Assignment-7

Course: SC-374

Computational and Numerical Methods

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Made by:

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Problem: 1

♦ **Statement:**

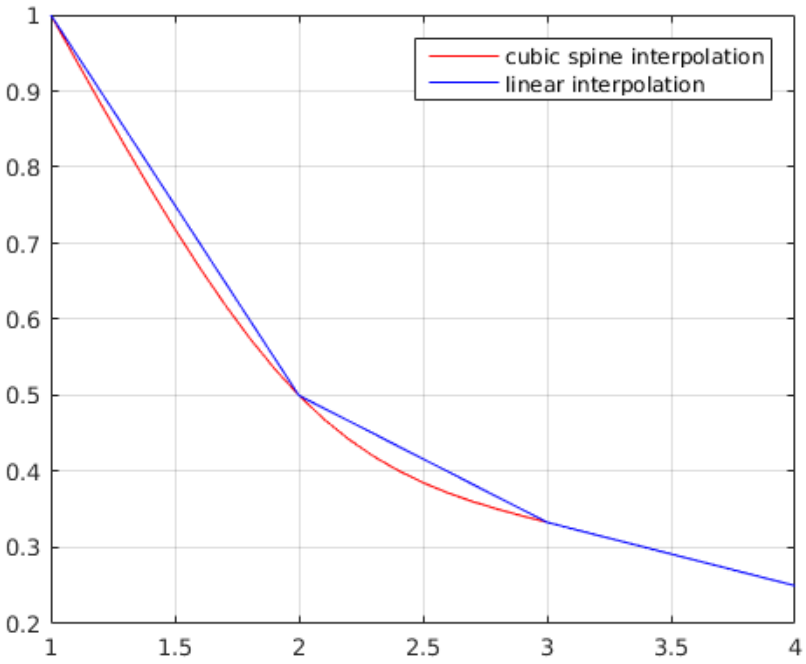
Carry out the cubic spline interpolation of the following data:

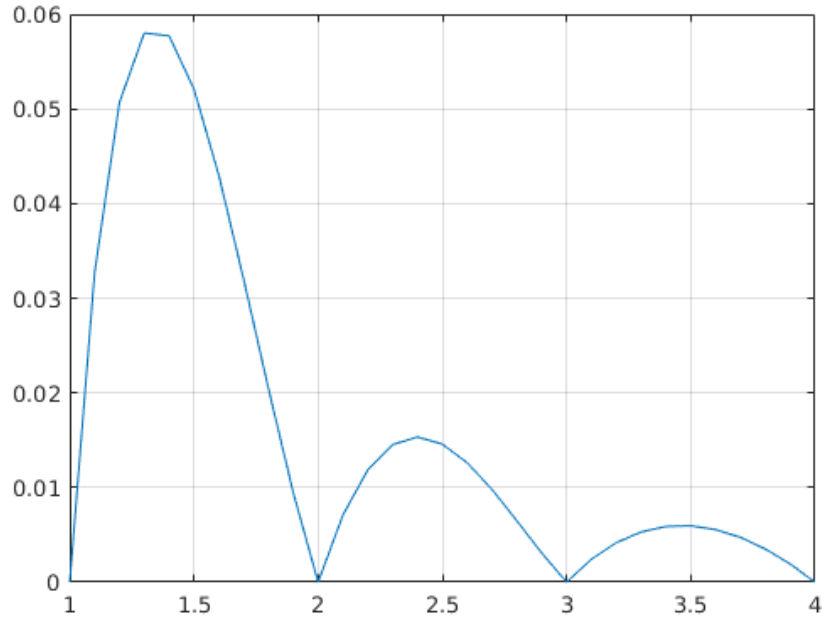
Points:

(1) (1,1), (2) (2,1/2) , (3) (3,1/3) , (4) (4,1/4).

Plot both the cubic and the piecewise linear interpolation together.

Estimate the error between the cubic interpolation and y=1/x.





