

Course/Branch: B Tech -I Years(OP1-OP14)

Subject Name : Engg. Mathematics-I

Subject Code : BAS-103

Q-1 : Apply the concept of matrices for solving linear simultaneous equations

Q-2 : Apply the concept of differentiation in successive derivatives, Leibnitz theorem, partial and total differentiation

Section - A (CO - I) # Attempt both the questions. Each question is of 10 marks

Q.1: Attempt any SIX questions (Short Answer Type). Each question is of 10 marks

a)	Show that the matrix $\begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ is unitary if $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$	(BKL 2 Level)
b)	The Eigen values of a matrix A are 2, 3, 1 then find the Eigen values of $A^{-1} + A^2$ .	(BKL 2 Level)
c)	Examine whether the vectors $X_1 = [3, 1, 1], X_2 = [2, 0, 1], X_3 = [4, 2, 1]$ are linearly independent.	(BKL 2 Level)
d)	Test for consistency of the following system of equations: $\begin{aligned} 2x - y + 3z &= 0 \\ -x + 2y + z &= 0 \\ 3x + y - 4z &= 0 \end{aligned}$	(BKL 2 Level)
e)	Two eigen values of the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ are equal to 1 each. Find the eigen values of $A^{-1}$ .	(BKL 2 Level)
f)	Find the rank of the following matrix using Echelon Form Method $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$	(BKL 2 Level)
g)	Find the Eigen value of the matrix $A = \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ corresponding to the eigen vector $\begin{bmatrix} 102 \\ 102 \end{bmatrix}$ .	(BKL 2 Level)

Q.2: Attempt any THREE questions (Medium Answer Type). Each question is of 6 marks

a)	For what values of a and b, the equations $x + 2y + 3z = 6$ , $x + 3y + 5z = 9$ , $2x + 5y + az = b$ have (i) no solution (ii) a unique solution (iii) more than one solution?	(BKL 2 Level)
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b)	Reduce the matrix $A$ to its normal form when $A = \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ Hence find the rank of $A$ .	(BKL:K3 Level)
c)	Find the eigen values and eigen vectors of matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$	(BKL:K3 Level)
d)	Using elementary row transformation, find the inverse of the following matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$	(BKL:K3 Level)
e)	If $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ (a) State Cayley-Hamilton theorem (b) Compute $A^{-1}$ (c) Evaluate $A^6 - 6A^5 + 9A^4 - 2A^3 - 12A^2 + 23A - 9I$ .	(BKL:K3 Level)

**Section - B (CO - 2) # Attempt both the questions # 30 Marks**

Q.3: Attempt any **SIX** questions (Short Answer Type). Each question is of two marks. (2 x 6 = 12 Marks)

a)	State Leibnitz Theorem for $n^{th}$ differential coefficient of the product of two functions. If $y^{\frac{1}{m}} + y^{\frac{1}{m}} = 2x$ , then find the relation in $y_2, y_1, y$ .	(BKL:K3 Level)
b)	If $u = x \sin^{-1}\left(\frac{y}{x}\right) + y \sin^{-1}\left(\frac{x}{y}\right)$ evaluate $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$	(BKL:K3 Lev)
c)	Find the first order partial derivatives of the function $z = y^x$ .	(BKL:K3 Level)
d)	Find the $n^{th}$ derivative of $y = x^2 \sin x$ .	(BKL:K3 Level)
e)	Find the $n^{th}$ derivative of $\frac{(5x+1)}{(3x-1)(2x+7)}$ .	(BKL:K3 Level)
f)	If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ , find the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y}$	(BKL:K3 Level)
g)	Find $n^{th}$ derivative of $y = \sin 2x \sin 3x$ .	(BKL:K3 Level)

Q.4: Attempt any **THREE** questions (Medium Answer Type). Each question is of 6 marks. (3 x 6 = 18 Marks)

a)	If $y = (\sin^{-1} x)^2$ , then find the relation in $y_{n+2}, y_{n+1}, y_n$ .	(BKL:K3 Level)
b)	Verify Euler's theorem for the function $z = \frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}$ .	(BKL:K3 Level)
c)	If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$ , then show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ .	(BKL:K3 Level)
d)	If $u = \operatorname{cosec}^{-1}\left(\frac{\frac{1}{x^2+y^2}}{\frac{1}{x^3+y^3}}\right)^{1/2}$ , Find $\left(x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}\right)$ .	(BKL:K3 Level)
e)	If $y = \sin(m \sin^{-1} x)$ , then find the relation in $y_{n+2}, y_{n+1}, y_n$ . and hence find $y_n$ at $x = 0$ .	(BKL:K3 Level)