Assignment-8

Course: SC-374

Computational and Numerical Methods

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Problem: 1

♦ **Statement:**

Carry out a cubic spline interpolation of the data provided in Question 3 on Polynomial Interpolation. Present your result by plotting the spline functions.

Points:

(1) (0,2) ,

(2) (1,2.1592) ,

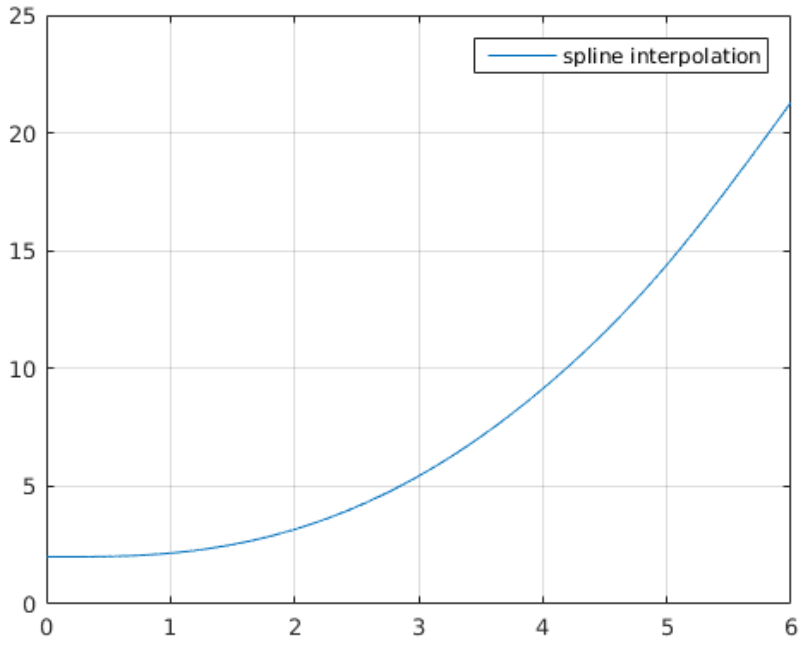
(3) (2,3.1697),

(4) (3,5.4332),

(5) (4,9.1441),

(6) (5,14.406) ,

(7) (6,21.303)



Problem: 2

♦ **Statement:**

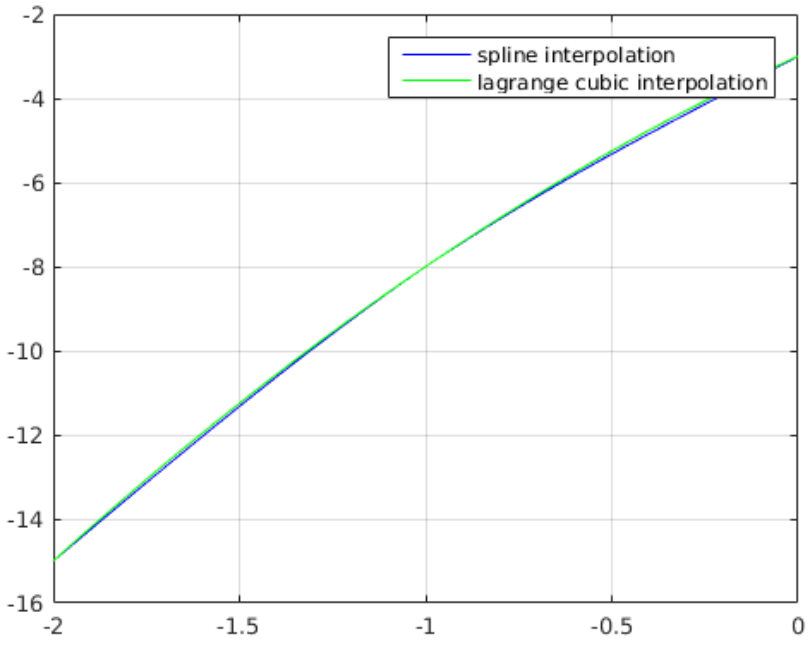
Carry out quadratic Lagrange and cubic spline interpolations of the data provided in Question 6 on Polynomial Interpolation. Present both results in a single plot.

Points:

(1) (-2,-15),

(2) (-1,-8) ,

(3) (0,-3)



Problem: 3

♦ **Statement:**

Tabulate the results of Questions 1 & 2 on Numerical Integration and Differentiation. For each function, tabulate n , $T_n(f)$ and $S_n(f)$in three columns.