Problem: 2

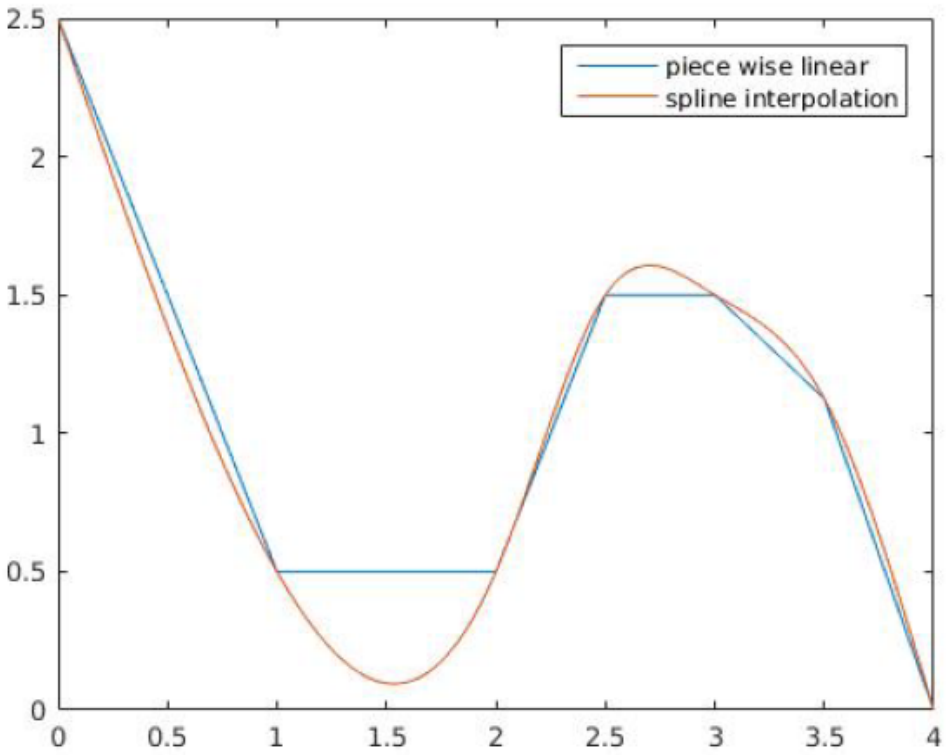
♦ **Statement:**

Carry out a cubic spline interpolation with all data points in the table provided in question 5 of set 6 above.

Points:

(1) (0,2.5), (2) (1,0.5) , (3) (2,0.5) , (4) (2.5,1.5) , (5) (3,1.5) , (6) (3.5,1.125) , (7) (4,0).

Plot the spline interpolation for all data points alongside the piecewise linear interpolation for comparison.



