

Intrinsic instabilities in Fermi glasses

Yat Fan Lau and Tai Kai Ng*

Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay Road, Kowloon, Hong Kong

We study in this paper the effect of weak, short-ranged repulsive interaction on disordered metals. Through analysing the interaction matrix elements between different eigenstates of the non-interacting and corresponding Hartree-Fock single-particle Hamiltonian, we argue that the Fermi glass state is unstable towards formation of magnetic moments for repulsive interaction and formation of local fermion pairs for attractive interaction as a result of localized single-particle eigenstates around the Fermi surface. Numerical simulations are performed to verify our analysis. We further propose within a Landau Fermi-liquid type framework that our result is applicable for general electronic systems with weak, short-ranged interaction as long as the quasi-particle states exist and are localized, suggesting that Fermi glass state is intrinsically unstable.

A. Introduction

Interacting disordered systems have been a major areas of research in condensed matter physics because of the many interesting physics associated with disordered systems, including Fermi Glass[1–4], Coulomb Glass[5–7], Wigner crystal[8, 9], the Many-Body Localized (MBL) states[10–12] and the anomalous metal states[13–15], etc. The problem is difficult theoretically because of the absence of an effective theoretical framework where interaction and disorder can be treated together systematically.

Based on an analogy with Fermi liquid theory Anderson proposed that for weak enough short-ranged repulsive interactions, the ground and low energy excited states of disordered fermion systems with localized single-particle states evolve continuously as the interaction is turned on[1, 2] and the ground state remains a paramagnetic insulator with localized quasi-particles (Fermi Glass) (see also Ref.[16]).

On the other hand, using a Hartree-Fock (HF) approximation, Milica *et al.* found that instability towards the formation of local magnetic moments occurs easily when repulsive interaction between electrons is turned on in the presence of disorder[17]. More recently, Pilati and Fratini[18] found in numerical Monte Carlo calculations that the instability towards the formation of local moments occurs when the Fermi surface crosses the mobility edge, i.e. when the electronic states on Fermi surface become localized.

In this paper, we study the effect of weak, short-ranged interaction on disordered electronic systems. We shall analyze carefully the interaction matrix elements between the eigenstates of the non-interacting and the corresponding Hartree-Fock single-particle Hamiltonians, and show that spontaneous formation of local magnetic moments for repulsive interactions and local fermion pairs for attractive interaction occurs as long as the single-particle eigenstates around the Fermi surface are localized. We further propose within a Fermi-liquid

type framework that our result remains robust for general short-ranged, repulsive interaction as long as quasi-particle states exist and are localized around the Fermi surface.

B. Interaction matrix elements

In general, the Hamiltonian of a disordered fermionic system can be written as $H = H_0 + H'$ where H_0 is the non-interacting disordered Hamiltonian and H' represents interaction between fermions. For concreteness, we consider a general lattice spin-1/2 fermion model of form

$$\begin{aligned} H_0 &= - \sum_{ij\sigma} t_{ij} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i\sigma} W_i n_{i\sigma} \\ H' &= \sum_{i\sigma j\sigma'} \frac{1}{2} U_{ij}^{\sigma\sigma'} n_{i\sigma} n_{j\sigma'}, \end{aligned} \quad (1)$$

where $c(c^\dagger)_{i\sigma}$ are spin- σ fermion operators on lattice site i , $\sigma = \pm \frac{1}{2}$. The first term in H_0 represents the hopping of fermions between different lattice sites and the second term represents an onsite disordered potential W_i which can be chosen to be a random variable distributed uniformly between $-W/2$ and $W/2$. H' describes the interaction between fermions. We consider short-ranged interactions in this paper where $U_{ij}^{\sigma\sigma'}$ is nonzero only when the distance $|\vec{r}_i - \vec{r}_j|$ is less than a few lattice sites.

The (single-particle) eigenstates of the non-interacting Hamiltonian H_0 is given by

$$\sum_j (h_{0ij} - \mu \delta_{ij}) \phi_k(\vec{r}_j) = \xi_k \phi_k(\vec{r}_i) \quad (2)$$

where $h_{0ij} = -t_{ij} + W_i \delta_{ij}$. The Hamiltonian can be written in the eigenstate representation of the fermion operators $c(c^\dagger)_{k\sigma}$ defined by

$$c_{i\sigma} = \sum_k \phi_k(\vec{r}_i) c_{k\sigma} \quad \text{and} \quad c_{i\sigma}^\dagger = \sum_k \phi_k^*(\vec{r}_i) c_{k\sigma}^\dagger. \quad (3)$$

Substituting the above representations into Eq.(1), we

* phtai@ust.hk

obtain the Hamiltonian in H_0 -eigenstate representation,

$$H = \sum_{k\sigma} \xi_k c_{k\sigma}^\dagger c_{k\sigma} + \frac{1}{2} \sum_{\substack{klpq \\ \sigma\sigma'}} U_{klpq}^{\sigma\sigma'} c_{k\sigma}^\dagger c_{l\sigma} c_{p\sigma'}^\dagger c_{q\sigma'} \quad (4)$$

where

$$U_{klpq}^{\sigma\sigma'} = \sum_{ij} U_{ij}^{\sigma\sigma'} \phi_k^*(\vec{r}_i) \phi_l(\vec{r}_i) \phi_p^*(\vec{r}_j) \phi_q(\vec{r}_j).$$

We emphasize that the indices k, l, p, q are eigenstate indices, and are not related to momenta of particles.

