PHYSICAL REVIEW C **88**, 055205 (2013)

Acoustic horizons in steady spherically symmetric nuclear fluid flows

Niladri Sarkar,[1,*] Abhik Basu,[1,†] Jayanta K. Bhattacharjee,[2,‡] and Arnab K. Ray[3,§]

[1] *Condensed Matter Physics Division, Saha Institute of Nuclear Physics, Calcutta 700064, India*

[2] *Harish Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad 211019, India*

[3] *Department of Physics, Jaypee University of Engineering and Technology, Raghogarh, Guna 473226, Madhya Pradesh, India*

(Received 3 June 2013; revised manuscript received 2 September 2013; published 18 November 2013)

We consider a hydrodynamic description of the spherically symmetric outward flow of nuclear matter, using a nuclear model that introduces a weakly dispersive effect in the flow. On the resulting stationary conditions of the flow, we apply an Eulerian scheme to derive a fully nonlinear equation of a time-dependent radial perturbation. In its linearized limit, with no dispersion, this equation implies the static acoustic horizon of an analog gravity model. This horizon also defines the minimum radius of the steady flow. We model the perturbation as a high-frequency traveling wave, in which the weak dispersion is taken iteratively. A WKB analysis shows that even arbitrarily small values of dispersion make the horizon fully opaque to any acoustic disturbance propagating against the bulk flow, with the amplitude and the energy flux of the radial perturbation decaying exponentially just outside the horizon. Nonlinear effects shift the horizon from its steady position.

DOI: 10.1103/PhysRevC.88.055205   PACS number(s): 24.10.Nz, 21.65.−f, 25.75.Ld, 46.15.Ff

I. INTRODUCTION

A wide variety of hydrodynamic flows, ranging from astrophysical flows to flows in kitchen sinks, shows the existence of an "acoustic horizon" [1–10]. Such a horizon is attained when the speed of the bulk flow matches the speed with which a relevant wave carries information through the medium (for example, sound waves or gravity waves). Passage of information is unidirectional across the horizon, as happens analogously in the case of black or white holes. This point of view, also known as analog gravity, leads to many interesting consequences in fluid flows. For instance, in a low-dimensional flow, with viscosity lending an additional effect, there is an abrupt increase in the depth of the fluid at the horizon—a phenomenon that is popularly known as the hydraulic jump [8,11–13].

In the distinctly different context of nuclear physics, in an energetic heavy-ion collision, a hot and expanding nuclear gas is formed [14]. Hydrodynamic features are known to be collectively important in such cases, and appropriate hydrodynamic descriptions were furnished both for the expansion stage of the fluid [14] and its compression [15]. In the former case, an analytical treatment was presented for the nonlinear hydrodynamic equations, describing a free isentropic expansion [14]. The hydrodynamic equations invoked in this approach were the familiar ones used in studies of radially outward flows in spherical symmetry, and later studies were also to pursue the same line [16]. One of the early investigations at low energies looked into the aftermath of nuclear collisions near the speed of sound [17], in which the possibility of soliton formation was considered, and so in due course dispersive terms were also required to be introduced in the hydrodynamic equations. The subject of solitons in nuclear reactions was more formally addressed in later works [18,19]. Likewise, the question of shock fronts was also accorded its rightful importance [20].

In nuclear fluids, hydrodynamic models with dispersion and viscous effects have been a regular topic of study over the past decade. A significant amount of work including dispersive effects in particular has been carried out by now, all of which required careful studies of suitable equations of state and setting up of hydrodynamic equations, leading to solitonic solutions in some cases [21–24]. Alongside dispersion, the importance of viscous hydrodynamics in the description of nuclear matter at extreme energy densities has not been overlooked either [25–29].

Given this overall background on hydrodynamics, in the present work we study the hydrodynamic aspects of nuclear matter from the perspective of analog gravity and acoustic horizons. For a radial outflow in spherical symmetry, we make use of standard hydrodynamic model equations, accommodating a weakly dispersive effect that, nevertheless, has a strong influence on the flow. It is interesting to note that the same influence may also be exerted by viscosity, which is a more natural attribute of a fluid (discussed at the end of Sec. V). We apply an Eulerian perturbation scheme on a steady extended flow and, including nonlinearity to any arbitrary order, we obtain the proper form of the metric of an acoustic horizon. We find that the acoustic horizon defines the minimum radius of the stationary flow. We also proceed to argue that nonlinearity has an adverse impact on the analogy of a static acoustic horizon. In the linear limit, we fashion the perturbation as a high-frequency traveling wave and see how small effects of dispersion influence the steady conditions. Working iteratively by having recourse to the WKB method, we show that dispersion reduces the amplitude and the energy flux of the radially propagating wave completely to zero.

To summarize the principal results of our work, we have demonstrated the existence of an acoustic horizon, specifically that of a white hole, in our chosen model of nuclear hydrodynamics (Sec. III), with no physical flow solution admitted within the horizon (Sec. IV). Linearized perturbations do not

[*] niladri.sarkar@saha.ac.in

[†] abhik.basu@saha.ac.in

[‡] jkb@hri.res.in

[§] arnab.kumar@juet.ac.in

destabilize either the stationary flow or the horizon, and at the horizon of the white hole, all acoustic signals are fully extinguished due to the dispersive effect in the linear regime (Sec. V and Appendix B). Nonlinearity, however, disturbs the precise condition of a stationary horizon (Sec. III and Appendix A).

II. THE HYDRODYNAMIC EQUATIONS

The hydrodynamic description that we have adopted here is relevant to high-energy impacts or collisions whose result is an outflow of the nuclear fluid [14]. The outward flow is described by a velocity field, v, and a baryonic density field, n. The latter is related to the mass density, ρ, by $\rho = Mn$, where M is the nucleon mass. The two fields, v and n, are coupled through two equations, one given by the condition of momentum balance and the other by the continuity equation. These conditions are further supplemented by an equation of state connecting n to the local pressure, P. For a perfect nuclear fluid, P is related to the enthalpy per nucleon, h, under isentropic conditions, by [21]

$$\nabla P = n \nabla h, \tag{1}$$

so that the condition for momentum balance can be set as

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{1}{M}\nabla h. \tag{2}$$

We now use a form of h, given as [21]

$$h = E(n_{\rm e}) + \frac{Mc_{\rm s}^2}{2n_{\rm e}^2}\left(3n^2 + n_{\rm e}^2 - 4nn_{\rm e}\right), \tag{3}$$

in which $c_{\rm s}$ is the speed of sound in the nuclear fluid and $n_{\rm e}$ is the equilibrium density about which the energy per nucleon, $E(n)$, is expanded in a Taylor series [21]. At this stage the hydrodynamics is both inviscid and nondispersive. We can bring in a viscous term from the usual expressions of the stress tensor. To introduce a dispersive term, a specific model is required. Such a model [21] provides a dispersive term through the zeroth order of the Taylor expansion, augmenting $E(n_{\rm e})$ in Eq. (3) by a nonlocal term, something that can be done only in terms of n, since $n_{\rm e}$ is spatially constant. This effectively amounts to crafting a small nonlocal effect about the usual baryon-vector meson local coupling, and this contribution is considered at the mean-field level. So with this understanding, we can write

$$E(n_{\rm e}) = \left(\frac{g_V^2}{2m_V^2}\right)n_{\rm e} + \frac{\chi(n_{\rm e})}{n_{\rm e}} + \left(\frac{g_V^2}{m_V^4}\right)\nabla^2 n. \tag{4}$$

Here $\chi(n_{\rm e})$ has a known form [21] and g_V is the coupling constant of the baryon–vector meson interaction, with m_V being the mass of the vector meson field.

