Assignment – 1, 2

Course: SC-374

Computational and Numerical Methods

Instructor: Prof. Arnab Kumar

Made by:

Yatin Patel – 201601454

Rutvik Kothari – 201601417

Assignment - 1

The Binary Search and Information Entropy

Average information content is given by the formula, $< I > = -k \sum_i P_i \log_2 P_i$, in which k is constant and P_i is the probability of an event.

(a) For a two-outcome problem (eg. a coin toss), Show that $< I >$ peaks at $P = \frac{1}{2}$.

$$f(p) = < I > = -k\ (plog_2(p) + (1-p)\log_2(1-p))$$

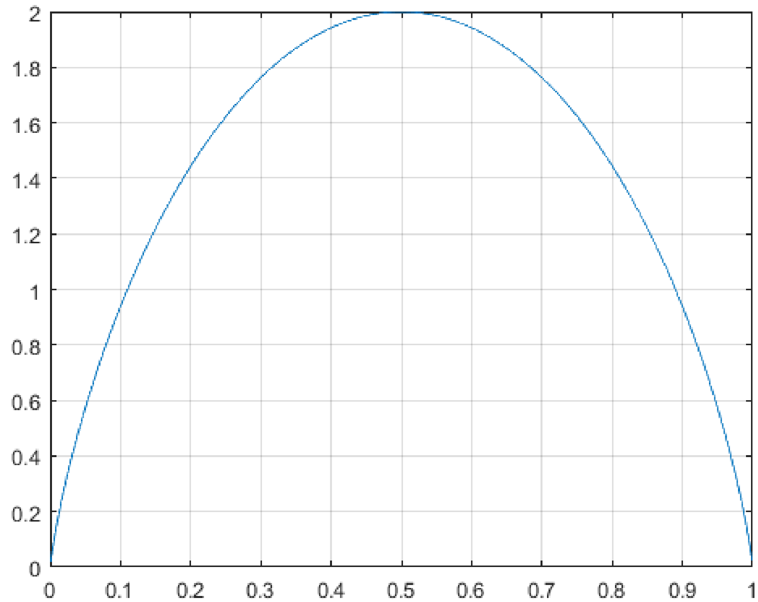
$$f'(p) = -k(\log_2 p + 1 - 1 - \log_2(1-p)\ )$$

$$f'(p) = 0\ for\ maximize,$$

$$\log_2 p = \log_2(1-p)$$

$$p = 1 - p$$

$$p = \frac{1}{2}$$



(b) Apply a very small perturbation as $P = \frac{1}{2} + \varepsilon$, in which $\varepsilon \ll \frac{1}{2}$. Show that in this perturbative approach $< I > = a - b\varepsilon^2$, with $a = k$ and $b = \frac{4k}{ln2}$.

$$< I > = -k\left(\left(\varepsilon + \frac{1}{2}\right)\ln\left(\varepsilon + \frac{1}{2}\right) + \left(\frac{1}{2} - \varepsilon\right)\ln\left(\frac{1}{2} - \varepsilon\right)\right)$$

$$< I > = -k\left(\left(\varepsilon + \frac{1}{2}\right)\ln\left(\frac{2\varepsilon + 1}{2}\right) + \left(\frac{1}{2} - \varepsilon\right)\ln\left(\frac{-2\varepsilon + 1}{2}\right)\right)$$

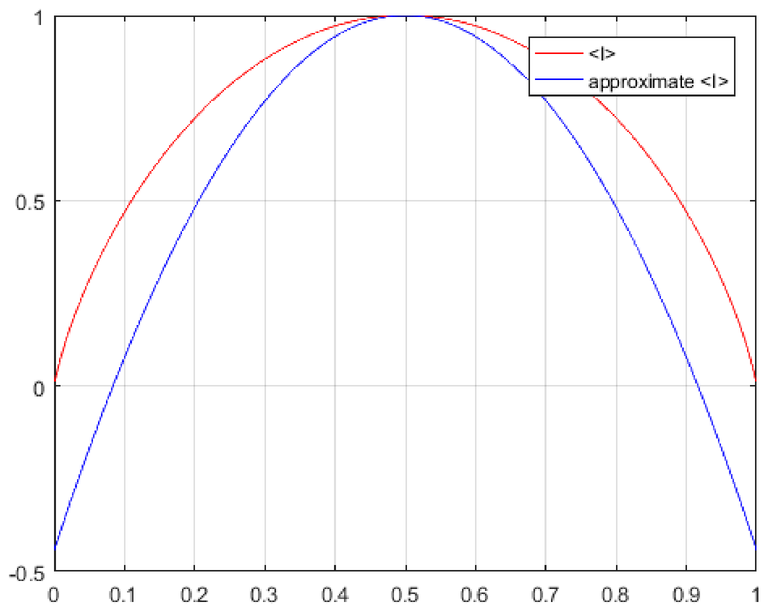
$$< I > = -k\left(\left(\varepsilon + \frac{1}{2}\right)\left(\frac{2\varepsilon}{\ln(2)} - 1\right) + \left(\frac{1}{2} - \varepsilon\right)\left(-\frac{2\varepsilon}{\ln(2)} - 1\right)\right)$$