Similarly, we can introduce eigenstates of the HF Hamiltonian, given by

$$H_{HF} = - \sum_{ij\sigma} t_{ij}^{HF} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i\sigma} W_i^{HF} n_{i\sigma} \quad (5a)$$

where

$$\begin{aligned} t_{ij}^{HF} &= t_{ij} + U_{ij}^{\sigma\sigma} \langle c_{j\sigma}^\dagger c_{i\sigma} \rangle \\ W_i^{HF} &= W_i + \sum_{j\sigma'} U_{ij}^{\sigma\sigma'} \langle n_{j\sigma'} \rangle, \end{aligned} \quad (5b)$$

where $\langle \dots \rangle$ denotes self-consistently determined ground state expectation value. We have assumed that $U_{ij}^{\uparrow\uparrow} = U_{ij}^{\downarrow\downarrow}$, $U_{ij}^{\uparrow\downarrow} = U_{ij}^{\downarrow\uparrow}$ and a non-magnetic HF ground state is formed in the self-consistent HF theory, with $\langle c_{j\uparrow}^\dagger c_{i\uparrow} \rangle = \langle c_{j\downarrow}^\dagger c_{i\downarrow} \rangle$ and $\langle n_{j\uparrow} \rangle = \langle n_{j\downarrow} \rangle$. (For attractive interaction, a Cooper instability occurs which shall be considered later.) The HF single-particle states are given by

$$\sum_j (h_{ij}^{HF} - \mu \delta_{ij}) \phi_\alpha^{HF}(\vec{r}_j) = E_k \phi_\alpha^{HF}(\vec{r}_i) \quad (6)$$

where $h_{ij}^{HF} = -t_{ij}^{HF} + W_i^{HF} \delta_{ij}$. We introduce also the HF-eigenstate representation of the fermion operators $c(c^\dagger)_{\alpha\sigma}$, given by

$$c_{i\sigma} = \sum_\alpha \phi_\alpha^{HF}(\vec{r}_i) c_{\alpha\sigma} \quad \text{and} \quad c_{i\sigma}^\dagger = \sum_\alpha \phi_\alpha^{HF*}(\vec{r}_i) c_{\alpha\sigma}^\dagger. \quad (7)$$

In the following, we shall analyze the interaction matrix elements $U_{klpq}^{\sigma\sigma'}$ between eigenstates $\phi_k(\vec{r}_i)$'s of H_0 and $U_{\alpha\beta\gamma\zeta}^{\sigma\sigma'}$ between eigenstates $\phi_\alpha(\vec{r}_i)$'s of H_{HF} . We shall see that our analysis gives the same result in both cases as long as the single-particle states are localized.

We first consider localized eigenstates $\phi_k(\vec{r}_i)$ of H_0 . In this case we may write

$$\phi_k(\vec{r}_i) \sim \frac{1}{L^{d/2}} e^{-\frac{|\vec{r}_i - \vec{x}_k|}{2L}} \quad \text{and} \quad |\phi_k(\vec{r}_i)|^2 \sim \frac{1}{L^d} e^{-\frac{|\vec{r}_i - \vec{x}_k|}{L}} \quad (8)$$

for $|\vec{r}_i - \vec{x}_k| \gg L$ where \vec{x}_k is the centre of the localized state k and L is the localization length. In this case, we expect there is small wavefunction overlap between states that are far from each other and the matrix element $U_{klpq}^{\sigma\sigma'}$

decays exponentially as the distance between the states $|\vec{x}_a - \vec{x}_b| \gg L$ for any $(a, b) = (k, l, p, q)$. The matrix element is sizable only if $|\vec{x}_a - \vec{x}_b| < L$ for all $(a, b) = (k, l, p, q)$. In this case the order of magnitude of $U_{klpq}^{\sigma\sigma'}$ can be estimated as

$$\begin{aligned} U_{klpq}^{\sigma\sigma'} &= \sum_{ij} U_{ij}^{\sigma\sigma'} \phi_k^*(\vec{r}_i) \phi_l(\vec{r}_i) \phi_p^*(\vec{r}_j) \phi_q(\vec{r}_j) \\ &\sim \left(\sum_j U^{\sigma\sigma'}(\vec{r}_i - \vec{r}_j) \right) \left(\sum_i \phi_k^*(\vec{r}_i) \phi_l(\vec{r}_i) \phi_p^*(\vec{r}_i) \phi_q(\vec{r}_i) \right) \\ &\sim \bar{U}^{\sigma\sigma'} \frac{1}{L^{2d}} (L^d) = \frac{1}{L^d} \bar{U}^{\sigma\sigma'} \end{aligned} \quad (9)$$

where $\bar{U}^{\sigma\sigma'} = \sum_j U^{\sigma\sigma'}(\vec{r}_i - \vec{r}_j)$ represents the ‘‘average strength \times the range of the interaction potential’’ and $\sum_i \phi_k^*(\vec{r}_i) \phi_l(\vec{r}_i) \phi_p^*(\vec{r}_i) \phi_q(\vec{r}_i) \sim L^d \sim$ volume where the wavefunctions overlap substantially. The estimation is valid as long as $L \gg$ range of the interaction potential.