Now with the help of Eqs. (3) and (4), we can substitute h in Eq. (2) to obtain

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v} = -\frac{c_{\rm s}^2}{2n_{\rm e}^2}(6n\nabla n - 4n_{\rm e}\nabla n) - \frac{g_V^2}{Mm_V^4}\nabla(\nabla^2 n), \tag{5}$$

which has the standard inertial and advective terms in the left-hand side. The first term in the right-hand side involves the gradient of the baryonic density and is analogous to the pressure term in standard hydrodynamics. The last term in Eq. (5) is the interaction term that describes the nonlocal coupling between the baryons and the vector meson field. This term acts in the manner of a dispersion and is of principal interest in our study. It was introduced in nuclear hydrodynamics to investigate the formation and propagation of solitons in nuclear matter [21]. We are interested not so much in propagating solitonic solutions as we are in knowing how this dispersion term affects, in a perturbative sense, the stationary solution that is yielded by the rest of the terms in Eq. (5). So we consider the dispersion term only in the regime of extremely weak baryon–vector meson interactions.

While Eq. (5) gives one condition for the dynamics of v and n, another condition is also required. This is provided by the continuity equation, going as

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = 0, \tag{6}$$

from whose stationary limit we obtain the condition of baryon conservation.

Considering now a spherically symmetric outward flow, Eq. (5) is recast as

$$\frac{\partial v}{\partial t} + \frac{\partial}{\partial r}\left[\frac{v^2}{2} + \frac{c_{\rm s}^2}{2n_{\rm e}^2}\left(3n^2 - 4n_{\rm e}n\right)\right] = -\zeta\frac{\partial}{\partial r}\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial n}{\partial r}\right)\right], \tag{7}$$

in which, we have set $\zeta = g_V^2/Mm_V^4$ for notational convenience, with $\zeta \longrightarrow 0$, when the baryon–vector meson interaction is treated as just a very small perturbative effect. Likewise, in spherically symmetric geometry, Eq. (6) is rendered as

$$\frac{\partial n}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(nvr^2) = 0. \tag{8}$$

Taken together, Eqs. (7) and (8) form a closed set to describe the coupled dynamics of the fields $v(r,t)$ and $n(r,t)$. Our subsequent analysis is based on these two equations only.

III. THE PERTURBATION AND THE ACOUSTIC HORIZON

Under steady conditions and with $\zeta \longrightarrow 0$, Eqs. (7) and (8) give the stationary fields, $v_0(r)$ and $n_0(r)$. About these stationary values, the perturbation schemes which we prescribe for both v and n, respectively, are $v(r,t) = v_0(r) + v'(r,t)$ and $n(r,t) = n_0(r) + n'(r,t)$, with the primed quantities representing time-dependent perturbations. Next, following an Eulerian perturbation treatment, adopted from the field of astrophysical accretion [30], we define a new variable, $f = nvr^2$, whose steady value, f_0, is a constant, governed by the condition

$$n_0 v_0 r^2 = f_0. \tag{9}$$

This fact can be verified easily from the stationary form of Eq. (8), from which we can, therefore, extract

$$\frac{\partial n'}{\partial t} = -\frac{1}{r^2}\frac{\partial f'}{\partial r}. \tag{10}$$

From the definition of f itself, we further derive

$$\frac{f'}{f_0} = \frac{n'}{n_0} + \frac{v'}{v_0} + \frac{n'}{n_0}\frac{v'}{v_0}, \tag{11}$$

in which we have maintained all admissible orders of nonlinearity.

Now, making use of Eq. (10) in Eq. (11), we get

$$\frac{\partial v'}{\partial t} = \frac{v}{f}\frac{\partial f'}{\partial t} + \frac{v^2}{f}\frac{\partial f'}{\partial r}. \tag{12}$$

It is worth stressing here that Eq. (12), which is fully nonlinear, and Eq. (10) together give a closed set of conditions by which we can represent n' and v' exclusively in terms of f'. We now need an independent condition, to which we can apply Eqs. (10) and (12), and just such a condition is afforded by Eq. (7). We take the second-order time derivative of Eq. (7), and then to it we apply the results implied by Eqs. (10) and (12), as well as the second-order time derivative of Eq. (12). Consequently we obtain

$$\frac{\partial}{\partial t}\left(h^{tt}\frac{\partial f'}{\partial t}\right)+\frac{\partial}{\partial t}\left(h^{tr}\frac{\partial f'}{\partial r}\right)+\frac{\partial}{\partial r}\left(h^{rt}\frac{\partial f'}{\partial t}\right)+\frac{\partial}{\partial r}\left(h^{rr}\frac{\partial f'}{\partial r}\right) = \zeta\frac{\partial}{\partial r}\left\{\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{\partial}{\partial r}\left(\frac{1}{r^2}\frac{\partial f'}{\partial r}\right)\right]\right\}, \tag{13}$$

in which

$$h^{tt} = \frac{v}{f}, \quad h^{tr} = h^{rt} = \frac{v^2}{f}, \quad h^{rr} = \frac{v}{f}(v^2 - a^2), \tag{14}$$

with

$$a^2 = 3c_s^2\frac{n}{n_e}\left(\frac{n}{n_e} - \frac{2}{3}\right). \tag{15}$$

This last expression further suggests that there shall be no acoustic propagation in the fluid if $n \leqslant 2n_e/3$. This limit on the steady value of the particle density defines a low point, below which the flow loses the character of a fluid continuum, with an acoustic propagation no longer possible. The factor of $2/3$ is simply due to the choice of the equation of state in Eq. (7).

It is particularly interesting to study Eq. (13) in the limit of $\zeta = 0$, i.e., when there is no nonlocal baryon–vector meson interaction. In this special case, not only do we get proper background stationary solutions out of Eqs. (7) and (8), but also, going by the symmetry of Eq. (13), we can recast it in a compact form as

$$\partial_\mu(h^{\mu\nu}\partial_\nu f') = 0, \tag{16}$$

with the Greek indices running from 0 to 1, under the equivalence that 0 stands for t and 1 stands for r. We see that Eq. (16), or equivalently, Eq. (13), is a nonlinear equation containing arbitrary orders of nonlinearity in the perturbative expansion. If, however, we work with a linearized equation, then $h^{\mu\nu}$, containing only the zeroth-order terms, can be read from the matrix

$$h^{\mu\nu} = \frac{v_0}{f_0}\begin{pmatrix} 1 & v_0 \\ v_0 & v_0^2 - a_0^2 \end{pmatrix}, \tag{17}$$

in which $a_0 \equiv a_0(r)$ is the steady value of a. A significant implication of the foregoing matrix is that under steady conditions, an acoustic disturbance in the fluid propagates with the speed, a_0, and its value is determined when $n = n_0$ in Eq. (15).

