Assignment-4

Course: SC-374

Computational and Numerical Methods

Instructor: Prof. Arnab Kumar

Made by:

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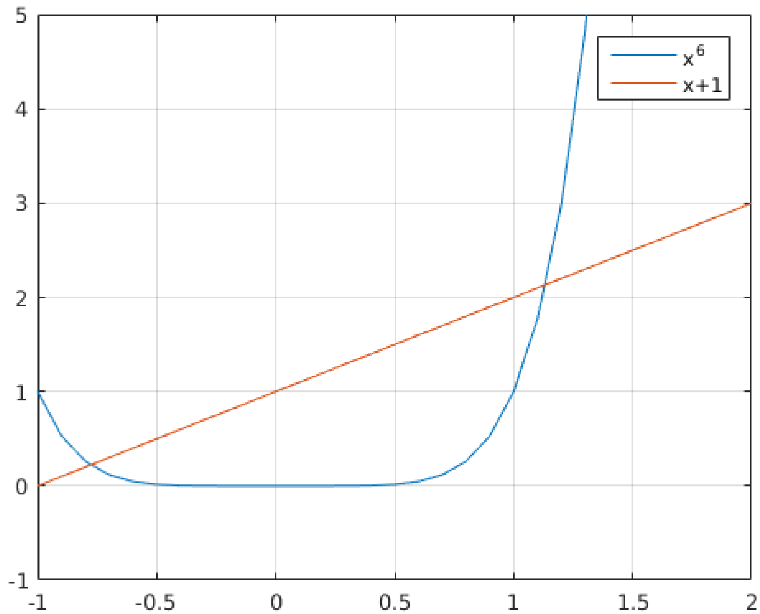
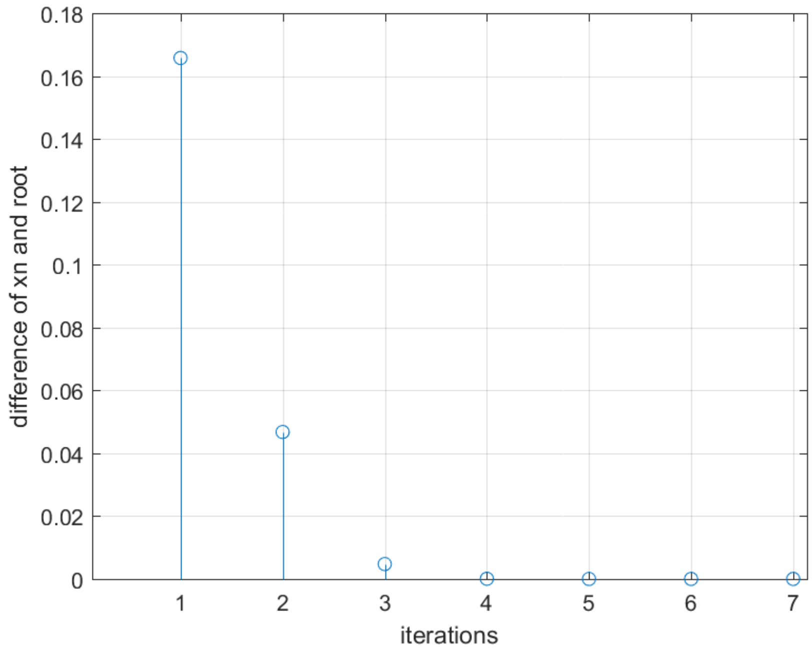
Problem: 1

♦ **Statement:**

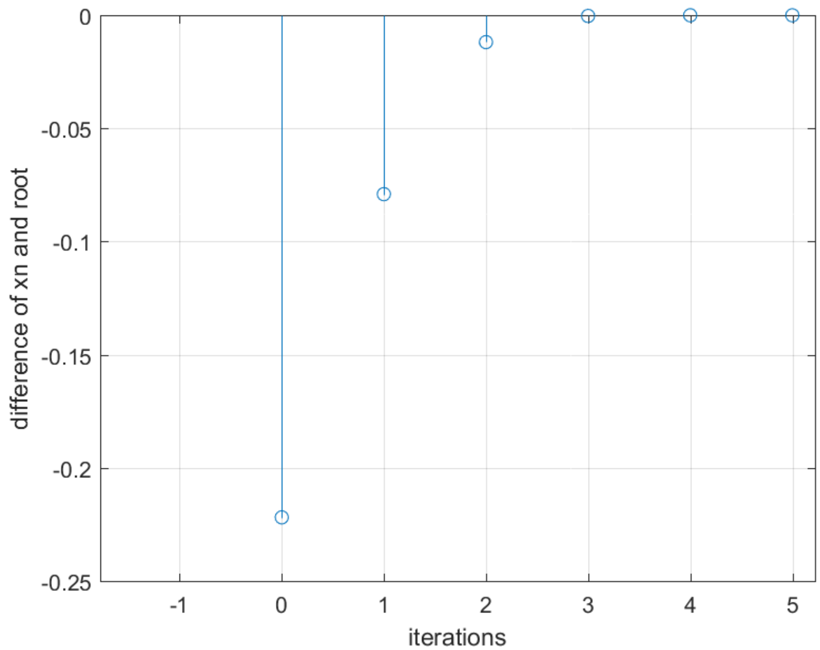
Write a code, applying the algorithm of the Newton-Raphson method to determine both the real roots of $\boldsymbol{f(x) = x^6 - x - 1 = 0}$.

♦ **Graphs:**





no	x_n	f_x_n	f_x_d	x_n_1
1	1.5	8.891	44.563	1.3
2	1.3	2.537	21.32	1.181
3	1.181	0.538	12.813	1.139
4	1.139	0.049	10.525	1.135
5	1.135	0.001	10.29	1.135



no	x_n	f_x_n	f_x_d	x_n_1
1	-1	1	-7	-0.857
2	-0.857	0.254	-3.776	-0.79
3	-0.79	0.033	-2.846	-0.778
4	-0.778	0.001	-2.714	-0.778
5	-0.778	0	-2.711	-0.778

♦ Observations:

Smallest Root which we are getting is at x = -0.7781 .

Largest Root which we are getting is at x = 1.1347 .