It is useful to look at the disorder-average of $U_{klpq}^{\sigma\sigma'}$ which is given by

$$\langle U_{klpq}^{\sigma\sigma'} \rangle_{dis} \sim \frac{\bar{U}^{\sigma\sigma'}}{L^d} \times P_{klpq},$$

where $P_{klpq} \sim (\frac{L^d}{V})^3$ is the probability of finding all four states within distance L from each other and V is the volume of the system[19].

Notice that for $k = l$ and $p = q$, $U_{kkpp}^{\sigma\sigma'} = \sum_{ij} U^{\sigma\sigma'}(\vec{r}_i - \vec{r}_j) |\phi_k(\vec{r}_i)|^2 |\phi_p(\vec{r}_j)|^2 \sim \frac{1}{L^d} \bar{U}^{\sigma\sigma'}$ and $\langle U_{kkpp}^{\sigma\sigma'} \rangle_{dis} \sim \frac{\bar{U}^{\sigma\sigma'}}{L^d} \times P_{kkpp} \sim (\frac{\bar{U}^{\sigma\sigma'}}{V})$ is much larger than $\langle U_{klpq}^{\sigma\sigma'} \rangle_{dis}$ because the probability of finding two states within distance L ($\sim L^d/V$) is much larger than the probability of finding four states within distance L ; i.e.,

$$\langle U_{kkpp}^{\sigma\sigma'} \rangle_{dis} \sim \left(\frac{\bar{U}^{\sigma\sigma'}}{V} \right) \gg \langle U_{klpq}^{\sigma\sigma'} \rangle_{dis} \sim \left(\frac{\bar{U}^{\sigma\sigma'}}{V} \right) \left(\frac{L^d}{V} \right)^2. \quad (10)$$

In particular, $\langle U_{kkkk}^{\sigma\sigma'} \rangle_{dis} \sim \frac{1}{L^d} \bar{U}^{\sigma\sigma'}$ as there is no probability correction factor associated with finding the same state k .

We now compare the above results with the case of extended states where

$$\phi_k(\vec{r}_i) \propto \frac{1}{\sqrt{V}} e^{iu_k(\vec{r}_i)} \quad \text{and} \quad |\phi_k(\vec{r}_i)|^2 \propto \frac{1}{V} \quad (11)$$

where $u_k(\vec{r}_i)$ is a real number function and we obtain $U_{klpq}^{\sigma\sigma'} \sim U_{kkpp}^{\sigma\sigma'} \sim U_{kkkk}^{\sigma\sigma'} \sim \bar{U}^{\sigma\sigma'}/V$.

Physically, a fermion in extended state k interacts with all other fermion states in the system with matrix element of order \bar{U}/V , and its properties are determined by the collective behaviour of the whole system. However, for localized states, a fermion in state k interacts only with states within a distance L , but with a much stronger

interaction $\sim \bar{U}/L^d$. Its behavior is determined by local fermion environment in this case.

It is quite obvious that our result is independent of the precise form of H_0 or the detailed behaviour of single-particle wavefunctions. It depends only on the assumption of localization of the single-particle wavefunction and the interaction potential is short-ranged. In particular, the same qualitative conclusion would be obtained for eigenstates $\phi_\alpha(\vec{r}_i)$ of H_{HF} as long as the eigenstates of H_{HF} are also localized. We caution here that the localization of eigenstates of H_0 does not necessarily imply that the corresponding eigenstates of H_{HF} are also localized.

To verify the above result, we perform a numerical simulation of $U_{kkpp}^{\uparrow\downarrow}$ where k and p are eigenstates of H_0 on a t - t' - W -Hubbard-model on a 60×60 square lattice,

$$H = -t \sum_{\langle ij \rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) - t' \sum_{\langle\langle ij \rangle\rangle \sigma} (c_{i\sigma}^\dagger c_{j\sigma} + h.c.) + \sum_{i\sigma} W_i n_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} \quad (12)$$

where the t , t' terms are the nearest and next nearest neighbor hopping, respectively. We choose $t = 1$, $t' = 0.6$ and $W = 9$ in our simulation, corresponding to an average localization length $L \sim 7.65$ [20]. The same set of parameters is used in all our numerical simulations below. The matrix element $U_{kkpp}^{\uparrow\downarrow}$ is computed as a function of distance d between two eigenstates k and p where d is defined as the separation between $\max(|\phi_k(\vec{r}_i)|)$ and $\max(|\phi_p(\vec{r}_i)|)$. The results are shown in Fig.1(a) where each data point corresponds to a randomly chosen pair of states (k, p) from 400 states near the band centre with energy range from $\sim -0.91t$ to $\sim 0.26t$ for five randomly chosen disorder configurations. The average $\langle U_{kkpp}^{\uparrow\downarrow} \rangle$ as function of distance is shown in Fig.1(b) where we find that although there exists large fluctuations in $U_{kkpp}^{\uparrow\downarrow}$, $\langle U_{kkpp}^{\uparrow\downarrow} \rangle$ decays exponentially as expected with a decay factor roughly equal to L .