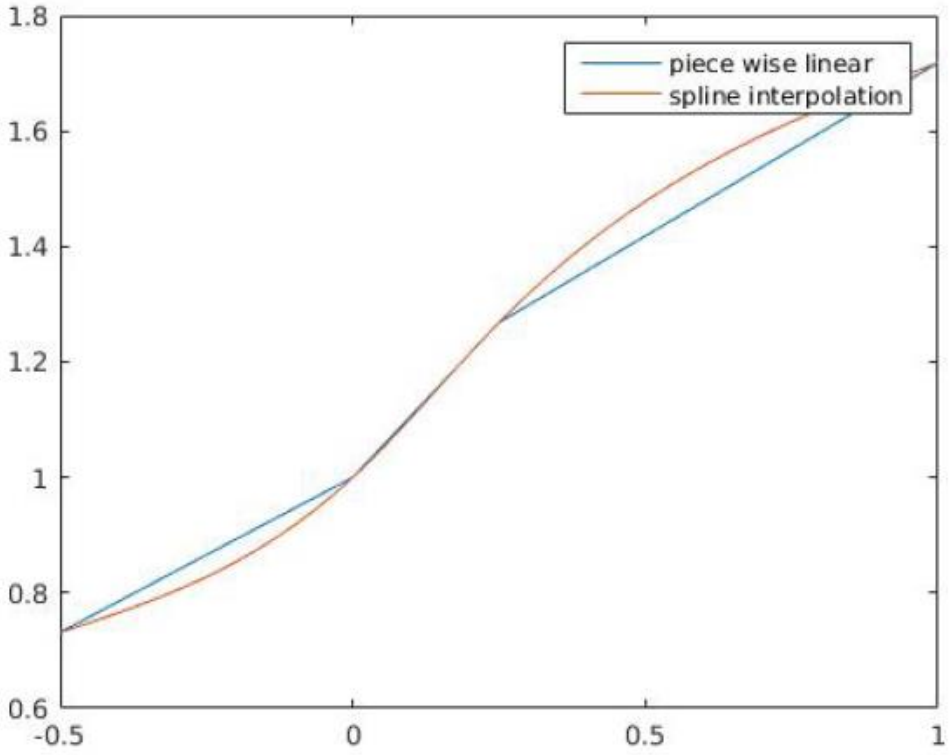
Problem: 3

♦ **Statement:**

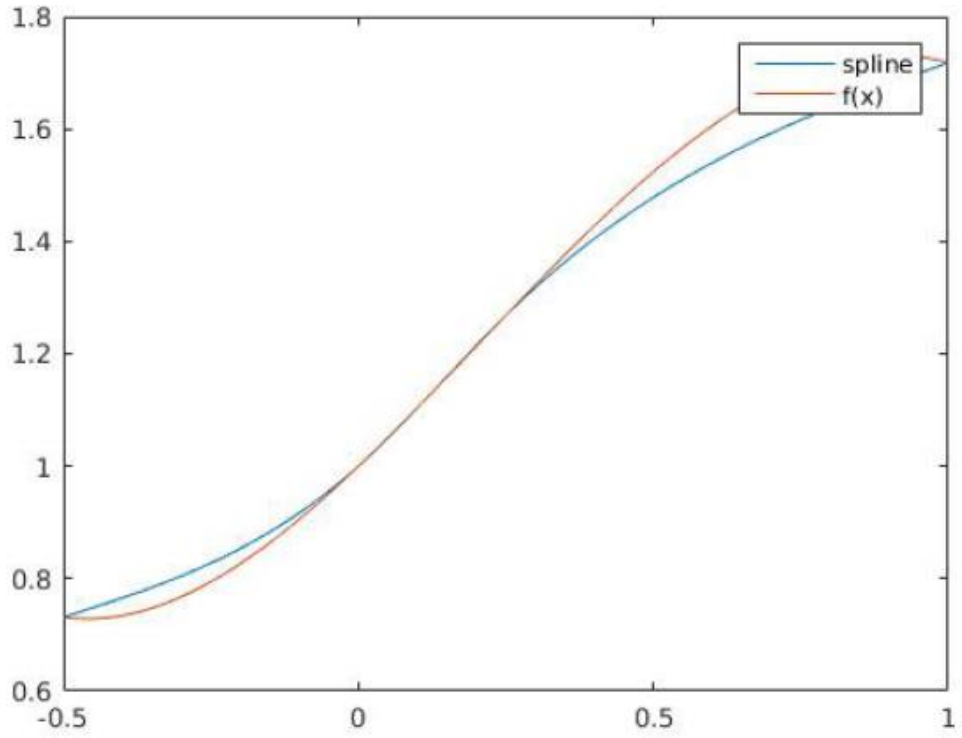
Given data points:

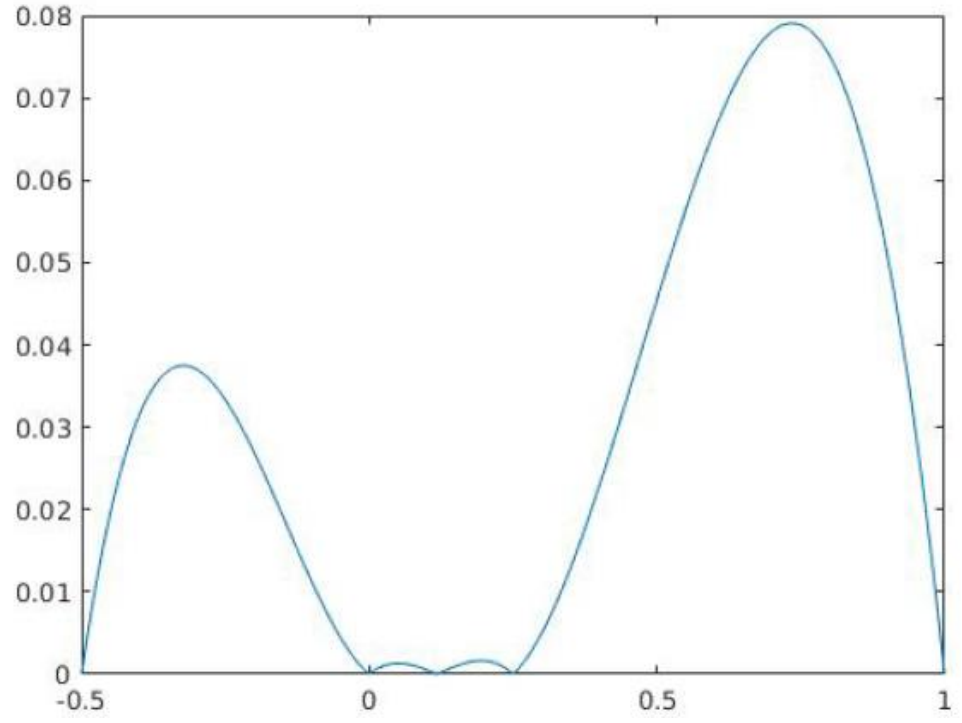
(1) (-0.5,0.731531), (2) (0.0,1.0) , (3) (0.25,1.2684) , (4) (1.00,1.718282)

(A) Perform both a piecewise linear interpolation and a cubic spline interpolation. plot both together in the same graph.



(B) Compare both the results of your cubic interpolation with the function $f(x) = e^x - x^3$. Estimate the error between the cubic interpolation function and f(x).


spline
f(x)
1.8
1.6
1.4
1.2
1
0.8
0.6
-0.5
0
0.5
1


0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.5
0
0.5
1

Problem: 4

♦ **Statement:**

Numerically verify Questions 1,2,3 & 4 in the theory exercises on spline interpolation. Graphically show the spline functions in every case.

(1) For three data points (0,1) , (1,1) and (2,5),

ANALYTICALLY :