Now, in Lorentzian geometry the d'Alembertian of a scalar field in curved space is obtained from the metric, $g_{\mu\nu}$, as

$$\Delta\varphi \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\, g^{\mu\nu}\partial_\nu\varphi), \tag{18}$$

where $g^{\mu\nu}$ is the inverse of the matrix, $g_{\mu\nu}$ [2,9]. We look for an equivalence between $h^{\mu\nu}$ and $\sqrt{-g}\, g^{\mu\nu}$ by comparing Eqs. (16) and (18) with each other, and we see that Eq. (16) gives an expression of f' that is of the type given by Eq. (18). In the linear order, the metrical part of Eq. (16), as Eq. (17) shows it, may then be extracted, and its inverse will indicate the existence of an acoustic horizon, when $v_0^2 = a_0^2$. In the case of a radially outflowing nuclear fluid, this horizon is due to an acoustic white hole. The radius of the horizon is the critical radius, r_c, that cannot be breached by any acoustic disturbance (carrying any kind of information) propagating against the bulk outflow, after having originated in the subcritical region, where $v_0^2 < a_0^2$ and $r > r_c$. Effectively, then, we can say that the flow of information across the acoustic horizon is unidirectional.

However, this discussion is valid only as far as the linear ordering goes. When nonlinearity is to be accounted for, then instead of Eq. (17), it is Eq. (14) that defines the elements, $h^{\mu\nu}$, depending on the order of nonlinearity that one wishes to retain (in principle one could go up to any arbitrary order). The first serious consequence of including nonlinearity is that the static description of $h^{\mu\nu}$, as stated in Eq. (17), will not suffice any longer. This view is in conformity with a numerical study [31], in which, for the case of spherically symmetric astrophysical accretion, it was shown that if the perturbations were to become strong, then acoustic horizons would suffer a radial shift about their previous static position, and the analogy between an acoustic horizon and the event horizon of a black hole (or a white hole) would appear limited. We have presented an analytical perspective of this argument in Appendix A.

IV. THE STATIONARY BACKGROUND SOLUTIONS

Carrying forward the understanding that with $\zeta = 0$, an acoustic horizon may be expected when $v_0^2 = a_0^2$, we now look for the stationary profile of the dispersion-free flow. To this end, apart from the steady continuity condition, as given by Eq. (9), we also need the Bernoulli equation, obtained from the stationary limit of Eq. (7). This gives

$$\frac{v_0^2}{2} + \frac{c_s^2}{2n_e^2}\left(3n_0^2 - 4n_e n_0\right) = E, \tag{19}$$

with E being the Bernoulli constant. It is expedient to work with dimensionless variables, under the scaling prescription, $x = n_0/n_e$, $y = v_0/c_s$, $u = a_0/c_s$, and $R = r/\sqrt{f_0(c_s n_e)^{-1}}$. In that case, Eqs. (9) and (19) can, respectively, be recast as

$$xyR^2 = 1 \tag{20}$$

and

$$y^2 + 3x^2 - 4x = B, \tag{21}$$

where $B = 2E/c_s^2$ is the scaled Bernoulli constant. The two foregoing equations provide a one-parameter family of solutions, upon either retaining y and eliminating x or vice versa. We choose the former approach, because the condition of the acoustic horizon, which we already know to be $v_0^2 = a_0^2$, is better understood in terms of the velocity of the flow. So making use of the stationary function of a_0^2, as given by Eq. (15), which we scale as $u^2 = x(3x - 2)$, we obtain

$$y_c^2 = u_c^2 = x_c(3x_c - 2), \tag{22}$$

where we have used the subscript "c" to label all values at the horizon. The radius of the horizon, in the scaled set of variables, is given by the condition $R_c^2 = (x_c y_c)^{-1}$. With the help of Eq. (22), we can, therefore, write

$$R_c^4 = \frac{1}{x_c^3(3x_c - 2)}. \tag{23}$$

Our next task would be to show that R_c has a fixed value. This can only happen when x_c has a fixed value, a condition that can be obtained by combining Eqs. (21) and (22) to yield

$$x_c^2 - x_c - (B/6) = 0. \tag{24}$$

With x_c having been fixed thus, R_c also becomes a fixed quantity. Keeping only the physically meaningful positive root of the discriminant of Eq. (24), we get $x_c = [1 + \sqrt{1 + (2B/3)}]/2$. We note that at the horizon, $x_c > 1$, i.e., $n_0 > n_e$.

Now we find the stationary velocity profile, $y \equiv y(R)$, which we obtain from Eqs. (20) and (21) as

$$y^4 - By^2 - \frac{4y}{R^2} + \frac{3}{R^4} = 0. \tag{25}$$

What we have got is the equation of a quartic polynomial, a mathematical condition that would have remained qualitatively unaltered if we had chosen the density profile, $x(R)$, for our study. Before we solve for $y(R)$, it would be instructive to look at the first derivative of y. This is given by

$$\frac{dy}{dR} = \frac{2(3 - 2yR^2)}{R^3(2y^3R^2 - ByR^2 - 2)}. \tag{26}$$

The turning point in the velocity profile occurs when the numerator in the right-hand side of the first derivative vanishes. Making use of Eq. (20), we find that the turning point corresponds to $x = 2/3$, which we recall to be the limiting value of x, below which the flow loses its continuum character. The singular point in the flow, when the denominator in the right-hand side of the derivative vanishes, offers a more interesting insight. In this case, going by Eqs. (20) and (21), we actually arrive at the condition $x^2 - x - (B/6) = 0$, which is precisely the condition that prevails at the horizon, as given by Eq. (24). The simple conclusion to draw here is that the velocity profile becomes singular at the horizon. Further, taking the reciprocal of Eq. (26) at the singular point, we get $dR/dy = 0$, which means that the horizon is also the minimum radial position that flow solutions may reach. No solution is admitted within the horizon.

Now, to know what the stationary velocity profile looks like, we first have to view Eq. (25) in the standard form of a quartic equation, $y^4 + 2A_3y^3 + A_2y^2 + 2A_1y + A_0 = 0$, noting the equivalence, $A_3 = 0$, $A_2 = -B$, $A_1 = -2/R^2$, and $A_0 = 3/R^4$. Evidently, Eq. (25) will yield four roots. These roots can be found analytically by using Ferrari's method of solving quartic equations. In order to do so, a term going like $(\Upsilon_1 y + \Upsilon_2)^2$ is to be added to both sides of Eq. (25), and then the resulting left-hand side is required to be a perfect square in the form $(y^2 + \kappa)^2$, so that the full equation will be rendered as $(y^2 + \kappa)^2 = (\Upsilon_1 y + \Upsilon_2)^2$. This will deliver three conditions going as $\kappa^2 = A_0 + \Upsilon_2^2$, $\Upsilon_1\Upsilon_2 = -A_1$, and $2\kappa = A_2 + \Upsilon_1^2$. Eliminating Υ_1 and Υ_2 from these three conditions will deliver an auxiliary cubic equation in κ going as $2\kappa^3 - A_2\kappa^2 - 2A_0\kappa + (A_2A_0 - A_1^2) = 0$, which, under the transformation, $\kappa = \Psi + (A_2/6)$, can be reduced to the canonical form of the cubic equation, $\Psi^3 + P\Psi + Q = 0$, with $P = -(A_2^2/12) - A_0$ and $Q = -(A_2^3/108) + (A_2A_0/3) - (A_1^2/2)$. Analytical solutions of the roots of the cubic equation in Ψ can be obtained by the application of the Cardano–Tartaglia–del Ferro method of solving cubic equations. This will lead to the solution