1. Effective grain picture

As we discussed, interactions between fermions is dominated by events within volume L^d . This leads to the effective grain picture where we may consider the system as composed of grains of size L^d interacting weakly with each other. Each grain contains $\mathcal{N}_G \sim \mathcal{N}(\frac{L^d}{V})$ particles, where \mathcal{N} is the total number of particles in the whole system. The average energy level spacing of states within each grain is given by $\langle \Delta E_G \rangle \sim W_B / \mathcal{N}_G \propto 1/(L^d N(0))$ where $N(0)$ is the density of states on the Fermi surface and W_B is the bandwidth. Notice that $\langle \Delta E_G \rangle$ is much larger than that average energy level spacing of the whole system given by $\langle \Delta E \rangle = W_B / \mathcal{N} \propto 1/(VN(0))$.

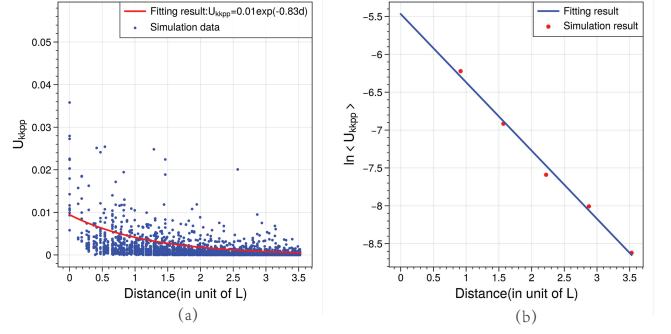


FIG. 1: (a) Simulation raw data for $U_{kkpp}^{\sigma\sigma'}$ as a function of distance where k, p are eigenstates of H_0 . (b) The average $\langle U_{kkpp}^{\sigma\sigma'} \rangle_{dis}$ as a function of distance

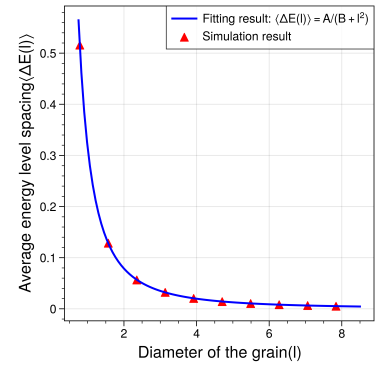


FIG. 2: The average energy level spacing $\langle \Delta E(l) \rangle_{dis}$ as a function of the grain diameter l (measured in unit of L).

To verify this result, we consider grains of size $\sim l^2$ within our 60×60 site lattice and compute the single-particle energy level spacing $\langle \Delta E(l) \rangle$ of the grains with different diameters l averaged over grains chosen randomly at different regions of our simulated sample in 15 disorder configurations. The results are shown in Fig.2. We see that the average level spacing decreases when l increases. The numerical results is fitted to a curve $\langle \Delta E(l) \rangle_{dis} \sim A/(B + l^2)$ with A, B the fitting parameters. When l reaches the system size, the average energy level spacing is $\langle \Delta E(l) \rangle_{dis} \sim 0.004$.

C. First-order perturbation and HF theory

For weak enough interaction we can study the system in first-order perturbation theory. In this case the wavefunctions of the system remain as eigenstates of H_0 and the H' only provides energy corrections to the eigenstates which are specified by the occupation numbers $\{n_{k\sigma}\}$ of

H_0 . The resulting first-order energy takes the form

$$E^{(1)} = \langle \Phi_0 | H | \Phi_0 \rangle = \sum_{k\sigma} \xi_k n_{k\sigma} + \frac{1}{2} \sum_{kp\sigma\sigma'} (U_{kkpp}^{\sigma\sigma'} - \delta_{\sigma\sigma'} U_{kppk}^{\sigma\sigma}) n_{k\sigma} n_{p\sigma'}. \quad (13)$$

It is interesting to note that the same form of energy is obtained in the HF theory where the many-body states $|\Phi_{HF}\rangle$ are formed by occupying the single-particle eigenstates of H_{HF} ($\{\phi_{\alpha\sigma}\}$) with energy

$$E_{HF} = \langle \Phi_{HF} | H | \Phi_{HF} \rangle = \sum_{\alpha\sigma} \epsilon_\alpha n_{\alpha\sigma} + \frac{1}{2} \sum_{\alpha\beta\sigma\sigma'} (U_{\alpha\alpha\beta\beta}^{\sigma\sigma'} - \delta_{\sigma\sigma'} U_{\alpha\beta\beta\alpha}^{\sigma\sigma}) n_{\alpha\sigma} n_{\beta\sigma'} \quad (14)$$

where $n_{\alpha\sigma}$'s are the occupation number of the single-particle state $\phi_{\alpha\sigma}$, ϵ_α is the expectation value of H_0 for the single-particle state $\phi_{\alpha\sigma}$ and $U_{\alpha\alpha\beta\beta}^{\sigma\sigma'} = \sum_{ij} U_{ij}^{\sigma\sigma'} |\phi_\alpha^{HF}(\vec{r}_i)|^2 |\phi_\beta^{HF}(\vec{r}_j)|^2$ [21].

As $E^{(1)}$ and E_{HF} has the same form of energy we shall simplify notation and use the same label k and p to denote single-particle eigenstates of both H_0 and H_{HF} in the following when we consider the energies required to add and remove particles from the ground states.