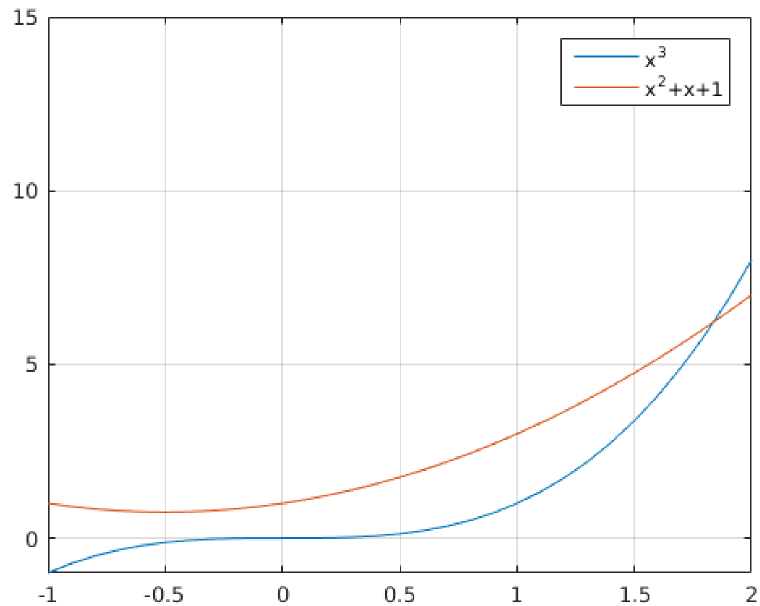
Problem: 2 & 3

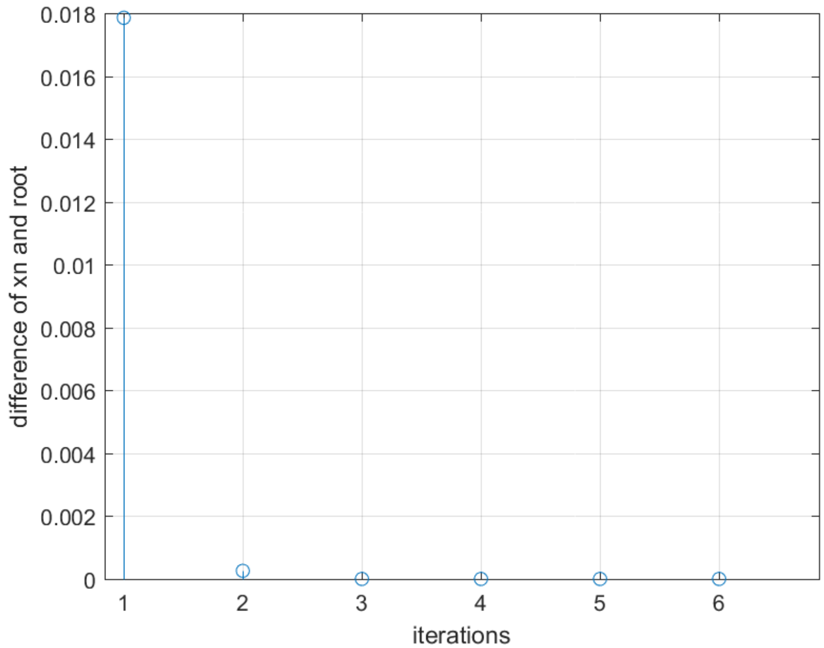
♦ **Statement:**

Use the bisection method to find the real roots of the following functions, using an error tolerance of € = 0.0001.

(A) $f(x) = x^3 - x^2 - x - 1 = 0$

♦ **Graphs:**





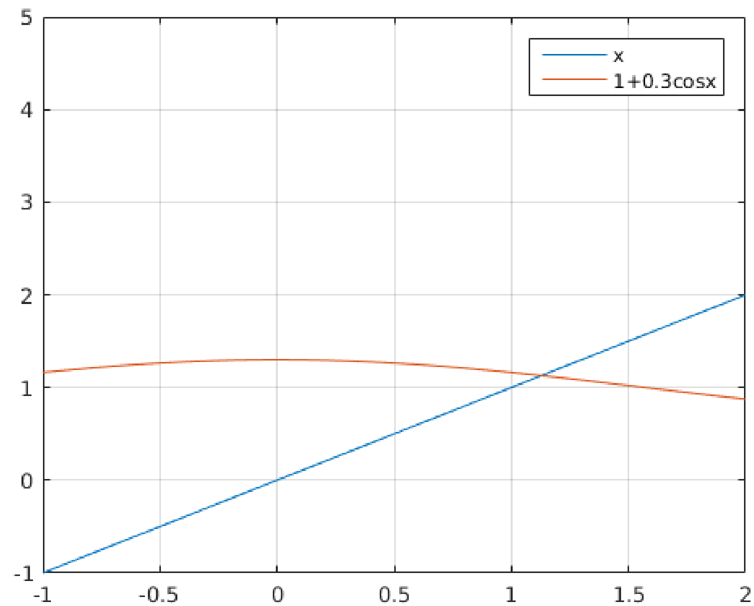
no	x_n	f_x_n	f_x_d	x_n_1
1	2	1	7	1.857
2	1.857	0.099	5.633	1.84
3	1.84	0.001	5.473	1.839
4	1.839	0	5.47	1.839

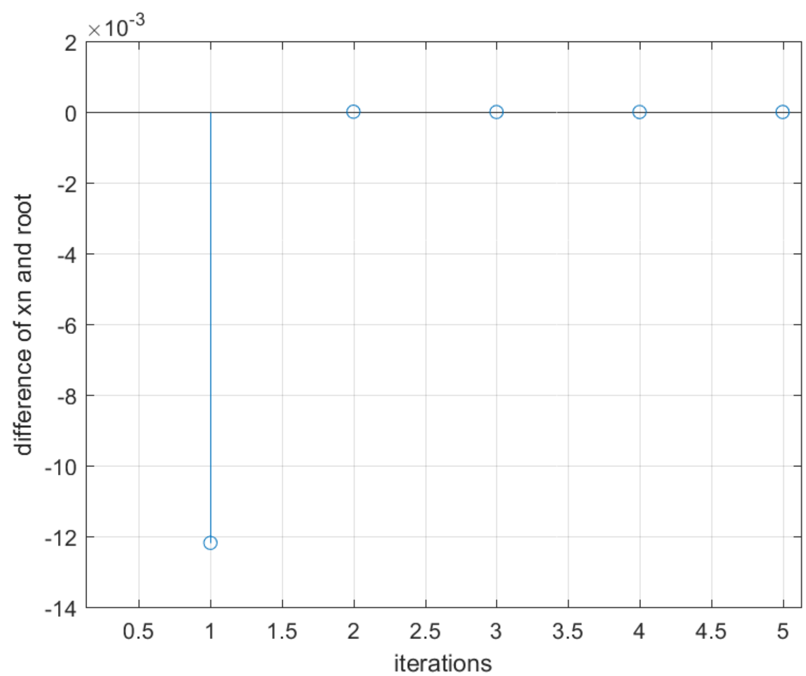
♦ **Observations:**

Root which we are getting is at x = 1.8393 .

(B) $f(x) = x - 1 - 0.3cosx = 0$

♦ **Graphs:**





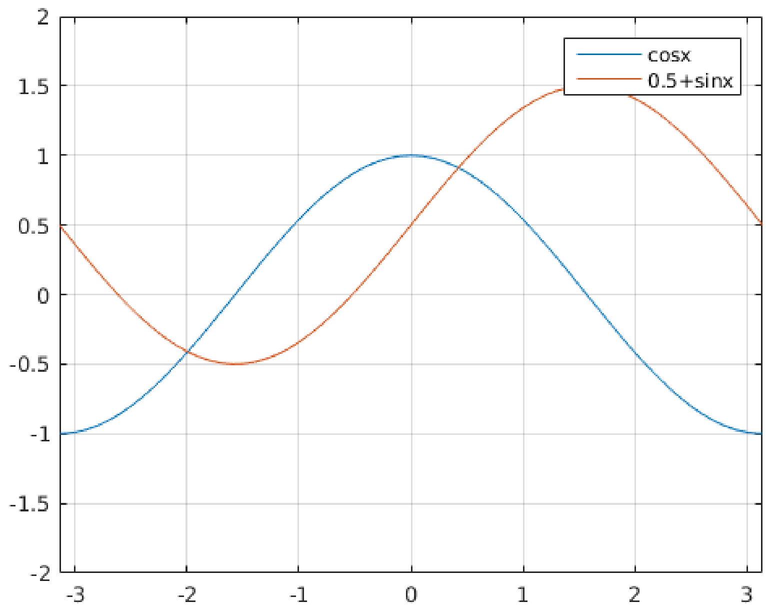
no	x_n	f_x_n	f_x_d	x_n_1
1	2	1.125	1.273	1.116
2	1.116	-0.015	1.27	1.128
3	1.128	0	1.271	1.128

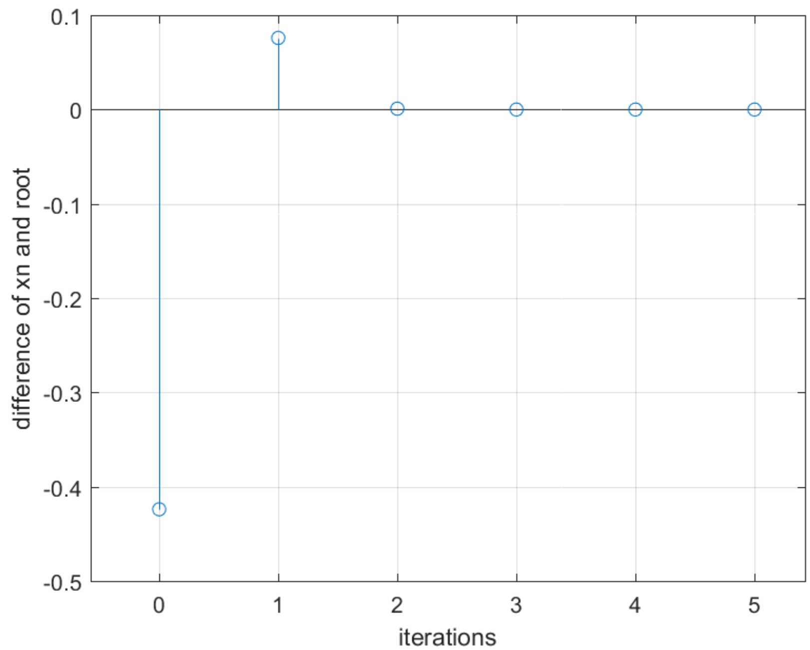
♦ Observations:

Root which we are getting is at x = 1.1284 .