M1=0

M2=6

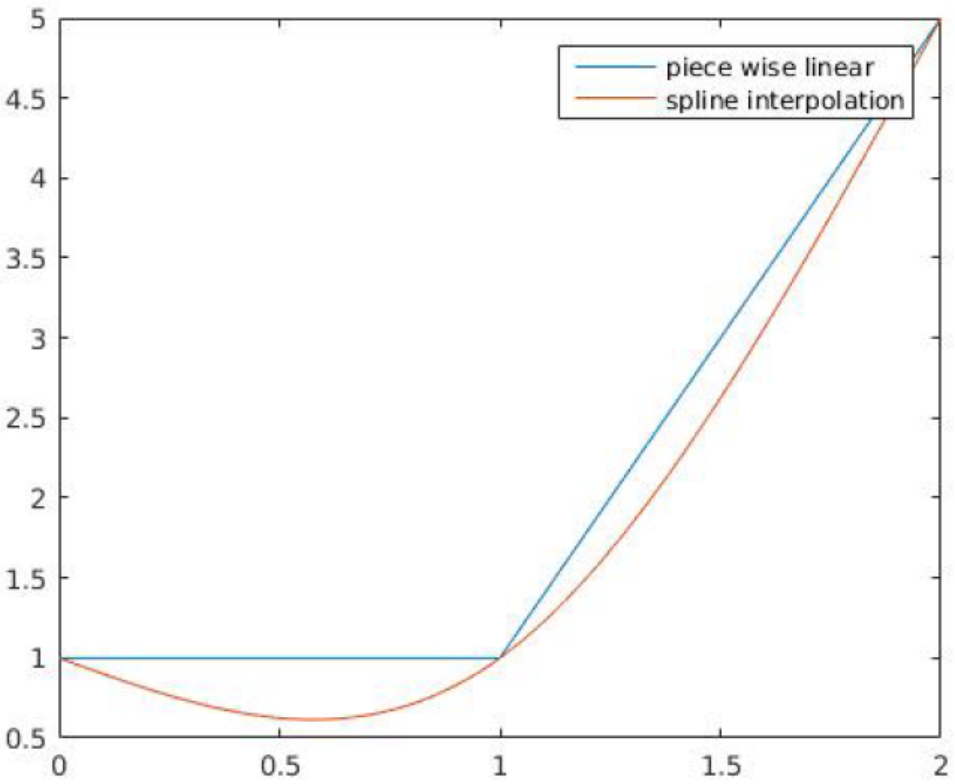
M3=0

FOR J=2, WE GOT C=1 AND D=0, EQUATION THAT WE HAVE GOT IS:

$S(x) = x^3-x+1$

FOR J=3, WE GOT C=0 AND D=5, EQUATION THAT WE HAVE GOT IS:

$S(x) = (2-x)^3 + 5x - 5$



(2) For the following data:

Points:

(1) (1,3) , (2) (2,1) , (3) (3,2) , (4) (4,3) , (5) (5,2).

Find both the piecewise linear interpolation function and natural cubic spline.

ANALYTICALLY :

M1=0

M2=4.6071

M3=-0.4286

M4=-2.8929

M5=0

FOR J=2, WE GOT C=3 AND D=0.2321, EQUATION THAT WE HAVE GOT IS:

$S(x) = 0.7679(x-1)^3 + 3(2-x) + 0.2321(x-1)$

FOR J=3,WE GOT C=0.2321 AND D=2.0714,EQUATION THAT WE HAVE GOT IS:

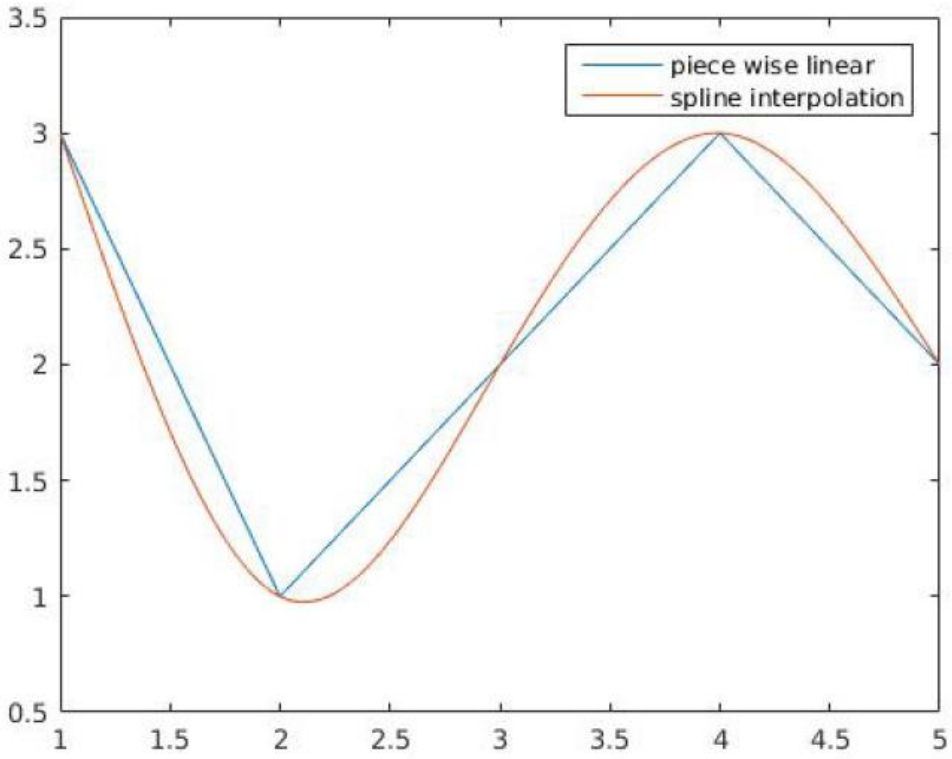
$S(x) = 0.7679(3-x)^3 - 0.0714(x-2)^3 + 0.2321(3-x) + 2.0714(x-2)$