In both cases the energy required to add one particle to an empty state k is

$$E_1(k) = \xi_k + \sum_{p\sigma'} (U_{kkpp}^{\sigma\sigma'} - \delta_{\sigma\sigma'} U_{kppk}^{\sigma\sigma}) n_{p\sigma'}. \quad (15a)$$

whereas the energy for adding two particles in states k and p is

$$E_2(k, p) = E_1(k) + E_1(p) + (U_{kkpp}^{\sigma\sigma'} - \delta_{\sigma\sigma'} U_{kppk}^{\sigma\sigma}). \quad (15b)$$

Notice that an extra interaction term between the two added particles appears in $E_2(k, p)$.

A necessary condition for ground state stability is that $E_1(k) > 0$ and $E_2(k, p) > 0$ when we add particles to any (unoccupied) states k, p in the ground state since otherwise we can lower the system's energy by adding one or two fermions to state k, p . Similarly, we expect $E_1(k) < 0$ and $E_2(k, p) < 0$ when we remove particles from any (occupied) states k, p in the ground state. We shall examine the stability of Fermi Glass state using these criteria in the following.

We note that as H_{HF} is determined self-consistently from $\{\phi_{\alpha\sigma}\}$, the values of ξ_α , $U_{\alpha\alpha\beta\beta}^{\sigma\sigma'}$ and $U_{\alpha\beta\beta\alpha}^{\sigma\sigma'}$ will be modified slightly when we add or remove a few particles to/from the system. We shall see that these small differences will not affect our conclusion as long as the qualitative behaviour of the states $\{\phi_{\alpha\sigma}\}$ are not modified by adding or removing a few particles to/from the system.

D. Instabilities of the Fermi glass ground state

The system's ground state $|\Omega_0\rangle$ is formed by doubly occupying the lowest $2N$ energy levels of H_0 (first order perturbation theory) or H_{HF} (in HF theory). We shall show now that this state is unstable towards formation of local fermion pairs for attractive interaction in first order perturbation and in HF theory as long as there exist a finite density states on the Fermi surface and the single-particle states we examine are localized. We start by adding a spin- σ fermion to a state k above the Fermi surface. We assume $E_1(k) > 0$ as otherwise the Fermi glass state is already unstable. Now we add another electron with spin $-\sigma$ to the same state k . Using Eq. (15b) the excitation energy in this case is

$$E_2(k, k) = 2E_1(k) + U_{kkkk}^{\uparrow\downarrow}.$$

For attractive interaction, $U_{kkkk}^{\uparrow\downarrow}$ is negative and if $2E_1(k) < |U_{kkkk}^{\uparrow\downarrow}|$, we have $E_2 < 0$ which violates the stability criteria and the ground state becomes unstable.

In this case the ground state of the system is formed by doubly occupying all states k with energy $E_1(k) < |U_{kkkk}^{\uparrow\downarrow}|/2$. A single-particle excitation on the Fermi surface of the new ground state will acquire an energy gap $\sim |U_{kkkk}^{\uparrow\downarrow}|/2$ although the energy for adding a pair of ($\uparrow\downarrow$) fermions on state k remains gapless.

Applying a similar analysis for repulsive interactions, it is easy to see that the Fermi glass ground state is unstable towards removing two fermions in a state k below the Fermi surface with $E_1(k) < 0$ if $E_2(k, k) = 2E_1(k) + U_{kkkk}^{\uparrow\downarrow} > 0$. In this case, we expect the real ground state is formed by doubly occupying all states k with $E_1(k) < -U_{kkkk}^{\uparrow\downarrow}/2$ and singly occupying states k with energy $-U_{kkkk}^{\uparrow\downarrow}/2 < E_1(k) < 0$, i.e., the ground state becomes *spin-polarized*.

The above considerations have physical implication if there exists finite density of fermion states with $2|E_1(k)| < |U_{kkkk}^{\uparrow\downarrow}|$. The smallest plausible value of $|E_1(k)|$ is given by $|E_1(k)| \sim$ energy level spacing of the bulk system $\sim 1/(VN(0))$ whereas $|U_{kkkk}^{\uparrow\downarrow}| \sim (\frac{\bar{U}^{\uparrow\downarrow}}{L^d})$ for localized state k (see Eqs.(9) and (10)), indicating that such instability occurs generally in first order perturbation or HF theory when both L and $N(0)$ are finite. The density of such states is of order $n \sim \frac{\bar{U}^{\uparrow\downarrow}}{2L^d} \times N(0)$. On the contrary, for extended systems $|U_{kkkk}^{\uparrow\downarrow}| \sim \frac{\bar{U}^{\uparrow\downarrow}}{V}$ and the instability occurs only when $\bar{U}^{\uparrow\downarrow} N(0) > 2$, i.e., the instability occurs only for strong enough interaction, which is essentially Stoner criterion.

We note that we have considered only one- and two-particle excitations occupying the same state k in our above stability analysis and further instabilities may also occur when we consider multiple particle excitations. As an example, we consider two particle

excitations occupying states k, p . In this case, new instability occurs if $U_{kk\sigma\sigma'}^{\sigma\sigma'} \sim U_{kppk}^{\sigma\sigma'} \geq \bar{U}^{\uparrow\downarrow}/L^d$. However, this happens only when states k and p are within distance L . The probability for this to occur is of order $\sim \frac{U}{L^d} \times N(0) \times L^d \ll 1$ for weak interaction.

