21-12-17.

# Unit-1 Theory Of Relativity.

→ This theory deals with the object that moves with high velocity. nearly equal to velocity of light.

Rest - Object which does not change its position w.r.to surroundings.

Motion - Object which change its position w.r.to surroundings.

**Frame of Reference:-**

It is the simplest coordinate system which is used to describe the motion of object.

eg:- Cartesian coordinate

There are two types of frame of Reference.

1) Inertial Frame of Reference

2) Non-Inertial F.O.Reference.

**Inertial F.O.R:-**

Any frame which is at rest (or) in uniform motion is called Inertial frame of Reference.

Acceleration is absent in Inertial frame of Reference.

eg:- Earth is considered as Inertial F.O.R.

→ The frame which obeys Newton's law are called Inertial frame of reference.

**Non-Inertial F.O.R:-**

The frames which does not obey Newton's law are considered to be Non-inertial frame of reference.

eg:- All accelerated frames/objects.

**Special Theory Of Relativity:-** (Einstein)

→ Special theory of Relativity tells us how measurement of space and time are effected due to relative motion between observer and what is being observed.

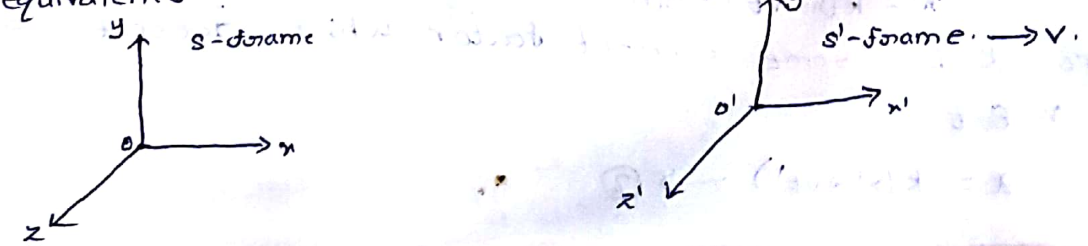
**Postulates Of Special Theory**

→ The laws of physics remains same/holds good in all inertial frame of reference.

→ The velocity of light (c) remains same in all inertial frames of reference.

**Galilean Transformation Equations:-**

Galilean transformations are used to convert measurements of one frame of reference to its equivalents in another frame of reference.



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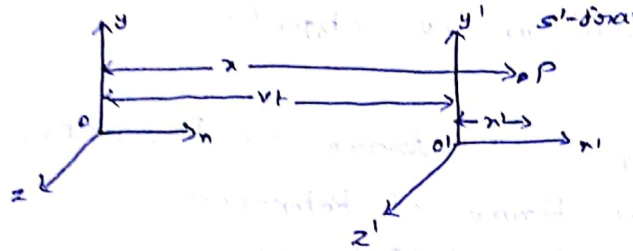
Consider two frames S-frame with coordinates x, y, z and t.
The coordinates of S' frame be x', y', z' & t'.
For any measurement, we consider S-frame is at rest and S'-frame moves with constant velocity.




$$\left.\begin{array}{l} x = x'+vt' \\ y = y' \\ z = z' \\ t = t'. \end{array}\right. \quad \left\{\begin{array}{l} x' = x - vt \\ y' = y \\ z' = z \\ t' = t \end{array}\right\} \text{ Galilean Transformations Eq}^{ns}.$$

$$\frac{dx'}{dt} = v_x' = \frac{dx}{dt} - v.$$

$$\boxed{v_x' = v_x - v}$$

$$\frac{dv_x'}{dt} = \frac{d}{dt}(v_x) - 0.$$

$$\boxed{a_x' = a_x.}$$

These equation could not be used for relativistic velocities.

Note:-
__Limitations of Galilean Transformation Eq^ns:-__

→ G.T.E applicable for $v << c$

→ It fails to obey second postulate of special theory of relativity.

i.e., $\boxed{\begin{array}{l} v_x' = v_x - v. \\ c' = c - v \end{array}}$

__Lorrentz Transformation Equations:-__

We have from GTEq

$$x' = x - vt$$

$$x' = k(x - vt) \rightarrow ①$$

where $k$ is some correct factor which doesnot depends on $x$ & $t$.

$$x = k(x' + vt') \rightarrow ②$$

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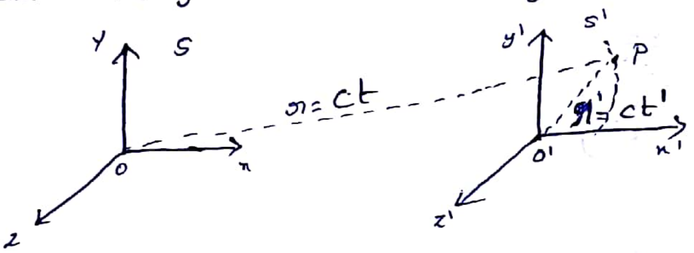
$$x = k\left[k(x-vt) + vt'\right]$$
$$x = k\left[kx - kvt + vt'\right]$$
$$x = k^2x - k^2vt + kvt'. \rightarrow (3)$$

$$kvt' = x - k^2x + k^2vt.$$
$$t' = \frac{-x(k^2-1)}{kv} + \frac{k^2vt}{kv}.$$
$$t' = k\left[t - \frac{(k^2-1)x}{k^2v}\right].$$

$$t' = k\left[t - bx\right] \rightarrow (4)$$

where $b = \frac{k^2-1}{k^2v}$.

Consider two frames S & S'. S' is moving with constant velocity 'v' w.r.t 'S', along +ve x-axis.




According to S-frame
$$r = ct.$$
$$r^2 = x^2+y^2+z^2$$
$$r^2 = c^2t^2$$
$$x^2+y^2+z^2 = c^2t^2.$$
$$x^2+y^2+z^2-c^2t^2 = 0 \rightarrow (5)$$

According to S'-frame.
$$x'^2+y'^2+z'^2-c^2t'^2 = 0 \rightarrow (6).$$

From eqns (5) & (6).
$$x^2+\cancel{y^2}+\cancel{z^2}-c^2t^2 = x'^2+\cancel{y'^2}+\cancel{z'^2}-c^2t'^2$$

w.k.t $y' = y$, $z' = z$,
$$x^2-c^2t^2 = x'^2-c^2t'^2.$$
$$x^2-c^2t^2 = \left[k(x-vt)\right]^2 - c^2\left[k(t-bx)\right]^2.$$
$$= k^2(x-vt)^2 - c^2k^2(t-bx)^2.$$
$$= k^2\left[x^2+v^2t^2-2xvt - c^2t^2 - c^2b^2x^2 + 2c^2bxt\right]$$
$$x^2-c^2t^2 = k^2x^2 + k^2v^2t^2 - 2k^2xvt - k^2c^2t^2 - k^2c^2b^2x^2 + 2k^2c^2bxt.$$

equating coef of $x^2$
$$1 = k^2 - k^2c^2b^2.$$
$$k^2(1-c^2b^2) = 1 \rightarrow (7)$$

coef of $t^2$
$$-c^2 = k^2v^2 - k^2c^2$$
$$-c^2 = -k^2\left[c^2-v^2\right] \rightarrow (8)$$

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Coeff of $xt$

$$0 = -2k^2v + 2k^2c^2b. \rightarrow (9)$$

From eq (8)

$$k^2 = \frac{c^2}{c^2 - v^2}$$

$$k = \frac{c}{\sqrt{c^2 - v^2}}$$

$$= \frac{\cancel{c}}{\cancel{c}\sqrt{1 - \left(\frac{v}{c}\right)^2}}$$

$$\boxed{k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \rightarrow (10)$$

From eq (7).

$$1 = \frac{c^2}{c^2 - v^2}\left(1 - c^2b^2\right).$$

$$\cancel{c^2} - v^2 = \cancel{c^2} - c^4b^2$$

$$v^2 = c^4b^2$$

$$b^2 = \frac{v^2}{c^4}.$$

$$\boxed{b = \frac{v}{c^2}}$$

$$x' = k(x - vt).$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$$t' = k(t - bx)$$

$$\boxed{t' = \frac{t - \left(\frac{v}{c^2}\right)x}{\sqrt{1 - v^2/c^2}}}$$

$$y' = y.$$

$$z' = z.$$

Inverse L.T.E:.

1) $x = \dfrac{x' + vt'}{\sqrt{1 - v^2/c^2}}$

2) $y = y'$

3) $z = z'$

4) $t = \dfrac{t' + (v/c^2)x'}{\sqrt{1 - v^2/c^2}}.$

$$k = \frac{1}{\sqrt{1 - v^2/c^2}} = \gamma.$$

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$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

If $v << c$.

$\boxed{x' = x - vt} \to$ G.T.E

Note:-

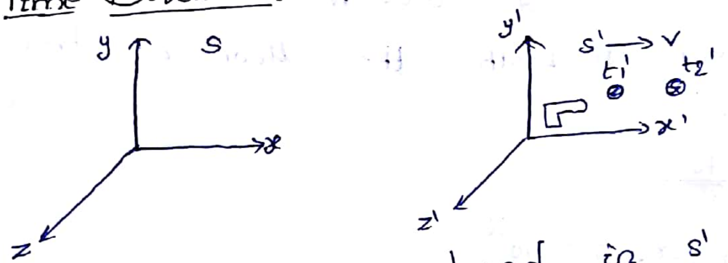
Lorentz Transformation Eq^ns can be applied to even low velocitie i.e.) $v << c$.

If $v << c$ $\frac{v^2}{c^2} \simeq 0$.

$\therefore x' = x - vt$.

At low velocities L.T Eq^ns could be reduced to G.T.E.

Time Dilation:- → Delay.



Consider a gun is placed in s' frame which is moving constant velocity 'v' w.r.to S-frame.

Let $t_1'$ & $t_2'$ be the time at which two shots are fired from the gun.

The time interval b/w the two shots as observed by observer in s' frame is

$$t_2' - t_1' = t_0. \text{ (Proper time interval)}$$

Let $t_1$ & $t_2$ be the time measured by the observer from S-frame.

The time interval b/w the two shots is

$$t_2 - t_1 = t. \text{ (Apparent time)}$$

We have from LTE

$$t_1 = \frac{t_1' + \left(\frac{v}{c^2}\right)x'}{\sqrt{1 - v^2/c^2}}.$$

$$t_2 = \frac{t_2' + \left(\frac{v}{c^2}\right)x'}{\sqrt{1 - v^2/c^2}}.$$

$t = t_2 - t_1$

$$= \frac{t_2' + \cancel{\left(\frac{v}{c^2}\right)x'} - t_1' - \cancel{\left(v/c^2\right)x'}}{\sqrt{1 - v^2/c^2}} = \frac{t_2' - t_1'}{\sqrt{1 - v^2/c^2}}.$$

$$\boxed{t = \frac{t_0}{\sqrt{1 - v^2/c^2}}} \quad t > t_0.$$

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The time difference between rest frame and moving frame is called time dilation.

Note:-
The clock in a moving frame appears to be clicks slowly.

Eg: Cosmic rays (Muon)

These rays are present above 6km from atmosphere.

Muon travel with a velocity $v = 2.998 \times 10^8$ m/s

Life span of Muon $t_0 = 2.2 \mu s$.

Muons can travel distance $d = v \times t_0$

$= 2.998 \times 10^8 \times 2.2 \times 10^{-6}$

$d = 0.66$ km.

According to observer on Earth, the Muon can travel the distance $d = v \times t$.

$$= v \times \frac{t_0}{\sqrt{1 - v^2/c^2}}$$

$$= 2.99 \times 10^8 \times \frac{2.2 \times 10^{-6}}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}}$$

$d \simeq 6$ km.

Problems:-

1) How fast must a spacecraft travel to the earth for each 2 days on the day in the spacecraft corresponding to earth.

A) Given:- $t_0 = 1$ days.

$t = 2$ days

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{2}$$

$$1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = 1 - \frac{1}{4} = \frac{3}{4}$$

$$v^2 = c^2 \times \frac{3}{4}$$

$$v = c \times \frac{\sqrt{3}}{2}$$

$$= 3 \times 10^8 \times \frac{\sqrt{3}}{2}$$

$v = 2.598 \times 10^8$ m/s

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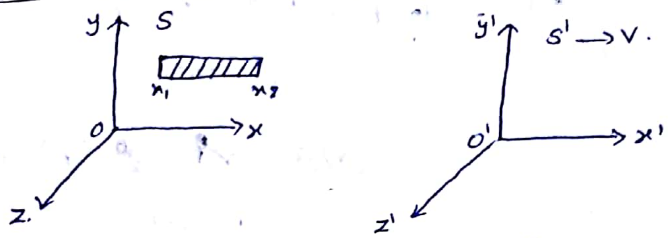
## Length Contraction :- Fitzerald Contraction.




Consider a rod is placed in S-frame which is at rest w.r.to s'-frame.

Let $x_1, x_2$ be the coordinates of the ends of the rod.

The length of the rod in s-frame is $x_2 - x_1 = l_0$. (proper length)

According to s'-frame let $x_1'$ & $x_2'$ be the coordinates of the ends of the rod.

∴ The length of the rod is $x_2' - x_1' = l$.

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \qquad x_2' = \frac{x_2 - vt}{\sqrt{1-v^2/c^2}}$$

$$x_2' - x_1' = l = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{l = \frac{l_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

$\therefore \cancel{l_0 > l}$

$$x_1 = \frac{x_1' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}} \qquad x_2 = \frac{x_2' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$l_0 = x_2 - x_1 = \frac{x_2' - x_1'}{\sqrt{1 - v^2/c^2}} = \frac{l}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

$$\boxed{l_0 = \frac{l}{\sqrt{1 - \frac{v^2}{c^2}}}} \qquad \boxed{l = l_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

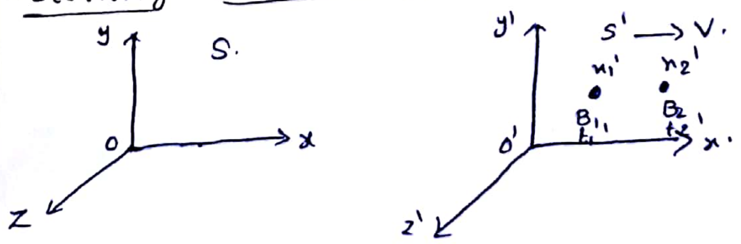
$\therefore l < l_0.$

Note :- 1) If $v << c$.

$\therefore l = l_0.$

2) Length contraction appears only in the direction of motion of object.

## Relativity Simultaneity :-




According to s' frame.

$t_2' - t_1' = 0$ (simultaneous.)

According to s frame the time at which bomb explodes $t_1$ & $t_2$ at places $x_1$ & $x_2$.

The time interval $t_2 - t_1 \neq 0$.

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$$\frac{t_2' + (\frac{v}{c^2}) x_2'}{\sqrt{1 - v^2/c^2}} - \left( \frac{t_1' + (v/c^2) x_1'}{\sqrt{1 - v^2/c^2}} \right) = \frac{t_2' - t_1' + (v/c^2)(x_1' - x_2')}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{(v/c^2)\ (x_2' - x_1')}{\sqrt{1 - v^2/c^2}}$$

$$\neq 0.$$

The event is simultaneous for one observer is not simultaneous for another observer who is moving relative to 1st observer.

1) A certain particle has a life time at $1 \times 10^{-7}$ sec when measured at rest how far does it go before decaying if its speed is 0.99c.

A) $t_0 = 1 \times 10^{-7}$ s

$v = 0.99c$

Distance $s = ?$

$t = ?$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{10^{-7}}{\sqrt{1 - (0.99)^2 c^2 / c^2}} = 7 \times 10^{-7} s$$

$$S = v \times t = 0.99c \times 7 \times 10^{-7} = 0.99 \times 3 \times 10^8 \times 7 \times 10^{-7}$$

$$= 21 \times 9.9.$$

$$= 207.9 \text{ m}.$$

21
×99
189
189
207.9

2) An aeroplane is flying at 300m/s. How much time must elapse before a clock in the aeroplane and one on the ground differ by 1 sec.

A) Given:-

$t - t_0 = 1s.$

$V = 300$ m/s.

$t = ?$

$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}} =$$

$$1 + \cancel{t_0} = \frac{\cancel{t_0}}{\sqrt{1 - \frac{300 \times 300}{9 \times 10^{16}}}}$$

The given velocity is very very less than velocity of light

Therefore relativistic approach cannot be applied.

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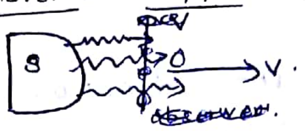
## Doppler Effect :-

The apparent change in frequency of light due to the relative motion between the observer & source. is called Doppler effect.

### 1) Transverse Doppler Effect :-

**Away from Source :**




Let $t_0$ be the time interval between the two ticks.
$t$ be the interval between the two ticks for observer.
Let $vt$ be distance covered / travelled by the light to reach the observer.
Time taken by the light to reach the observer is $\left(\frac{vt}{c}\right)$.
Total time taken by light to reach observer when he is in motion.

$$T = t + \frac{vt}{c}$$

$$= t\left(1 + \frac{v}{c}\right)$$

$$T = \frac{t_0}{\sqrt{1 - v^2/c^2}}\left(1 + \frac{v}{c}\right)$$

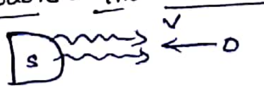
Frequency $\nu = \frac{1}{T} = \frac{\sqrt{1 - v^2/c^2}}{t_0\left(1 + \frac{v}{c}\right)}$.

$$\nu = \nu_0 \frac{\sqrt{1 - v^2/c^2}}{\left(1 + \frac{v}{c}\right)}$$

$$= \frac{\nu_0 \cdot \cancel{\sqrt{1 + v/c}} \cdot \sqrt{1 - \frac{v}{c}}}{\cancel{\sqrt{1 + v/c}} \cdot \sqrt{1 + \frac{v}{c}}}$$

$$\boxed{\nu = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}}}.$$

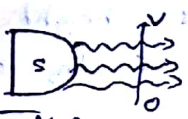
**Towards the Source :**




$$T = t - \frac{vt}{c}.$$

$$\boxed{\nu = \nu_0 \sqrt{\frac{1 + v/c}{1 - v/c}}}.$$

**Ler to the source :-**




$$T = t$$

$$T = \frac{t_0}{\sqrt{1 - v^2/c^2}}. \Rightarrow \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0}$$

$$\boxed{\nu = \nu_0 \cdot \sqrt{1 - v^2/c^2}}$$

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Note:-

1) When observer is moving away from source the apparent frequency decreases. i.e, $\nu < \nu_0$

2) When observer is moving towards the source the apparent frequency increases. i.e, $\nu > \nu_0$

3) When observer is moving $\perp$er to the direction of light the apparent frequency decreases. i.e, $\nu < \nu_0$.

Application Of Doppler Effect:-

eg: Expansion Of Universe:-

Problem:-

1) A rocket is 100m long on the ground when it is in flight its length is 99m to an observer on the ground what is its speed?

A) Given:- $l = 99m.$

$l_0 = 100\ m.$

$$l = l_0 \cdot \sqrt{1 - v^2/c^2}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{99}{100}$$

$$1-\frac{v^2}{c^2} = \left(\frac{99}{100}\right)^2.$$

$$\frac{v^2}{c^2} = 1-\left(\frac{99}{100}\right)^2.$$

$$\frac{v}{c} = \sqrt{1-\left(\frac{99}{100}\right)^2}.$$

$$\frac{v}{c} = \frac{\sqrt{100^2-99^2}}{100}.$$

$$\frac{v}{c} = \frac{\sqrt{199}}{100}$$

$$v = \frac{3\times10^8 \times \sqrt{199}}{100}$$

$$= 3\times10^6 \times \sqrt{199}.$$

$$= 42.32\times10^6\ m/s.$$

$$V = 0.423\times10^8\ m/s.$$

2) An astronaut whose height on the earth is 6ft is flying parallel to the axis of spacecraft moving 0.9c relative to earth? What is its height measured by an observer in the same spacecraft & by observer on earth.

A) Height $l = l_0\sqrt{1-\frac{v^2}{c^2}} : 6\times\sqrt{1-(0.9)^2}$

measured by observer on earth = $6\times0.435 = 2.61$ ft.

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Height measured by the another observer in the space craft, astronaut's height = 6ft (∵ same frame of ref).

3) A star is moving away from the earth at a speed of $6\times10^7$ m/s. By how much the yellow spectral line of wave length 580 nm emitted by the star is red shifted.

A) $V = n_0\lambda_0$ $\lambda = ?$

$$n_0 = \frac{V}{\lambda_0} = \frac{6\times10^7}{580\times10^{-9}}$$

$$= \frac{6\times10^7\times10^8}{58}$$

$n_0 = 0.103\times10^{15}$ Hz.

$n_0 = 1.03\times10^{14}$ Hz.

$$n = n_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

$$= 1.03\times10^{14}\sqrt{\frac{1-\frac{6\times10^7}{3\times10^8}}{1+\frac{6\times10^7}{3\times10^8}}}$$

$$= 1.03\times10^{14}\sqrt{\frac{1-0.2}{1+0.2}}$$

$$= 1.03\times10^{14}\times\frac{0.894}{1.095}$$

$n = 0.84\times10^{14}$.

$V = n\lambda$.

$$\lambda = \frac{V}{n} = \frac{6\times10^7}{0.84\times10^{14}}$$

$$= \frac{6\times10^{-7}}{0.84}$$

$= 7.14\times10^{-7}$

$\lambda = 714$ nm.

4) A distant galaxy moving away from earth such a high speed that blue-hydrogen line at a wave length 434 nm is recorded as 600nm in red orange of spectrum. what is speed of galaxy?

A) Given: $\lambda_0 = 434$ nm.

$\lambda = 600$ nm.