FOR J=4, WE GOT C=2.0714 AND D=3.4821, EQUATION THAT WE HAVE GOT IS:

$S(x) = -0.0714(4-x)^3 - 0.4822(x-3)^3 + 2.0714(4-x) + 3.4821(x-3)$

FOR J=5, WE GOT C=3.4821 AND D=2, EQUATION THAT WE HAVE GOT IS:

$S(x) = -0.4822(5-x)^3 + 3.8421(5-x) + 2(x-4)$



(3) For the following data:

Points:

(1) (0,0) , (2) (1/2,1/4) , (3) (1,1) , (4) (2,-1) , (5) (3,-1).

Find both the piecewise linear interpolation function and natural cubic spline.

ANALYTICALLY :

M1=0

M2=5.4286

M3=-9.7143

M4=5.4286

M5=0

FOR J=2, WE GOT C=0 AND D=0.0476, EQUATION THAT WE HAVE GOT IS:

$S(x) = 1.8095(x)^3 + 0.0476(x)$

FOR J=3,WE GOT C=0.0476 AND D=2.8095,EQUATION THAT WE HAVE GOT IS:

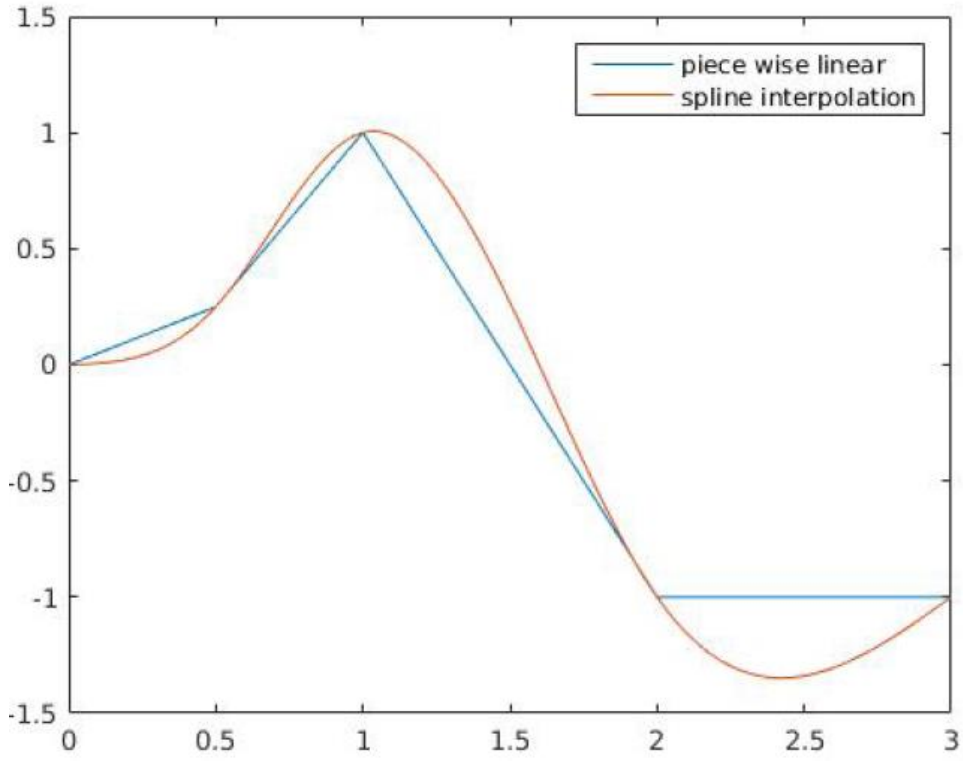
$S(x) = 1.8095(1-x)^3 - 3.2381(x-1/2)^3 + 0.0476(1-x) + 2.8095(x-1/2)$

FOR J=4, WE GOT C=2.6190 AND D=-1.9048, EQUATION THAT WE HAVE GOT IS:

$S(x) = -1.619(2-x)^3 + 0.9048(x-1)^3 + 2.6190(2-x) - 1.9048(x-1)$

FOR J=5, WE GOT C=-1.9048 AND D=-1, EQUATION THAT WE HAVE GOT IS:

$S(x) = 0.9048(3-x)^3 - 1.9048(3-x) - (x-2)$



(4) For the following data:

Points:

(1) (0,1.4) , (2) (1,0.6) , (3) (2,1) , (4) (2.5,0.65) , (5) (3,0.6) , (6) (4,1).