To confirm our result we perform a Monte-Carlo simulation of the ground state of the repulsive t - t' - W -Hubbard-model (Eq.(12)) in first-order perturbation theory where the ground state occupation numbers $\{n_{k\sigma}\}$ are determined by minimizing $E^{(1)}$ with respect to $\{n_{k\sigma}\}$. The results are shown in Fig.3(a) where we plot the average occupation numbers $\langle n_k \rangle_{dis} = \langle n_{k\uparrow} + n_{k\downarrow} \rangle_{dis}$ as a function of energy of the states k and the data points are fitted by $\langle n_k \rangle_{dis} = \frac{2}{e^{\beta(\xi_k - \mu)} + 1}$ with β, μ being fitting parameters (note that β is not the temperature here). We obtain $\beta^{-1} \sim 0.045U$ for small U in our simulation. In this simulation, we choose $\mathcal{N} = 3000$ and perform 10^5 Monte Carlo steps for each disorder configuration for 15 disorder configurations. The instability of the Fermi glass ground state towards formation of local magnetic moments in the regime $UN(0) \ll 1$ is clear. We find that the number of polarized spins increases linearly with U for small U but increases faster for larger U (Fig.3(b)), indicating that instabilities associated with more than one single-particle states become important when U increases.

Our results are supported by an earlier numerical work which employs quantum Monte Carlo simulation on a continuous-space Hamiltonian for repulsive Fermi gas in which the system is subjected to a correlated speckle disorder[18]. It was shown that the system strongly favors the paramagnetic-to-ferromagnetic transition when the Fermi surface approaches the vicinity of the mobility edge. Our analysis suggests that the physical mechanism behind this and earlier HF calculations[17] is the enhancement of the interaction matrix elements when quasi-particle states become localized.

We emphasize that this localization-induced ferromagnetic state is very different from Stoner ferromagnets because of the absence of Stoner instability in the weak-interaction limit $UN(0) \ll 1$. The preformed local magnetic moments which exist at temperature $T \leq U/L^d$ interacts with each other with an effective interaction of order $J_{eff} \sim Ue^{-l/L}$, where $l \sim (n^{-1/d})$ is the average distance between preformed local moments and $J_{eff} \sim Ue^{-(\frac{2}{UN(0)})^{1/d}}$. A localization-induced ferromagnetic state is expected to form only at temperature $T \leq J_{eff}$. In the limit $L^{-d} \gg e^{-(\frac{2}{UN(0)})^{1/d}}$, there is a wide temperature range $U/L^d > T > J_{eff}$ where free local magnetic moments exist and will have a strong effect on the magnetic and transport properties of the system. The electronic transport is described by a Unitary ensemble[22] in this regime and weak localization effect will be suppressed[23].

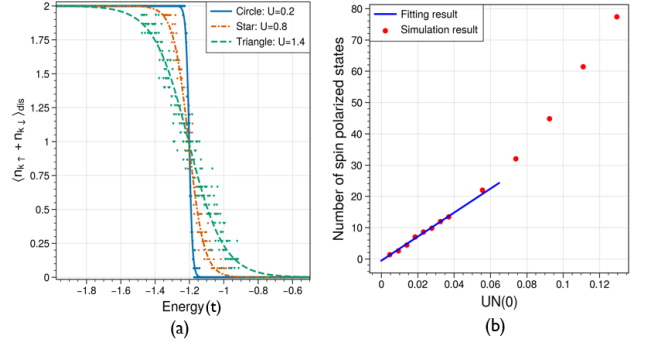


FIG. 3: (a) The average distribution $\langle n_k \rangle_{dis} = \langle n_{k\uparrow} + n_{k\downarrow} \rangle_{dis}$ over 15 disorder configurations for 3 different values of U , (b) The corresponding number of polarized spins as a function of $UN(0)$, where $N(0)$ is the density of states at Fermi surface

For attractive interaction, it is expected that a superconducting ground state will be formed for arbitrary weak interaction in a self-consistent BCS mean-field theory[24] and the effect of preformed local fermion (Cooper) pairs occupying the same state k will be superseded. We show in a separate paper that the superconductor spectral gap is enhanced by these preformed local fermion pairs in a modified BCS theory[25].

E. Beyond perturbation and Hartree-Fock analysis

The localization-induced instability of the Fermi glass state in first-order perturbation and HF theory lead us to consider the general situation of disordered spin-1/2 fermion systems with short-ranged repulsive interaction. Following a Fermi-liquid type analysis, Fleishman and Anderson [26] argued that for weak, short-ranged interaction, the quasi-particle states on the Fermi surface can be described by an effective single-particle Hamiltonian with a generic form $H_{eff} = H_0 + \Sigma(\mu)$, where $\Sigma(\mu)$ is a random Hermitian matrix representing the interaction-induced self-energy on the Fermi surface and $\text{Im}\Sigma(\mu) = 0$ (stability of Fermi surface). The eigenstates of H_{eff} represent the “wave-function” of quasi-particles on the Fermi surface and are localized if H_{eff} is “random” enough in three dimensions (Fermi glass).