$$\Psi = \left(-\frac{Q}{2} + \sqrt{\mathcal{D}}\right)^{1/3} + \left(-\frac{Q}{2} - \sqrt{\mathcal{D}}\right)^{1/3}, \tag{27}$$

with the discriminant, $\mathcal{D}$, having been defined by $\mathcal{D} = (Q^2/4) + (P^3/27)$. The sign of $\mathcal{D}$ is crucial here. If $\mathcal{D} > 0$, then there will be only one real root of Ψ, given directly by Eq. (27). On the other hand, if $\mathcal{D} < 0$, then there will be three real roots of Ψ, all of which, under a new definition, $\vartheta = \arccos[-Q/\sqrt{-4(P/3)^3}]$, can be expressed in a slightly modified form as

$$\Psi_j = 2\sqrt{\frac{-P}{3}}\cos\left[\frac{\vartheta + 2\pi(j-1)}{3}\right], \tag{28}$$

with the label j taking the values $j = 1, 2, 3$ for the three distinct roots. Of these three roots, the one corresponding to $j = 1$ continues smoothly over to the root given by Eq. (27). So this root, Ψ_1, is the one of our choice. Once Ψ is known thus, it is a simple task thereafter to find κ, Υ_1, and Υ_2, all of which depend on R and B. With this having been accomplished, all the four roots of y in Eq. (25) can be obtained by solving the two distinct quadratic equations, $y^2 + \kappa = \pm(\Upsilon_1 y + \Upsilon_2)$. Since we are concerned with a physical outflow with positive values of the velocity profile, we choose the upper sign, and in consequence, the two roots we get are

$$y = \frac{1}{2}\left[\Upsilon_1 \pm \sqrt{\Upsilon_1^2 - 4(\kappa - \Upsilon_2)}\right]. \tag{29}$$

As soon as y is obtained for a value of R, we can get $x(R)$ from Eq. (20). Thereafter, $u(R)$ also becomes known.

The overall behavior of the stationary background flow is best comprehended from Fig. 1, in which, both $y(R)$ and $u(R)$ have been plotted. Either function has got two branches, corresponding to the two signs of the discriminant in Eq. (29). That there are two solutions of y for every value of R, is actually a consequence of the invariance of Eqs. (20) and (21) under the transformation, $y \longrightarrow -y$. A similar symmetry exists in the mathematical problem of stationary

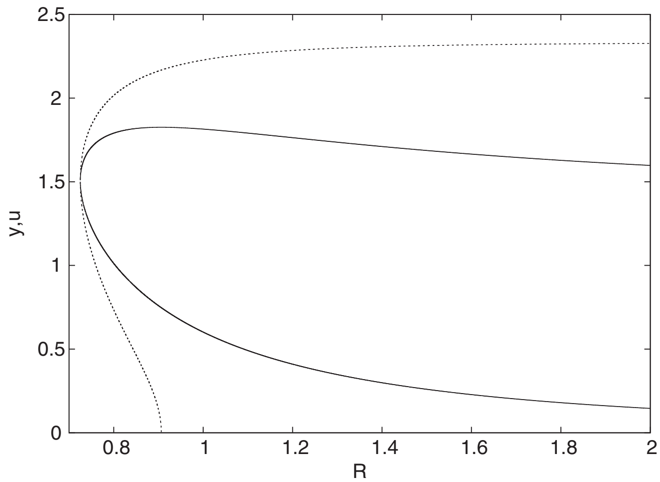

FIG. 1. The plot of the velocity solution, $y(R)$, as Eq. (29) gives it, is depicted by the inner continuous curve (for $B = 2$). There are two branches of this curve, the upper one corresponding to the positive sign of the discriminant in Eq. (29) and the lower one corresponding to the negative sign. Both branches meet at $y \simeq 1.5$, where the flow becomes singular. The radial coordinate of this point is the minimum radius of the flow, as well as the radius of the acoustic horizon. There is a maximum point in the upper branch, when $x = 2/3$. The outer envelope of a dotted curve shows the two branches of $u(R)$, the speed of acoustic propagation. There are two branches of this curve as well, both meeting $y(R)$ at the horizon. The upper flat branch of u traces the acoustic profile that goes with the lower branch of the velocity profile, y. The lower branch of the acoustic profile, going with the upper branch of y, vanishes abruptly when $x = 2/3$. This is the same position where the upper branch of y reaches its maximum. So, beyond this point, the upper branch of y loses the nature of a hydrodynamic continuum, with no acoustic propagation being possible in it. The lower branch of y maintains its hydrodynamic features globally, and decays asymptotically as $y \sim R^{-2}$.

accretion or wind [32]. Both the branches of y and u meet at $y = u \simeq 1.5$, where the flow becomes singular and where the discriminant of Eq. (29) vanishes. The radial coordinate of this point is the minimum radius of the flow and, as we are already aware, the radius of the acoustic horizon. This effectively means that the flow starts from the acoustic horizon itself, with no physical flow (solution of y) being able to penetrate the horizon. While this may seem like a surprising result, precedence of it exists in the two-dimensional ideal shallow-water flow [12]. To understand this condition mathematically, we need to realize that in Eq. (21), there is no strong attractor term, dependent on the radial coordinate, that will enable the flow to originate as deep as possible. In contrast, in stationary astrophysical accretion, gravity furnishes just such a term and makes the flow attain as small values of the radial coordinate as would be feasible under a given inner boundary condition [30,33,34]. One can also test that when gravity is "switched off" in the momentum balance equation of the stationary accretion or wind problem, then the global flow features become qualitatively similar to what has been shown in Fig. 1, with velocity solutions of either the accretion branch or the wind branch surviving only outside the acoustic horizon. Apropos of our present problem on nuclear fluid flows, we conjecture that nuclear forces, which are strongly attractive on small length scales, could be a candidate mechanism for a stationary flow solution to physically breach the acoustic horizon.

V. TRAVELING WAVES AND DISPERSION

So far our discussion has been restricted to the case of $\zeta = 0$. However, if the coupling term governing the nonlocal baryon–vector meson interaction is revived even to a very small extent, i.e., when weak dispersion is accounted for in Eq. (13), then the symmetric structure implicit in Eq. (16) will be disrupted. Consequently, the precise condition of an acoustic horizon will be lost. We note that this disruptive effect is analogous to that caused by viscosity (dissipation) in the flow for the case of the hydraulic jump [8,13] or that due to the coupling of the flow with the geometry of space-time in general relativistic spherical accretion [35]. Nevertheless, the most important feature to emerge from the analogy of a white hole horizon shall remain qualitatively unchanged, namely, an acoustic disturbance propagating upstream from the subcritical flow region, where $v_0^2 < a_0^2$, encounters an insurmountable obstacle when $v_0^2 = a_0^2$.