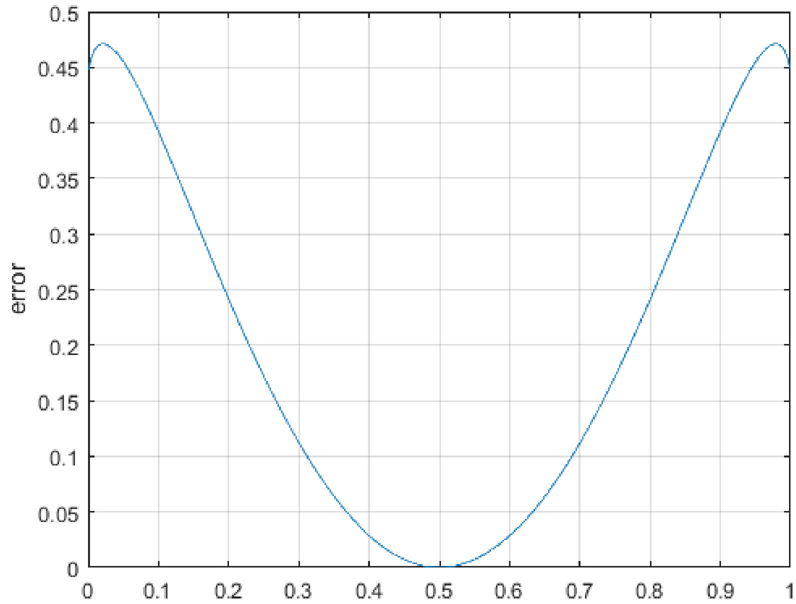
$$< I > = -k\left(\frac{4\varepsilon^2}{\ln(2)} - 1\right)$$

$$< I > = k - \frac{4k}{\ln(2)}\varepsilon^2$$

$$a = k \; and \; b = \frac{4k}{\ln(2)}$$

(c) Plot $< I >$ versus P for both the actual function and the approximate function together and then compare the graph for closeness on the line $< I > = 0$. For plotting choose $k = 1$.





Assignment - 2

An Astrophysical Inflow

In the problem of spherically symmetric astrophysical accretion, interstellar fluid matter (a very thin gas) travels a great distance (almost from infinity) along radial lines and falls on to a massive star (or a neutron star or even a black hole) located at the origin of coordinates. The star can be treated as a point-like particle, and the rate of the fluid flow (matter flowing in unit time) on to it is given as

$$\dot{m} = \frac{\pi G^2 M^2 \rho_\infty}{c_s^3(\infty)} \left(\frac{2}{5-3\gamma}\right)^{\frac{5-3\gamma}{2\gamma-2}}$$

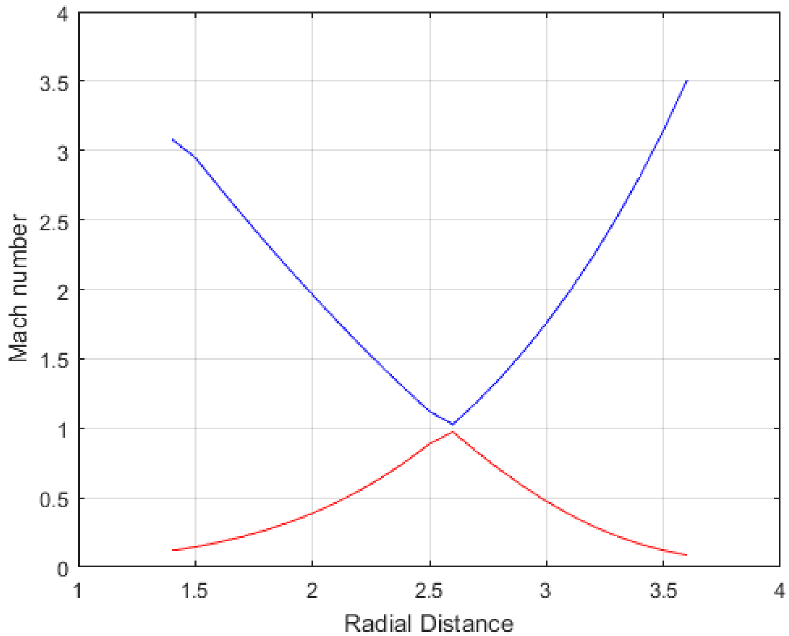
in which G is Newton's universal gravitational constant, M is the mass of the central astrophysical object, ρ_∞ is the constant density of the gas at infinity, cs(∞) is the speed of sound at infinity, and γ is a dimensionless number called the polytropic exponent ($1 \le \gamma \le 5/3$). The velocity of the fluid flow v, as a function of the radial distance from the centre r, is given by the equation

$$f(v,r) = \frac{v^2}{2} + n\left(\frac{\dot{\mu}}{vr^2}\right)^{\frac{1}{n}} - \frac{GM}{r} - nc_s^2(\infty) = 0$$

$$\dot{\mu} = \left(\frac{\dot{m}}{4}\pi\rho_\infty\right)c_s^{2n}$$

$$n = \frac{1}{\gamma - 1}$$

Solve Eq.(1) by the bisection method to find v(r), using the values M = 2 × 1030 kg, cs(∞) = 10 km s−1, ρ∞ = 10−21 kg m−3 and n = 2.5. These values are typical of accretion of the interstellar medium on to a star. Each value of r in Eq.(1) will give a set of two real and physical roots of v. The plot of v(r) is shown in Fig. 1, in which v is scaled as the Mach number, v(r)/cs(r). Obtain a similar plot.



Mach number
Radial Distance
0
0.5
1
1.5
2
2.5
3
3.5
4
1
1.5
2
2.5
3
3.5
4