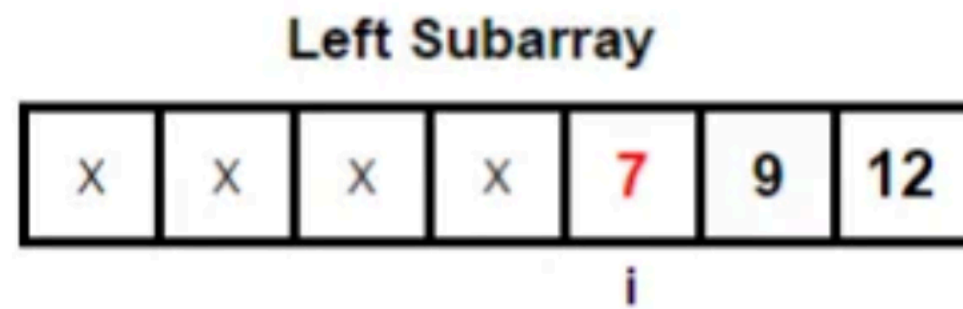
As $7 > 5$, (7, 5) forms an inversion pair.

Also as left subarray is sorted, it is obvious that elements 9 and 12 will also form inversion with element 5. i.e (9, 5) and (12, 5). So we can say that for element 5, total number of inversions are 3 which is exactly equal to number of elements left in the left subarray.

Left Subarray						
x	x	x	x	7	9	12
				i		

Right Subarray					
x	x	x	6	8	10
			j		

Similarly as $7 > 6$, (7, 6), (9, 6) and (12, 6) forms an inversion.



As $7 < 8$, no inversion is formed

Left Subarray

x	x	x	x	x	9	12
---	---	---	---	---	---	----

i

Right Subarray

x	x	x	x	8	10
---	---	---	---	---	----

j

As $9 > 8$, (9, 8) and (12, 8) forms an inversion

Left Subarray

x	x	x	x	x	9	12
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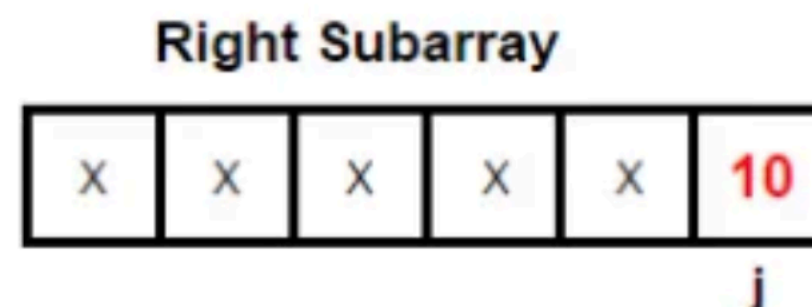
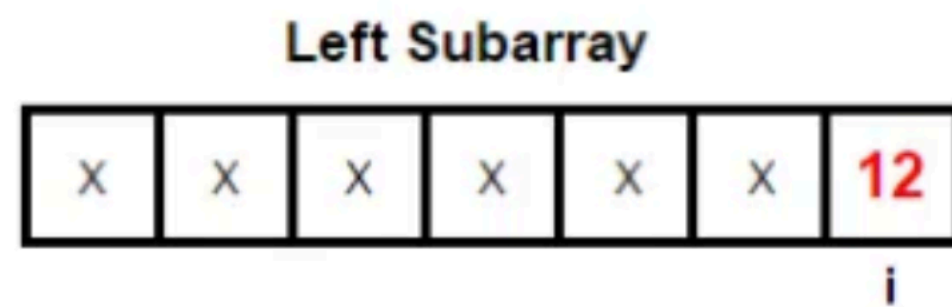
i

Right Subarray

x	x	x	x	x	10
---	---	---	---	---	----

j

As $9 < 10$, no inversion is formed



As $12 > 10$, (12, 10) forms an inversion

- Time complexity will be $O(n \log n)$ with auxiliary space $O(n)$