We take up Eq. (13) in its linearized limit, which means that all $h^{\mu\nu}$ are to be read from Eq. (17). However, now we also account for dispersion in Eq. (13), with $\zeta \neq 0$. We then treat the perturbation as a high-frequency traveling wave, whose wavelength, λ, is much less than the natural length scale in the fluid system, r_c, the radius of the acoustic horizon, i.e., $\lambda \ll r_c$. Usually linear perturbations do not destabilize the stationary background, especially when there are no source-like terms in the right-hand side of Eq. (13). However, the merest presence of such terms gives rise to varying physical behavior of the linear perturbation. For instance, in the case of the shallow-water circular hydraulic jump, a viscosity-dependent source term causes a large divergence in the amplitude of the perturbation in the vicinity of the horizon [8]. In contrast, the curvature of space-time in general relativistic spherical accretion has a strongly stabilizing effect on the perturbation [35]. Along these lines, we now ask how the stationary background flow is affected by the dispersive source-like term in the right hand side of Eq. (13). The background flow of interest for us is represented by the lower branch of the stationary velocity function in Fig. 1. Hydrodynamic features are globally preserved along this branch, starting from the horizon to arbitrarily large outer radii of the flow. Under these specifications, we use a solution of f', in which we separate spatial and temporal dependence by

$$f'(r,t) = \exp[is(r) - i\omega t], \tag{30}$$

with the understanding that ω is much greater than any characteristic frequency of the system. Applying the foregoing solution to Eq. (13), and on multiplying the resulting expression throughout by $(f_0/f')v_0^{-1}$, we arrive at

$$-\omega^2 + 2v_0\omega\frac{ds}{dr} + \left(v_0^2 - a_0^2\right)\left[i\frac{d^2s}{dr^2} - \left(\frac{ds}{dr}\right)^2\right] - 2i\omega\frac{dv_0}{dr} + \frac{i}{v_0}\frac{d}{dr}\left[v_0\left(v_0^2 - a_0^2\right)\right]\frac{ds}{dr}$$
$$= \zeta n_0\left\{\left[i\frac{d^4s}{dr^4} - 4\frac{ds}{dr}\frac{d^3s}{dr^3} - 6i\left(\frac{ds}{dr}\right)^2\frac{d^2s}{dr^2} - 3\left(\frac{d^2s}{dr^2}\right)^2 + \left(\frac{ds}{dr}\right)^4\right]\right.$$
$$\left. - \frac{4}{r}\left[i\frac{d^3s}{dr^3} - 3\frac{ds}{dr}\frac{d^2s}{dr^2} - i\left(\frac{ds}{dr}\right)^3\right] + \frac{8}{r^2}\left[i\frac{d^2s}{dr^2} - \left(\frac{ds}{dr}\right)^2\right] - i\frac{8}{r^3}\frac{ds}{dr}\right\}. \quad (31)$$

It is clear from Eqs. (30) and (31) that s should have both real and imaginary components. Therefore, we write $s(r) = \alpha(r) + i\beta(r)$, and viewing this solution along with Eq. (30), we can see that while α contributes to the phase of the perturbation, β contributes to its amplitude.

Our approach to obtaining solutions of both α and β is by the WKB analysis of Eq. (31) for high-frequency traveling waves. However, a look at Eq. (31) also reveals that the highest orders in it are quartic. The saving grace is that all orders higher than the second order are dependent on ζ, whose involvement has been designed in our analysis to be very feeble anyway. We exploit this fact by first setting $\zeta = 0$ in the right-hand side of Eq. (31) and then we solve the second-order differential equation implied by the ζ-independent left-hand side of this equation. To stress this special case, we also modify our solution as, $s_0(r) = \alpha_0(r) + i\beta_0(r)$. Using this in Eq. (31), in which now $\zeta = 0$, we first separate the real and the imaginary parts, which are then individually set equal to zero. The WKB prescription stipulates that $\alpha_0 \gg \beta_0$. Going by this requirement, we therefore collect only the real terms which do not contain β_0 and solve a resulting quadratic equation in $d\alpha_0/dr$ to obtain

$$\alpha_0 = \int \frac{\omega}{v_0 \mp a_0}\, dr. \quad (32)$$

Likewise, from the imaginary part, in which we need to use the solution of α_0, we obtain

$$\beta_0 = \tfrac{1}{2}\ln(v_0 a_0) + C, \quad (33)$$

with C being a constant of integration.

It should be pertinent now to perform a self-consistency check on α_0 and β_0, according to the condition of the WKB analysis that $\alpha_0 \gg \beta_0$. First, we note that with regard to the frequency, ω, of the high-frequency traveling waves, α_0 (containing ω) is of a leading order over β_0 (containing ω^0). Next, on very large scales of length, i.e., $r \longrightarrow \infty$, the background velocity goes asymptotically as $v_0 \longrightarrow 0$, and the corresponding speed of acoustic propagation, a_0, approaches a constant asymptotic value. In that case $\alpha_0 \sim \omega r$, from Eq. (32). Moreover, on similar scales of length, going by Eqs. (9) and (33), we see that $\beta_0 \sim \ln r$. Further, near the acoustic horizon, where $v_0 \simeq a_0$, for the wave that goes against the bulk flow with the speed, $v_0 - a_0$, we obtain large values of α_0. Using all of these facts together, we see that our solution scheme is very much in conformity with the WKB prescription.

Thus far we have worked with $\zeta = 0$ (absence of dispersion). To know how dispersion affects the traveling wave, we now need to find a solution of s from Eq. (31), with $\zeta \neq 0$. To this end we adopt an iterative approach, with an imposition of the condition that ζ has a very small value. To put this in a properly quantified perspective, we first find a scale of ζ. A simple dimensional analysis enables us to set this scale as $\zeta_s = c_s^2 r_c^2/n_e$. We then write $\zeta = \eta\zeta_s$, where η is a dimensionless parameter that tunes the numerical value of ζ. In observance of the condition of our iterative method, we require that $\eta \ll 1$.