$V = ?$

$$n = n_0\sqrt{\frac{1-v/c}{1+v/c}}$$

$$\lambda_0 = \lambda\sqrt{\frac{1-v/c}{1+v/c}}$$

$$\frac{434}{600} = \sqrt{\frac{1-v/c}{1+v/c}} \Rightarrow \frac{1-v/c}{1+v/c} = \left(\frac{434}{600}\right)^2$$

$$\frac{c-v}{c+v} = \left(\frac{434}{600}\right)^2 = (0.723)^2 = 0.522$$

$c-v = (c+v)\times0.522$

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$$c - 0.522c = v + 0.522v.$$

$$0.478c = 1.522v.$$

$$v = \frac{0.478c}{1.522} = 0.314c. = 9.4 \times 10^7 m/s.$$

5) A meter stick is moving w.r.t an observer appears 500 mm long. What is its relativistic speed ? How long does it take to pass the observer when meter stick is moving ||el to the direction of its motion.

A) $l = 500mm = 0.5m$ $l_0 = 1m.$

$$l = l_0\sqrt{1 - v^2/c^2}$$

$$\left(\frac{0.5}{1}\right)^2 = 1 - \frac{v^2}{c^2}.$$

$$\frac{v^2}{c^2} = 1 - 0.25$$

$$\frac{v}{c} = \sqrt{0.75}$$

$$v = \frac{\sqrt{3}}{2}c.$$

$$v = 2.6 \times 10^8 m/s.$$

$$T = \frac{D}{V} = \frac{1}{2.6 \times 10^8} = 0.384 \times 10^{-8} = 3.84 ns.$$

6) A person standing on a platform observes that a train moving with velocity 0.6c. Takes 1sec to pass by him. Rest length Find length of train as seen by the person. of the train.

A) $D = T \times V$

$$= 1 \times 0.6 \times 3 \times 10^8 = 1.8 \times 10^8 m. = \text{length of train as seen by observer}$$

$$l = l_0\sqrt{1 - \frac{v^2}{c^2}}.$$

$$\frac{1.8 \times 10^8}{\sqrt{1 - \frac{(0.6)^2c^2}{c^2}}} = l_0.$$

$$l_0 = 2.25 \times 10^8 m.$$

7) An astronaut is standing in a spacecraft ||el to its direction of motion. An observer on the earth observes that space craft speed is 0.6c and astronaut height is 1.3m. What is the astronaut's height as measured in the space craft.

A) Given:- $l = 1.3m.$ $l_0 = ?$

$$l_0 = \frac{1.3}{\sqrt{1 - (0.6)^2}} = 1.625m.$$

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8) If a rod is appear shrunk by half along its direction of motion at what speed it should travel.

A) $l = l_0\sqrt{1-v^2/c^2}$

$$\frac{l_0}{2} = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$$\frac{1}{4} = 1-\frac{v^2}{c^2}.$$

$$\frac{v^2}{c^2} = \frac{3}{4}.$$

$$v = \frac{\sqrt{3}}{2}c.$$

$$= 0.866c.$$

9) A woman leaves the earth in a spacecraft that make a round trip to nearest star 4 light year distance at a speed of 0.9C. How much younger is she upon her return than her own twin sister who remained behind on earth.

A) $D = T \times V.$

$$T = \frac{D}{V} = \frac{4}{0.9} = 4.44 \text{ yrs.}$$

$$~~= \frac{4}{0.9\times 3\times10^8}~~$$

$$~~= \frac{4}{2.7}\times10^{-8}~~$$

$$~~= 1.48\times10^{-8} = t.~~$$

$$~~t_0 = t\sqrt{1-v^2/c^2}.~~$$

$$~~t_0 = t \times 0.43 = 0.645\times10^{-8}\text{ sec.}~~$$

$$t_0 = 2t.$$

$$t_0 = t\sqrt{1-v^2/c^2}$$

$$t_0 = 4.44\times0.43 = 1.91 \text{ yrs}$$

$$~~t_0 = 2t = 2\times4.44 = 8.88.~~$$

$$~~\Delta t = 8.88 - 4.44 = 4.44 \text{ yrs.}~~$$

Woman is 1.91 yrs younger to her twin sister.

## Velocity Addition:-

Consider two frames S & S'. Let $V_x, V_y, V_z$ be the components of velocity observed by observer from S-frame $V_x', V_y', V_z'$ be velocity of components observed by observer from S' frame.

From L.T.E, we have

$$x = \frac{x'+vt'}{\sqrt{1-v^2/c^2}} \Rightarrow dx = \frac{dx'+vdt'}{\sqrt{1-v^2/c^2}}.$$

$$t = \frac{t'+\left(\frac{v}{c^2}\right)x'}{\sqrt{1-v^2/c^2}} \Rightarrow dt = \frac{dt'+(v/c^2)dx'}{\sqrt{1-v^2/c^2}}.$$

$$\frac{dx}{dt} = \frac{dx'+vdt'}{\cancel{\sqrt{1-v^2/c^2}}} \times \frac{\cancel{\sqrt{1-v^2/c^2}}}{dt'+\left(\frac{v}{c^2}\right)dx'}$$

$$V_x = \frac{dx'+vdt'}{dt'+\left(\frac{v}{c^2}\right)dx'}.$$

5-1-18.

$$V_x = \frac{\left(\frac{dx_1'}{dt'}\right) + V}{1 + \left(\frac{V}{c^2}\right)\frac{dx'}{dt'}}.$$

w.r.t s-frame.

$$\boxed{V_x = \frac{V_x' + V}{1 + \left(\frac{V}{c^2}\right) V_x'}}$$

S'-Frame.

$$\boxed{V_x' = \frac{V_x - V}{1 - \left(\frac{V}{c^2}\right) V_x}}$$

We know that $dy = dy'$.

$$V_y = \frac{dy'}{dt} = \frac{dy' \sqrt{1 - v^2/c^2}}{dt' + (v/c^2) dx'}$$

$$V_y = \frac{\left(\frac{dy'}{dt'}\right)\sqrt{1 - v^2/c^2}}{1 + \left(\frac{V}{c^2}\right)\frac{dx'}{dt'}}.$$

$$\boxed{V_y = \frac{V_y' \sqrt{1 - v^2/c^2}}{1 + \left(\frac{V}{c^2}\right) V_x'}}$$

$$\boxed{V_z = \frac{V_z' \sqrt{1 - v^2/c^2}}{1 + \left(\frac{V}{c^2}\right) V_x'}}$$

$$\rightarrow V_x = \frac{V_x' + V}{1 + \left(\frac{V}{c^2}\right) V_x'}$$

$V_x' = c.$

$$V_x = \frac{c + V}{1 + \left(\frac{V}{c^2}\right) \times c}$$

$$= \frac{c + V}{c + V} \times c.$$

$V_x = c$

This proves velocity of light remains constant.

1) A person on a rocket travelling at a speed of 0.5c with respect to earth observes a meteroid coming from behind and passing him at a speed of 0.5c. How fast is the meteroid is moving w.r.to the earth.

A) Given: $V_x' = 0.5c$.

Velocity of person $V = 0.5c$.

$$V_x = \frac{V_x' + V}{1 + \left(\frac{V}{c^2}\right) V_x'} = \frac{0.5c + 0.5c}{1 + \frac{0.5 \times 0.5 \times c^2}{c^2}} = \frac{1c}{1 + .25} = \frac{3 \times 10^8}{1.25} \text{ m/s.}$$

$$= \frac{3 \times 100}{125} \times 10^8$$

$V_x = 2.4 \times 10^8$ m/s.

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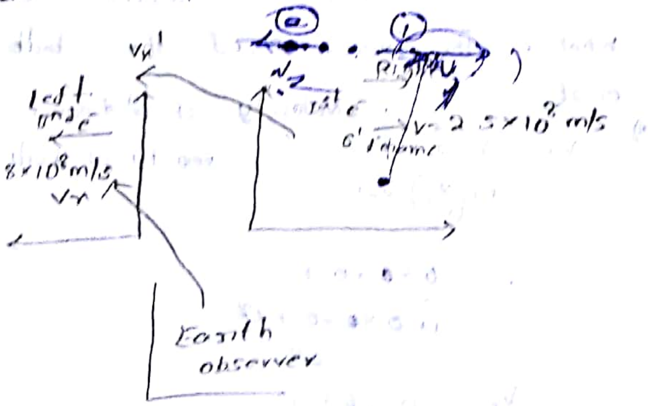
2) An electron moving to the right with a speed of $2.5 \times 10^8$ m/s passes an e⁻ moving to the left with a speed of $2.8 \times 10^8$ m/s. Find the speed of one relative to other.

A) $\cancel{V_x' = V_x - V_R}$.




$$V_x' = \frac{V_x - V}{1 - \left(\frac{V}{c^2}\right)(V_x)}$$

$$= \frac{-5.3 \times 10^8}{1 + \frac{2.5 \times 2.8 \times 10^{16}}{3 \times 3 \times 10^{16}}}$$

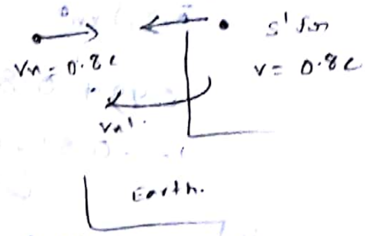
$$= \frac{\times 10^8 \times 9}{9 + 2.5 \times 2.8}$$

$$= \frac{-5.3 \times 9 \times 10^8}{16}$$

$\cancel{V_x' = 1.35 \times 10^8 \text{ m/s}}$

$V_x' = 2.9 \times 10^8$ m/s.

3) 2 particles come towards each other with a speed of 0.8c w.r.t earth what is their relative speed.




A) $V_x' = \frac{V_x - V}{1 - \left(\frac{V}{c^2}\right) V_x}$. V is −ve.

$$= \frac{V_x - V}{1 - \frac{(-0.8c) \times 0.8c}{c^2}}$$

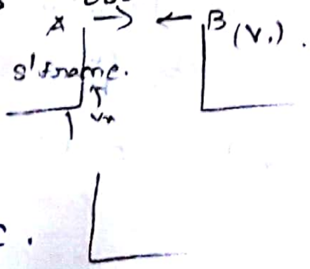
$$= \frac{0.8c - (-0.8c)}{1 + 0.64}$$

$$= \frac{1.6c}{1.64}$$

$$= \frac{1.6 \times 3 \times 10^8}{1.64}$$

$V_x' = 2.92 \times 10^8$ m/s $= 0.97c$.

4) 2 space crafts A & B are moving in opposite directions. An observer on the earth measures the speed of A is 0.75c & speed of B 0.85c. Find the velocity of B as observed by the observe on the space craft A.




V is −ve.

$$V_x' = \frac{V_x - V}{1 - \left(\frac{V}{c^2}\right) V_x} = \frac{0.75c + 0.85c}{1 - \frac{(-0.85)c \cdot 0.75c}{c^2}}$$

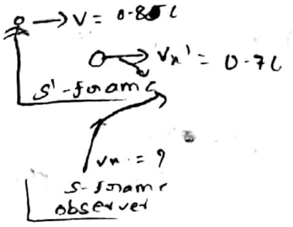
$$= \frac{1.6c}{1.637} = 0.977c.$$

8-1-18.

5) A motor cycle moving with a speed of 0.85c passes an observer at rest. If the rider tosses a ball in the forward direction with a speed 0.7c relative himself. What is the speed of the bulb relative to the observer at rest.

A) Velocity of $S'$ frame $v = 0.8c$.

Velocity of ball $v_x' = 0.7c$.



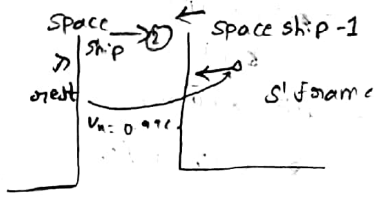

$$v_x = \frac{v_x' + v}{1 + \left(\frac{v}{c^2}\right) v_x'}$$

$$= \frac{0.85 + 0.7}{1 + \frac{0.85 \times 0.7\, c^2}{c^2}}$$

$$v_x = 0.961\, c$$

6) Two spaceships are heading directly towards each other at 0.8c. At what speed must a gun shot fired from the 1st space ship to approach the other at 0.99c. as seen by the 2nd ship.

A) $v_x = 0.99c$ (w.r.t 2nd spaceship)

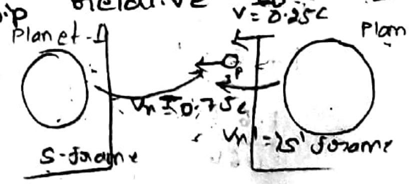



$$v_x' = \frac{v_x - v \to (-ve)}{1 - \left(\frac{v}{c^2}\right) v_x}$$

$$= \frac{(0.99 + 0.8)c}{1 - \frac{(-0.8) \times 0.99 \times c^2}{c^2}}$$

$$v_x' = 0.998 \cdot c$$

7) Two planets are heading directly towards each other at 0.25c. A space ship sent from one plane approaches the second at 0.75c. as seen by the 2nd planet. What is the velocity of space ship relative to 1st planet




A) $v_x = 0.75c$

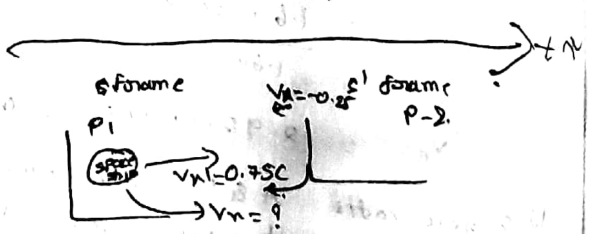
$v = -0.25c$

$$v_x' = \frac{0.75 + 0.25\, c}{1 - \frac{(-0.25) \times 0.75\, c^2}{c^2}}$$

$$= \frac{1}{1 + \frac{1}{4} \times \frac{3}{4}}\, c$$

$$= \frac{16}{19}\, c$$

$$v_x' = 0.842c.$$




$$v_x = \frac{v_x' + v}{1 + \frac{v}{c^2} v_x} = \frac{0.75 - 0.25}{1 - \frac{0.25}{c^2} \times 0.75c^2}$$

$$v_x = 0.615c.$$

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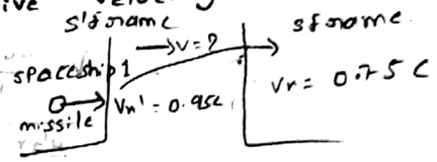
11-1-2018.

8) When a missile is shot from one spaceship towards another. It leaves the first one at $0.95c$ and approaches the other at $0.75c$. What is the relative velocity of 2 ships

A)




$$V_x' = \frac{V_x - v}{1 - \left(\frac{v}{c^2}\right) V_x}$$

$$0.95c = \frac{0.75c - v}{1 - \frac{(0.75c)\, v}{c^2}}$$

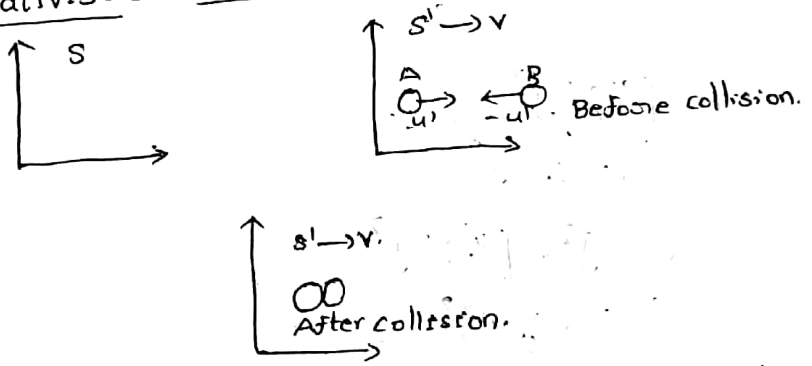
$$0.95c = \frac{0.75c^2 - vc}{c - 0.75v}$$

$$0.95c^2 - 0.7125cv = 0.75c^2 - vc$$

$$0.2c^2 = -0.2875vc$$

$$v = \frac{0.2}{0.2875} = -0.695c$$

## Relativistic Mass :-




Consider a collision between 2 identical bodies A & B each of mass 'm' and moving with velocity u' in S' frame.

According to law of conservation of momentum

$$mu' - mu' = m(u' - u') = 0 \longrightarrow ①$$

For S-frame let $m_1$ & $m_2$ be the masses of A & B respectively. $u_1$ & $u_2$ be the velocities of A & B.

According to law of momentum

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, V \longrightarrow ②$$

We have $u_1 = \dfrac{u' + v}{1 + \left(\frac{v}{c^2}\right) u'}$ , $u_2 = \dfrac{-u' + v}{1 - \left(\frac{v}{c^2}\right) u'}$

$$② \Rightarrow m_1 \left[\frac{u' + v}{1 + \frac{v}{c^2} u'}\right] + m_2 \left[\frac{-u' + v}{1 - \frac{v}{c^2} u'}\right] = (m_1 + m_2)\, V$$

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$$m_1\left[\frac{u'+v}{1+\frac{v}{c^2}u'}-v\right]=m_2\left[v-\frac{-u'+v}{1-\frac{v}{c^2}u'}\right]$$

$$\frac{m_1}{m_2}=\frac{v-\frac{-u'+v}{1-\frac{v}{c^2}u'}}{\frac{u'+v}{1+\frac{v}{c^2}u'}-v}$$

$$=\frac{\frac{\cancel{v}-\cancel{\frac{v^2}{c^2}u'}+u'-\cancel{v}}{1-\frac{v}{c^2}u'}}{\frac{u'+\cancel{v}-\cancel{v}-\frac{v^2}{c^2}u'}{1+\frac{v}{c^2}u'}}=\frac{u'-\cancel{\frac{v^2}{c^2}u'}}{1-\frac{v}{c^2}u'}\times\frac{1+\frac{v}{c^2}u'}{u'-\cancel{\frac{v^2}{c^2}u'}}$$

$$\boxed{\frac{m_1}{m_2}=\frac{1+(v/c^2)u'}{1-\left(\frac{v}{c^2}\right)u'}}\longrightarrow (3)$$

Let

$$1-\frac{u_1^2}{c^2}=1-\frac{\left[\frac{u'+v}{1+\frac{v}{c^2}u'}\right]^2}{c^2}$$

$$1-\frac{u_1^2}{c^2}=1-\frac{\left[\frac{u'+v}{c}\right]^2}{\left[1+\frac{v}{c^2}u'\right]^2}$$

$$=\frac{\left[1+\frac{v}{c^2}u'\right]^2-\left[\frac{u'+v}{c}\right]^2}{\left[1+\frac{v}{c^2}u'\right]^2}$$

$$=\frac{1+\frac{v^2}{c^4}u'^2+\cancel{\frac{2vu'}{c^2}}-\left(\frac{u'^2}{c^2}-\frac{v^2}{c^2}-\cancel{\frac{2vu'}{c^2}}\right)}{\left[1+\frac{v}{c^2}u'\right]^2}$$

$$=\frac{\left(1-\frac{u'^2}{c^2}\right)-\frac{v^2}{c^2}\left(1-\frac{u'^2}{c^2}\right)}{\left[1+\frac{v}{c^2}u'\right]^2}$$

$$1-\frac{u_1^2}{c^2}=\frac{\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left[1+\frac{v}{c^2}u'\right]^2}\longrightarrow (4)$$

Similarly

$$1-\frac{u_2^2}{c^2}=\frac{\left(1-\frac{u'^2}{c^2}\right)\left(1-\frac{v^2}{c^2}\right)}{\left[1-\frac{v}{c^2}u'\right]^2}\longrightarrow (5)$$

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(5)/(1)
$$\frac{1-u_2^2/c^2}{1-u_1^2/c^2} = \frac{\left[1+\frac{v}{c^2}u'\right]^2}{\left[1-\frac{v}{c^2}u'\right]^2} \longrightarrow (6).$$

Compare eqns (2) & (6).

$$\frac{m_1}{m_2} = \frac{\sqrt{1-u_2^2/c^2}}{\sqrt{1-u_1^2/c^2}}.$$

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_0.$$

$$m_1\sqrt{1-\frac{u_1^2}{c^2}} = m_0, \quad m_2\sqrt{1-\frac{u_2^2}{c^2}} = m_0.$$

$$m_1 = \frac{m_0}{\sqrt{1-\frac{u_1^2}{c^2}}} \qquad m_2 = \frac{m_0}{\sqrt{1-\frac{u_2^2}{c^2}}}.$$

In general

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}} \longrightarrow \text{relativistic mass.}$$

$\boxed{m > m_0}$

→ The body which is in motion appears to be heavier.