Assuming the validity of a Fermi-liquid type phenomenology we can write down a corresponding Landau energy functional describing low-energy excitations in the Fermi glass state, where

$$\delta E_{LD}(\{\delta n_{k\sigma}\}) = \sum_{k\sigma} \xi_k \delta n_{k\sigma} + \frac{1}{2} \sum_{k\sigma; k'\sigma'} f_{k\sigma; k'\sigma'} \delta n_{k\sigma} \delta n_{k'\sigma'} \quad (16)$$

where $\delta n_{k\sigma}$ represents the fluctuation of quasi-particle occupation number $n_{k'\sigma'}$ on the Fermi surface, ξ_k is the single quasi-particle energy and $f_{k\sigma; k'\sigma'}$ is the Landau interaction. Comparing Eq. (16) with Eq. (13) and Eq.

(14), we find that $\xi_k = E_1(k)$ and

$$f_{k\sigma;p\sigma'} = U_{kkpp}^{\sigma\sigma'} - \delta_{\sigma\sigma'} U_{kppk}^{\sigma\sigma}$$

in first-order perturbation or HF theory, with k and p denoting the eigenstates of H_0 or H_{HF} . In Landau Fermi liquid theory language, the instability of the Fermi glass state in first-order perturbation or HF theory is associated with a singular nature of forward scattering amplitude $f_{k\sigma;k-\sigma}$ which is of order $\frac{\bar{U}}{L^d}$. More generally, it is well known that singular forward scattering term $f_{k\sigma;k\sigma'}$ which remains finite when system volume $V \rightarrow \infty$ leads to instability of the Fermi liquid state as long as the density of states on the Fermi surface $N(0)$ is finite [27–31] and our result from first order perturbation and HF theory is just another example demonstrating this general mechanism.

More generally, we expect that the singular forward scattering amplitude we observe in our analysis is a general property of localized quasi-particle states in systems with weak, short-ranged interaction. For localized quasi-particles k and p , $f_{k\sigma;p\sigma'}$ should not depend on the system volume and can only depend on the distance d between the two localized states when $d \gg L$ if interaction does not induce long-ranged effective interactions between quasi-particles. In particular, for $k = p$, the only length scale which is available is the localization length and we anticipate that $f_{k\sigma;k-\sigma}$ can only scale with the size of the quasi-particle wavefunction $\sim 1/L^d$, and instability always occur if both L and the density states $N(0)$ are finite, as demonstrated in our analysis.

Summarizing, based on analysis of interaction matrix elements in first-order perturbation and Hartree-Fock theory, we show the existence of instability of Fermi surface towards formation of local magnetic moments for short-ranged repulsive interaction (and formation of local Fermion pair for attractive interaction) if the eigenstates on the Fermi surface are localized. We further

propose within a Landau Fermi-liquid type framework that our result is applicable for general electronic systems with weak, short-ranged interaction as long as the quasi-particle states exist and are localized, suggesting that Fermi glass states are intrinsically unstable.

We caution that in general whether quasi-particles are localized is not determined by H_0 or H_{HF} , but by the more general $H_{\text{eff}} = H_0 + \Sigma(\mu)$. In particular, our perturbation or HF approach cannot determine with certainty whether the quasi-particles are localized for a given Hamiltonian. There is also the possibility that only part of the quasi-particle states are localized around the Fermi surface. Our present analysis cannot resolve these possible scenario.

We also note that we have assumed implicitly that $U_{kkkk}^{\sigma(-\sigma)}$'s are of the same sign as k changes when we refer to “attractive” or “repulsive” interaction. This is the case for Hubbard interaction we discuss in this paper. For general form of interaction $U_{ij}^{\sigma\sigma'}$ this assumption may not be correct and both polarized spins and local fermion pairs may exist together in the ground state (see also [32]). In this case, the system becomes frustrated and spin glass or superconducting glass states may occur. We shall address these more exotic possibilities in future papers.

Finally we note that our analysis can be generalized to systems with spin-orbit coupling and the similar conclusion will be reached as long as inversion symmetry is not destroyed as the Kramer’s degeneracy will be preserved (see for example [33]).

ACKNOWLEDGMENTS

The authors acknowledge helpful comment from Zvi Ovadyahu on our manuscript. The project is supported by a special funding support from the School of Science, the Hong Kong University of Science and Technology.