(C) $f(x) = cosx - sinx - 0.5 = 0$

♦ Graphs:





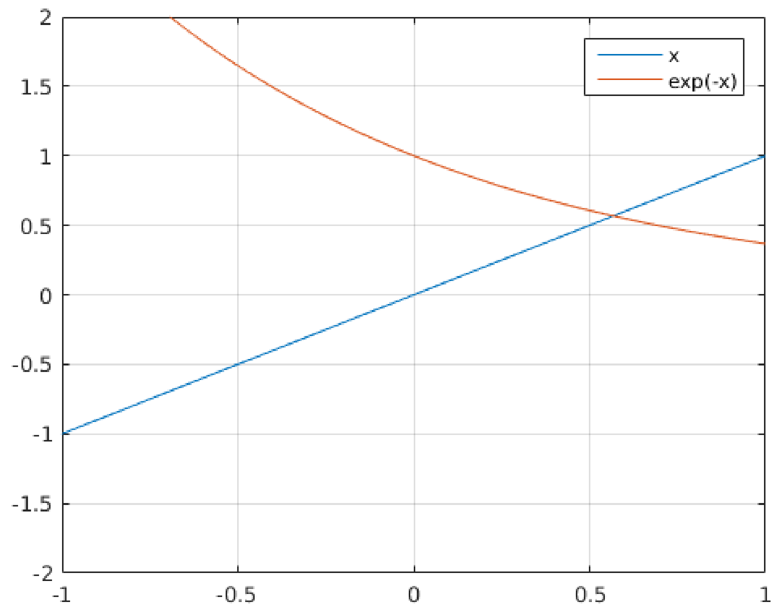
no	x_n	f_x_n	f_x_d	x_n_1
1	0	0.5	-1	0.5
2	0.5	-0.102	-1.357	0.425
3	0.425	-0.001	-1.323	0.424
4	0.424	-0	-1.323	0.424

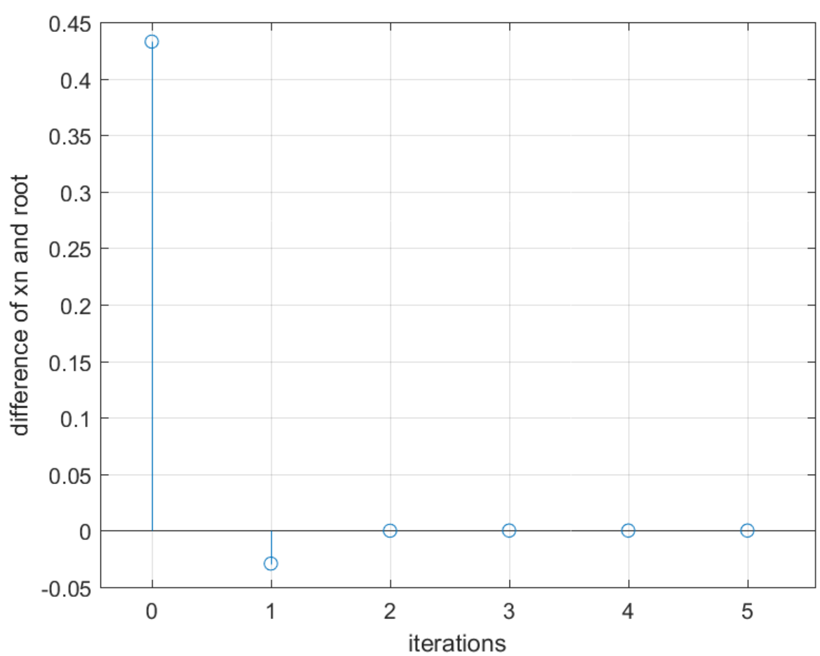
♦ Observations:

Root which we are getting is at x = 0.4241 .

(D) $f(x) = x - e^{-x} = 0$

♦ Graphs:





no	x_n	f_x_n	f_x_d	x_n_1
1	1	0.632	1.368	0.538
2	0.538	-0.046	1.584	0.567
3	0.567	-0	1.567	0.567

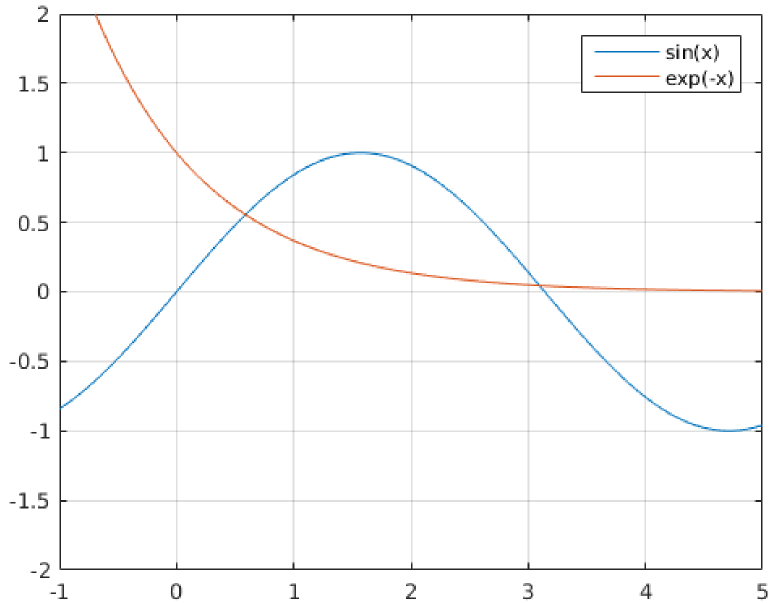
4	0.567	-0	1.567	0.567

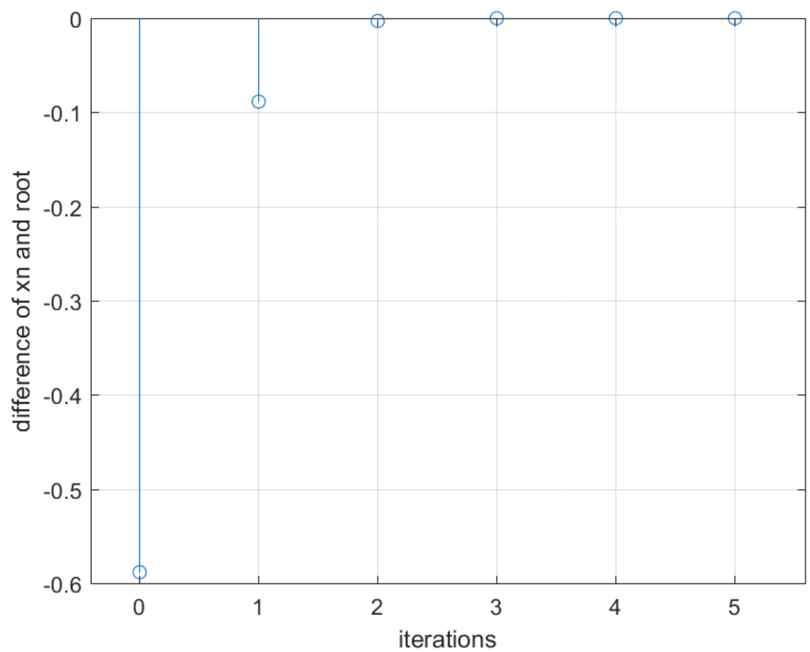
♦ Observations:

Root which we are getting is at x = 0.5672 .