## Relativistic Momentum :-

We have momentum $P = mv$.

$$P = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \cdot v.$$

$$\boxed{P = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}}$$

## Relativistic Force :-

$$F = \frac{dP}{dt}.$$

$$= \frac{d}{dt}\left[\frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= m_0 \frac{d}{dt}\left[\frac{v}{\sqrt{1-\frac{v^2}{c^2}}}\right]$$

$$= \frac{m_0}{\left(1-\frac{v^2}{c^2}\right)^{3/2}} \frac{dv}{dt}.$$

$$\boxed{F = \frac{m_0 a}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}}$$

$$\frac{\sqrt{1-\frac{v^2}{c^2}}\cdot 1 - v\cdot\frac{1}{2\sqrt{1-\frac{v^2}{c^2}}}\cdot\frac{-2v}{c^2}}{1-\frac{v^2}{c^2}}$$

$$\frac{2\left(1-\frac{v^2}{c^2}\right)c^2 - 2v^2}{2\sqrt{1-\frac{v^2}{c^2}}\left(1-\frac{v^2}{c^2}\right)}$$

$$\frac{2c^2 - 2v^2 - 2v^2c^2}{c^2\cdot 2\sqrt{1-\frac{v^2}{c^2}}\left(1-\frac{v^2}{c^2}\right)}$$

$$\frac{c^2 - v^2 - v^2c^2}{2\sqrt{1-\frac{v^2}{c^2}}\left(1-\frac{v^2}{c^2}\right)}$$

$$\frac{c^2 - v^2 - v^2c^2}{(c^2-v^2)^{3/2}}.$$

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$$a = \frac{dv}{dt}$$

$$\frac{\left(\sqrt{1-\frac{v^2}{c^2}}\right) a - v \cdot \frac{1}{2\sqrt{1-\frac{v^2}{c^2}}} \cdot 2v}{\left(1-\frac{v^2}{c^2}\right)}$$

$$\boxed{a = \frac{F(1 - v^2/c^2)^{\frac{3}{2}}}{m_0}}$$ Relativistic Acceleration.

$$a = \frac{F}{m_0}(1 - v^2/c^2)^{\frac{3}{2}}$$

$$= \frac{\quad}{m_0^2}$$

## Relativistic Kinetic Energy:

We have $F = \frac{dp}{dt} = \frac{d}{dt}(mv)$

In relativistic theory both m & v are variable

$$F = m\frac{dv}{dt} + v\frac{dm}{dt}.$$

Change in K.E = work done.

$$d(KE) = \text{Force} \times \text{displacement} = F \times dx$$

$$d(KE) = \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right) \cdot dx.$$

$$= m\,dv\frac{dx}{dt} + v\,dm\left(\frac{dx}{dt}\right)$$

$$d(KE) = m\,dv \times v + v^2 dm. \longrightarrow ①$$

We have $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$.

squaring on both sides.

$$m^2 = \frac{m_0^2}{1-\frac{v^2}{c^2}}.$$

$$m^2 = \frac{m_0^2 c^2}{c^2 - v^2}.$$

$$m^2c^2 - m^2v^2 = m_0^2c^2.$$

$$m^2c^2 = m^2v^2 + m_0^2c^2.$$

differentiate

$$c^2 \times \cancel{2m}\,dm = \cancel{2m}\,dm\,v^2 + \cancel{2}v\,dv \cdot m^{\cancel{2}} + 0.$$

$$c^2 dm = v^2 dm + mv\,dv \longrightarrow ②$$

Compare eqns ① & ②

$$d\,KE = c^2 dm$$

Integrate $$\int_0^{E_K} d\,KE = \int_{m_0}^{m} c^2\,dm.$$

$$\boxed{E_K = (m - m_0)c^2}$$

$$E_K = mc^2 - m_0c^2$$

$$\boxed{E_K = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2}$$

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Total Energy $E = E_K + E_0$

$$= \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \cancel{m_0c^2} + \cancel{m_0c^2}$$

$$\boxed{E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}}$$

## Relation Between Total Energy & Momentum :-

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$P = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

$$E^2 - p^2c^2 = \frac{m_0^2c^4}{1-\frac{v^2}{c^2}} - \frac{m_0^2v^2c^2}{1-\frac{v^2}{c^2}}$$

$$E^2 - p^2c^2 = \frac{m_0^2c^2(c^2-v^2)}{1-\frac{v^2}{c^2}}$$

$$= \frac{m_0^2c^2\cancel{(c^2-v^2)}}{\cancel{c^2-v^2}} \times c^2$$

$$\boxed{E^2 - p^2c^2 = m_0^2c^4}$$

## Massless Particle :-

Consider a particle of mass 'm' moving with a velocity 'v'.

when mass $m=0$, velocity $v<<c$

$p = mv = 0$ } massless Particle does not exists.

$E = mc^2 = 0$

**Relativistic :-** $\boxed{v \simeq c} \rightarrow$ condition for massless particle exist

If $m_0 = 0$

$$P = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0}{0}$$

$$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{0}{0}$$

eg:- Photon.

$$E^2 = p^2c^2 + m_0^2c^4$$

$$E^2 = p^2c^2$$

$$E = pc$$

$$\boxed{p = \frac{E}{c} = \frac{h\nu}{c}}$$

18-1-2018.

$E = mc^2 = 9.1\times10^{-31}\times9\times10^{16} = 81.9\times10^{-15}$ J.

In eV $= \frac{81.9\times10^{-15}}{1.6\times10^{-19}} = 51.18\times10^{4}$ eV.

$= 0.511$ MeV

$E = 0.511$ MeV → rest energy of electron

→ Mass of Proton $= 1.67\times10^{-27}$ kg.

Rest energy of ~~elec~~ proton $= \frac{1.67\times10^{-27}\times9\times10^{16}}{1.6\times10^{-19}}$

$\approx 9.39\times10^{8}$

$= 939$ MeV.

→ Momentum of $\bar{e}$ $\quad P = M_e V = 9.1\times10^{-31}\times0.5c$

$= 9.1\times10^{-31}\times0.5\times3\times10^{8}$

$P = 14.25\times10^{-23}$

$E = PC$

$P = \frac{E}{c} = \frac{MeV}{c}$ ⇒ momentum.

$E = mc^2$

$m = \frac{E}{c^2} = \frac{MeV}{c^2}$ ⇒ mass

1) At what speed does K.E of a particle equal to its rest energy.

A) $E_K = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2.$ $\qquad E_0 = m_0c^2.$

Given:- $E_K = E_0$,

$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 = m_0c^2$

$\frac{\cancel{m_0c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = 2\cancel{m_0c^2}$

$1-\frac{v^2}{c^2} = \frac{1}{4}$

$\frac{v^2}{c^2} = \frac{3}{4}$

$v = \frac{\sqrt{3}}{2}c. = 2.598\times10^{8}$ m/s.

2) An $\bar{e}$ has a K.E of 0.1 MeV find its speed according to classical & relativistic. $m_e = 9.1\times10^{-31}$

A) Given :- $E_K = 0.1$ MeV.

$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 = 0.1$ MeV.

$\frac{0.511}{\sqrt{1-\frac{v^2}{c^2}}} = 0.1 + 0.511 = 0.611$

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$\sqrt{1-\frac{v^2}{c^2}} = \frac{0.511}{0.611}$.

$\boxed{1\,MeV = 931.5\,J}$

$1-\frac{v^2}{c^2} = 0.699$

0.511 - $3\times10^3$
0.1 - ?

$\frac{v^2}{c^2} = 0.3$.

$\frac{v}{c} = \sqrt{0.3}$

$v = 0.548c. = 1.644\times10^8\,m/s.$

According to classical Mechanics.

$E_K = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2E_K}{m}} = \sqrt{\frac{0.2\times1.6\times10^{-19}\times10^6}{9.1\times10^{-31}}}$.

$v =$ ~~$4.688\times10^4\,m/s$~~ $1.875\times10^8\,m/s$.

3) A particle 'S' has K.E 20 times its rest energy Find the speed of particle interms of c.

A) $E_K = 20E_0$.

$\frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2 = 20m_0c^2$.

$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 21$.

$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{21}$.

$1-\frac{v^2}{c^2} = 2.26\times10^{-3}$.

$\frac{v^2}{c^2} = 1-2.26\times10^{-3}$

$\frac{v}{c} = \sqrt{1-2.26\times10^{-3}}$

$v = 0.998c$.

4) How much work must be done to increase the speed of an $\bar{e}$ from $1.2\times10^8$ to $2.4\times10^8\,m/s$.

A) Given :- . Work done = change in K.E. $= \Delta E_K$.

$= E_{K_2} - E_{K_1}$

$= \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - \cancel{m_0}c^2 - \frac{m_0c^2}{\sqrt{1-\frac{v_1^2}{c^2}}} + \cancel{m_0}c^2$

$= m_0c^2\left[\frac{1}{\sqrt{1-\frac{0.8\times0.8}{3\times3}}} - \frac{1}{\sqrt{1-\frac{0.4\times0.4}{3\times3}}}\right]$

$= 0.511\left(\frac{1}{\sqrt{1-0.64}} - \frac{1}{\sqrt{1-0.16}}\right)$

$= 0.511\left[\frac{1}{0.6} - \frac{1}{\sqrt{0.84}}\right]$

$W = 0.294\,MeV$.

19-1-2018.

5) What is the energy of a photon whose momentum same as that of proton whose k.E is 10 mev.

A) Given:-

$P_{photon} = P_{proton}$

$E_{K(proton)} = 10\,Mev.$

$E_{photon} = ?$

$$E_{K \cdot proton} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} - m_0c^2.$$

$$\frac{10\times10^{6}\times 1.6\times10^{-19}}{1.67\times10^{-27}\times 9\times10^{16}} = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$$

$$\frac{10\times10^{6}\times10^{-19}\times10^{11}}{1.67\times9} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1$$

$$\frac{10^{-1}}{1.67\times9} + 1 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{1.0066.}$$

$$1-\frac{v^2}{c^2} = \frac{1}{1.0133} = 0.9868.$$

$$\frac{v^2}{c^2} = 0.0132.$$

$$\frac{v}{c} = 0.1148.$$

$$\therefore v = 0.434\times10^{8}\ m/s.$$

Momentum of proton $p = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$,

$$= \frac{1.67\times10^{-27}\times 0.434\times10^{8}}{\sqrt{1-\frac{0.434\times0.434}{9}}}$$

$$P. = 0.74\times10^{-19}\ kg\text{-}m/s$$

Energy of photon. $E = PC.$

$$= 0.74\times10^{-19}\times3\times10^{8}.$$

$$= \frac{2.22\times10^{-11}\ J.}{1.6\times10^{-19}.}$$

$$= 1.38\times10^{8}.$$

$$E = 138\ MeV.$$

6) Find the speed & momentum Gev/c of a proton whose total energy is 3.5 GeV/c.

17-1-18.

A) $E = 3.5\,GeV.$

$m_0 = 0.938\ GeV/c^2.$

$\cancel{E = Pc.}$

$\cancel{P = \frac{E}{c}.}$

$\cancel{= \frac{3.5}{3\times10^8}.}$

$\cancel{P = 1.16\times10^{-8}}$

$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}.$ $3.5\,\cancel{GeV} = \frac{0.938\,\cancel{GeV}/\cancel{c^2}\times\cancel{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$

$\sqrt{1-\frac{v^2}{c^2}} = \frac{0.938\times\cancel{9\times10^{16}}}{3.5}$

$\sqrt{1-\frac{v^2}{c^2}} = 0.268$

$1-\frac{v^2}{c^2} = 0.0718$

$\frac{v^2}{c^2} = 0.928.$

$\frac{v}{c} = 0.963$

$\boxed{v = 0.963\ c.}$

$P = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}.$

$= \frac{0.938\ \frac{GeV}{c^2}\times0.963\cancel{c}.}{\sqrt{1-\frac{(0.963c)^2}{c^2}}}$

$P = 3.35\ \frac{GeV}{c}.$

7) Find the T.E of a neutron whose momentum is $1.2\,GeV/c.$

A) Given :- $P = 1.2\,GeV/c.$

$m_0 = 0.938\ GeV/c^2.$

$E = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}.$

$P = \frac{m_0v}{\sqrt{1-\frac{v^2}{c^2}}}$

$1.2\ \frac{\cancel{GeV}}{\cancel{c}} = \frac{0.938\ \cancel{GeV}/c^2\times v}{\sqrt{1-\frac{v^2}{c^2}}}$

$\frac{c^2\sqrt{1-\frac{v^2}{c^2}}}{v} = \frac{0.938}{1.2} = 0.781.$

$\frac{c^2}{v^2}\left[1-\frac{v^2}{c^2}\right] = 0.611.$

$\frac{c^2}{v^2} - 1 = 0.611.$

$\frac{c^2}{v^2} = 1.611.$

$v^2 = \frac{9\times10^{16}}{1.611} = 5.586\times10^{16}.$

$v = 2.36\times10^8\ m/s.$

$E = \frac{0.938}{\sqrt{1-\frac{2.36\times2.36}{9}}} = 1.519\ GeV.$

11-1-18

8) Find the P & V of $e^-$ having K.E of 10 meV. the rest energy of $e^-$ is 0.512 mev.

A)

$$E_k = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) m_0 c^2.$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 = \frac{10}{0.512}$$

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 20.53125$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{20.53125}$$

$$\frac{v^2}{c^2} = 0.997.$$

$$v = 0.998\,C.$$

$$\cancel{P = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$E = E_k + E_0 = 10 + 0.512$$

$$E = 10.512 \text{ Mev.}$$

$$E^2 = p^2c^2 + m_0^2c^4.$$

$$p^2 = \frac{E^2 - m_0^2c^4}{c^2}$$

$$p^2 = \frac{(10.512)^2 (\text{Mev})^2 - (0.512 \text{ Mev})^2}{c^2}$$

$$P = 10.49 \frac{\text{Mev}}{C}.$$

9) A particle has K.E of 62 Mev and P of 335 $\frac{\text{MeV}}{c}$ find the mass in $\frac{\text{MeV}}{c^2}$ & speed

A) Given:- $E_K = 62$ mev  $P = 335$ Mev/c.

$$E^2 = p^2c^2 + E_0^2$$

$$(E_K + E_0)^2 = p^2c^2 + E_0^2.$$

$$E_K^2 + \cancel{E_0^2} + 2E_KE_0 = p^2c^2 + \cancel{E_0^2}$$

$$62^2 + 2\times 62 \times E_0 = (335)^2$$

$$E_0 = \frac{335^2 - 62^2}{124}$$

$$E_0 = 874 \text{ Mev}$$

$$E_0 = m_0c^2.$$

$$m_0 = \frac{E_0}{C^2} = 874 \frac{\text{Mev}}{c^2}.$$

$$E = E_K + E_0 = 936 \text{ Mev.} = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{874}{936}$$

$$1 - \frac{v^2}{c^2} = 0.871.$$

$$v^2/c^2 = 0.128. \Rightarrow v = 0.357\,C.$$

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22-1-2018

# Formulae :-

## 1) Galilean Transformations :-

$$\left.\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\} \text{G.T.E} \qquad \left.\begin{aligned} x &= x' + vt' \\ y &= y' \\ z &= z' \\ t &= t' \end{aligned}\right\} \text{I.G.T.E.}$$

## 2) Lorentz Transformations :-

$$\left.\begin{aligned} x' &= \frac{x - vt}{\sqrt{1-\frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \left(\frac{v}{c^2}\right)x}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}\right\} \text{L.T.E}$$

$$\left.\begin{aligned} x &= \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}} \\ y &= y' \\ z &= z' \\ t &= \frac{t' + \left(\frac{v}{c^2}\right)x'}{\sqrt{1-\frac{v^2}{c^2}}} \end{aligned}\right\} \text{I.L.T.E.}$$

## 3) Time Dilation :-

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$t > t_0$

$t$ = Apparent time measured from S-Frame.

$t_0$ = Proper time measured from S'-Frame.

## 4) Length Contraction :-

$$l = l_0\sqrt{1-\frac{v^2}{c^2}}$$

$\therefore l < l_0$.

$l$ = app. length (from S'-Frame) moving frame

$l_0$ = proper length (from rest frame(S))

## 5) Doppler Effect :-

~~$\nu_{away}$ =~~

$$\boxed{\nu_{away} = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}}}$$

v = velocity of body

$$\boxed{\nu_{toward} = \nu_0 \sqrt{\frac{1 + v/c}{1 - v/c}}}$$

$$\boxed{\nu_{\perp er} = \nu_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

## 6) Velocity Addition :-

S-Frame.

$$\boxed{v_x = \frac{v_x' + v}{1 + \left(\frac{v}{c^2}\right) v_x'}}$$

S'-frame.

$$\boxed{v_x' = \frac{v_x - v}{1 - \left(\frac{v}{c^2}\right) v_x}}$$

$$v_y = \frac{v_y'\sqrt{1 - v^2/c^2}}{1 + \left(\frac{v}{c^2}\right) v_x'} \qquad v_z = \frac{v_z'\sqrt{1 - v^2/c^2}}{1 + \left(\frac{v}{c^2}\right) v_x'}$$

7) <u>Relativistic Mass</u>:-

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad m > m_0.$$

8) <u>Relativistic Momentum</u>:-

$$P = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}.$$

9) <u>Relativistic Force</u>:-

$$\cancel{F = \frac{m_0 a}{\sqrt{1-\frac{v^2}{c^2}}}} \qquad \boxed{F = \frac{m_0 a}{\left(1-\frac{v^2}{c^2}\right)^{3/2}}}$$

10) <u>Relativistic Acceleration</u>:-

$$a = \frac{F\left(1-v^2/c^2\right)^{3/2}}{m_0}.$$

11) <u>Relativistic K.E</u>:-

$$F = \frac{dP}{dt}$$

$$\boxed{E_K = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} - m_0 c^2}$$

12) <u>Relation B/W T.E & Momentum</u>:-

$$E_T = \frac{m_0 c^2}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \text{Rest energy } E = m_0 c^2$$

$$\boxed{E^2 - P^2 c^2 = m_0^2 c^4}$$

13) <u>Massless Particle</u>:-

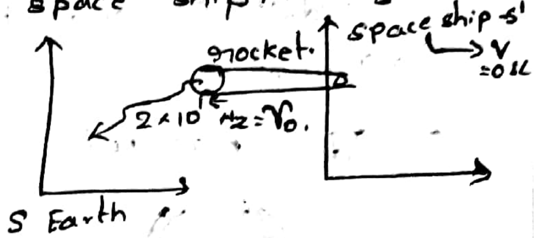
Photon. $E_{photon} = PC.$

$$P = \frac{E}{c} = \frac{h\nu}{c}.$$

rest energy of $e^-$ = 0.511 MeV.

" " proton = 939 MeV.

1) A space ship moving with a velocity $0.6C$ w.r.t an observer on the earth. A rocket is fired from the space ship in the opposite direction. Rocket emits a signal with frequency of $= 2\times10^{15}$ Hz. is observed by the observed on the earth as $1.5\times10^{15}$ Hz. what is the velocity of rocket w.r.t space ship.




A)
$$\nu' = \nu\sqrt{\frac{1-v/c}{1+v/c}}$$

$$\frac{1.5\times10^{15}}{2\times10^{15}} = \sqrt{\frac{1-v/c}{1+v/c}}$$

$$0.5625 = \frac{1-v/c}{1+v/c}$$

$$0.5625 + 0.5625\frac{v}{c} = 1-\frac{v}{c}$$

$$1.5625\frac{v}{c} = 1-0.5625$$

$$\frac{v}{c} = \frac{0.4375}{1.5625}$$

$$\frac{v}{c} = 0.28$$

$$v = 0.28C = v_x$$

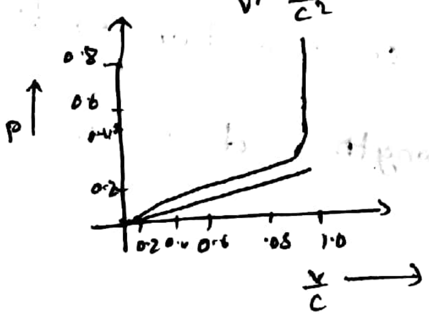
$$\cancel{v_x' = \frac{v_x - v}{1-\left(\frac{v}{c^2}\right)v_x} = \frac{0.6-(-0.28)c}{1-\left(\frac{-0.28c}{c^2}\right)0.6c}}$$

$$\cancel{v_x' = 0.75C}$$

$$v_x' = \frac{0.28C - 0.6C}{1-\frac{0.6c\times0.28c}{c^2}} = -0.38C$$

Relation Between Linear momentum & Relativistic momentum varying with velocity :-

$P = mv$ $\quad P = \frac{m_0 v}{\sqrt{1-\frac{v^2}{c^2}}}$




$$E^2 - p^2c^2 = m_0^2c^4,$$

$$E^2 - p^2c^2 = \text{constant}.$$

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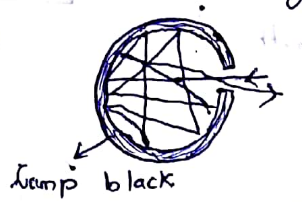
2-2-2018

# Unit-II Modern Physics

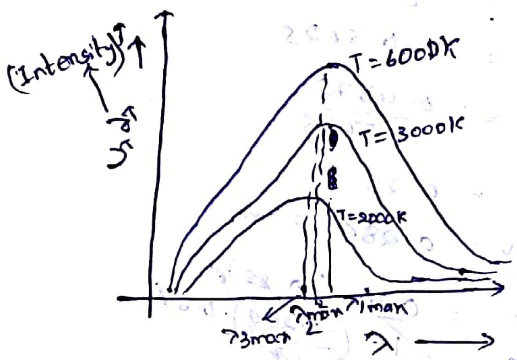
## Black body Radiation :-

**Black body** :- A object/body which perfectly absorbs all types of radiation incident on it and perfectly emits the radiation at thermal equilibrium.

→ Ideal black body doesnot exist.




Black-body Radiation Spectrum




$\lambda_{max} \propto \frac{1}{T}$

$\lambda_{max} \times T$ = a constant ⇒ Weins displacement law

$= 2.989 \times 10^{-3}$ mK.

$U_\lambda d\lambda$ → Amount of radiation emitted by the black body per unit area per second → density of radiation / Intensity.

Black-body Radiation spectrum means a graph which represents variation of Intensity as a function of wavelength (or) frequency.

## Characteristics Of Black-body Radiation :-

→ As temperature increases the amount of radiation emitted increases.

→ As temperature increases the wavelength decreases

## Wein's Displacement Law :-

$\lambda_{max} \propto \frac{1}{T}$

$\lambda_{max} T$ = a constant ⇒ Wein's displacement law

$= 2.989 \times 10^{-3}$ mK

## Rayleigh - Jeans Law :-

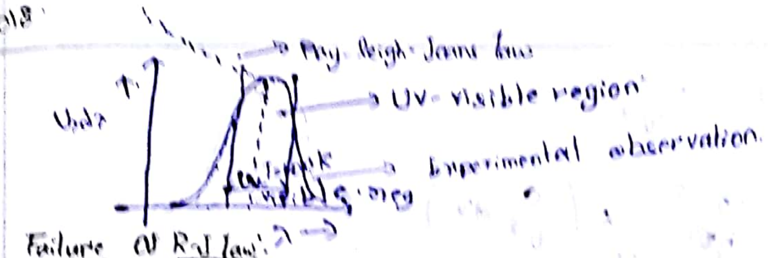
According to law of equi-partition energy, the energy associated with an oscillator is $kT$

$d\lambda$ = change in wave length

$\boxed{U_\nu d\nu = \frac{8\pi kT \nu^2 d\nu}{c^3}}$ $\boxed{U_\lambda d\lambda = \frac{8\pi kT d\lambda}{\lambda^4}}$ → R-J law.