Find both the piecewise linear interpolation function and natural cubic spline.

ANALYTICALLY :

M1=0

M2=2.6788

M3=-3.5154

M4=2.5344

M5=0.5766

M6=0

FOR J=2, WE GOT C=1.4 AND D=0.1535, EQUATION THAT WE HAVE GOT IS:

$S(x) = 0.4465(x)^3 + 1.4(1-x) + 0.1535x$

FOR J=3,WE GOT C=0.1535 AND D=1.5859,EQUATION THAT WE HAVE GOT IS:

$S(x) = 0.4465(2-x)^3 - 0.5859(x-1)^3 + 0.1535(2-x) + 1.5859(x-1)$

FOR J=4, WE GOT C=2.2929 AND D=1.0888, EQUATION THAT WE HAVE GOT IS:

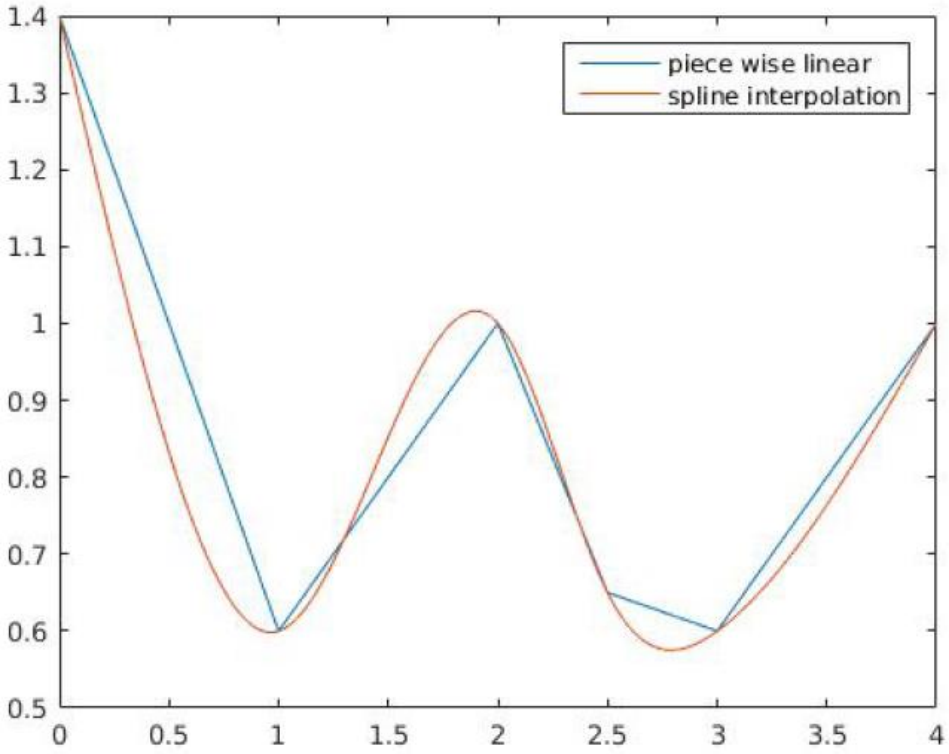
$S(x) = -0.5859((5/2)-x)^3 + 0.8448(x-2)^3 + 2.2929((5/2)-x) - 1.0888(x-2)$

FOR J=5, WE GOT C=1.0888 AND D=1.1519, EQUATION THAT WE HAVE GOT IS:

$S(x) = 0.8448(3-x)^3 - 0.1925(x-(5/2))^3 + 1.0888(3-x) + 1.1519(x-(5/2))$

FOR J=6, WE GOT C=0.5037 AND D=1, EQUATION THAT WE HAVE GOT IS:

$S(x) = 0.0963(4-x)^3 + 0.5037(4-x) + (x-3)$


piece wise linear
spline interpolation
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4