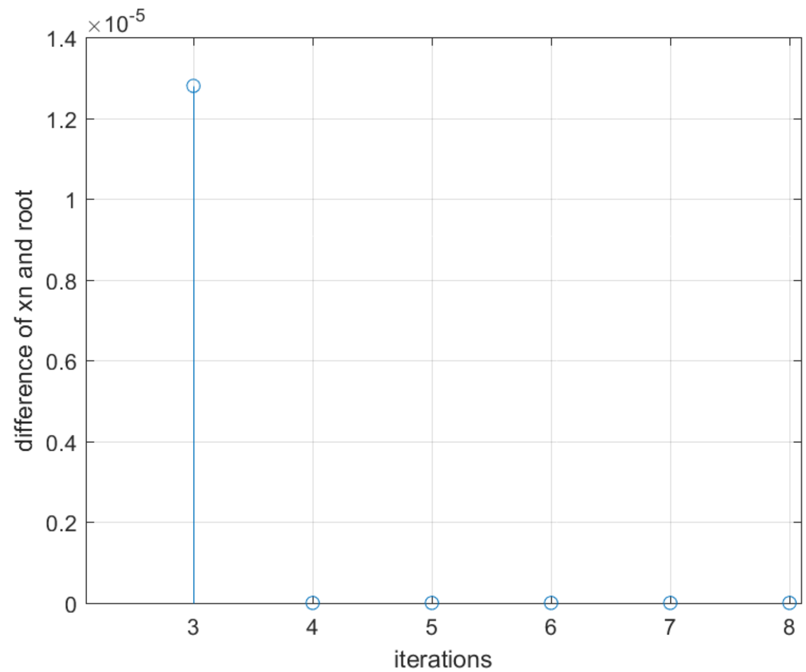
(E) $f(x) = e^{-x} - sinx = 0$

♦ Graphs:





no	x_n	f_x_n	f_x_d	x_n_1
1	0	1	-2	0.5
2	0.5	0.127	-1.484	0.586
3	0.586	0.004	-1.39	0.589
4	0.589	0	-1.387	0.589



no	x_n	f_x_n	f_x_d	x_n_1
1	4	0.775	0.635	2.78
2	2.78	-0.292	0.873	3.114
3	3.114	0.017	0.955	3.096
4	3.096	0	0.954	3.096

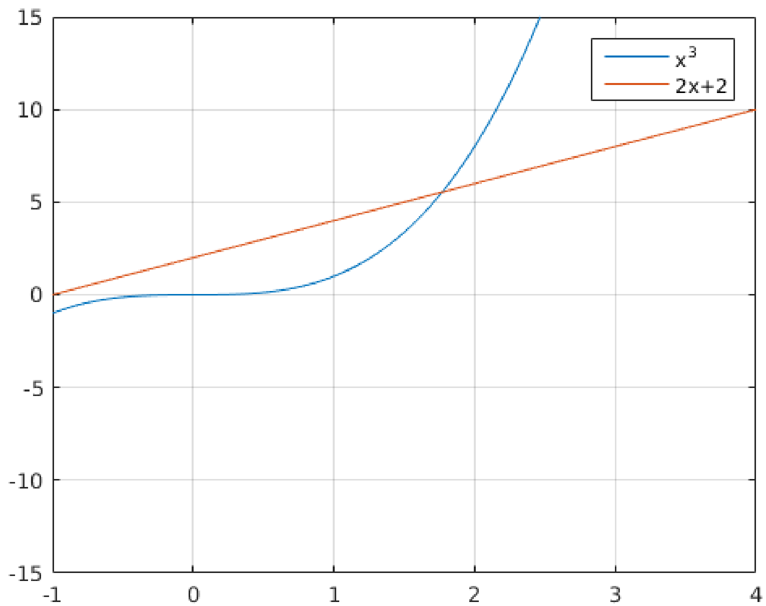
♦ Observations:

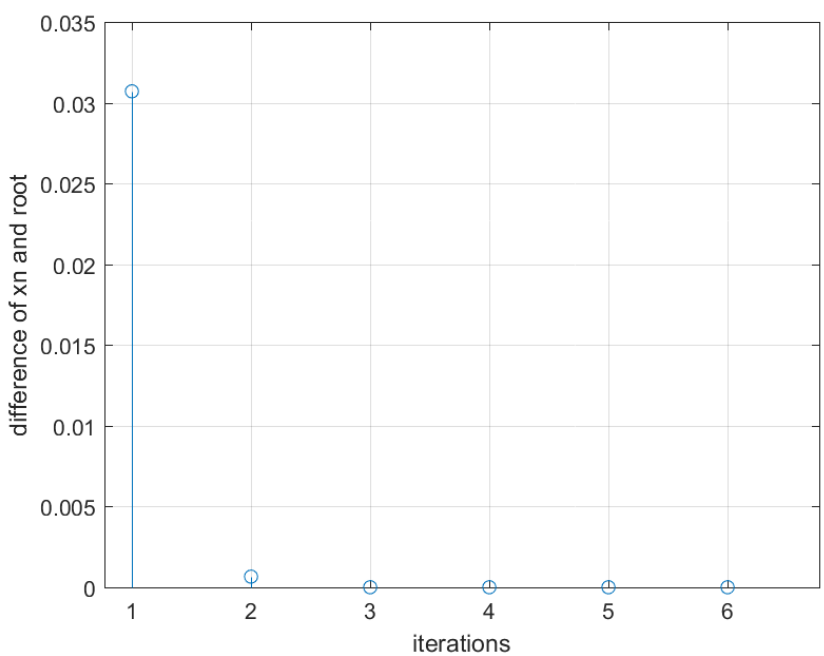
Root which we are getting is at x = 0.5885 .

Root which we are getting is at x = 3.0964 .

(F) $f(x) = x^3 - 2x - 2 = 0$

♦ Graphs:





no	x_n	f_x_n	f_x_d	x_n_1
1	2	2	10	1.8
2	1.8	0.232	7.72	1.77
3	1.77	0.005	7.398	1.769