$k = 1.38 \times 10^{-23}$ J/k.

2-2-2018




Failure of R-J law:

→ Rayleigh-Jeans law fails to explain the shorter wavelength region of black-body radiation spectrum.

→ Ultraviolet Catastrophe :- According to R-J law black body emits maximum energy in the shorter wavelength region but practical observations shows that black-body emits maximum energy in the UV-visible region. The discrepancy between experimental & theoretical observations is termed as ultraviolet - Catastrophe.

Max Planck Quantum Theory:

According to Planck, the energy or radiation emitted by black body is not continuous. It is emitted in the form of small packets of energy called quanta/photon.

photon/quanta/ $E = h\nu$

$$\Delta E = E_2 - E_1$$

$$\boxed{\nu = \frac{\Delta E}{h}}$$

Planck's Law :-

→ Black body consists of large no of oscillators. Energy associated with this $E = nh\nu$ $n = 0, 1, 2, 3, \ldots$

→ The frequency associated during absorption and emission is given by

$$\nu = \frac{\Delta E}{h} = \frac{E_2 - E_1}{h}.$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda kT} - 1)} d\lambda}$$

$c = \nu\lambda$

$\lambda = \frac{c}{\nu}$.

In terms of frequency

$$\cancel{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda kT} - 1)} d\lambda}$$

$$U_\lambda d\lambda = \frac{8\pi hc}{\frac{c^5}{\nu^5}(e^{\frac{hc}{ckT}\times\nu} - 1)} d\lambda$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi h\nu^5}{c^4 (e^{\frac{h\nu}{kT}} - 1)} d\lambda}$$

2 5-2-2018.

$$d\lambda = d\left(\frac{c}{\nu}\right) = -\frac{c}{\nu^2}d\nu.$$

$$U_\lambda d\nu = \frac{8\pi h c \nu^5}{c^5\left(e^{\frac{h\nu}{kT}}-1\right)} \times \left(\frac{c}{\nu^2}d\nu\right)$$

$$\boxed{U_\nu d\nu = \frac{8\pi h \nu^3}{c^3\left(e^{\frac{h\nu}{kT}}-1\right)}d\nu}$$

Case (i) :- For longer wave length region

If $\lambda$ is very large, ~~$e^{\frac{hc}{\lambda kT}}$~~ $\frac{hc}{\lambda kT} \ll 1$.

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$e^x = 1 + x.$$

$$x = \frac{hc}{\lambda kT}$$

$$e^x = e^{\frac{hc}{\lambda kT}} = 1 + \frac{hc}{\lambda kT}.$$

$$U_\lambda d\lambda = \frac{8\pi h \cancel{c}}{\lambda^{\cancel{5}4}\left(\cancel{1} + \frac{h\cancel{c}}{\lambda kT} - \cancel{1}\right)}d\lambda$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi kT}{\lambda^4}d\lambda} \rightarrow \text{R-J law.}$$

Case (2) :- For shorter wave length region

If $\lambda$ is very small $\frac{hc}{\lambda kT}$ is very large

i.e., $e^{\frac{hc}{\lambda kT}} \gg 1. \Rightarrow e^{\frac{hc}{\lambda kT}} - 1 = e^{\frac{hc}{\lambda kT}}$.

$$\boxed{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5 e^{\frac{hc}{\lambda kT}}}d\lambda}.$$

$$\boxed{U_\lambda d\lambda = 8\pi hc\, \lambda^{-5} e^{-\frac{hc}{\lambda kT}} d\lambda.} \rightarrow \text{Weins distribution law.}$$

According to planck's Law average energy

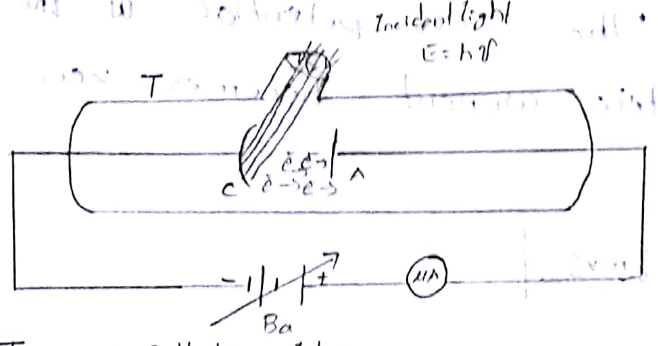
$$\boxed{\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{kT}}-1}}$$

9-2-2018

## Particle Nature Of Light :- Experiment Observation :-

**Photo Electric Effect :-** It is the evidence for light behaves as particles.

The phenomena of emission of $e^-$'s from a metallic surface when light of suitable frequency or wavelength incident on it. The electrons which are emitted are called photoelectrons. The photo electron constitute a current is called photoelectric current.




T ⟶ Cathode ray tube
C ⟶ Cathode
A ⟶ Anode

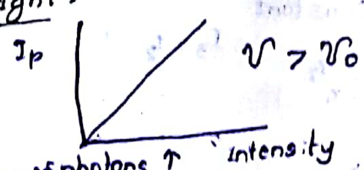
→ Photoelectric current depends on
(1) Frequency of Incident light.
(2) Intensity of Incident light.
(3) Potential difference b/w Cathode and Anode
(4) Nature of photo sensitive cathode.

**Frequency Of Incident Light :-**

→ **Threshold Frequency ($\nu_0$) :-**
The minimum frequency at which photoelectric effect takes places is called threshold frequency.

Note :- (1) Photoelectric current increases with increase in frequency of incident light before attaining saturation state
(2) As frequency increases the kinetic energy of photoelectrons increases.

→ **Intensity Of Incident Light :-**



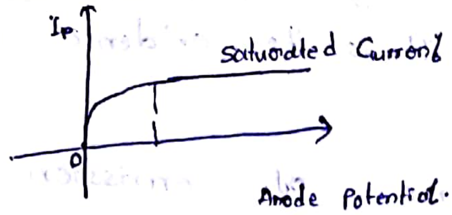

Note :-
As intensity increases no. of photons ↑ thereby the photoelectric current increase.

→ [illegible]

9-2-18

## Potential Difference B/W Cathode And Anode :-

① 

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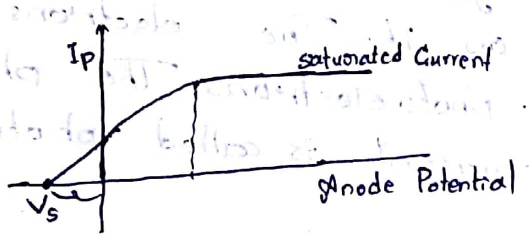
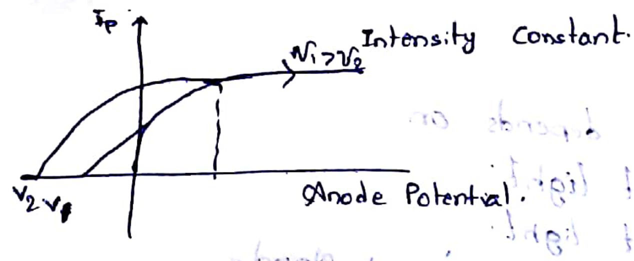


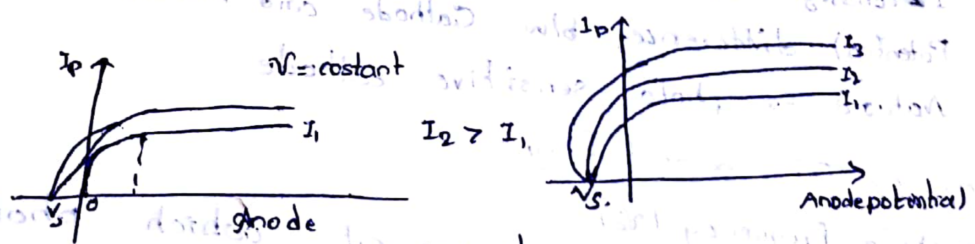

**Stopping Potential($V_s$)**:- The −ve potential of the anode at which photoelectric current becomes zero is called stopping potential.

$$\boxed{e V_s = \frac{1}{2} m v^2_{max}}$$

③ 



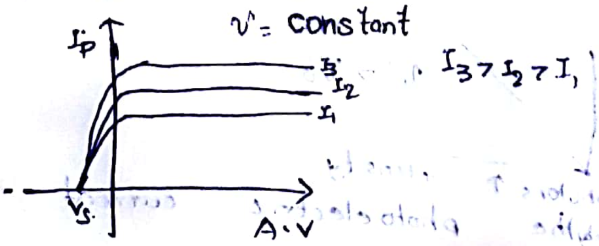

④ 




At constant frequency, intensity and photo electric current are directly proportional.

As Intensity increases the stopping potential remains constant.

At constant intensity stopping potential is directly proportional to frequency of light.



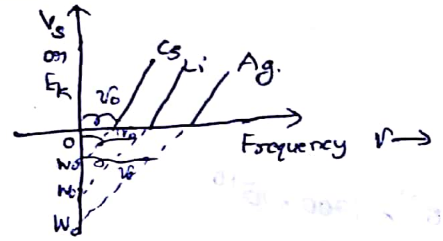
10-2-2018

Nature Of Photosensitive Metal:

Li, Cu, Ag, Cs.

Work Function: The minimum amount of energy required to eject the $\bar{e}$ from metal surface is called work function.

$$\boxed{W = h\nu_0}$$

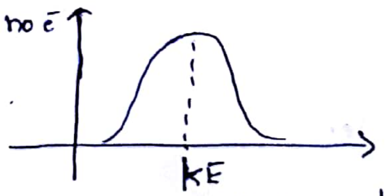



Einstein's Explanation Of Photoelectric Effect:

According to einstein light consists of pockets of energy called photon (or) quanta. When photon from incident light collides with the $\bar{e}$ in the target (or) cathode material, photon transfers its complete energy to the electron. With increase in the energy, the $\bar{e}$ ejected from the metallic surface i.e.,

$$\boxed{E = W + E_k}$$

$$\boxed{h\nu = h\nu_0 + \frac{1}{2} m V^2_{max}} \rightarrow \text{Einsteins photo eletric Equation.}$$




Einstein showed that particle particle collision between $\bar{e}$ and photon this shows that particle nature of light.

Laws Of Photoelectric Effect:-

*. Photoelectric effect is instantaneous process ($\sim 10^{-9}s$)

*. Photoelectric current depends on intensity of incident light and independent of frequency of incident light.

*. Stopping potential is directly proportional to frequency of incident light. and independent of intensity.

*. The total energy $E = W + E_K$ (or) $h\nu = h\nu_0 + \frac{1}{2}mv^2_{max}$.

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10-2-2018.

Problems:

1) The work function of Tungsten is 5.4 ev when a light of wavelength 170 nm is incident on it surface ēs with energy 1.7 ev are ejected. Estimate the planck's constant.

A) Given:- $W = 5.4 ev$

$\lambda_0 = 170 nm$

$E_K = 1.7 ev$

$E = W + E_K$

$= 5.4 + 1.7$

$\frac{hc}{\lambda} = 7.1 ev.$

$h = \frac{7.1 \times 1.6 \times 10^{-19} \times 1700 \times 10^{-10}}{3 \times 10^{8}}$

$= \frac{71 \times 16 \times 10^{\cancel{0}} \times 17 \times 10^{-37}}{3}$

$= 6437.3 \times 10^{-37}$

$h = 6.437 \times 10^{-34}$ J-s.

2) The $V_s$ for ēs emitted from a metal is 1v for light of wavelength 2500 A°. Calculate the work function is ēs

A) Given:- $V_s = 1v$.

$E = W + E_K$

$W = E - E_K$

$= \frac{hc}{\lambda} - 1ev.$

$= \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{2500 \times 10^{-10}}$

$= \frac{6625 \times 3 \times 10^{-34} \times 10^{16} \times 10^{-2}}{25 \times 1.6 \times 10^{-19}} - 1ev$

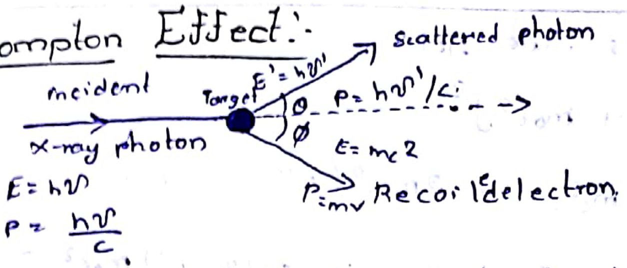
$= 4.96 ev - 1.$

$W = 3.96 ev.$

3) A metal surface has a photoelectric cut off wave length 325.6 nm. What is $V_s$ for incident light of $\lambda$ 259.8 nm

12-2-2018

# Compton Effect:-




$E' < E$

$\lambda' > \lambda$

When an X-ray photon interact with an $\bar{e}$ in the target material scattering of photon takes place. This scattered photon has higher wavelength than the incident photon. This phenomenon is called compton effect. The change in wavelength between incident photon & scattered photon is called compton shift.

Compton shift $\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}[1 - \cos\theta]}$

Consider an X-ray photon collides with free $\bar{e}$ in the target material transfer of energy takes place incident photon and the $\bar{e}$ in target material. After collision the photon comes out as a scattered photon with an angle θ with the direction of incident photon. After absorbing energy from incident photon, the electron in target material comes out as a recoiled electron with an angle 'φ' with the direction of incident photon.

Apply law of conservation of energy.

$$h\nu + m_0c^2 = h\nu' + mc^2 \longrightarrow (1)$$

Apply law of conservation of momentum

$$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\theta + mv\cos\phi$$ (Along direction of incident photon.)

$$0 + 0 = \frac{h\nu'}{c}\sin\theta - mv\sin\phi \longrightarrow (3).$$

(2) ⇒ $h\nu - h\nu'\cos\theta = mvc\cos\phi. \longrightarrow (4)$

(3) ⇒ $h\nu'\sin\theta = mvc\sin\phi \longrightarrow (5)$

Squaring & adding eqns (4) & (5), we get.

$$m^2v^2c^2(\sin^2\phi + \cos^2\phi) = (h\nu)^2 + (h\nu'\cos\theta)^2 - 2h^2\nu\nu'\cos\theta + h^2\nu'^2\sin^2\theta$$

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2[\cos^2\theta + \sin^2\theta] - 2h^2\nu\nu'\cos\theta$$

$$m^2v^2c^2 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu'\cos\theta \longrightarrow (6)$$

eqn (1) ⇒ $mc^2 = h(\nu - \nu') + m_0c^2.$

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Squaring on both sides, we get

$$m^2c^4 = [h(\nu-\nu') + m_0c^2]^2$$

$$m^2c^4 = [(h(\nu-\nu'))^2 + m_0^2c^4 + 2m_0c^2(h(\nu-\nu'))]$$

$$m^2c^4 = h^2\nu^2 + h^2\nu'^2 - 2h^2\nu\nu' + m_0^2c^4 + 2h(\nu-\nu')m_0c^2 \rightarrow (7)$$

(7) − (6)

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4 + 2h(\nu-\nu')m_0c^2 - 2h^2\nu\nu' + 2h^2\nu\nu'\cos\theta$$

$$m^2c^2(c^2-v^2) = 2h^2\nu\nu'(\cos\theta-1) + m_0^2c^4 + 2h(\nu-\nu')m_0c^2$$

We have $E = p^2c^2 + m_0^2c^4$.

$$(mc^2)^2 = (mv)^2c^2 + m_0^2c^4$$

$$m^2c^4 = m^2v^2c^2 + m_0^2c^4$$

$$m^2c^4 - m^2v^2c^2 = m_0^2c^4$$

$$m^2c^2[c^2-v^2] = m_0^2c^4 \longrightarrow (9)$$

$$\cancel{m_0^2c^4} = 2h^2\nu\nu'(\cos\theta-1) + \cancel{m_0^2c^4} + 2h(\nu-\nu')m_0c^2$$

$$\cancel{2}h(\nu-\nu')m_0c^2 = \cancel{2}h^2\nu\nu'(1-\cos\theta)$$

$$(\nu-\nu')m_0c^2 = h\nu\nu'(1-\cos\theta)$$

$$\left(\frac{c}{\lambda} - \frac{c}{\lambda'}\right)m_0\cancel{c^2} = h \times \frac{\cancel{c^2}}{\lambda\cdot\lambda'}(1-\cos\theta)$$

$$\frac{c(\lambda'-\lambda)}{\cancel{\lambda\lambda'}}m_0 = \frac{h}{\cancel{\lambda\lambda'}}(1-\cos\theta)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1-\cos\theta)}$$

Compton Shift $\boxed{\Delta\lambda = \lambda'-\lambda = \frac{h(1-\cos\theta)}{m_0c}}$

i) When $\theta = 0^\circ$ $\Delta\lambda = 0$.

ii) When $\theta = 90^\circ$, $\Delta\lambda = \frac{h}{m_0c} = 2.42\times10^{-12}$ m.

iii) When $\theta = 180^\circ$, $\Delta\lambda = \frac{2h}{m_0c} = 4.84\times10^{-12}$ m.

iii) When $\theta = 270^\circ$ $\Delta\lambda = 2.42\times10^{-12}$ m

iv) When $\theta = 360^\circ$ $\Delta\lambda = \frac{h}{m_0c} = 0$.

$\Delta\lambda$ = max for $\theta = 180^\circ$

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## i Direction of Recoiled Electron :-

Using eq$^{ns}$ (4) & (5)

$$h\nu - h\nu'\cos\theta = mvc\cos\phi.$$

$$h\nu'\sin\theta = mvc\sin\phi.$$

$$\tan\phi = \frac{h\nu'\sin\theta}{h(\nu - \nu'\cos\theta)}$$

$$\boxed{\phi = \tan^{-1}\left[\frac{\nu'\sin\theta}{\nu - \nu'\cos\theta}\right]}$$

Problem :-

1) Find $\Delta\lambda$ of scattered x-ray photon when it is scattered through an angle of $60^\circ$ by a free $e^-$.

A) Given :- $\theta = 60^\circ$.

$$\Delta\lambda = \frac{h}{m_0 c}[1-\cos\theta]$$

$$= \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}[1-\cos60^\circ]$$

$$= \cancel{0.121\times10} \; 1.212\times10^{-12} = 1.21\ \text{pm}.$$

2) x-ray with wave length of $1A^\circ$ scattered from carbon block. The scattered radiation is $90^\circ$ to incident beam.

cal. a) compton shift

b) K-E of recoild $e^-$.

A) a) Given :- $\lambda = 1A^\circ$.

$\theta = 90^\circ$

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}[1-\cos\theta]$$

$10^{-3}\times10^{-8}$

$= 10^{-11}$.

$$\lambda' = \lambda + \frac{h}{m_0 c}(1-\cos\theta)$$

$$= 1\times10^{-10} + \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}[1-\cos90^\circ]$$

$$= 1A^\circ + 0.242\times10^{-11}.$$

$$\lambda' = 1.024A^\circ$$

$$\Delta\lambda = \lambda' - \lambda = 1.024 - 1 = 0.024A^\circ.$$

(b) Kinetic energy of $e^-$ = Energy of Incident photon - energy of scattered photon

$$K.E = \frac{hc}{\lambda} - \frac{hc}{\lambda'}.$$

$$= \frac{6.62\times10^{-34}\times3\times10^{8}}{1\times10^{-10}} - \frac{6.62\times10^{-34}\times3\times10^{8}}{1.024\times10^{-10}}$$

$$= 4.65\times10^{-17}\ J.$$

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3) X-ray photon of energy 0.124 MeV undergoes compton scattering through 180°. cal. energy & wavelength of scattered photon.

A).
$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}[1-\cos\theta]$$

$\lambda$ in $A^\circ = \frac{12400}{0.124 \times 10^6}$

$= 0.1 A^\circ$

$$\lambda' = \lambda + \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^8} \times 2[1-(-1)]$$

$= 0.1A^\circ + 0.024 A^\circ$

$\lambda' = ~~0.124~~ A^\circ$ 0.148

Energy of scattered photon $= \frac{12400}{0.124}$ $= \frac{12400}{0.148} = 0.0837$ Mev.

4) Cal. velocity of photo e emitted from metal surface whose $W_0$ is 1.5 ev. & $\lambda$ of incident light is $4\times10^{-7}$ m.

A) $E = W + K.E.$

$\frac{1}{2}mv^2 = E - W.$

$= 3.1 - 1.5$

$\frac{1}{2}mv^2 = 1.6$ ev

$v^2 = \frac{3.2}{m}$

$v^2 = \frac{3.2\times1.6\times10^{-19}}{9.1\times10^{-31}}$

$v = 7.5\times10^5$ m/s.

5) Stopping potential 4.6v was absorbed for a light of $\nu = 2\times10^{15}$ Hz when the frequency of light was changed $4\times10^{15}$ Hz the stopping potential changed to 12.9 v. Cal. (h)

A) $E_1 = W_0 + KE$ $\quad E_2 = W_0 + KE$

$E_1 - KE_1 = E_2 - KE_2$

$h\nu_1 - 4.6ev = h\nu_2 - 12.9ev.$

$h(\nu_1 - \nu_2) = -12.9 + 4.6$

$h = \frac{-8.3}{(2-4)\times10^{15}}$

$= 4.15\times10^{-15} \times 1.6\times10^{-19}$

$h = 6.64\times10^{-34}$ J-s

6) X-ray of $\lambda$ 0.5 A°, undergo compton scattering. cal. a) $\lambda'$ of scattered photon b) K.E of scattering e in ev.