(1A)

no	T_n	S_n	err_T_n	err_S_n
2	26.516	22.715	25.214	21.413
4	3.2491	-4.5067	1.9467	5.8091
8	1.6245	1.083	0.32213	0.21938
16	1.3757	1.2928	0.07333	0.0096047
32	1.3203	1.3018	0.017919	0.00055133
64	1.3068	1.3024	0.0044549	3.3112e-05
128	1.3035	1.3024	0.0011127	1.4172e-06
256	1.3027	1.3024	0.00027858	5.5311e-07
512	1.3025	1.3024	7.0152e-05	6.7609e-07
1024	1.3024	1.3024	1.8051e-05	6.8377e-07

(1B)

no	T_n	S_n	err_T_n	err_S_n
2	0.33839	0.28452	0.052674	0.0011962
4	0.29879	0.28559	0.013077	0.00012145
8	0.28897	0.2857	0.0032607	1.1513e-05
16	0.28653	0.28571	0.00081457	8.227e-07
32	0.28592	0.28571	0.00020378	1.8363e-07
64	0.28577	0.28571	5.1152e-05	2.7643e-07
128	0.28573	0.28571	1.3002e-05	2.8488e-07
256	0.28572	0.28571	3.4647e-06	2.8564e-07
512	0.28572	0.28571	1.0804e-06	2.8571e-07
1024	0.28571	0.28571	4.844e-07	2.8571e-07

(1C)

no	T_n	S_n	err_T_n	err_S_n
2	2.1667	2.6251	0.17311	0.28533
4	2.2687	2.3027	0.071099	0.037094
8	2.3323	2.3535	0.0074958	0.013705
16	2.3378	2.3397	0.0019534	0.00010593
32	2.3393	2.3398	0.00048916	1.08e-06
64	2.3396	2.3398	0.00012234	6.7432e-08
128	2.3397	2.3398	3.0588e-05	4.2169e-09
256	2.3398	2.3398	7.6473e-06	2.6359e-10
512	2.3398	2.3398	1.9118e-06	1.6475e-11
1024	2.3398	2.3398	4.7796e-07	1.029e-12

(2A)

no	T_n	S_n
2	2.5	1.6667
4	1.2548	0.83977
8	0.88943	0.76763
16	0.88623	0.88516
32	0.88623	0.88623
64	0.88623	0.88623
128	0.88623	0.88623
256	0.88623	0.88623
512	0.88623	0.88623
1024	0.88623	0.88623

(2B)

no	T_n	S_n
2	2.1865	2.1958
4	2.1775	2.1744
8	2.1751	2.1743
16	2.1745	2.1743
32	2.1743	2.1743
64	2.1743	2.1743
128	2.1743	2.1743
256	2.1743	2.1743
512	2.1743	2.1743
1024	2.1743	2.1743

Problem: 4

♦ **Statement:**

Tabulate the results of Questions 6 on Numerical Integration and Differentiation. For each function, present all your results in a table with increasing h. Compare the accuracy of the numerical derivative with the analytical derivative.

(A) Analytical value of derivative = 0.5

$$f(x) = \arctan(\mathrm{x}^2 - \mathrm{x} + 1)$$

$$f'(x) = \frac{2x - 1}{1 + (x^2 - x + 1)}$$

$$x = 1, f'(x) = \frac{1}{2}$$

no	forward	central	err_forward	err_central
0.1	0.52086	0.49586	0.020855	0.0041443
0.05	0.51146	0.49896	0.01146	0.0010403
0.025	0.50599	0.49974	0.0059897	0.00026033
0.0125	0.50306	0.49993	0.0030599	6.5099e-05
0.00625	0.50155	0.49998	0.0015462	1.6276e-05

(B) Analytical value of derivative = 0.5

$$f(x) = \arctan(100\mathrm{x}^2 - 199\mathrm{x} + 100)$$

$$f'(x) = \frac{200x - 199}{1 + (100x^2 - 199x + 100)}$$

$$x = 1, f'(x) = \frac{1}{2}$$

no	forward	central	err_forward	err_central
0.1	3.4098	0.20029	2.9098	0.29971
0.05	2.5941	0.39043	2.0941	0.10957
0.025	1.6757	0.46978	1.1757	0.030224
0.0125	1.1093	0.49226	0.60933	0.0077385
0.00625	0.80839	0.49805	0.30839	0.0019461