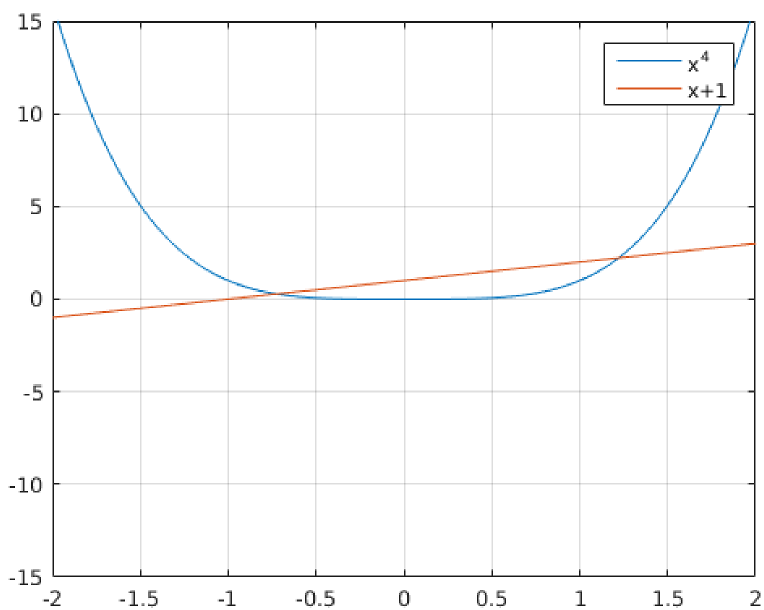
4	1.769	0	7.391	1.769

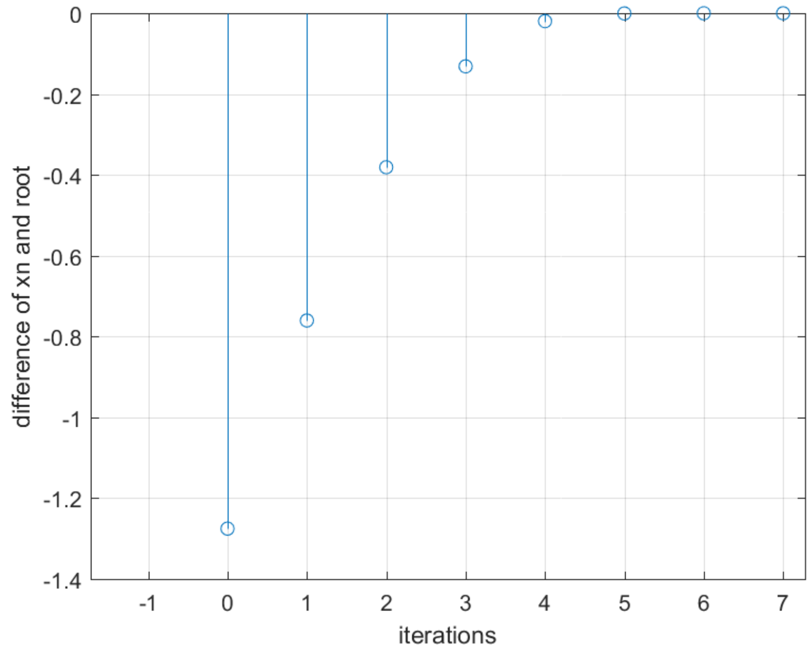
♦ **Observations:**

Root which we are getting is at x = 1.7693 .

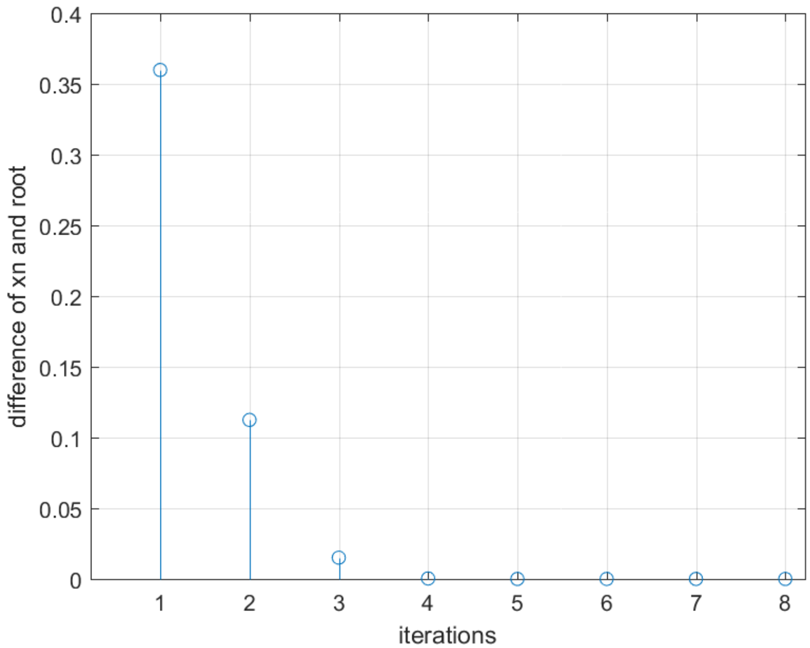
(G) $f(x) = x^4 - x - 1 = 0$

♦ **Graphs:**





no	x_n	f_x_n	f_x_d	x_n_1
1	-2	17	-33	-1.485
2	-1.485	5.346	-14.095	-1.106
3	-1.106	1.6	-6.405	-0.856
4	-0.856	0.392	-3.508	-0.744
5	-0.744	0.05	-2.647	-0.725
6	-0.725	0.001	-2.524	-0.724
7	-0.724	0	-2.521	-0.724



no	x_n	f_x_n	f_x_d	x_n_1
1	2	13	31	1.581
2	1.581	3.662	14.797	1.333
3	1.333	0.826	8.478	1.236
4	1.236	0.096	6.549	1.221
5	1.221	0.002	6.282	1.221
6	1.221	0	6.277	1.221

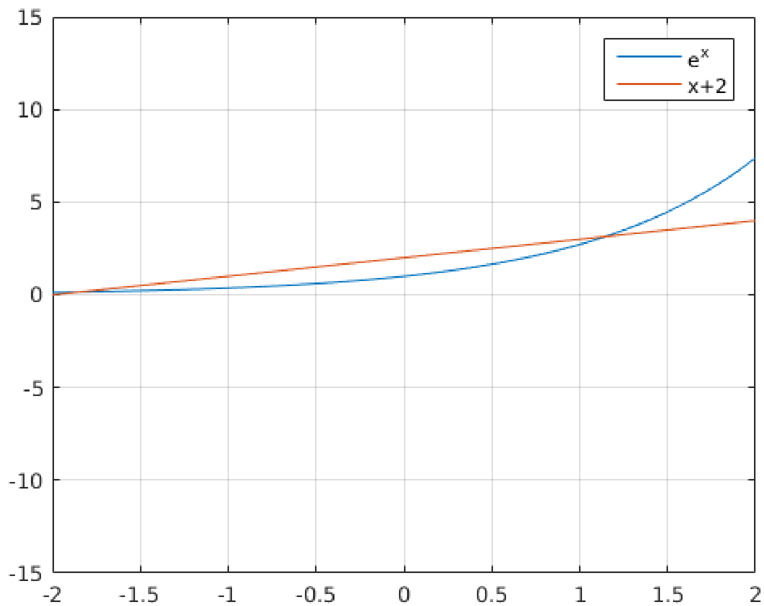
♦ Observations:

Smallest Root which we are getting is at x = -0.7245 .

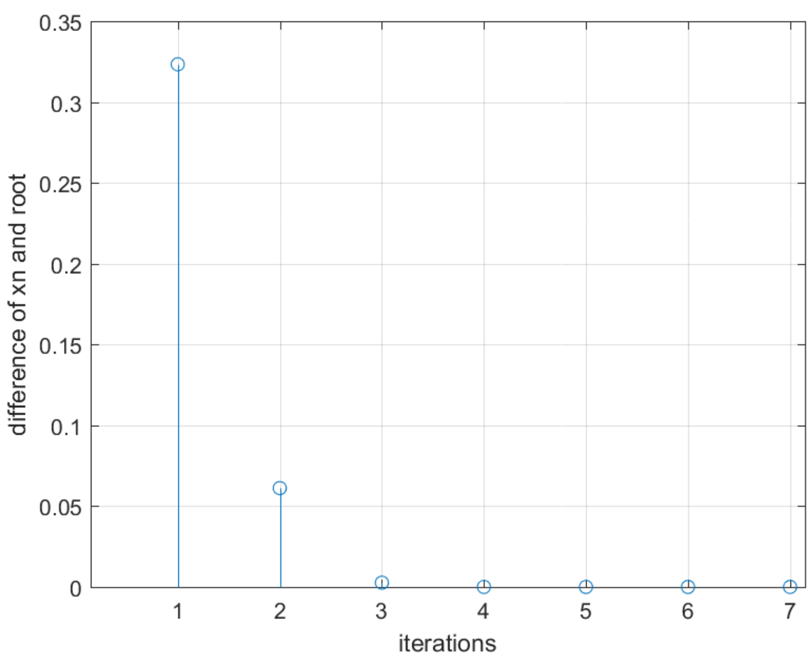
Largest Root which we are getting is at x = 1.2207 .