16-2-2018

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}[1-\cos\theta].$$

$$\lambda' = \lambda + \frac{h}{m_0 c}[1-\cos\theta]$$

$$= 0.5 + \frac{6.62\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}\left[1-\left(-\frac{1}{2}\right)\right]$$

$$= 0.5 + \frac{6.62\times10^{-34}\times3}{9.1\times10^{-31}\times3\times10^{8}\times2}$$

$$= 0.5 + 3.63\times10^{-12}.$$

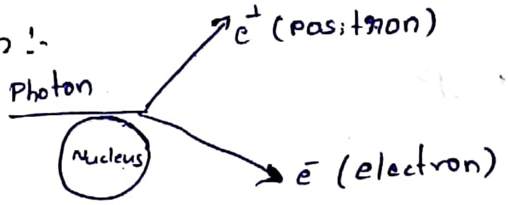
$$= 0.5 + 0.036.$$

$$\therefore \lambda' = 0.536\ A^{\circ}.$$

$$K.E = E_{incident} - E_{scattered}.$$

$$\boxed{K.E = 1665.6\ e.v.}$$

## Pair Production :-




When a photon interacts with nucleus of an element. emission of $\bar{e}$ & $e^{+}$(positron) takes place.

$$\text{photon} \xrightarrow[\text{nucleus}]{} e^{+} + e^{-}.$$

During this interaction law of conservation of charge is satisfied. The presence of nucleus is required to satisfy the law of conservation of energy & momentum because nucleus is very much heavier compared to photon.

The minimum energy of incident photon required for the pair production is 1.02 Mev. If any excess energy of photon is converted to K.E of $\bar{e}$ & ($e^{+}$)positron

→ Each $\bar{e}$ & $e^{+}$ have minimum energy of 0.51 Mev.

## Pair Annihilation :-

$$\bar{e} + e^{+} \longrightarrow \gamma + \gamma$$

$$1.02\,\text{Mev} \longrightarrow 0.51\,\text{Mev} + 0.51\,\text{Mev}.$$

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16-2-2018

## X-rays:-

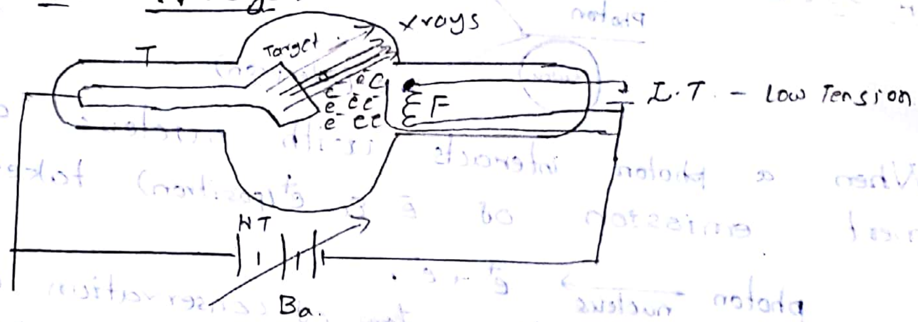
X-rays were discovered Roentgen in 1895, whose wavelength l'iew b/w 0.01nm to 10nm.

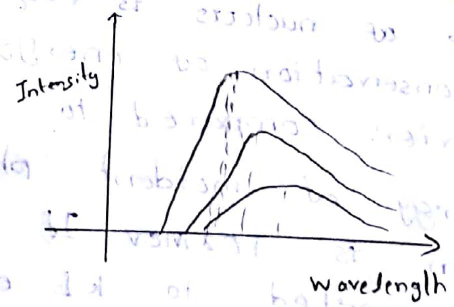
When high energetic $\bar{e}s$ collides with the $\bar{e}s$ in the target material photons (or) radiation of high frequency are emitted. These photons of unknown nature are called X-rays.

## Characteristics Of Xays:-

→ X-rays are electromagnetic radiation.
→ X-rays travel in st. lines.
→ They have high penetrating power.
→ They are not effected by electric & magnetic fields.
→ They produce phosphorescence in some metal surface

## Production Of X-rays:-




$\lambda \longrightarrow$

$$\lambda_{min} \propto \frac{1}{V}$$

$$(K.E)_{max} = eV$$

$$h\nu_{max} = eV$$

$$\frac{hc}{\lambda_{min}} = eV$$

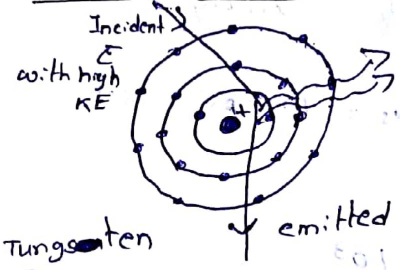
$$\lambda_{min} = \frac{hc}{eV}$$

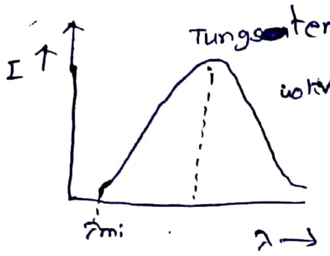
$$\boxed{\lambda_{min} = \frac{1.24 \times 10^{-6}}{V} \ V\cdot m}$$

22-2-2018

**Spectrum:-** It is arrangement of wavelength or frequency increasing order is called spectrum.

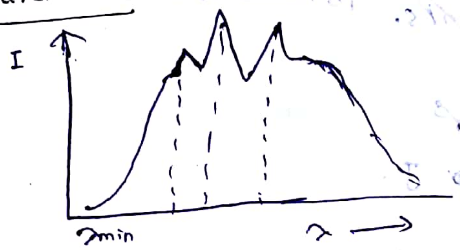
**Continuous X-ray Spectrum:-**

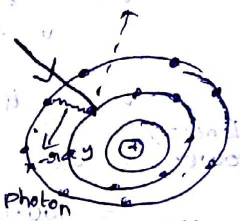



Continuous x-rays are the resultant of interaction b/w accelerated e with the outer ē clouds.

**Characteristic X-ray Spectrum:-**

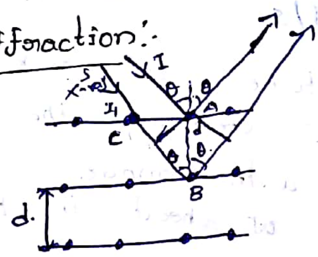



Characteristic x-rays are produced due to chipping out of core ēs of the target atoms.

**Wavelength of X-rays:-**

**Bragg's Diffraction:-**




$BC = d\sin\theta \qquad BD = d\sin\theta$

Path difference b/w two x-rays : $d\sin\theta + d\sin\theta = 2d\sin\theta$

Path difference $= n\lambda$

$\boxed{n\lambda = 2d\sin\theta} \longrightarrow$ Bragg's eq$^n$.

$$\boxed{\lambda = \frac{2d\sin\theta}{n}}$$

1) Find the $\lambda_{min}$ of x-rays produced by an x-ray tube operated on 1000 keV.

A) $\lambda_{min} = \dfrac{12400\,A^0}{(eV)}$

$= \dfrac{12400}{10^6}$

$= 0.0124\,A^0$

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2) The P.d. across X-ray tube is 100keV and current through it is 5mA. Cal. max speed of X-rays produced & rate of production heat at the target if only 0.1% of energy is converted into X-rays.

A) Given:- $V = 100kV$.

stopping potential $eV_s = 100keV = K.E$

$$K.E = eV_s$$

$$\frac{1}{2}mv^2 = 100 \times 10^3$$

$$v^2 = \frac{2 \times 100 \times 10^3 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}$$

$$v = \sqrt{\frac{2 \times 100 \times 10^3 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}}$$

$v = \cancel{4.66 \times 10^{17}\ m/s}$. $187.5 \times 10^6 m/s$.

Total Energy Power $= V \times I$

$= 100 \times 10^3 \times 5 \times 10^{-3}$

$= 500\ J.$

Heat produced $= \frac{99.9}{100} \times$ (Total energy)

$= 499.5\ J/s.$

3) The p.d across an X-ray tube is 50 kV & I through it is 2.5mA. Cal. 1) no. of $e^-$ striking the anode per second

2) speed with which they strike with anode

3) approx. rate of production of heat in the anode.

A) Given:- $I = 2.5mA$

$$\frac{q}{t} = 2.5 \times 10^{-3}$$

$$q = 2.5 \times 10^{-3} \times 1.$$

$$q = 2.5 \times 10^{-3}\ C$$

No. of Es striking anode/s $n\bar{e} = q$.

$$n = \frac{q}{e}$$

$$= \frac{2.5 \times 10^{-3}}{1.6 \times 10^{-19}}$$

$n = 1.56 \times 10^{16}\ \bar{e}/s.$

ii) $\frac{1}{2}mv^2 = eV.$

$$v^2 = \frac{1.6 \times 10^{-19} \times 50 \times 10^3 \times 2}{9.1 \times 10^{-31}}$$

$$v = \sqrt{\frac{1.6 \times 10^{-19} \times 100 \times 10^3}{9.1 \times 10^{-31}}} = 1.32 \times 10^8\ m/s!$$

23-2-2018.

iii) Heat energy produced $= V \times I$

$= 50 \times 10^{3} \times 2.5 \times 10^{-3}$

$= 125$ J.

4) The spacing b/w planes of NaCl crystal is $2.82 A^{\circ}$. It is found that 1st order Bragg reflection occurs at an angle of $10^{\circ}$. What is $\lambda$ of X-rays?

a)
$$n\lambda = 2d\sin\theta.$$

$$\lambda = \frac{2d\sin\theta}{n}$$

$$= \frac{2 \times 2.282 \times \sin 10^{\circ}}{1}$$

$$\lambda = 0.97 A^{0}$$

5) Find the smallest glancing angle at which $K_\alpha$ line emitted from Mo of $\lambda$ $0.7 A^{0}$ from calcite crystal of spacing $3.036 A^{\circ}$. at what angle will 3rd reflection takes place.

a) Given :-
$\lambda = 0.7 A^{0}$
$d = 0.36 A^{0}$

$$n\lambda = 2d\sin\theta.$$

$$\sin\theta = \frac{n\lambda}{2d}$$

$$= \frac{1 \times 0.7 \times \cancel{10^{-10}}}{2 \times \cancel{3}0.36 \times \cancel{10^{-10}}}$$

$$\theta = 6.62^{\circ}.$$

$$\sin\theta = \frac{3\lambda}{2d}$$

$$= \frac{3 \times 0.7}{2 \times 3.036}$$

$$\theta = 20.23^{\circ}.$$

6) A photon of energy 2.6 Mev undergoes pair production by emitting $\bar{e}$ & $e^{+}$ (positron). Cal. K.E of $\bar{e}$ & $e^{+}$.

a) Given:- $E_{min}$ to take pair production $= 1.02$ Mev.

Remain energy $= 2.6 - 1.02 = 1.58$ Mev $=$ Total K.E of $e^{+}$ & $e^{-}$

K.E of $\bar{e} = \frac{1.58}{2} = 0.79$ Mev.

K.E of $e^{+} = 0.79$ Mev.

$$h\nu \longrightarrow \underset{0.51 + 0.51}{\bar{e} + e^{+}} \longrightarrow 1.02 \text{ Mev.}$$

23-2-2015

7) The K.E of $\bar{e}$ is 0.6 Mev & K.E of $e^+$ 0.5 Mev undergoes pair annihilation with production 2 photons. Cal. energy of each photon.

A) $\bar{e} + \bar{e} \longrightarrow \nu_1 + \nu_2$.

$0.51 \quad 0.51$

Total K.E of $e^+$ & $\bar{e}$ = $K.E_{\bar{e}} + K.E\, e^+$

$= 0.5 + 0.6$

$= 1.1$ Mev.

Total Energy = $E_{min}$ + Total K.E

$= 1.02 + 1.1$

$= 2.12$ Mev.

This energy is shared b/w 2 photons.

Energy of each photon = $\frac{T.E}{2}$

$= \frac{2.12}{2}$

$= 1.06$ Mev.

Total energy of $\bar{e}$ = rest mass Energy + KE

$= 0.51 + 0.6 = 1.11$ Mev.

Total energy of $e^+$ = rest mass energy + KE

$= 0.51 + 0.5 = 1.01$ Mev.

Total energy of $\bar{e}$ & $e^+$ = 2.12 Mev $\longrightarrow \nu + \nu$

energ of each $\nu$ = $\frac{2.12}{2} = 1.06$ Mev.

8) The $\lambda$ of X-ray 0.7 A°. X-rays are scattered from carbon block through an angle of 90°. whose interplanar distance is 3.13 A°. Cal. angular separation in the $1^{st}$ order b/w the modified & unmodified rays.

$\lambda = 0.7A^0$

A) $\Delta\lambda = \lambda' - \lambda = \frac{h}{m_0 c}[1-\cos\theta]$.

$\lambda' = \lambda + \frac{h}{m_0 c}[1-\cos 90^\circ]$

$= 0.7 + \frac{6.62\times 10^{-34}}{9.1\times 10^{-31}\times 3\times 10^8}$

$= 0.7A^0 + 0.024\,A^0$

$\lambda' = 0.724A^0$

$n\lambda' = 2d\sin\theta$

$\sin\theta' = \frac{n\lambda'}{2d} = \frac{1\times 0.724}{2\times 3.13}$

$\theta' = 6.64^0$

Angular separation = $\theta - \theta' = 90 - 6.64 = 83.36^0$

1-3-2018.

# Dual Nature Of Matter

## Wave nature of Matter:-

Matter Waves: All moving particles are associated with waves, these waves are called matter waves or der Broglie waves.

→ Wave length of matter waves depends on mass & velocity of the particle.

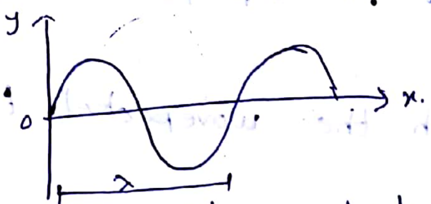
→ These waves are not electromagnetic.

$$\boxed{\lambda = \frac{h}{p}}$$

$$\boxed{\lambda = \frac{h}{mv}}$$

$$\boxed{\lambda_e = \frac{h}{\sqrt{2meV}}} \boxed{= \frac{12.27}{\sqrt{V}} (A^\circ)}$$

## Phase Velocity ($V_{ph}$):-




The velocity with which the angular position of matter wave changes is called phase velocity($V_{ph}$)

$$\boxed{V_{ph} = \frac{\omega}{k}}$$

where $\omega$ = angular frequency = $2\pi n$.

$k$ = Wave no (or) propagation constant

$$k = \frac{2\pi}{\lambda}.$$

$$\omega = 2\pi\nu = \frac{2\pi E}{h}.$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h}.$$

$$V_{ph} = \frac{2\pi E}{h \times 2\pi p} \times h$$

$$\boxed{V_{ph} = \frac{E}{p}}, \quad V_{ph} = \frac{h\nu}{mv} = \quad V_{ph} = \frac{mc^2}{mv}$$

$$\boxed{V_{ph} = \frac{c^2}{v}} \Rightarrow V_{ph} > c^2.$$

$v$ → velocity of particle.

1-3-2018

$V = \nu\lambda$.

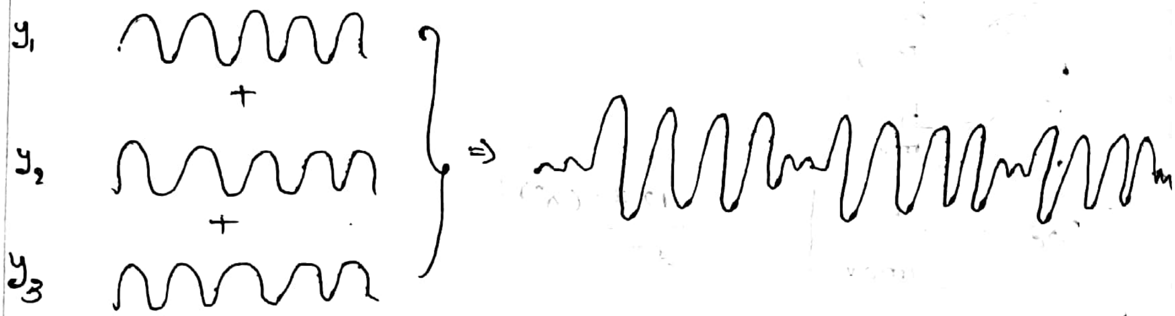
$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$.

$k = \frac{2\pi}{\lambda}$. $\therefore \lambda = \frac{2\pi}{k}$,

$V = \frac{\omega}{2\pi} \times \frac{2\pi}{k}$

$$\boxed{V_p = \frac{\omega}{k}}$$

Group Velocity ($v_g$):-




Wave pocket is the result of superposition of matter waves

The velocity with which the wavepocket travels is called group velocity.

$y = y_1 + y_2$.

$y_1 = A\sin(\omega t - kx)$

$y_2 = A\sin[(\omega + \Delta\omega)t - (k + \Delta k)x]$

$$\boxed{V_{group} = \frac{\Delta w}{\Delta k}}$$

Heisenberg's Uncertainity Principle :-

The amplitude of matter waves tells about the probability of finding the position of particle at any instant of time.

→ "It is impossible to measure both position and momentum of a particle in a matter wave accurately and simultaneously."

2-3-2018

$$\boxed{1)\ \Delta x \cdot \Delta P \geq \frac{h}{4\pi}} \boxed{\geq \frac{\hbar}{2}} \qquad \hbar = \frac{h}{2\pi}.$$

$$\Delta x \propto \frac{1}{\Delta P}.$$

$\Delta x \longrightarrow$ Error involved in the measurment of position of a particle.

$\Delta P \longrightarrow$ Error in momentum measurement.

$$\boxed{2)\ \Delta E \cdot \Delta t \geq \frac{h}{4\pi}}$$

$$\boxed{3)\ \Delta L \times \Delta \theta \geq \frac{h}{4\pi}}.$$

**Application Of Uncertainity Principle:** inside the nucleus ?

1) Why $\bar{e}$ is not present inside the nucleus

A) Assume that $\bar{e}$ is present inside the measurment of then the error involved in the nucleus is $\Delta x \leq 10^{-15}$ m. position of $\bar{e}$ inside the nucleus

According to Heisenberg

$$\Delta x \cdot \Delta P \geq \frac{h}{4\pi}.$$

$$\Delta P \geq \frac{h}{4\pi \Delta x}$$

$$\geq \frac{6.62 \times 10^{-34}}{4 \times \pi \times 10^{-15}}$$

$$\cancel{\Delta P \geq 3.94 \times 10^{-17}}$$

$$\Delta P \geq 5.27 \times 10^{-20}.$$

$$E^2 = P^2c^2 + m_0^2c^4.$$

$$E^2 \geq (5.27 \times 10^{-20})^2 \times (3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4.$$

$$E^2 \geq 2.49 \times 10^{-22}.$$

$$E \geq 1.58 \times 10^{-11}\ J.$$

$$E \geq 98\ MeV.$$

Experimental observations reveal that $\bar{e}$ posses 3 to 4 MeV energy. $\therefore$ Electron is not present inside the nucleus.

3-3-2018.

## Wave Function ($\psi$):

The function which gives the state of a particle (position, momentum energy etc) ~~of a particle~~ at any instant of time is called wave function.

$\psi$ is complex quantity, it has both real and imaginary parts.

Imaginary part doesnot have any physical significance therefore $\psi$ itself doesnot have any physical significance.

Instead of $\psi$ we use $|\psi|^2$.

$$\boxed{P = \int_{-\infty}^{+\infty} |\psi|^2 \, dv = 1.}$$ probability density.

$$\psi^2 = |\psi \psi^*|$$

### Conditions of Well behaved wave Functions:

→ $\psi$ must be single valued everywhere.

→ $\psi$ must be continuous

→ $\psi$ must have finite value.

Such $\psi$ is acceptable function (or) Eigen function.

### Schrodinger Wave Equations:

### Time Dependent Wave Equation:

We have $y = A e^{-i(\omega t - kx)} \longrightarrow ①$

$$\omega = 2\pi n = 2\pi \nu = 2\pi \times \frac{E}{h} = \frac{E}{\hbar}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

$$① \Rightarrow y = A e^{-\frac{i}{\hbar}(Et - Px)}$$

For matter waves

$$\psi = A e^{-i/\hbar (Et - Px)} \longrightarrow ②$$

Diff. eq② w.r.t 'x' twice.

$$\frac{\partial \psi}{\partial x} = A e^{-i/\hbar (Et - Px)} \left[\frac{i}{\hbar} P\right]$$

$$\frac{\partial^2 \psi}{\partial x^2} = A e^{-i/\hbar [Et - Px]} \left[\frac{i}{\hbar} P\right]^2 = \psi \left[\frac{i}{\hbar} P\right]^2$$

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$$\frac{\partial^2\psi}{\partial x^2} = \frac{-p^2\psi}{\hbar^2} \quad [\because i^2 = -1]$$

$$p^2\psi = -\hbar^2 \frac{\partial^2\psi}{\partial x^2} \longrightarrow (3)$$

Diff eqn (3) w.r.to 't'

$$\psi = A e^{-\frac{i}{\hbar}[Et - Px]}$$

$$\frac{\partial\psi}{\partial t} = A e^{-\frac{i}{\hbar}[Et-Px]} \left[-\frac{i}{\hbar} E\right]$$

$$\frac{\partial\psi}{\partial t} = \psi\left[\frac{-i}{\hbar} E\right]$$

$$E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t} \longrightarrow (4)$$

Total Energy of a particle $E = KE + PE$.

$$E = \frac{p^2}{2m} + V. \quad [V \text{ is } P.E].$$

$$E\psi = \frac{p^2\psi}{2m} + V\psi \longrightarrow (5)$$

Substitute (3) & (4) in (5) eq

$$-\frac{\hbar}{i}\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi.$$

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi.}$$

For 3-d:

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x^2} \frac{\partial^2\psi}{\partial y^2} \frac{\partial^2\psi}{\partial z^2}\right] + V\psi}$$

For free particle $V = \cancel{\phi}\, 0$ [P.E is '0']

For free particle P.E i.e, $V = 0$

$\therefore$ Schrodinger wave eq for free particle

$$\boxed{i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}}$$

<u>Schrodinger</u> <u>Time</u> <u>Independent</u> <u>Wave</u> <u>Eq</u>$^n$:-

We have for matter waves

$$\psi = A e^{-i/\hbar[Et - Px]} \longrightarrow (1)$$

$$\psi = A e^{-i/\hbar(Et)} \cdot e^{i/\hbar(Px)}$$

$$\longrightarrow (2)$$

Let $\tilde{\psi} = A e^{i/\hbar(Px)}$

$$\therefore \psi = \tilde{\psi} e^{-i/\hbar(Et)} \longrightarrow (3)$$

~~Diff eq (3) w.r.t~~ We have $E\psi = -\frac{\hbar}{i}\frac{\partial\psi}{\partial t}$.