-
- [1] PW Anderson. Comments solid state phys., 2. 1970.
 - [2] Robert Freedman and JA Hertz. *Physical Review B*, 15(4):2384, 1977.
 - [3] Fahad Mahmood, Dipanjan Chaudhuri, Sarang Gopalakrishnan, Rahul Nandkishore, and NP Armitage. *Nature Physics*, 17(5):627–631, 2021.
 - [4] RN Bhatt and TV Ramakrishnan. *Journal of Physics C: Solid State Physics*, 17(24):L639, 1984.
 - [5] Al L Efros and Boris I Shklovskii. *Journal of Physics C: Solid State Physics*, 8(4):L49, 1975.
 - [6] AL Efros. *Journal of Physics C: Solid State Physics*, 9(11):2021, 1976.
 - [7] Zvi Ovadyahu. *Comptes Rendus Physique*, 14(8):700–711, 2013.
 - [8] Falson Joseph et al. *Nature Materials*, 21(3):311–316, 2022.
 - [9] Hossain Md S. et al. *Proceedings of the National Academy of Sciences*, 117(51):32244–32250, 2020.
 - [10] Rahul Nandkishore and David A Huse. *Annu. Rev. Condens. Matter Phys.*, 6(1):15–38, 2015.
 - [11] Dmitry A Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn. *Reviews of Modern Physics*, 91(2):021001, 2019.
 - [12] Dmitry A Abanin and Zlatko Papić. *Annalen der Physik*, 529(7):1700169, 2017.
 - [13] B Spivak, P Oreto, and SA Kivelson. *Physical Review B*, 77(21):214523, 2008.
 - [14] Aharon Kapitulnik, Steven A Kivelson, and Boris Spivak. *Reviews of Modern Physics*, 91(1):011002, 2019.
 - [15] Ziqiao Wang, Yi Liu, Chengcheng Ji, and Jian Wang. *Reports on Progress in Physics*, 2023.
 - [16] Denis M Basko, Igor L Aleiner, and Boris L Altshuler. *Annals of physics*, 321(5):1126–1205, 2006.
 - [17] Milica Milovanović, Subir Sachdev, and RN Bhatt. *Physical review letters*, 63(1):82, 1989.
 - [18] S Pilati and E Fratini. *Physical Review A*, 93(5):051604,

- 2016.
- [19] Rigorously speaking $P_{klpq} = (\frac{L^d}{V})^3 \times C_{klpq}$ where C_{klpq} is an order $O(1)$ function that depends on the energy of the four states. (See Patrick A Lee and TV Ramakrishnan. *Reviews of modern physics*, 57(2):287, 1985).
 - [20] For localized states we estimated the localization length from the average inverse participation ratio (IPR) over all states k for 15 disorder configurations: $L \sim \langle (\sum_i |\phi_k(\vec{r}_i)|^4)^{-1/2} \rangle_{k, \text{dis}}$ (See Franz Wegner. *Zeitschrift für Physik B Condensed Matter*, 36(3):209–214, 1980).
 - [21] Tai-Kai Ng. *Introduction to classical and quantum field theory*. Wiley-VCH, 2009.
 - [22] YY Atas, Eugene Bogomolny, O Giraud, and G Roux. *Physical review letters*, 110(8):084101, 2013.
 - [23] Shinobu Hikami, Anatoly I Larkin, and Yosuke Nagaoka. *Progress of Theoretical Physics*, 63(2):707–710, 1980.
 - [24] Michael Ma and Patrick A Lee. *Physical Review B*, 32(9):5658, 1985.
 - [25] Yat Fan Lau and Tai Kai Ng. *arXiv preprint arXiv:2311.14914*, 2023.
 - [26] L Fleishman and PW Anderson. *Physical Review B*, 21(6):2366, 1980.
 - [27] Yasuhiro Hatsugai and Mahito Kohmoto. *Journal of the Physical Society of Japan*, 61(6):2056–2069, 1992.
 - [28] Ganapathy Baskaran. *Modern Physics Letters B*, 5(09):643–649, 1991.
 - [29] Takahiro Morimoto and Naoto Nagaosa. *Scientific reports*, 6(1):19853, 2016.
 - [30] Tai-Kai Ng. *arXiv preprint arXiv:1910.06602*, 2019.
 - [31] Philip W Phillips, Luke Yeo, and Edwin W Huang. *Nature Physics*, 16(12):1175–1180, 2020.
 - [32] Subir Sachdev. *Philosophical Transactions of the Royal Society of London. Series A: Mathematical, Physical and Engineering Sciences*, 356(1735):173–195, 1998.
 - [33] Steven M Girvin and Kun Yang. *Modern condensed matter physics*. Cambridge University Press, 2019.

Appendix A: Some details of our numerical calculations

In our simulation, we adopted a periodic boundary condition for the (60×60) square lattice. We set the disorder strength $W/t = 9$ which is comparable to the bandwidth of the energy spectrum($\sim 8.8t$) at $W = 0$.

The density of states(DOS) per unit volume(area in 2D) with different disorder strengths for $t' = 0.6t$ is shown in Fig. 1. The large peak in the DOS reflects the existence of Van Hove's singularity in the $W = 0$ limit.

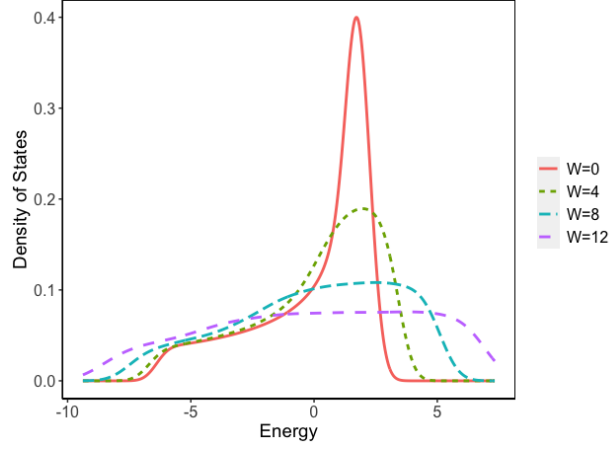


FIG. 1: DOS per unit volume for different disorder strengths. We have set $t' = 0.6t$ and taken average over 15 disorder configurations