(H) $f(x) = e^x - x - 2 = 0$

♦ Graphs:



15
10
5
0
-5
-10
-15
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
e^x
x+2



0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
1
2
3
4
5
6
7
difference of xn and root
iterations

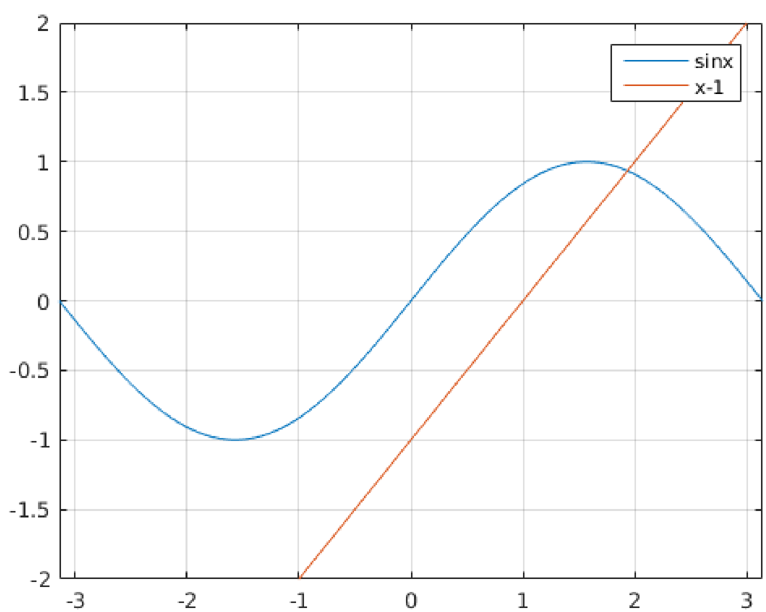
no	x_n	f_x_n	f_x_d	x_n_1
1	2	3.389	6.389	1.47
2	1.47	0.878	3.347	1.207
3	1.207	0.137	2.345	1.149
4	1.149	0.006	2.154	1.146
5	1.146	0	2.146	1.146

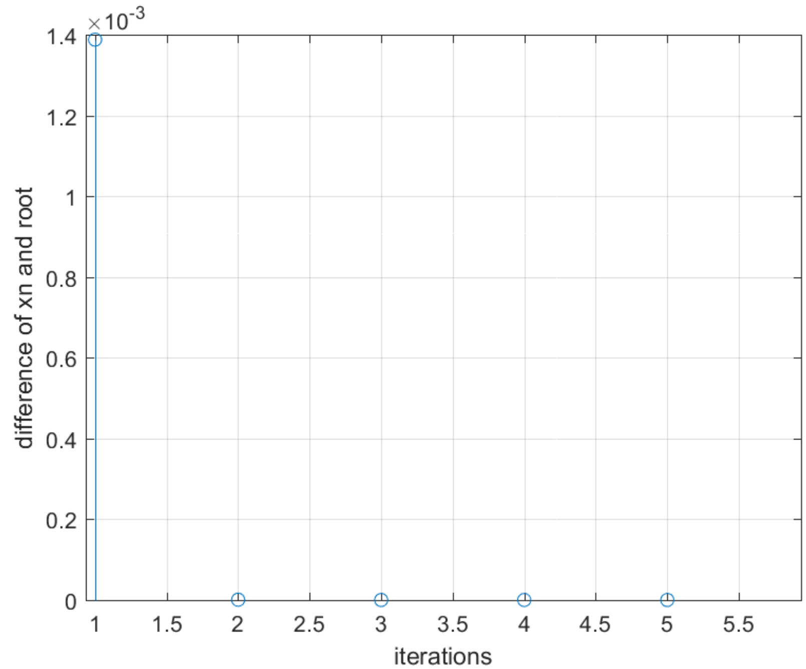
- **Observations:**

Root which we are getting is at x = 1.1462 .

$$(\boldsymbol{I})\ \boldsymbol{f(x) = 1 - x + sinx = 0}$$

- **Graphs:**





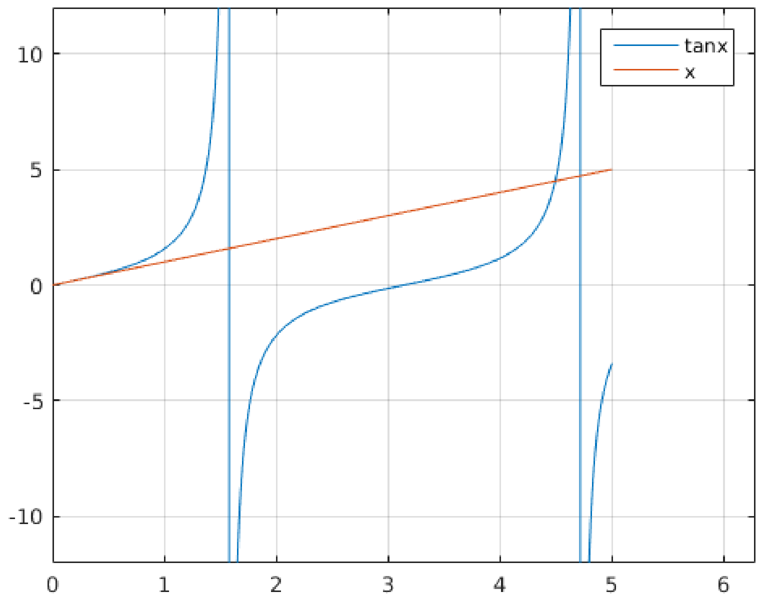
no	x_n	f_x_n	f_x_d	x_n_1
1	2	-0.091	-1.416	1.936
2	1.936	-0.002	-1.357	1.935
3	1.935	-0	-1.356	1.935

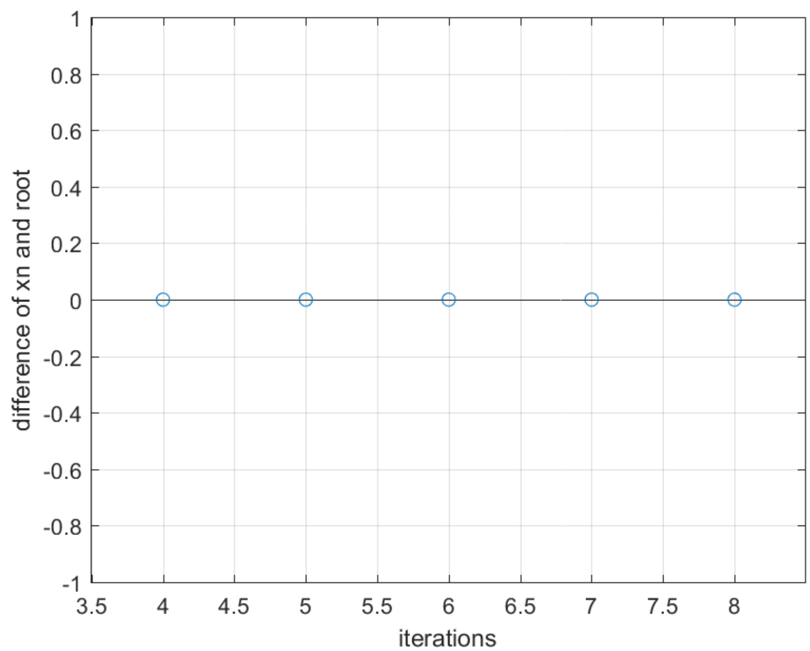
♦ **Observations:**

Root which we are getting is at x = 1.9345 .