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We have $E = PE + KE$

$$E = \frac{p^2}{2m} + V$$

Multiply with $\psi$

$$E\psi = \frac{p^2}{2m}\psi + V\psi$$

$$E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

we have $\psi = \psi\, e^{i/\hbar(px)}$

$$E\psi\, e^{-i/\hbar(Et)} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left[\psi\, e^{-i/\hbar(Et)}\right] + V\psi\, e^{-i/\hbar(Et)}$$

Divide with $e^{-i/\hbar(Et)}$

$$E\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + (E-V)\psi = 0.$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0.}$$ → Time independent wave eq$^n$

For 3-dimension.

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}$$

$$\boxed{\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0}$$

For free particle $V = 0$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\psi = 0}$$

1) The deBroglie $\lambda$ of an $\bar{e}$ is accelerated under a p.d of $V = 100V$

Cal. '$\lambda$'

A) $\cancel{\lambda = \frac{12400}{E\,in\,(eV)}(A^\circ)}$

$\cancel{= \frac{12400}{100\,eV}\,A^\circ}$

$\cancel{\lambda_e = 124\,A^\circ}$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$= \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times1.6\times10^{-19}\times100}}$$

$$\lambda = 1.24\,A^\circ$$

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2) (a). $p$ & $\lambda$ associated with an $\bar{e}$ whose KE 1.5 keV

A) $\lambda = \frac{h}{\sqrt{2m(eV)}}$

$= \frac{h}{\sqrt{2m\,KE}}$

$= \frac{6.62 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1.5 \times 10^{3}}}$

$\lambda = 0.31\,A^{\circ}$

$p = \frac{h}{\lambda} = 2.08 \times 10^{-23}\ kg-m/s.$

3) A positron with K.E 3 MeV collides with an $\bar{e}$ moving in the opposite direction at 1 MeV and anhilliated. Two photons are produced comment on the energies of photons.

A) Total K.E of $\bar{e}$ & $e^{+}$ = 4 MeV

For pair anihilation req energy = 1.02 MeV — wrong

$\frac{\cancel{2.98}\ 1.49}{2}$

Remaining energy = 4 − 1.02 = 2.98 MeV.

For Particles :-

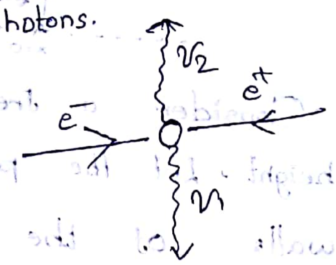
Let $E_1$ & $E_2$ be the energies of $\bar{e}$ & $e^{+}$.

$P_1$ & $P_2$ be the momentum of $\bar{e}$ & $e^{+}$.

For $\gamma$ - photons :

Let $E_1'$ & $E_2'$ be the energies of 2 photons.

Let $P_1'$ & $P_2'$ be the momentum of 2 photons.




$E_1 = 1 + 0.511 = 1.511$ MeV

$E_2 = 3 + 0.511 = 3.511$ MeV.

Applying Law of conservation of energy

$E_1' + E_2 = E_1' + E_2'$. $\longrightarrow$ (1)

Momentum by law

$P_1 - P_2 = P_1' - P_2'$ ($\because$ $\bar{e}$ & $e^{+}$ are in opposite direction)

$(P_1 - P_2)\,c = P_1'c - P_2'c$

$(P_1 - P_2)c = E_1' - E_2' \longrightarrow$ (2)

$E_2' = E_1' - (P_1 - P_2)c \longrightarrow$ (3).

(3) in (1).

$E_1 + E_2 = E_1' + E_1' - (P_1 - P_2)\,c.$

$E_1 + E_2 = 2E_1' - (P_1 - P_2)\,c.$

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$$E_1' = \frac{E_1+E_2+(P_1-P_2)c}{2}$$

We have $E^2 = P^2c^2 + m_0^2c^4$.

$$P_1^2c^2 = E_1^2 - m_0^2c^4.$$

$$P_1c = \sqrt{(3.511)^2 - (0.511)^2}$$

$$P_1 = 3.47 \text{ MeV}/c.$$

$$P_2 = 1.421 \text{ MeV}/c.$$

$$E_1' = \frac{3.511+1.511+(3.47-1.42)}{2}$$

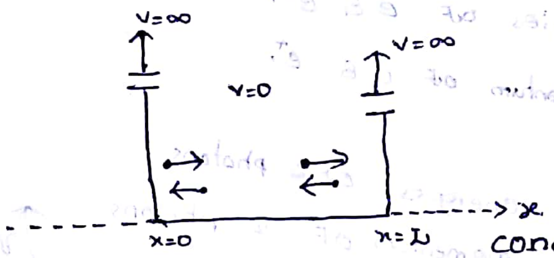
$$E' = 3.53 \text{ MeV}$$

$$E_2' = 3.511 + 1.511 - 3.536$$

$$E_2' = 1.49.$$

## Applications Of Schrodinger's Wave Equation:-

### Particle in a potential Well Of Infinite Height :- (1-dimension)




confined

Consider a free particle ~~confined~~ in a potential well of height. Let the particle be moving between the walls of the potential well.

The boundary conditions of the particle are

$V=0$ for $x>0$ & $x<L$.

$V=\infty$ for $x \le 0$ & $x \ge L$.

$\psi = 0$ for $x \le 0$ & $x \ge L$.

We have Schrodinger wave eqn (Time independent)

$$\frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

For Free particle $V=0$.

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$$\therefore \frac{\partial^2\psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \longrightarrow (1)$$

$k^2 = -\frac{8\pi^2 mE}{h^2}$

$$\frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0 \longrightarrow (2)$$

where $k^2 = \frac{8\pi^2 mE}{h^2}$.

Solution for eq (2).

$$\psi = C\cos kx + D\sin kx \longrightarrow (3)$$

where C & D are constants

To evaluate C and D values, apply boundary conditions

$\psi = 0$ at $x = 0$

(3) $\Rightarrow$ $0 = C\cos k(0) + D\sin k(0)$

$$\boxed{C = 0}$$

$\psi = 0$ at $x = L$

(3) $\Rightarrow$ $0 = C\cos kL + D\sin kL$

$0 = 0 + D\sin kL$

$D\sin kL = 0$

$D \neq 0$, $\sin kL = 0$

$kL = n\pi$

$$\boxed{k = \frac{n\pi}{L}}$$

(3) $\Rightarrow$ $\psi = D\sin\frac{n\pi}{L}x \longrightarrow (4)$

To find value of D, apply normalization condition

$$\int_{-\infty}^{\infty} |\psi|^2 dx = 1.$$

$$\int_{x=0}^{x=L} |\psi|^2 dx = 1.$$

$$\int_0^L \left[D\sin\left(\frac{n\pi x}{L}\right)\right]^2 dx = 1.$$

$$\int_0^L D^2\sin^2\left(\frac{n\pi x}{L}\right) dx = 1.$$

$$\sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$\int_0^L D^2 \times \frac{1}{2}\left[1 - \cos\frac{2\pi n x}{L}\right] dx = 1.$$

$$\frac{D^2}{2}\left[\int_0^L 1\,dx - \int_0^L \cos\left(\frac{2n\pi x}{L}\right)\right] = 1$$

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$$\frac{D^2}{2}\left[(x)_0^L - \left(\frac{L}{2n\pi}\right) \times \sin\left(\frac{2n\pi x}{L}\right)_0^L\right] = 1.$$

$$\frac{D^2}{2}[L - 0] = 1.$$

$$\frac{D^2}{2} = \frac{1}{L}$$

$$D^2 = \frac{2}{L}$$

$$\boxed{D = \sqrt{\frac{2}{L}}}$$

eqn (4) $\Rightarrow$

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$$

$$\boxed{\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}$$

Wave function or Eigen func$^n$.

We have $k = \frac{n\pi}{L}$.

from (2)

$$\sqrt{\frac{8\pi^2 mE}{h^2}} = \frac{n\pi}{L}$$

$$\frac{8\cancel{\pi^2} mE}{h^2} = \frac{n^2 \cancel{\pi^2}}{L^2}$$

$$\boxed{E_n = \frac{n^2 h^2}{8mL^2}}$$ Energy eigen value.

When $n=1$, ground state.

$$E_1 = \frac{1^2 \times (6.62 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times L^2}$$

When $n=2$, first excited state

$$E_2 = 4E_1$$

when $n=3$

$$E_3 = 9E_1$$

$\vdots$

16-3-2018.

## Wave Function:-

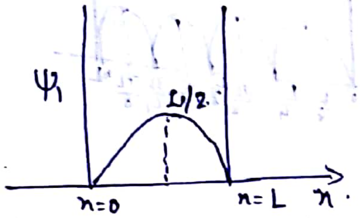
$$\psi_n = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

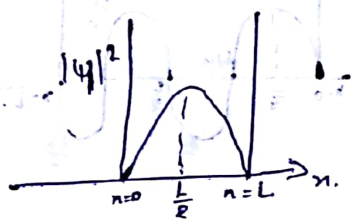
For $n=1$

$$\psi_1 = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$$

$\psi_1$ is maximum, when $x = \frac{L}{2}$,

$\psi_1$ is minimum, when $x = 0, L$



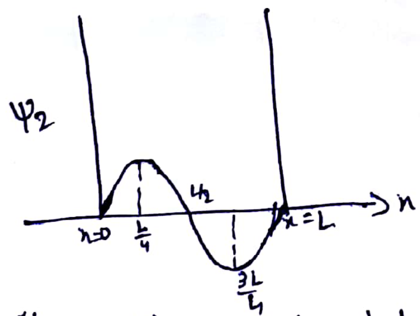



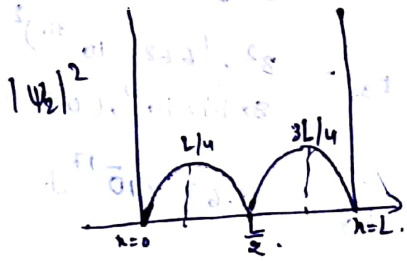

For first excited state (n=2)

$$\psi_2 = \sqrt{\frac{2}{L}} \sin\left(\frac{2\pi x}{L}\right)$$

$\psi_2$ is max, when $x = \frac{L}{4}, \frac{3L}{4}$

$\psi_2$ is min, when $x = 0, \frac{L}{2}, L$.



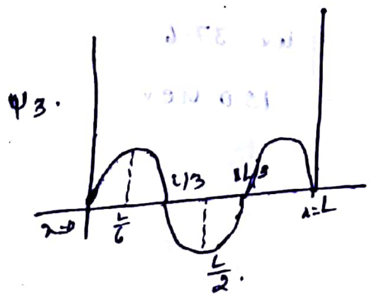



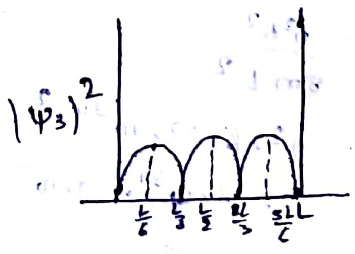

For the 2nd excited state (n=3)

$$\psi_3 = \sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)$$

$\psi_3$ is max, when $x = \frac{L}{6}, \frac{L}{2}, \frac{5L}{6}$

$\psi_3$ is min, when $x = 0, \frac{L}{3}, \frac{2L}{3}, L$.



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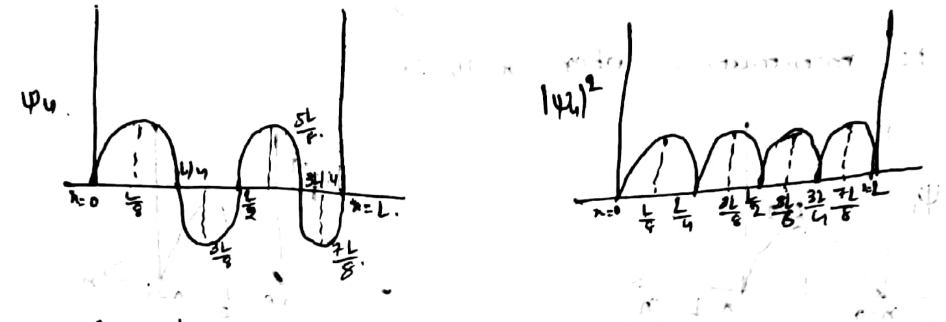
For the 3rd excited state ($n=4$)

$$\psi_4 = \sqrt{\frac{2}{L}} \sin\left(\frac{4\pi x}{L}\right)$$

$\psi_4$ is max when $x = \frac{L}{8}, \frac{3L}{8}, \frac{5L}{8}, \frac{7L}{8}$

$\psi_4$ is min when $x = \frac{L}{2}, \frac{L}{4}, \frac{3L}{4}, L$.




1) An $\bar{e}$ is bound in 1-dimensional potential well of width 0.18nm find its energy value in the 2nd excited state.

A) Given:

$L = 0.18\,nm$.

$n = 3$ (2nd E.S).

$m_e = 9.1\times10^{-31}\,kg$.

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_3 = \frac{3^2\times(6.62\times10^{-34})^2}{8\times9.1\times10^{-31}\times(0.18\times10^{-9})^2}$$

$E_3 = 1.67\times10^{-17}\,J.$

$E_3 = 104.6\,ev$

2) An $\bar{e}$ is bound in -1dimension potential well of width $1A^\circ$. Find its energy valu in the ground state & 1st excited state.

A)

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_1 = \frac{1^2\times(6.62\times10^{-34})^2}{8\times9.1\times10^{-31}\times(1\times10^{-10})^2}$$

$$= \frac{6.01\times10^{-18}}{1.6\times10^{-19}}$$

$E_1 = 37.6\,ev$

$E_2 = 2^2\times E_1$

$= 4\times 37.6$

$= 150.4\,ev.$

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3) Cal Zero point energy of an $e^-$ in a box of width 10 Å.

A) Zero point energy n = 1. (or) groun

$$E_1 = \frac{1^2 \times (6.62\times10^{-34})^2}{8\times 9.1\times10^{-31}\times(10\times10^{-10})^2}$$

$= 0.376$ eV.

4) An $e^-$ is moving in an 1-dimension potential well of width 2 Å. What is the probability of finding an $\bar{e}$ in $x=0$, & $x = 1\times10^{-10}$ m in $2^{nd}$ second E.S ?

A) Given: $L = 2\times10^{-10}$ m.

Probability density $P = \int_{-\infty}^{+\infty} |\psi|^2 dx$

For one-dimensional potential the wave function of an electron

$$\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$

n = 3

$$P = \int_{x=0}^{x=1\times10^{-10}} \left[\sqrt{\frac{2}{L}} \sin\left(\frac{3\pi x}{L}\right)\right]^2 dx$$

$$= \frac{2}{L}\int_0^{10^{-10}} \sin^2\frac{3\pi x}{L}\, dx.$$

$$= \frac{2}{L}\int_0^{10^{-10}} \left[\frac{1-\cos\frac{6\pi x}{L}}{2}\right] dx.$$

$$= \frac{1}{L}\int_0^{10^{-10}} 1\,dx - \int_0^{10^{-10}} \cos\frac{6\pi x}{L}\, dx.$$

$$= \frac{1}{L}\left[10^{-10}-0\right] - \left[\frac{\sin\frac{6\pi x}{L}}{\frac{6\pi}{L}}\right]_0^{10^{-10}}$$

$$= \frac{1}{2\times10^{-10}}\left[10^{-10}\right] - \frac{2\times10^{-10}}{6\pi}\left[\sin\left(\frac{6\pi\times10^{-10}}{2\times10^{-10}}\right)\right]$$

$P = 0.5.$

5) A particle is moving in an 1-dimensional potential well of width 1 Å. Cal probability of finding the particle when it is least energy state. n = 1 [G.E.S]

A) Probability density $P = \int_{-\infty}^{+\infty} |\psi|^2 dx$

$$P = \int_0^{10^{-10}} \left[\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)\right]^2 dx.$$

$$= \frac{2}{L}\int_0^{10^{-10}} \sin^2\left(\frac{n\pi x}{L}\right) dx.$$

$$= \frac{2}{10^{-10}\times 2}\left[\int_0^{10^{-10}} 1\,dx - \int_0^{10^{-10}} \frac{\cos\left(\frac{2\pi x}{L}\right)}{\frac{2\pi}{L}}\right]$$

$$= \frac{10^{-10}}{10^{-10}} - \left[\frac{\sin\frac{2\pi x}{10^{-10}}}{\frac{2\pi}{10^{-10}}}\right]_0^{10^{-10}}$$

P = 1.

19-3-2018

c) A proton is confined to move in a 1-dimensional box of width 0.2nm cal. Least energy of and compare to that of $e^-$

$$E_p = \frac{1 \times (6.62\times10^{-34})^2}{8\times 1.6\times10^{-27}\times (0.2\times10^{-9})^2}$$

A)
$$E_0 \propto \frac{1}{m}$$

$$\frac{E_1}{E_2} = \frac{m_2}{m_1}$$

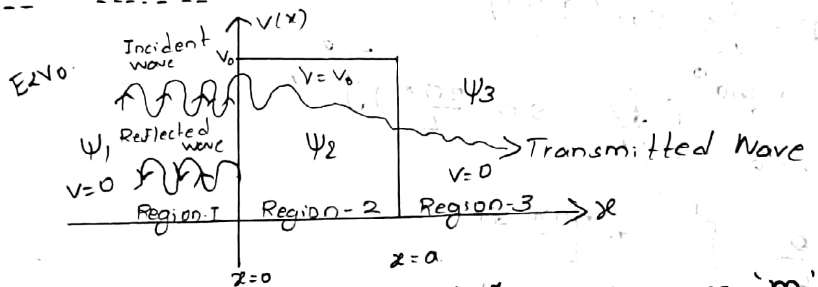
$$\frac{E_p}{E_e} = \frac{m_e}{m_p}$$

$$E_p = \frac{9.1\times10^{-31}}{1.6\times10^{-27}} \times E_e$$

$$E_p = 5.68\times10^{-4}\, E_e$$

## Tunneling Effect :-

### Particle Strikes A Potential Barrier Of Finite Height :-




Consider a stream of particles of mass 'm' incident on a potential barrier of finite height ($V_0$) the boundary conditions of the particles are

$V=0$ when $x<0$ and $x>a$.

$V=V_0$ when $x>0$ and $x<a$.

If energy of the particles $E>V_0$, then the particle can cross over the barrier.

If energy of the particle $E<V_0$; then particles reflected back according to classical mechanics.

But according to quantum mechanics, even though $E<V_0$, there is a finite probability of tunneling of the particles through the barrier. This phenomenon is called "Tunneling effect". This tunneling of particles could be explained by the wave nature of particles.

eg:- Emission of β-particles from radioactive elements,

Tunneling diode

22-3-2018

Schrodinger's Wave eqn for region - I.

$$\frac{\partial^2\psi_1}{\partial x^2} + \frac{2m}{\hbar^2} E\psi_1 = 0$$

$$\frac{\partial^2\psi_1}{\partial x^2} + k^2\psi_1 = 0 \longrightarrow (1) \quad \text{where} \quad k^2 = \frac{2mE}{\hbar^2}$$

For Region- 2 :-

$$\frac{\partial^2\psi_2}{\partial x^2} + \frac{2m}{\hbar^2} (E-V_0)\psi_2 = 0$$

$$\frac{\partial^2\psi_2}{\partial x^2} - \frac{2m}{\hbar^2} (V_0-E)\psi_2 = 0$$

$$\frac{\partial^2\psi_2}{\partial x^2} - \alpha^2\psi_2 = 0 \longrightarrow (2) \quad \text{where} \quad \alpha^2 = \frac{2m}{\hbar^2}(V_0-E)$$

For Region -3 :-

$$\frac{\partial^2\psi_3}{\partial x^2} + \frac{2m}{\hbar^2} (E)\psi_3 = 0$$

$$\frac{\partial^2\psi_3}{\partial x^2} + k^2\psi_3 = 0 \longrightarrow (3)$$

Solutions of eqns (1), (2) & (3) are

$$\psi_1 = e^{ikx} + Ae^{-ikx} \longrightarrow (4)$$

$$\psi_2 = Be^{\alpha x} + Ce^{-\alpha x} \longrightarrow (5)$$

$$\psi_3 = De^{ikx} + \cancel{Ee^{-ikx}} \longrightarrow (6)$$

In region (1) $e^{ikx}$ represents incident wave and $e^{-ikx}$ represents reflected wave.

In region (2) $Be^{\alpha x}$ represents exponentially increasing wave, $Ce^{-\alpha x}$ represents exponentially decreasing wave.

