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Q1 # Sample $(x_1, x_2, x_3 \ldots x_n)$ mean $= \theta_1$ & var $= \theta_2$
(Normal distribution)

$L = \prod f(x_i | \theta)$

$$L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{-\left(\frac{(x-\theta_1)^2}{2\theta_2}\right)}$$

take log both sides

$$\ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = -\frac{n}{2} \ln(2\pi\theta_2) - \frac{1}{2\theta_2} \sum_{i=1}^{n} (x_i - \theta_1)^2$$

differentiate wrt θ_1 & θ_2 & then equate to 0

for θ_1:

$$\frac{\partial}{\partial\theta_1} \ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = \frac{1}{\theta_2} \sum_{i=1}^{n} (x_i - \theta_1) = 0$$

$$\sum_{i=1}^{n} (x_i - \hat{\theta}_1) = 0$$

$$\hat{\theta}_1 = \frac{1}{n} \sum_{i=1}^{n} x_i$$

MLE for θ_1 is sample mean

for θ_2:

$$\frac{\partial}{\partial\theta_2} \ln L(\theta_1, \theta_2 | x_1, x_2 \ldots x_n) = -\frac{n}{2\theta_2} + \frac{1}{2\theta_2^2} \sum (x_i - \hat{\theta}_1)^2 = 0$$

$$-\frac{n}{2\hat{\theta}_2} + \frac{1}{2\hat{\theta}_2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2 = 0$$

$$\frac{n}{2\hat{\theta}_2} = \frac{1}{2\hat{\theta}_2^2} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

$$\hat{\theta}_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{\theta}_1)^2$$

MLE for θ_2 is sample variance.

Q2 Bernoulli distribution

parameter $\rightarrow \theta \in \Theta = (0,1)$ unknown

$\rightarrow m$ (known +ve $\mathbb{Z}$)

likelihood of function is $\rightarrow$

$$L(\theta | x_1, x_2 \ldots x_n) = \prod_{i=1}^{n} P(X_i = x_i | \theta)$$

since X_i follows ~~benno~~ bernoulli distribution,

$$P(X_i = x_i | \theta) = \theta^{x_i} (1-\theta)^{m-n_i} \quad \text{for each } i$$

Taking log on both sides

$$\ln L(\theta | x_1, x_2 \ldots x_n) = \sum_{x_i=1}^{n} \ln\left(\theta^{x_i} (1-\theta)^{m-n_i}\right)$$

$$= \sum_{i=1}^{m} \left(x_i \ln\theta \; (m-x_i) \ln(1-\theta)\right)$$

differentiate wrt θ

$$\frac{\partial}{\partial\theta}\left(\ln L(\theta | x_1, x_2 \ldots x_n)\right)\Big] = 0$$

$$\sum_{i=1}^{n} \left(\frac{x_i}{\theta} - \frac{m - x_i}{1-\theta}\right) = 0$$

$$\sum_{i=1}^{n} \frac{x_i}{\theta} = mn - \sum_{i=1}^{n} \frac{x_i}{1-\theta}$$

$$\theta = \frac{\sum x_i}{nm}$$

$$\boxed{\hat{\theta}_{MLE} = \sum_{i=1}^{n} \frac{x_i}{n.m}}$$