(J) $f(x) = x - tanx = 0$

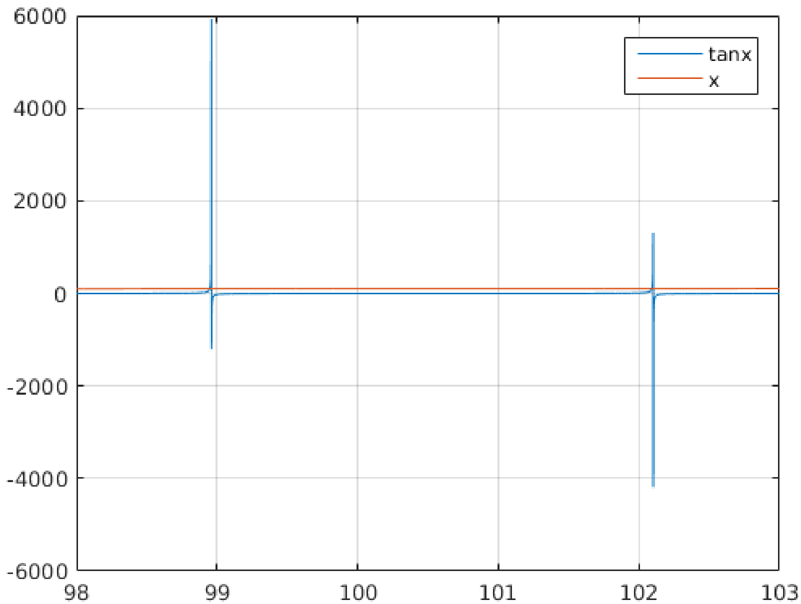
♦ **Graphs:**

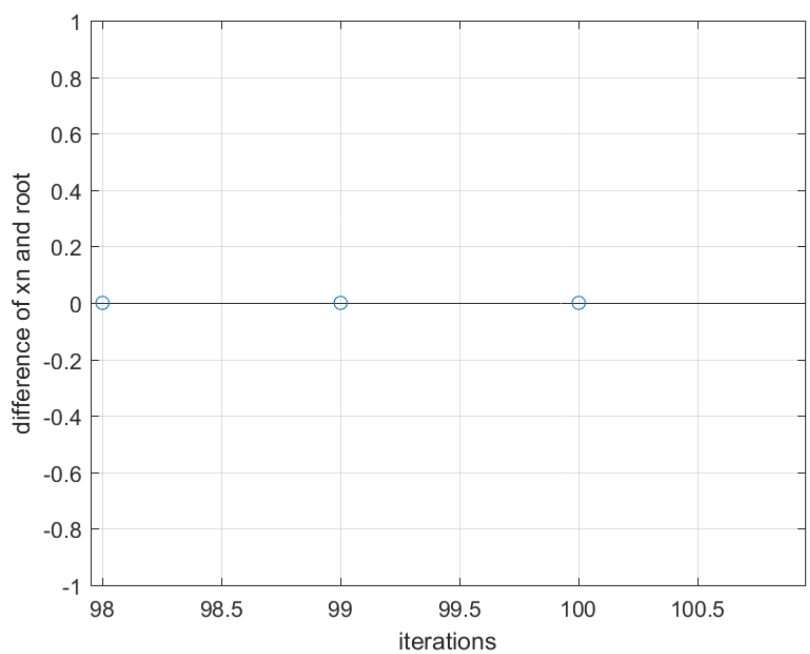




no	x_n	f_x_n	f_x_d	x_n_1
1	4.5	-0.137	-21.505	4.494
2	4.494	-0.004	-20.23	4.493

3	4.493	-0	-20.191	4.493





no	x_n	f_x_n	f_x_d	x_n_1
1	98.95	0.611	-9670.496	98.95

♦ **Observations:**

Smallest non-zero positive Root which we are getting is at x = 4.4934 .

Root closest to x = 100 , which we are getting is at x = 98.9501 .

<u>Problem: 4</u>

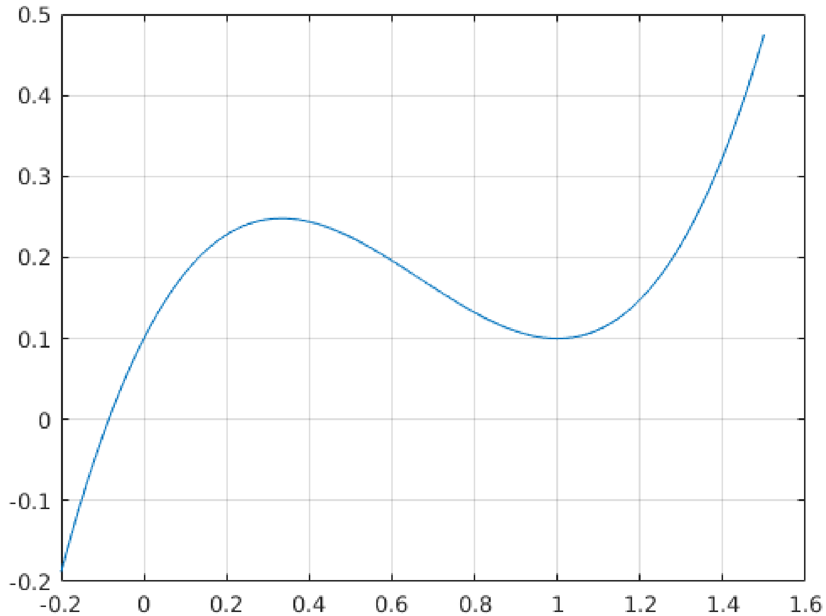
♦ **Statement:**

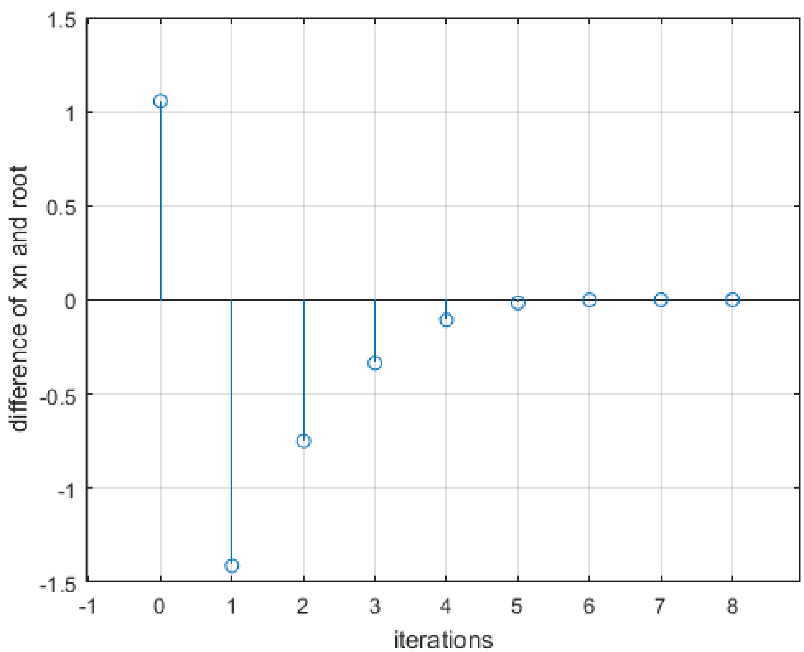
The function y = f(x) = a + x(x-1)^2 , with 0 <=a <=0.1.When a!=0 , there is only one real root of f(x)=0, with root being negative. Analytically check how many roots are obtained for a=0 , and what is the nature of the roots. Thereafter using the Newton – Raphson method , test for the convergence towards the negative real root , through a suitably chosen a values right down to a=0 . In every case your initial guess value should be slightly larger than 1. say 1.01 . For every value of a check how quickly the convergence happens.