$Ce^{-\alpha x}$ represents transmitted wave.

In region (3) $De^{ikx}$ represents transmitted wave.

I, R, T are coefficients of incident, Reflected & transmitted waves.

$R + T = 1$

$T = 1 - R$

$T = |D|^2 = |D^*D|$

In order to calculate constants A, B, C, D we have to apply boundary conditions.

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At $x=0$, $\psi_1 = \psi_2$.

$$\frac{\partial \psi_1}{\partial x} = \frac{\partial \psi_2}{\partial x}.$$

$$e^{ikx} + Ae^{-ikx} = Be^{\alpha x} + Ce^{-\alpha x}. \rightarrow (7)$$

$$e^{ik(0)} + Ae^{-ik(0)} = Be^{\alpha(0)} + Ce^{-\alpha \times 0}$$

$$1 + A = B + C. \rightarrow (8)$$

Diff (7) w.r.t $x$.

$$ike^{ikx} - Aike^{-ikx} = B\alpha e^{\alpha x} - C\alpha e^{-\alpha x}.$$

at $x=0$

$$ik - ikA = B\alpha - C\alpha. \rightarrow (9)$$

II boundary Condition

At $x=a$, $\psi_2 = \psi_3$.

$$\frac{\partial \psi_2}{\partial x} = \frac{\partial \psi_3}{\partial x}.$$

$$Be^{\alpha a} + Ce^{-\alpha a} = De^{ika} \rightarrow (10).$$

$$\alpha Be^{\alpha a} - \alpha Ce^{-\alpha a} = ikDe^{ika} \rightarrow (11)$$

On solving eqns (8) & (9) we get

$$B = \frac{D}{2\alpha}(\alpha + ik)e^{ika - \alpha a} \rightarrow (12)$$

$$C = \frac{D}{2\alpha}(\alpha - ik)\, e^{ika + \alpha a} \rightarrow (13)$$

Using eqns (12) & (13) we get

$$A = \frac{D(\alpha + ik)}{\alpha - ik} e^{ika - \alpha a} - \frac{\alpha + ik}{\alpha - ik}.$$

Substituting B & C values in (10) we get

$$D = \frac{2ik\alpha e^{-ika}}{(\alpha^2 - k^2)\sinh(\alpha a) - 2i\alpha k \cosh(\alpha a)}$$

Transmission probability

$$T = |D|^2 = |D^* D|$$

$$\boxed{T = \frac{4k^2\alpha^2}{(\alpha^2 - k^2)\sinh^2(\alpha a) + 4\alpha^2 k^2 \cosh^2(\alpha a)}}$$

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$\alpha a \gg 1$ & $\sinh(\alpha a) = \cosh(\alpha a) = \frac{1}{2} e^{\alpha a}$

$$\boxed{T = \frac{16k^2\alpha^2 e^{-2\alpha a}}{(\alpha^2+k^2)^2}}$$

Substitute values of $\alpha$ & $k$

$$T = \frac{16E(V_0-E)}{V_0^2} \cdot e^{-2\alpha a}$$

$$\frac{16E(V_0-E)}{V_0^2} \ll 1.$$

$$T \simeq e^{-2\alpha a.}$$

$$\boxed{T = e^{-2a\sqrt{\frac{2m}{\hbar^2}(V_0-E)}}}$$

1) A beam of $\bar{e}s$ is incident on a barrier six 6ev high and 0.2nm wide. Find the energy they should have if 1% of are get through the barrier.

S) Given:- $V_0 = 6ev.$

$a = 0.2nm$

$T = 1\%$

$E = ?$

$$T = e^{-2a\sqrt{\frac{2m}{\hbar^2}(V_0-E)}}$$

S.B.S

$$T^2 = e^{-4a^2\left(\frac{2m}{\hbar^2}\times(V_0-E)\right)}$$

$$2\log T = -4a^2\left(\frac{2m}{\hbar^2}\right)\times V_0-E)\log_e e$$

$$V_0 - E = \frac{2\log 0.01}{-4\times(0.2)^2\times10^{-18}} \times \frac{h^2}{2\times9.1\times10^{-31}\times4\pi^2.}$$

$$6ev - E = \frac{2\log 0.01 \times (6.6\times10^{-34})^2}{-4\times0.04\times10^{-18}\times 2\times9.1\times10^{-31}\times4\pi^2.}$$

$$6ev - E = \frac{1.52\times10^{-19}.}{1.6\times10^{-19}.}$$

$$6ev - E = 0.947$$

$$E = 6 - 0.947$$

$$E = 5.05ev.$$

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$$\frac{1}{100} \simeq e^{-2\times 0.2\times 10^{-9}\frac{\sqrt{2\times 9.1\times 10^{-31}\times(6ev-E)}}{\frac{6.6\times 10^{-34}}{2\pi}}}$$

$$\ln\left(\frac{1}{100}\right) = -2\times 0.2\times 10^{-9}\frac{\sqrt{2\times 9.1\times 10^{-31}\times(6ev-E)}\times 2\pi}{6.6\times 10^{-34}}$$

$$\sqrt{2\times 9.1\times 10^{-31}\times(6ev-E)} = \frac{\ln\left(\frac{1}{100}\right)\times 6.6\times 10^{-34}}{2\pi\times -2\times 0.2\times 10^{-9}}$$

$$2\times 9.1\times 10^{-31}(6ev-E) = \left[\frac{\ln\left(\frac{1}{100}\right)\times 6.6\times 10^{-34}}{2\pi\times -2\times 0.2\times 10^{-9}}\right]^2$$

$$6ev-E = 5.022\ ev.$$

$$E = 0.97\ ev.$$

2) An $e$ beam of energy 0.4ev are incident on a barrier of 3ev high and 0.1nm wide Find 'T'?

A)

$$T = e^{-2a\sqrt{\frac{2m}{\hbar^2}(V_0-E)}}$$

$$= e^{-2\times 0.1\times 10^{-9}\sqrt{\frac{2\times 9.1\times 10^{-31}\times(3-0.4)\times 1.6\times 10^{-19}}{6.6\times 10^{-34}}}\times 2\pi}$$

$$T = 0.19.$$

3) Same as protons

A)

$$T = e^{-2a\frac{\sqrt{2m(V_0-E)}}{h}\times 2\pi}$$

$$= e^{-2\times 0.1\times 10^{-9}\times 2\pi\sqrt{\frac{2\times 1.67\times 10^{-27}\times 2.6}{6.6\times 10^{-34}}}}$$

$$= 1.5\times 10^{-34}.$$

## Operators & Expectation Values:

$\hat{G}(x)\psi_n = G_n\psi_n$.

Real no (or) Eigen Value.

| Physical Quantity | Symbol | Operator. |
|---|---|---|
| Position | $\hat{x}$ | $x$. |
| Momentum | $\hat{P}$ | $\frac{\hbar}{j}\frac{\partial}{\partial x}$. |
| Kinetic Energy | $\hat{E}_K$ | $-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}$. |
| Potential Energy | $\hat{U}$ | $U$. |
| Total Energy | $\hat{E}$ | $i\hbar\frac{\partial}{\partial t}$. |

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eg:- $\psi = e^{2x}$ $\quad \frac{\partial^2}{\partial x^2} \longrightarrow$ Operator.

$$\frac{\partial^2}{\partial x^2}\psi = \underset{\substack{\downarrow \\ \text{Eigen value.}}}{4} e^{2x}.$$

Expectation Values:-

Expectation Value = Average Value.

$$\langle x \rangle = \frac{\int_{-\infty}^{\infty} x|\psi|^2 dx}{\int_{-\infty}^{\infty} |\psi|^2 dx}$$

denominator = 1 for normalization

$$\boxed{\langle x \rangle = \int_{-\infty}^{\infty} x|\psi|^2 dx.}$$

$$\boxed{\langle G(x) \rangle = \int_{-\infty}^{\infty} G(x)|\psi|^2 dx}$$

1) Find the expectation value of position of a particle whose wave function $e^{-x^2}$.

A) $\langle x \rangle = \int_{-\infty}^{\infty} x|e^{-x^2}|^2 dx.$

$$= \int_{-\infty}^{\infty} x\, |e^{-2x^2}|\, dx$$

$$= x\int_{-\infty}^{\infty} |e^{-2x^2}|\, dx.$$

$$= \cancel{x}\times\left[\frac{e^{-2x^2}}{-4\cancel{x}}\right]_{-\infty}^{\infty}$$

$\because \int_{-\infty}^{+\infty} x|\psi|^2 dx$

$$\langle x \rangle = \int_{-\infty}^{+\infty} x\, e^{-2x^2} dx.$$

$$\langle x \rangle = \int_{-\infty}^{0} x\, e^{-2x^2} dx + \int_{0}^{+\infty} x e^{-2x^2} dx$$

$$= \int_{-\infty}^{0} x\, e^{-2x^2} dx - \int_{+\infty}^{0} x e^{-2x^2} dx$$

$$\langle x \rangle = 0$$

2) Find the expectation value of position of a particle in one dimensional potential well of width 'L'.

A) $\langle x \rangle = \int_{-\infty}^{\infty} x|\psi|^2 dx.$

$$= \int_0^L x\cdot\left(\frac{2}{L}\right)\sin^2\left(\frac{n\pi x}{L}\right)dx.$$

$$= \frac{2}{L}\int_0^L x\sin^2\left(\frac{n\pi x}{L}\right)dx.$$

$$= \frac{2}{L}\left[\sin^2\left(\frac{n\pi x}{L}\right)\int_0^L x\,dx - \int_0^L 1\times\int_0^L\left(\frac{1-\cos\left(\frac{2n\pi x}{L}\right)}{2}\right)dx.\right.$$

$$= \frac{2}{L}\left[\frac{L^2}{2}\right] - \int_0^L \frac{1}{2}x + \int_0^L\left[\frac{\sin\left(\frac{2n\pi x}{L}\right)}{2\times 2n\pi}\times L\right]_0^L dx.$$

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$$= L - \frac{L^2}{2} \cancel{- \left( \frac{\cos\left(\frac{2n\pi x}{L}\right)}{4n\pi \times \frac{2n\pi}{L}} \right)\Bigg|_0^L}$$

$$= L - \frac{L^2}{2}.$$

$$\langle x \rangle = \int_{-\infty}^{\infty} x\, |\psi|^2 dx.$$

$$\langle x \rangle = \int_0^L x \left[ \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \right]^2 dx.$$

$$= \int_0^L x \times \frac{2}{L} \sin^2\left(\frac{\pi x}{L}\right) dx$$

$$= \frac{2}{L} \int_0^L x \left( \frac{1 - \cos\left(\frac{2\pi x}{L}\right)}{2} \right) dx.$$

$$= \frac{1}{L} \int_0^L x - x \cos\left(\frac{2\pi x}{L}\right) dx$$

$$= \frac{1}{L} \left[ \int_0^L x\, dx - \int_0^L x \cos \frac{2\pi x}{L}\, dx \right]$$

$$= \frac{1}{L} \left[ \frac{x^2}{2}\Big|_0^L - \left( x \sin\left(\frac{2\pi x}{L}\right)\right)\Big|_0^L - \int_0^L 1 \times \sin\frac{2\pi x}{L} \times \frac{L}{2\pi} dx \right.$$

$$= \frac{1}{L} \left[ \frac{L^2}{2} - \left( 0 - \frac{L}{2\pi} \times \left( -\cos \frac{2\pi x}{L} \right)\Big|_0^L \times \frac{L}{2\pi} \right] \right.$$

$$= \frac{1}{L} \left[ \frac{L^2}{2} - \left( 0 - \frac{L^2}{2\pi} [-1+1] \right) \right]$$

$$= \frac{1}{L} \times \frac{L^2}{2}$$

$$= \frac{L}{2}.$$

3) Find the expectation value of momentum of a particle in one dimensional potential well of width 'L'

A) Given:- $\psi = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right)$

$$P = \frac{\hbar}{i} \frac{\partial \psi}{\partial x} = \frac{\hbar}{i} \cos\left(\frac{\pi x}{L}\right) \times \frac{\pi}{L} \times \sqrt{\frac{2}{L}}$$

$$\langle p \rangle = \int_0^L \frac{\hbar \pi}{i L} \cos\left(\frac{\pi x}{L}\right) \left[ \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \right]^2 dx.$$

$$= \int_0^L \frac{\hbar \pi}{i L} \cos\left(\frac{\pi x}{L}\right) \left[ \frac{2}{L} \sin^2 \frac{\pi x}{L} \right]$$

$$= \frac{2 \hbar \pi \sqrt{2}}{i L^2 \sqrt{L}} \int_0^L \cos\frac{\pi x}{L} \left[ \frac{1 - \cos\frac{2\pi x}{L}}{2} \right] dx$$

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$$= \frac{\sqrt{2}\hbar\pi}{iL^2\sqrt{L}}\left[\int_0^L \cos\frac{\pi x}{L}dx - \int_0^L \cos\frac{\pi x}{L}\cdot\cos\frac{2\pi x}{L}dx\right]$$

$$= \frac{\sqrt{2}\hbar\pi}{iL^2\sqrt{L}}\left[\left[\frac{L}{\pi}\sin\frac{\pi x}{L}\right]_0^L - \left(\frac{1}{2}\left(\int_0^L \cos\frac{3\pi x}{L}dx + \int_0^L \cos\left(\frac{\pi x}{L}\right)dx\right)\right)\right]$$

$$= \frac{\sqrt{2}\hbar\pi}{iL^2\sqrt{L}}\left[0 - \left(\frac{1}{2}\frac{L}{3\pi}\sin\frac{3\pi x}{L}\right)_0^L + \frac{1}{2}\times\frac{L}{\pi}\sin\left(\frac{\pi x}{L}\right)_0^L\right]$$

$$= \frac{\sqrt{2}\hbar\pi}{iL^2\sqrt{L}}\left[0 - 0 + 0\right].$$

$$= 0.$$

$\therefore \langle P \rangle = 0.$

$$|\psi|^2 = \psi\psi^*$$

$$\langle P \rangle = \int_{-\infty}^{+\infty} \psi\, p\, \psi^*\, dx.$$

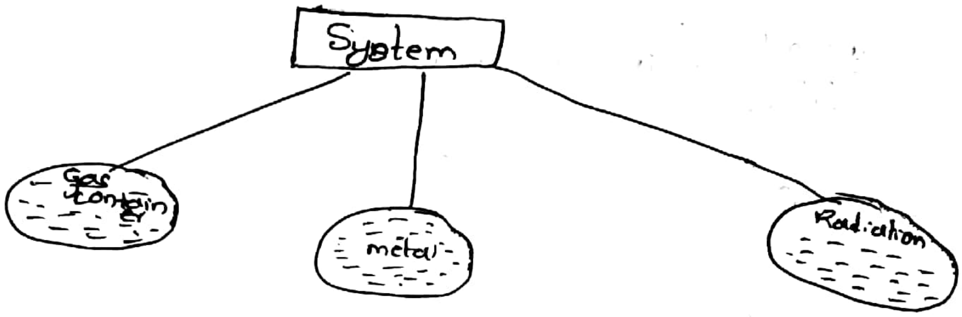
$$\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}\right)x \cdot \frac{\hbar}{i}\frac{\partial}{\partial x}\left[\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}\right)x\right]$$

$$\frac{2}{a}.$$

2-4-2018.

# Statistical Mechanics.

Statistical Mechanics deals with the system consists of very large no. of particles.
It gives the relatⁿ b/w the macroscopic behaviour of the system and microscopic behaviour of individual particles in that system.




We have 3 statistical distributions.

1) Maxwell's - Boltzmann Distribution.
2) Bose - Einstein Distribution.
3) Fermi - Dirac Distribution.

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| Maxwell-Boltzmann Distribution | Bose-Einstein Distribution | Fermi-Dirac Distribution |
|---|---|---|
| 1. It is classical mechanics system. | 1. It is Quantum mechanical system | 1. It is quantum mechanical system. |
| 2. It consists of identical and distinguishable molecules. | 2. Particles should be identical & indistinguishable. | 2. Particles should be identical & indistinguishable. |
| 3. Particles / Molecules have any spin. | 3. They have integral spin $n = 0, 1, 2, 3, \ldots$ | 3. They have half (or) half odd integral spins |
| 4. These molecules does not obey Pauli's exclusion principle | 4. They does not obey pauli's exclusion principle. | 4. They obey Pauli's exclusion principle. |
| eg:- Ideal gas molecules. | eg:- photons, phonons | eg:- Free electrons in metals |
| $\boxed{f(x) = A e^{-E/kT}}$ | $\boxed{f(x) = \frac{1}{e^{E/kT} - 1}}$ | $\boxed{f(x) = \frac{1}{e^{E-E_F/kT} + 1}}$ |

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## Maxwell-Boltzmann Statistics / Distribution :-

Let N be no. of gas molecules in the system.

$n(E)dE$ be no of molecules having energy E to $E+dE$.

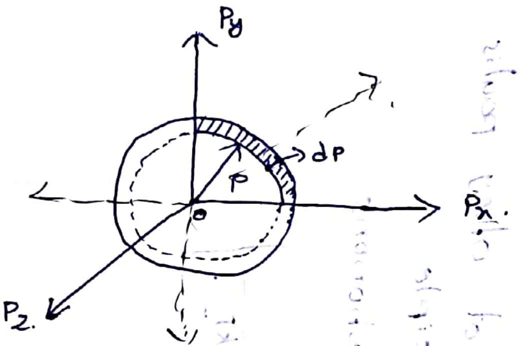
$g(E)dE \longrightarrow$ no. of energy states in the energy range E to $E+dE$.

$f(E) \longrightarrow$ distribution of gas molecules on different energy states.

$$\boxed{n(E)dE = g(E)\,dE \times f(E)}$$

## Density Of States [g(E)] :-

Consider momentum space in which $P_x, P_y$ and $P_z$ are the momentum coordinates along x-axis, y-axis and z-axis respectively.




No. of momentum states in the momentum range $p$ to $p+dp$ = $g(p)\,dp$.

$g(p)dp \propto$ volume of spherical shell of radius p and thickness dp.

$$g(p)dp \propto 4\pi p^2 dp$$

$$g(p)dp = B p^2 dp \longrightarrow ①$$

W.k.T $\quad P = \sqrt{2mE} \Rightarrow p^2 = 2mE$

Differentiating

$$2p\,dp = 2m\,dE$$

$$dp = \frac{m\,dE}{p}$$

$$dp = \frac{m\,dE}{\sqrt{2mE}} \longrightarrow ②$$

Substitute ② in ①.

$$① \Rightarrow g(E)\,dE = B \times 2mE \times \frac{m\,dE}{\sqrt{2mE}}$$

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$$\boxed{g(E)dE = B\sqrt{2}\, m^{3/2} \sqrt{E}\, dE.} \longrightarrow ④$$

Now $n(E)dE = g(E)dE \times f(E)$.

$$= B\sqrt{2}\, m^{3/2}\sqrt{E}\, dE \times A e^{-E/kT}$$

$$= AB\sqrt{2}\, m^{3/2} \sqrt{E}\, e^{-E/kT}\, dE.$$

$$\boxed{n(E)dE = C\sqrt{E}\, e^{-E/kT}\, dE} \longrightarrow ⑤$$

Calculate 'C' value.

$$\sqrt{E}\,\frac{e^{-E/kT}}{\frac{-1}{kT}} + \int \frac{1}{2\sqrt{E}}\,\frac{e^{-E/kT}}{\frac{+1}{kT}}$$

$$N = \int_0^\infty n(E)dE \qquad = \sqrt{E}\,kT e^{-E/kT} + \frac{kT}{2}\int$$

$$= C\int_0^\infty \sqrt{E}\, e^{-E/kT} dE.$$

$$\boxed{\int_0^\infty \sqrt{x}\, e^{-ax} dx = \frac{1}{2a}\sqrt{\frac{\pi}{a}}.}$$

$$N = C\,\frac{kT}{2}\sqrt{\pi kT} \times \frac{n}{\pi}.$$

$$\boxed{C = \frac{2\pi N}{(\pi kT)^{3/2}}}$$

Substitute $c$ in eq ⑤.

$$\boxed{n(E)dE = \frac{2\pi N}{(\pi kT)^{3/2}}\sqrt{E}\, e^{-E/kT}\, dE.}$$

Expression for molecular energy distribution.

## Total Energy :-

$$E_T = \int_0^\infty n(E)dE \times E$$

$$= \int_0^\infty \frac{2\pi N}{(k\pi T)^{3/2}}\sqrt{E}\, e^{-E/kT}\, dE \times E.$$

$$= \frac{2\pi N}{(\pi kT)^{3/2}}\int_0^\infty E^{3/2} e^{-E/kT} dE \qquad \int_0^\infty x^{3/2} e^{-ax} dx = \frac{3}{4a^2}\sqrt{\frac{\pi}{a}}$$

$$= \frac{2\pi N}{(\pi kT)^{3/2}}\,\frac{3\times}{4} k^2T^2\sqrt{\pi kT}.$$

$$\boxed{E_T = \frac{3}{2}NkT}$$

## Average Energy ($\bar{E}$):-

$$\bar{E} = \frac{\text{Total energy of all molecules}}{\text{total no. of molecules}}$$

$$= \frac{3/2\, NkT}{N}$$

$$\boxed{\bar{E} = 3/2\, kT}$$

6-4-2018.