Our next step is to take up Eq. (31) in its full form (with $\zeta \neq 0$ now) and propose a solution for it as $s = s_0 + \delta s_1$, with δ being another dimensionless parameter like η, obeying the same requirement, i.e., $\delta \ll 1$. Therefore, in the right-hand side of Eq. (31) all terms which carry the product, $\eta\delta$, can be safely neglected as being very small. This, in keeping with the principle of our iterative treatment, will effectively mean that all the surviving dispersion-related terms in the right-hand side of Eq. (31) will go as ηs_0. Further, by the WKB analysis, we have also assured ourselves that $\alpha_0 \gg \beta_0$. We can afford to ignore all the β_0-dependent terms as well, in the right-hand side of Eq. (31), when we compare them with all the terms containing α_0. Finally, the most dominant α_0-dependent real term to stand out in the right-hand side of Eq. (31) is of the fourth degree, $(d\alpha_0/dr)^4$. Preserving only this term in the right-hand side of Eq. (31) and extracting only the β-independent real terms from the left-hand side, we are left with a simple quadratic equation,

$$\left(v_0^2 - a_0^2\right)\left(\frac{d\alpha}{dr}\right)^2 - 2v_0\omega\frac{d\alpha}{dr} + \left[\omega^2 + \eta\zeta_s n_0\left(\frac{d\alpha_0}{dr}\right)^4\right] = 0, \quad (34)$$

to solve for α. Under the provision of $\eta \ll (\lambda/r_c)^2$, which is well in accordance with our requirement that η may be arbitrarily small, we obtain a solution of α, going as

$$\alpha \simeq \alpha_0 \mp \frac{1}{2}\eta\zeta_s\omega^3 \int \frac{n_0}{a_0(v_0 \mp a_0)^4}\, dr. \quad (35)$$

Similarly, to solve for β, we extract all the imaginary terms from the left-hand side of Eq. (31), and noting that the most dominant contribution to the imaginary terms in the right-hand side comes from the cubic-order terms involving α_0, we are required to solve the equation,

$$2\left[v_0\omega-\left(v_0^2-a_0^2\right)\frac{d\alpha}{dr}\right]\frac{d\beta}{dr}-2\omega\frac{dv_0}{dr}+\frac{1}{v_0}\frac{d}{dr}\left[v_0\left(v_0^2-a_0^2\right)\frac{d\alpha}{dr}\right]=4\eta\zeta_s\frac{n_0}{r}\left(\frac{d\alpha_0}{dr}\right)^3\left\{1-\frac{3}{2}\frac{d[\ln(d\alpha_0/dr)]}{d(\ln r)}\right\},\quad(36)$$

from which, on using Eq. (35), once again under the condition that $\eta\ll(\lambda/r_c)^2$, we obtain

$$\beta\simeq\beta_0-\eta\zeta_s\omega^2\mathcal{H}(r),\quad(37)$$

with $\mathcal{H}$ to be expressed fully as

$$\mathcal{H}(r)=\frac{1}{4}\frac{n_0}{a_0^2}\left[\frac{v_0^2-a_0^2}{(v_0\mp a_0)^4}\right]\mp 2\int\frac{n_0}{ra_0(v_0\mp a_0)^3}\left\{1+\frac{3}{2}\frac{d[\ln(v_0\mp a_0)]}{d(\ln r)}\right\}dr.\quad(38)$$

The significant aspects of both Eqs. (35) and (37) are that in the former, the correction to the zero-dispersion condition is of the order of ω^3 (an odd order contributing to the phase), and in the latter a similar correction is of the order of ω^2 (an even order contributing to the amplitude). Now, noting that α_0 is of the order of ω and β_0 is of the order of ω^0, the corrections to the zero-dispersion terms in both Eqs. (35) and (37) appear, in the high-frequency regime, to be dominant over their respective zeroth orders. This, however, is not really the case, as we shall argue. We have obtained the results given by Eqs. (35) and (37) under the restriction that $\eta\ll(\lambda/r_c)^2$. Once we view the wavelength, λ, as $\lambda(r)=2\pi(v_0\mp a_0)/\omega$, we immediately see that the combination of $\eta\omega^2$ in Eqs. (35) and (37) reduces both the correction terms on α_0 and β_0 to be subleading to their respective zeroth-order terms. While this appears to be true over most of the spatial range of the flow, an exception is to be made in the close neighbourhood of the acoustic horizon, where $v_0=a_0$. In this region, looking at Eq. (37) in particular, we see that the correction on β_0 diverges, while β_0 itself remains finite. Hereafter, we are interested primarily in the correction on β_0.

Going back to Eq. (30), we are able to write $f'(r,t)=e^{-\beta}\exp(i\alpha-i\omega t)$, from which, by extracting the amplitude part only and also making use of Eqs. (33) and (37), we get

$$|f'(r,t)|\simeq\frac{\tilde{C}}{\sqrt{v_0a_0}}\exp[\eta\zeta_s\omega^2\mathcal{H}(r)],\quad(39)$$

where $\tilde{C}$ is a constant. The influence of dispersion on the amplitude can be seen from Eq. (38), which, for a wave moving upstream against the bulk outward flow, indicates a divergence at the acoustic horizon, where $v_0=a_0$. However, a careful examination shows that if the wave moves towards the acoustic horizon from the subcritical region, where $v_0<a_0$, then the divergence in $\mathcal{H}$ carries a negative sign. Raised to an exponent, as indicated by Eq. (39), this will mean that the amplitude of the wave will decay to zero at the acoustic horizon, which behaves like an impervious wall to any signal approaching it from the subcritical region. The energy flux of the perturbation also behaves in a manner similar to its amplitude, as we show in Appendix B. Contrary to the wave traveling inwards, the solution of the perturbation traveling outwards with the bulk flow must originate at or just outside the horizon itself, since no steady background solutions are admitted within the horizon. So, taking all these facts together, we say that the passage of information at the horizon is unidirectional, and the horizon may be viewed analogously as a white hole. This point of view is, however, different from the analog of the Hawking radiation in the problem of an acoustic metric, in which, without the kind of dispersive correction that we have used here, an acoustic signal is allowed to cross the acoustic horizon with a finite and nonzero (but spatially decreasing) amplitude [1,4,10]. At the horizon of a general relativistic black hole, a similar feature is seen on carrying out a WKB-type analysis of incoming and outgoing probability amplitudes [36]. In contrast, in our study, not only does the amplitude of the perturbation drop sharply to zero at the horizon, making the horizon fully opaque to acoustic signals, but also physical flow solutions meet a dead end at an infinitely rigid horizon surface. Going by the former observation, our system is more akin to a classical black hole without any analog of the Hawking radiation.

In passing we also examine the possibilities presented by viscosity, although it has a very feeble presence in the type of nuclear fluid that we are studying. Nevertheless, if we had included viscous effects in our study, then the right-hand side of Eq. (5) would have contained terms like $\eta^\star\nabla^2\mathbf{v}$ and $[(\eta^\star/3)+\zeta^\star]\nabla(\nabla\cdot\mathbf{v})$, where $\eta^\star$ and $\zeta^\star$ are the first and second coefficients of viscosity, respectively [37]. For the case of a compressible, irrotational, and spherically symmetric flow, the differential operators in the two viscosity terms assume identical forms, bearing only the radial variation of the velocity field, $v(r,t)$ [38]. Taking the time derivative of these terms, and then making use of Eq. (12) in the context of our perturbation scheme, will involve viscosity in the field equation of $f'(r,t)$, with the stationary coefficients of the viscosity terms containing second-order spatial derivatives of $v_0(r)$. Near the horizon, dv_0/dr approaches very high values, and the viscosity-dependent terms, making dominant contributions to the perturbation, shall emerge with d^2v_0/dr^2 and $(dv_0/dr)^2$. This will be similar to the dispersion-related term in the right-hand side of Eq. (36), and so we can say that viscosity will enter the amplitude of the perturbation at the same order as dispersion.