f(x) = a + x(x-1)^2

(A) for a=0.1,

♦ **Graphs:**





no	x_n	f_x_n	f_x_d	x_n_1
1	1.01	0.05	0.02	-1.458

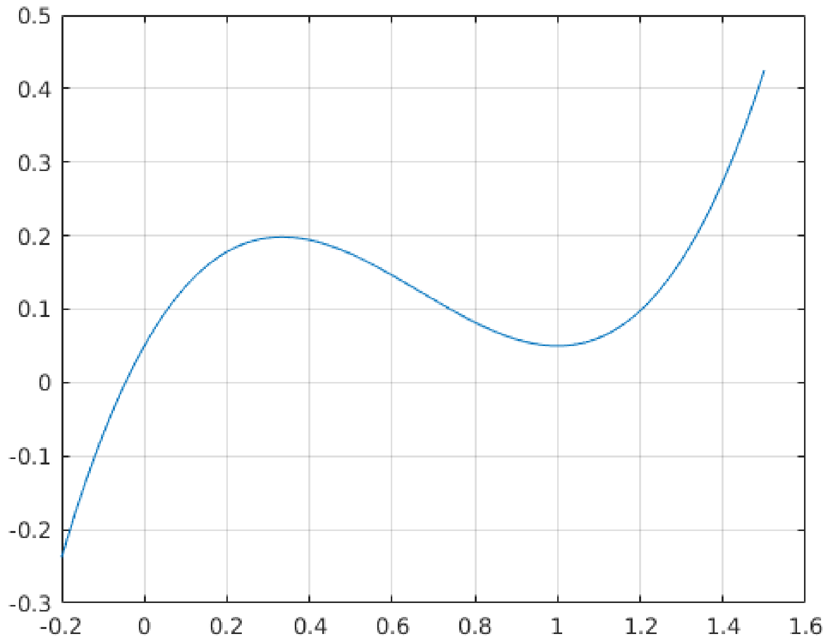
2	-1.458	-8.759	13.21	-0.795
3	-0.795	-2.511	6.075	-0.382
4	-0.382	-0.678	2.963	-0.153
5	-0.153	-0.153	1.681	-0.062
6	-0.062	-0.02	1.258	-0.046
7	-0.046	-0.001	1.191	-0.046
8	-0.046	-0	1.189	-0.046

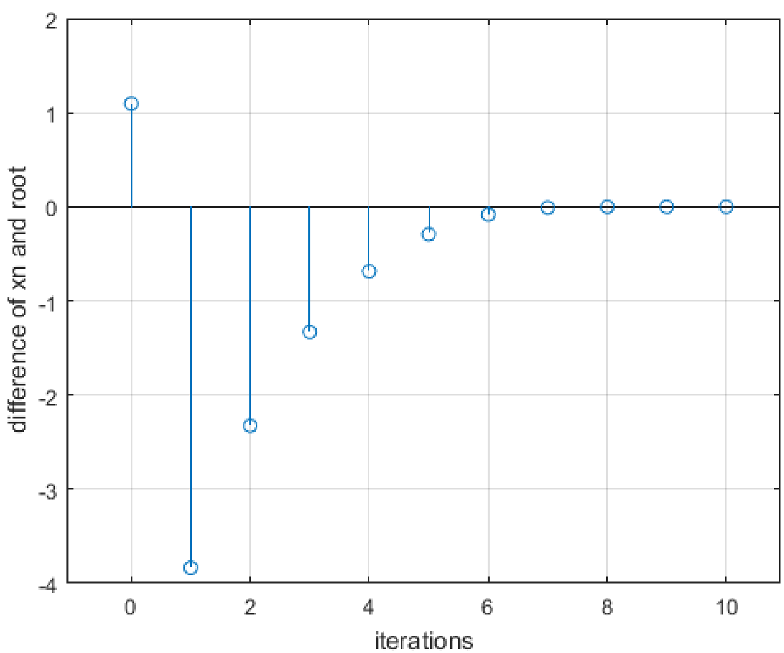
♦ Observations:

Root which we are getting is at x = -0.0850.

(A) **For a=0.05,**

♦ Graphs:





no	x_n	f_x_n	f_x_d	x_n_1
1	1.01	0.1	0.02	-3.921
2	-3.921	-94.857	62.809	-2.411

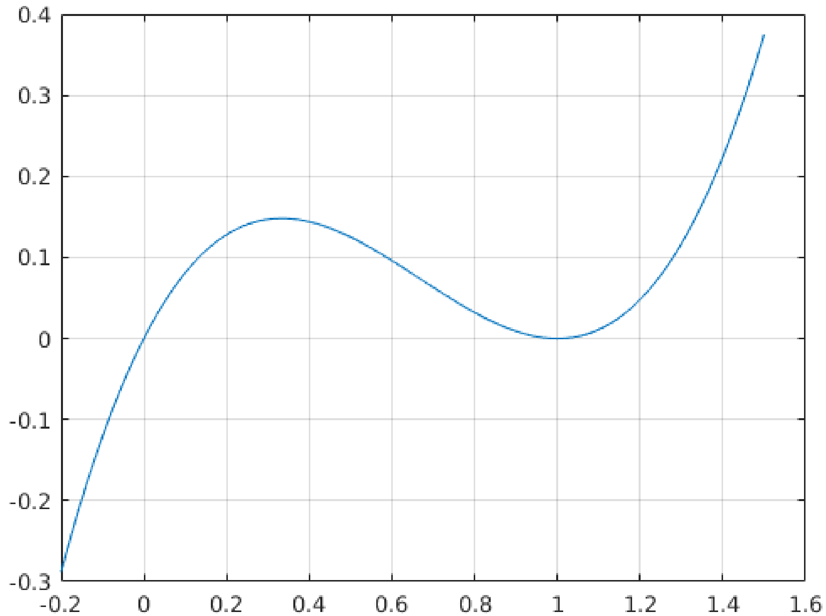
3	-2.411	-27.947	28.08	-1.416
4	-1.416	-8.16	12.674	-0.772
5	-0.772	-2.322	5.874	-0.376
6	-0.376	-0.613	2.93	-0.167
7	-0.167	-0.128	1.753	-0.094
8	-0.094	-0.013	1.404	-0.085
9	-0.085	-0	1.362	-0.085
10	-0.085	-0	1.361	-0.085

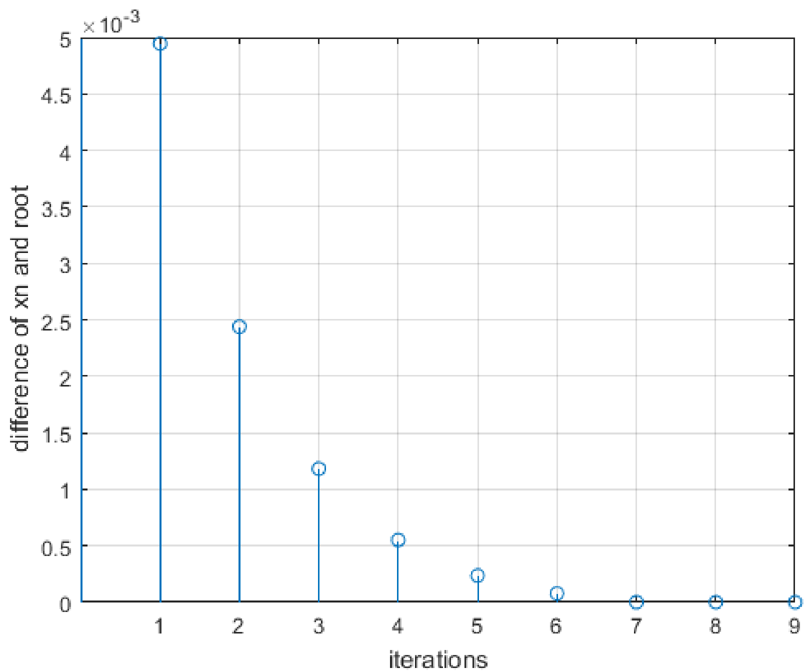
♦ Observations:

Root which we are getting is at x = -0.0457.

(A) for a=0,

♦ Graphs:





no	x_n	f_x_n	f_x_d	x_n_1
1	1.01	0	0.02	1.005
2	1.005	0	0.01	1.003

3	1.003	0	0.005	1.001
4	1.001	0	0.003	1.001
5	1.001	0	0.001	1
6	1	0	0.001	1
7	1	0	0	1

- **Observations:**

  Root which we are getting is at x = 1.0001.