## Molecular Speed Distribution:

We have $n(E)dE = \frac{2\pi N}{(\pi kT)^{3/2}} \sqrt{E}\, e^{-E/kT} dE.$

$$E = \frac{1}{2}mv^2$$

$$dE = \frac{1}{2}m \times 2v\,dv.$$

$$dE = mv\,dv.$$

$$\therefore n(v)dv = \frac{2\pi N}{(\pi kT)^{3/2}} \left[\frac{1}{2}mv^2\right]^{\frac{1}{2}} e^{-\frac{mv^2}{2kT}} \times mv\,dv.$$

$$\boxed{n(v)\,dv = 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}} dv}$$

## Most Probable Velocity ($v_p$):-

$$\frac{d}{dv}[n(v)] = 0$$

$$\frac{d}{dv}\left[4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-\frac{mv^2}{2kT}}\right] = 0.$$

$$4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} \frac{d}{dv}\left[v^2 e^{-\frac{mv^2}{2kT}}\right] = 0$$

$v\left[2 - v^2\frac{m}{kT}\right]$

$$4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}\left[2v e^{-\frac{mv^2}{2kT}} + v^2 e^{-\frac{mv^2}{2kT}} \times -\frac{2mv}{2kT}\right] = 0$$

$$4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}}\left[2v - \frac{mv^3}{kT}\right] = 0$$

$$2v - \frac{mv^3}{kT} = 0$$

$$2v = \frac{mv^{2}}{kT}$$

$$v^2 = \frac{2kT}{m}$$

$$\boxed{v_p = \sqrt{\frac{2kT}{m}}}$$

$v = v_p$ when $\frac{d}{dv}(n(v)) = 0$

## Average Velocity ($\bar{v}$ or $v_m$):-

Average Velocity = $\dfrac{\text{total velocity of all the molecules}}{\text{total no. of molecules.}}$

$$\bar{v} = \frac{\int_0^\infty n(v)dv \times v}{N}$$

$$= \frac{1}{N}\int_0^\infty 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} v^3\,dv.$$

$$= \cancel{\frac{1}{N}\,\frac{3}{4}\left(\frac{mv}{2kT}\right)^2 \sqrt{\frac{\pi}{mv^2}2kT}}$$

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$$= \frac{1}{N} \int_0^\infty 4\pi N \left(\frac{m}{2\pi kT}\right)^{3/2} e^{-\frac{mv^2}{2kT}} v^3 dv.$$

$$\boxed{\int_0^\infty x^3 e^{-ax^2} dx = \frac{1}{2a^2}}$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \int_0^\infty v^3 e^{\frac{-mv^2}{2kT}} dv.$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \times \frac{1}{2 \times \frac{m^2}{4k^2T^2}}.$$

$$= 4\pi \left(\frac{m}{2\pi kT}\right)^{3/2} \times \frac{2k^2T^2}{m^2}$$

$$= 4\pi^{1-\frac{3}{2}} m^{\frac{3}{2}-2} k^{2-\frac{3}{2}} T^{2-\frac{3}{2}} \times 2^{1-\frac{3}{2}}$$

$$= \sqrt{16}\, m^{-\frac{1}{2}} \times \sqrt{k} \times \sqrt{T} \times 2^{-\frac{1}{2}} \times \pi^{-\frac{1}{2}}.$$

$$\boxed{\bar{v}_{avg} = \sqrt{\frac{8kT}{\pi m}}}$$

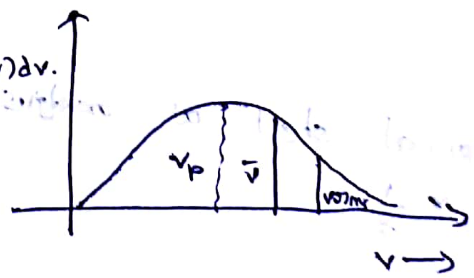
## Root mean Square Velocity ($v_{rms}$):-

$$\frac{3}{2} kT = E = \frac{1}{2} m\bar{v}^2.$$

$$\sqrt{\bar{v}^2} = \sqrt{\frac{3kT}{m}}.$$

$$\boxed{v_{rms} = \sqrt{\frac{3kT}{m}}}$$

## Graphical Representation:-




## Relation Between $v_p$, $\bar{v}$, $v_{rms}$:-

$$v_p : \bar{v} : v_{rms} = \sqrt{2} : \sqrt{\frac{8}{\pi}} : \sqrt{3}.$$

$$= 1.414 : 1.595 : 1.732.$$

$$\boxed{v_p : \bar{v} : v_{rms} = 1 : 1.128 : 1.224}$$

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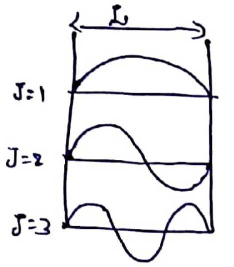
## Bose - Einstein Statistics :-

Consider J- space in which $J_x$, $J_y$ and $J_z$ along x, y and z-axis. If we give values to $J_x$, $J_y$, $J_z$ results 1 J value.

We have to calculate such J points for unit volume in J-space i.e., equal to the number of standing waves per unit volume

$g(\lambda)d\lambda \longrightarrow$ no. of standing waves in the wavelength range $\lambda$ to $\lambda+d\lambda$.

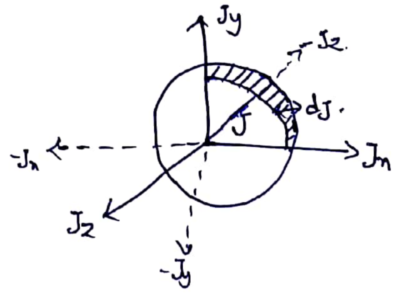
$$g(\lambda)d\lambda = g(J)dJ.$$




$$L \propto \frac{\lambda}{2}.$$

$$L = J\frac{\lambda}{2}.$$

$$J = \frac{2L}{\lambda}$$




$g(J)dJ =$ Volume of spherical shell of radius J and thickness dJ $\times \frac{1}{8}$

$$= 4\pi J^2 dJ \times \frac{1}{8}$$

$$g(J)dJ = \frac{1}{2}\pi J^2 dJ$$

$$g(J)dJ = \frac{1}{2}\pi J^2 dJ \times 2$$ ( standing waves have 2 direction of polarization)

$$g(J)dJ = \pi J^2 dJ$$

We have $J = \frac{2L}{\lambda}$ $\quad J^2 = \frac{4L^2}{\lambda^2}.$

$$dJ = 2L\, d\left(\frac{1}{\lambda}\right)$$

$$dJ = -\frac{2L}{\lambda^2}d\lambda = \frac{2L}{\lambda^2}d\lambda$$

$$\therefore g(\lambda)d\lambda = \pi \times \frac{4L^2}{\lambda^2} \times \frac{2L}{\lambda^2}d\lambda$$

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$$\boxed{\therefore g(\lambda)d\lambda = \frac{8\pi L^3}{\lambda^4} d\lambda.}$$

According to law of equipartition energy, the energy associated with each degree of freedom is $\frac{1}{2}kT$. since standing waves have 2 degrees of freedom

$$E = \frac{1}{2}kT \times 2 = kT.$$

No. of standing waves per unit volume

$$g(\lambda)d\lambda = \frac{8\pi}{\lambda^4} d\lambda \quad (L^3 = 1 \text{ for unit volume})$$

Energy associated with standing waves per unit volume

$U_\lambda d\lambda \longrightarrow$ Energy density

$$\boxed{U_\lambda d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda} \longrightarrow \text{R-J law.}$$

**Planck's Formula :-**

Avg. Energy of an oscillator $E = h\nu = \frac{hc}{\lambda}$.

B.E distribution $f(E) = \frac{1}{e^{E/kT} - 1}$

$$U_\lambda d\lambda = \frac{8\pi}{\lambda^4} \frac{hc}{\lambda} \left( \frac{1}{e^{hc/\lambda kT} - 1} \right) d\lambda$$

$$\boxed{U_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda}$$

**Wein's Displacement Law :-**

$\lambda_{max} T =$ a constant $= 2.989 \times 10^{-3}$.

$$\frac{d}{d\lambda}(U_\lambda d\lambda) = 0 \quad \text{for} \quad \lambda = \lambda_{max}.$$

$$\frac{d}{d\lambda}(U_\lambda) = \frac{8\pi hc \times -5}{\lambda^6} \cdot \frac{1}{e^{\frac{hc}{\lambda kT}} - 1} + \frac{8\pi hc}{\lambda^5} \left( \frac{-e^{\frac{hc}{\lambda kT}} \times}{(e^{hc/\lambda kT} - 1)^2} \frac{hc}{kT} \right)$$

$$= \frac{8\pi hc \times -5}{\lambda^6} \left[ e^{\frac{hc}{\lambda kT}} - 1 \right]^{-1} - \frac{8\pi h^2 c^2}{\lambda^5 kT} \frac{e^{hc/\lambda kT}}{(e^{hc/\lambda kT} - 1)^2}$$

$$= \frac{8\pi hc}{\lambda^5} \left[ -\frac{5}{\lambda} \left[ e^{\frac{hc}{\lambda kT}} - 1 \right]^{-1} - \frac{hc}{kT} \times \frac{e^{hc/\lambda kT}}{(e^{hc/\lambda kT} - 1)^2} \right]$$

$$= \frac{8\pi hc}{\lambda^5} \left[ \frac{-\frac{5}{\lambda}(e^{hc/\lambda kT} - 1) - \frac{hc}{kT} e^{hc/\lambda kT}}{(e^{hc/\lambda kT} - 1)^2} \right]$$

$$= \frac{8\pi hc}{\lambda^5 (e^{hc/\lambda kT} - 1)^2} \left[ \frac{5}{\lambda} - \frac{5}{\lambda}(e^{hc/\lambda kT}) - \frac{hc}{\lambda kT} \cdot e^{hc/\lambda kT} \right.$$

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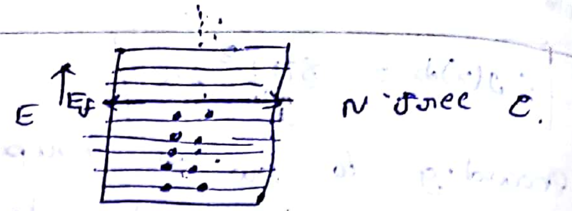
## Fermi - Dirac Statistics:-




$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

$E_F$ - Fermi Energy

The highest occupied energy state at T = 0K is called fermi energy.

① If $E < E_F$ at T = 0K

$$f(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

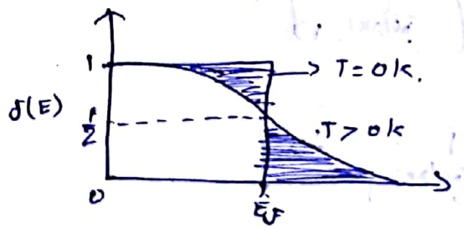
$$= \frac{1}{e^{-\infty} + 1} = \frac{1}{0+1} = 1.$$

② If $E > E_F$ at T = 0K.

$$f(E) = \frac{1}{e^{\infty} + 1} = \frac{1}{\infty + 1} = \frac{1}{\infty} = 0$$

③ $E = E_F$, T > 0K.

$$f(E) = \frac{1}{2}.$$




## Density Of States [g(E) dE]:-

Energy of a particle in a potential well is

$$E = \frac{n^2 h^2}{8mL^2}$$

For 3-dimensional metal, the energy of an $\bar{e}$ is.

$$E_n = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2) \rightarrow ①$$ where $n_x, n_y, n_z$ are +ve integers along x, y and z-axis.

Let $R^2 = n_x^2 + n_y^2 + n_z^2$

Put $E_0 = \frac{h^2}{8mL^2}$

① ⇒ $E = E_0 R^2 \rightarrow ②$.

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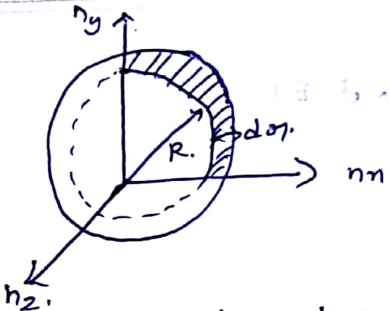


No. of points in the spherical shell of radius R and thickness dR is equalent to no. of energy states

$g(E)dE = \frac{1}{8}$ volume of a spherical shell of radius R and thickness dR.

$$= \frac{1}{8} \times 4\pi R^2 \times dR.$$

$$g(E)dE = \frac{1}{2} \times \pi R^2 dR. \times 2$$

Each energy states having 2 e's spinup & spindown.

$$\therefore g(E)dE = \pi R^2 dR \longrightarrow (3)$$

We have $E = E_0 R^2$.

$$R^2 = \frac{E}{E_0}.$$

$$2RdR = \frac{dE}{E_0}.$$

$$dR = \frac{dE}{2RE_0}$$

$$dR = \frac{dE}{2\left(\frac{E}{E_0}\right)^{1/2} E_0}$$

$$(3) \Rightarrow g(E)dE = \pi \times \frac{E}{E_0} \frac{dE}{2\left(\frac{E}{E_0}\right)^{1/2} E_0}.$$

$$= \pi \times \frac{\sqrt{E}}{\frac{h^2}{8mL^2}} \times \frac{dE \times \sqrt{h^2/8mL^2}}{2 \times \frac{h^2}{8mL^2}}$$

$$= \frac{\pi \times \sqrt{E}}{h^{4}3} \times \frac{64m^2L^4{}^3}{2} \times \frac{\cancel{h}\, dE}{\sqrt{8m}\,\cancel{L}}$$

$$g(E)dE = \frac{8\sqrt{2}\pi\, m^{3/2} L^3 \sqrt{E}\, dE}{h^3}$$

Per unit volume $L^3 = 1$.

$$\boxed{g(E)dE = \frac{8\sqrt{2}\pi\, m^{3/2}\sqrt{E}\, dE}{h^3}}$$

 

## Expression for Fermi Energy:-

We have $n(E)\,dE = g(E)dE \times f(E)$

Total N free e's.

$$N = \int_0^{E_F} n(E)\,dE$$

$$= \int_{E=0}^{E=E_F} g(E)dE \times f(E)$$

$$= \int_0^{E_F} \frac{8\sqrt{2}\pi m^{3/2}\sqrt{E}\,dE}{h^3} \times 1 \quad (\text{since } f(E)=1 \text{ at } T=0k)$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \int_0^{E_F} \sqrt{E}\,dE$$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} \left[\frac{E^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^{E_F}$$

$$N = \frac{16\sqrt{2}\pi m^{3/2}}{3h^3} E_F^{3/2}.$$

$$E_F^{3/2} = \frac{3Nh^3}{16\sqrt{2}\pi m^{3/2}}.$$

$$E_F = \left[\frac{3Nh^3}{16\sqrt{2}\pi m^{3/2}}\right]^{\frac{2}{3}}.$$

$$\boxed{E_F = \frac{h^2}{8m}\left(\frac{3N}{\pi}\right)^{2/3}.}$$

## Expression for Total Energy:-

We have $n(E)dE = g(E)\,dE \times f(E)$

$$= \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{\frac{1}{2}}dE \times \frac{1}{e^{(E-E_F)/kT}+1}$$

In terms of Fermi energy

$$n(E)dE = \frac{\left(\frac{3}{2}N\right) E_F^{-3/2} \sqrt{E}\,dE}{e^{(E-E_F)/kT}+1}.$$

$$E_T = \int_0^{E_F} n(E)dE \times E.$$

$$= \int_0^{E_F} \left(\frac{3}{2}N\right) E_F^{-3/2} \sqrt{E}\,dE \times E$$

at $T=0k$ $f(E) = \frac{1}{e^{(E-E_F)/kT}} = 1$

$$E_T = \left(\frac{3}{2}N\right) E_F^{-3/2} \int_0^{E_F} E^{3/2}\,dE.$$

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$$E_T = \left(\frac{3}{2}N\right) E_F^{-3/2} \int_0^{E_F} E^{3/2}\, dE$$

$$= \left(\frac{3}{2}N\right) E_F^{-3/2} \times \frac{2}{5} E_F^{5/2}$$

$$\boxed{E_T = \frac{3}{5} N E_F}$$

Average energy $\bar{E} = \frac{E_T}{N} = \frac{\frac{3}{5} N E_F}{N}$

$$\boxed{\bar{E} = \frac{3}{5} E_F}$$

Fermi Temperature ($T_F$):-

→ The temperature at which average thermal energy of free $\bar{e}$s equal to Fermi energy.

At $T = T_F$ $kT_F = E_F$

$$\boxed{T_F = \frac{E_F}{k}}$$

Fermi Velocity ($V_F$):- The velocity of free $\bar{e}$s which occupies fermi level is called fermi velocity.

$$\frac{1}{2} m V_F^2 = E_F.$$

$$V_F^2 = \frac{2E_F}{m}$$

$$\boxed{V_F = \sqrt{\frac{2E_F}{m}}}$$

1) The fermi level in Ag is 5.5ev at 0k calculate the no. of free $\bar{e}$s per unit volume and probability of occupation for $\bar{e}$s with energy 5.6ev in Ag at same temperature.

A) Given:- $E_F = 5.5ev.$

$T = 0K.$ $E = 5.6ev.$

$\cancel{F_T = 3/2}$

$N = ?$

$$E_F = \left(\frac{h^2}{8m}\right)\left(\frac{3N}{\pi}\right)^{2/3}.$$

$$5.5 = 5.98 \times 10^{-38} \left(\frac{3N}{\pi}\right)^{\frac{2}{3}}$$

$$\left(\frac{3N}{\pi}\right)^{\frac{2}{3}} = \frac{5.5}{5.98 \times 10^{-38}}$$

$$N = \left(\frac{5.5}{5.98 \times 10^{-38}}\right)^{\frac{3}{2}} \times \frac{\pi}{3} = 5.9 \times 10^{28}/m^3.$$

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$E = 5.6 ev$ at $T = 0k$.

$E_F = 5.5 ev$

Probability, $f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$

$= \frac{1}{e^{(5.6-5)/kT}+1}$

2) Cal. $E_F$ for a metal at 0k. $N = 5.86 \times 10^{28} / m^3$.

A) $E_F = \left(\frac{h^2}{8m}\right)\left(\frac{3N}{\pi}\right)^{\frac{2}{3}}$

$= \frac{(6.6 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31}} \times \left(\frac{3 \times 5.86 \times 10^{28}}{\pi}\right)^{\frac{2}{3}}$

$E_F = 5.5 ev$

3) The fermi level in Ag is 5.5ev what is the energy for which probability of occupancy at 300k is 0.99

A) Given:- $E_F = 5.5ev$. $E = ?$

$T = 300k$

$f(E) = 0.99$.

$f(E) = \frac{1}{e^{(E-E_F)/kT}+1}$

$0.99 = \frac{1}{e^{(E-5.5)/k \times 300}+1}$.

$e^{(E-5.5)/1.38 \times 10^{-23} \times 300} = \frac{1}{0.99} - 1$.

$e^{(E-5.5)/1.38 \times 10^{-23} \times 300} = \frac{0.01}{0.99} = \frac{1}{99}$.

$\frac{(E-5.5)}{1.38 \times 10^{-23} \times 300}$ ~~log~~ $\ln_e^e = \ln\left(\frac{1}{99}\right)$

$E - 5.5 = 1.38 \times 10^{-23} \times 300 \times \ln\left(\frac{1}{99}\right)$

$= \frac{-1.9 \times 10^{-20} J}{1.6 \times 10^{-19}}$

$E = -0.11 + 5.5$

$E = 5.38 ev$

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4) Find the temp at which there 1% probability a state with energy 0.5ev above the fermi energy is occupied.

A) Given:- $E-E_F = 0.5ev$

$T = ?$

$$f(E) = \frac{1}{e^{(E-E_F)/KT} + 1}$$

$$\frac{1}{100} = \frac{1}{e^{(E-E_F)/KT}+1}$$

$$e^{(0.5)/KT} = \frac{1}{100} - 1.$$

$$\cancel{= \frac{-99}{100}}$$

$$\cancel{e^{(0.5)/1.38\times10^{-23}} = -0.99.}$$

$$\cancel{\frac{0.5}{1.38\times10^{-23}\times T} \ln e = -\ln(0.99)}$$

$$\cancel{T = \frac{0.5 \times 1.6\times10^{-19}}{1.38\times10^{-23}\times -(\ln 0.99)}}$$

$$e^{(0.5)/KT} = 100-1.$$

$$e^{(0.5)/KT} = 99.$$

$$\frac{0.5}{1.38\times10^{-23}\times T} = \ln 99.$$

$$T = \frac{0.5}{1.38\times10^{-23}\times \ln 99}.$$

$T = 1261.5K.$

5) Cal. Peak wavelength in the spectrum of radiation from a black body at a temp of 500°c.

A) $\lambda_{max} \times T = a.$

$$\lambda_{max} = \frac{a}{T} = \frac{2.889\times10^{-3}}{773.}$$

$$= 3.738\times10^{-6}.$$

$\lambda_{max} = 3738$ nm.

6) Cal. temp at which the rms velocity of $O_2$ is 490 m/s.

A) $m = 32\times1.66\times10^{-27}$

$$V_{rms} = \sqrt{\frac{3kT}{m}}$$

$$V_{rms}^2 = \frac{3kT}{m}$$

$$T = \frac{V_{rms}^2 \times m}{3k.}$$

$$= \frac{490^2 \times 32 \times 1.66\times10^{-27}}{3\times1.38\times10^{-23}} = 308k.$$

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7) Find rms $O_2$ at $0°c$.

A) $$V_{rms} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 273}{32 \times 1.66 \times 10^{-27}}}$$

$V_{rms} = 461.26$ m/s.

8) Show that $V_{rms}$ of gas molecule is 9% greater than $V_{avg}$.

A) $$V_{rms} = \sqrt{\frac{3kT}{m}}.$$

$$V_{avg} = \sqrt{\frac{8kT}{\pi m}}.$$

$$\frac{V_{rms}}{V_{avg}} = \sqrt{\frac{3 \times \pi}{8}}$$

$$\frac{V_{rms}}{V_{avg}} = 1.085.$$

$$\frac{V_{rms}}{V_{avg}} - 1 = 1.085 - 1.$$

$$\frac{\Delta V_{rms}}{V_{avg}} = 0.085.$$

$\Delta V_{rms} \times 100 = 8.54\%$.

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