VI. CONCLUDING REMARKS

In this work we have studied nuclear fluids from a hydrodynamic perspective, with baryon-vector meson interactions bringing dispersion as a novelty to the standard hydrodynamics. Not accounting for dispersion, the hydrodynamic flow yields an analog metric, in the likeness of what is seen for a scalar field in curved space-time. Extending this point of view, we have found a critical horizon in the flow of a nuclear fluid, a feature that is reminiscent of a white hole. So, for a nuclear fluid flowing radially outwards, the horizon will be an opaque barrier to a wave propagating radially inwards through the fluid. This effect becomes particularly pronounced when dispersion forces the amplitude of an acoustic signal to suffer a much stronger decay than what the simple hydrodynamics might admit, and this is in stark contrast to the Hawking radiation in an analog black hole, where the horizon is not entirely opaque to an outgoing signal originating within the horizon.

With dispersion incorporated, however, the symmetric form of an analog metric is lost. This is similar to the way in which the coupling of the flow and the geometry of Schwarzschild space-time adversely affects the clearcut horizon condition obtained otherwise in the Newtonian construct of space and time [35]. Qualitatively speaking, the same behavior is also exhibited by viscous dissipation [8]. The influence of dispersive effects on the amplitude of high-frequency traveling waves can also be compared to the way viscous dissipation can act under similar circumstances. In the low-dimensional problem of the shallow-water hydraulic jump, viscosity is known to enhance the amplitude of a high-frequency traveling wave, as it moves against a radially outward bulk flow and approaches the acoustic horizon of an analog white hole from the subcritical flow region [8]. This is completely contrary to the way in which dispersion decays the amplitude of a wave that arrives at the acoustic horizon from the subcritical region. One way or the other, we realize now that dissipation [8], the geometry of curved space-time [35], nonlinearity [31], and dispersion all appear to disturb the symmetric structure of an acoustic metric.

An integral aspect of hydrodynamics is the equation of state, by which the pressure term in the momentum balance condition is closed. Depending on the nature of physical problems, the equation of state is prescribed variously. For instance, in astrophysical fluids, the standard formula is polytropic [33], while in the shallow-water hydraulic jump, a linear equation of state is applied [12]. The equation of state that we have used, specialized for our study of a nuclear fluid flow [21], is given by a composite function, bearing a linear term and a second-order term of the baryonic density. Hydrodynamic features hold true when the latter term is effective and break down under the dominance of the linear term. This is an unusual physical aspect designed into this problem only and is not seen when the equation of state is set by a single power-law term, as is usually the case. In a more general sense, however, as long as the equation of state, regardless of its particular form, provides a condition for an acoustic propagation, the mathematical procedure leading to an analog metric and an acoustic horizon remains universal.

ACKNOWLEDGMENTS

One of the authors (A.B.) wishes to thank the Max-Planck-Society (Germany) and the Department of Science and Technology/Indo-German Science and Technology Centre (India) for partial financial support through the Partner Group programme (2009). Comments from an anonymous referee have helped in improving this work.

APPENDIX A: NONLINEARITY AND THE ACOUSTIC HORIZON

At the end of Sec. III we indicated that the acoustic horizon would get displaced under nonlinear effects. We can now verify this contention analytically by confining our mathematical treatment to the second order of nonlinearity. All of the nonlinearity in Eq. (16) is contained in the metric elements, $h^{\mu\nu}$, involving the exact field variables, v, a, and f, as opposed to containing only their stationary background counterparts [2,3]. This is indeed going into the realm of nonlinearity, because v and a depend on f, while f is related to f' as $f = f_0 + f'$.

If we restrict ourselves to the second order of nonlinearity, we see that $h^{\mu\nu}$ in Eq. (14) will bear primed quantities in their first power only. Taken together with Eq. (13), this will, in effect, preserve all the terms which are nonlinear in the second order. So we carry out the necessary expansion of $v = v_0 + v'$, $n = n_0 + n'$, and $f = f_0 + f'$ in Eq. (14) up to the first order only. The next process to perform is to substitute both n' and v' with f'. To make this substitution possible, first we make use of Eq. (11) to represent v' in terms of n' and f'. While doing so, the product term of n' and v' in Eq. (11) is to be ignored, because including it will bring in the third order of nonlinearity. Once v' has been substituted in this manner, we have to write n' in terms of f'. This can be done by invoking the linear relationship suggested by Eq. (10), with the reasoning that if n' and f' are both separable functions of space and time, with the time part being exponential (all of which are standard mathematical prescriptions in perturbative analysis), then

$$\frac{n'}{n_0} = \sigma(r)\frac{f'}{f_0}, \tag{A1}$$

with σ being a function of r only. We self-consistently support this mathematical argument in Appendix B. The exact functional form of σ will be determined by the way the spatial part of f' is set up, but when n', v', and f' are all real fluctuations, σ should likewise be real.

Following all of these tedious but straightforward algebraic details, the nonstationary features of the elements, $h^{\mu\nu}$, in Eq. (16), can finally be expressed entirely in terms of f' as

$$\begin{aligned} h^{tt} &\simeq \frac{v_0}{f_0}\left(1+\epsilon\xi^{tt}\frac{f'}{f_0}\right), \quad h^{tr} \simeq \frac{v_0^2}{f_0}\left(1+\epsilon\xi^{tr}\frac{f'}{f_0}\right), \\ h^{rt} &\simeq \frac{v_0^2}{f_0}\left(1+\epsilon\xi^{rt}\frac{f'}{f_0}\right), \quad h^{rr} \simeq \frac{v_0}{f_0}\left(v_0^2-a_0^2\right)+\epsilon\frac{v_0^3}{f_0}\xi^{rr}\frac{f'}{f_0}, \end{aligned} \tag{A2}$$

in all of which ϵ has been introduced as a nonlinear "switch" parameter to keep track of all the nonlinear terms. When $\epsilon = 0$, only linearity remains. In this limit we return to the familiar linear result implied by Eq. (17). In the opposite extreme, when $\epsilon = 1$, in addition to the linear effects, the second order of nonlinearity becomes activated in Eq. (16), and the stationary position of an acoustic horizon gets disturbed due to the nonlinear ϵ-dependent terms. A look at h^{rr} in Eq. (A2) makes this fact evident. We also note that Eq. (A2) contains the factors, $\xi^{\mu\nu}$, which are to be read as

$$\xi^{tt} = -\sigma, \quad \xi^{tr} = \xi^{rt} = 1 - 2\sigma,$$
$$\xi^{rr} = 2 - 3\sigma\left[1 + \left(\frac{n_0}{n_e}\right)^2\left(\frac{c_s}{v_0}\right)^2\right], \tag{A3}$$

and all of which are just stationary quantities.

APPENDIX B: ENERGY FLUX OF THE TRAVELING WAVE

What we have seen as regards the amplitude of the traveling wave in Sec. V can also be seen in the energy flux associated with the traveling perturbation. We note first that the kinetic energy per unit volume of the flow is

$$\mathcal{E}_{\rm kin} = \tfrac{1}{2}M(n_0 + n')(v_0 + v')^2, \tag{B1}$$

and the internal energy per unit volume is

$$\mathcal{E}_{\rm int} = M\left[n_0\varepsilon + n'\frac{\partial}{\partial n_0}(n_0\varepsilon) + \frac{n'^2}{2}\frac{\partial^2}{\partial n_0^2}(n_0\varepsilon)\right], \tag{B2}$$

where ε is the internal energy per unit mass [37]. In both of the foregoing expressions of energy, the zeroth-order terms refer to the background flow, and the first-order terms can be made to disappear upon performing a time averaging. Thereafter, the time-averaged total energy in the perturbation, per unit volume of fluid, is to be obtained by summing the second-order terms in $\mathcal{E}_{\rm kin}$ and $\mathcal{E}_{\rm int}$. All of these terms will go either as n'^2 or v'^2, or a product of n' and v'. Our next task would be to represent both n' and v' in terms of f', and then use Eq. (39) to substitute f'. For this purpose, making use of Eqs. (10), (30), and (32), we get

$$\frac{n'}{n_0} \simeq \frac{v_0}{v_0 \mp a_0}\frac{f'}{f_0}. \tag{B3}$$

We can see now that this result is precisely what Eq. (A1) implies. Going further, we make use of Eq. (B3) in Eq. (11), ignoring the product of n' and v' in the latter, to obtain

$$\frac{v'}{v_0} \simeq \mp\frac{a_0}{v_0 \mp a_0}\frac{f'}{f_0}. \tag{B4}$$

Once we have two relations explicitly connecting n' and v' with f', as implied by Eqs. (B3) and (B4), we can derive the time-averaged total energy per unit volume as

$$\mathcal{E}_{\rm tot} = \frac{Mn_0v_0^2a_0^2}{2(v_0 \mp a_0)^2}\left[1 \mp 2\frac{v_0}{a_0} + \frac{n_0}{a_0^2}\frac{\partial^2(n_0\varepsilon)}{\partial n_0^2}\right]\frac{\langle f'^2\rangle}{f_0^2}. \tag{B5}$$

The energy flux, $\mathcal{F}$, associated with the spherical wavefront, traveling with the speed $(v_0 \mp a_0)$ is $\mathcal{F} = 4\pi r^2\mathcal{E}_{\rm tot}(v_0 \mp a_0)$. Accounting for a factor of $1/2$ due to the time-averaging of the phase part of f'^2, the flux can be written as

$$\mathcal{F} = \frac{2\pi\tilde{C}^2M}{f_0}\left\{\mp 1 - \frac{a_0}{2(v_0 \mp a_0)}\left[1 - \frac{n_0}{a_0^2}\frac{\partial^2(n_0\varepsilon)}{\partial n_0^2}\right]\right\} \times \exp[2\eta\zeta_s\omega^2\mathcal{H}(r)]. \tag{B6}$$

We realize immediately that near the acoustic horizon, the dispersion-dependent exponential factor in Eq. (B6) will have very much the same effect on the energy flux of the perturbation as it has on its amplitude, $|f'|$, given by Eq. (39).

[1] W. G. Unruh, Phys. Rev. Lett. **46**, 1351 (1981).
[2] M. Visser, Class. Quantum. Grav. **15**, 1767 (1998).
[3] R. Schützhold and W. G. Unruh, Phys. Rev. D **66**, 044019 (2002).
[4] T. K. Das, Class. Quantum. Grav. **21**, 5253 (2004).
[5] G. Volovik, JETP Lett. **82**, 624 (2005).
[6] T. K. Das, N. Bilić, and S. Dasgupta, J. Cosmol. Astropart. Phys. 06 (2007) 009.
[7] N. Roy and A. K. Ray, Mon. Not. R. Astron. Soc. **380**, 733 (2007).
[8] A. K. Ray and J. K. Bhattacharjee, Phys. Lett. A **371**, 241 (2007).
[9] C. Barceló, S. Liberati, and M. Visser, Living Rev. Relativity **14**, 3 (2011).
[10] S. J. Robertson, J. Phys. B: At. Mol. Opt. Phys. **45**, 163001 (2012).
[11] E. J. Watson, J. Fluid Mech. **20**, 481 (1964).
[12] T. Bohr, P. Dimon, and V. Putkaradze, J. Fluid Mech. **254**, 635 (1993).
[13] S. B. Singha, J. K. Bhattacharjee, and A. K. Ray, Eur. Phys. J. B **48**, 417 (2005).
[14] J. Bondorf, S. I. A. Garpmann, and J. Zimanyi, Nucl. Phys. A **296**, 320 (1978).
[15] G. F. Chapline, M. J. Johnson, E. Teller, and M. S. Weiss, Phys. Rev. D **8**, 4302 (1973).
[16] P. Csizmadia, T. Csörgő, and B. Lukács, Phys. Lett. B **443**, 21 (1998).
[17] G. N. Fowler, S. Raha, N. Stelte, and R. M. Weiner, Phys. Lett. B **115**, 286 (1982).
[18] S. Raha and R. M. Weiner, Phys. Rev. Lett. **50**, 407 (1983).
[19] E. F. Hefter, S. Raha, and R. M. Weiner, Phys. Rev. C **32**, 2201 (1985).
[20] A. M. Gleeson and S. Raha, Phys. Rev. C **21**, 1065 (1980).
[21] D. A. Fogaca and F. S. Navarra, Phys. Lett. B **639**, 629 (2006).
[22] D. A. Fogaca and F. S. Navarra, Phys. Lett. B **645**, 408 (2007).
[23] D. A. Fogaca, L. G. Ferreira Filho, and F. S. Navarra, Phys. Rev. C **81**, 055211 (2010).
[24] D. A. Fogaca, F. S. Navarra, and L. G. Ferreira Filho, Phys. Rev. D **84**, 054011 (2011).
[25] P. Romatschke and U. Romatschke, Phys. Rev. Lett. **99**, 172301 (2007).
[26] K. Dusling and D. Teaney, Phys. Rev. C **77**, 034905 (2008).
[27] M. Luzum and P. Romatschke, Phys. Rev. C **78**, 034915 (2008).
[28] M. Luzum and P. Romatschke, Phys. Rev. Lett. **103**, 262302 (2009).
[29] K. Dusling, T. Epelbaum, F. Gelis, and R. Venugopalan, Nucl. Phys. A **850**, 69 (2011).
[30] J. A. Petterson, J. Silk, and J. P. Ostriker, Mon. Not. R. Astron. Soc. **191**, 571 (1980).
[31] P. Mach and E. Malec, Phys. Rev. D **78**, 124016 (2008).

[32] A. R. Choudhuri, *The Physics of Fluids and Plasmas: An Introduction for Astrophysicists* (Cambridge University Press, Cambridge, 1999).
[33] H. Bondi, Mon. Not. R. Astron. Soc. **112**, 195 (1952).
[34] A. K. Ray and J. K. Bhattacharjee, Phys. Rev. E **66**, 066303 (2002).
[35] T. Naskar, N. Chakravarty, J. K. Bhattacharjee, and A. K. Ray, Phys. Rev. D **76**, 123002 (2007).
[36] P. Mitra, Phys. Lett. B **648**, 240 (2007).
[37] L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Butterworth-Heinemann, Oxford, 1987).
[38] A. K. Ray, Mon. Not. R. Astron. Soc. **344**, 1085 (2003).