Assignment – 1, 2 ,3 ,4 ,5

Course: SC-374

Computational and Numerical Methods

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Assignment - 1

The Binary Search and Information Entropy

Average information content is given by the formula, $<I> = -k\sum iPilog2Pi$, in which k is constant and Pi is the probability of an event.

(a) For a two-outcome problem (eg. a coin toss), Show that $<I>$ peaks at $P=12$.

$$fp= <I> = -k\ (plog2p+1-plog2 1-p)$$

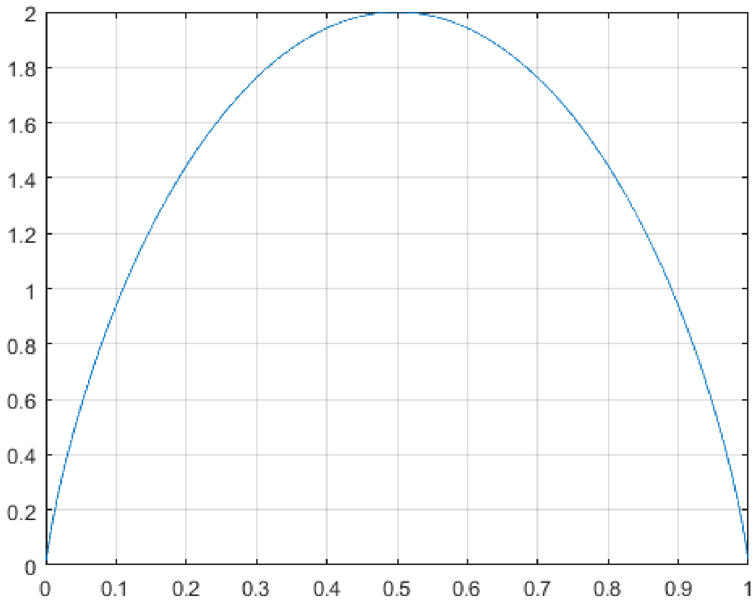
$$f'p= -k(log2 p+1-1-log2 1-p\ )$$

$$f'p=0 \text{ for maximize,}$$

$$log2 p=log2 1-p$$

$$p=1-p$$

$$p=12$$



(b) Apply a very small perturbation as P= ½ + e , in which e<<½ .Show that perturbation approach <I> = a-be^2 , where a=k and b=(4k)/ln2.

$$<I> = -k\,(\varepsilon+12\ln\varepsilon+12+12-\,\varepsilon\ln 12-\varepsilon)$$

$$<I> = -k\,(\varepsilon+12\ln 2\varepsilon+12+12-\varepsilon\ln -2\varepsilon+12)$$

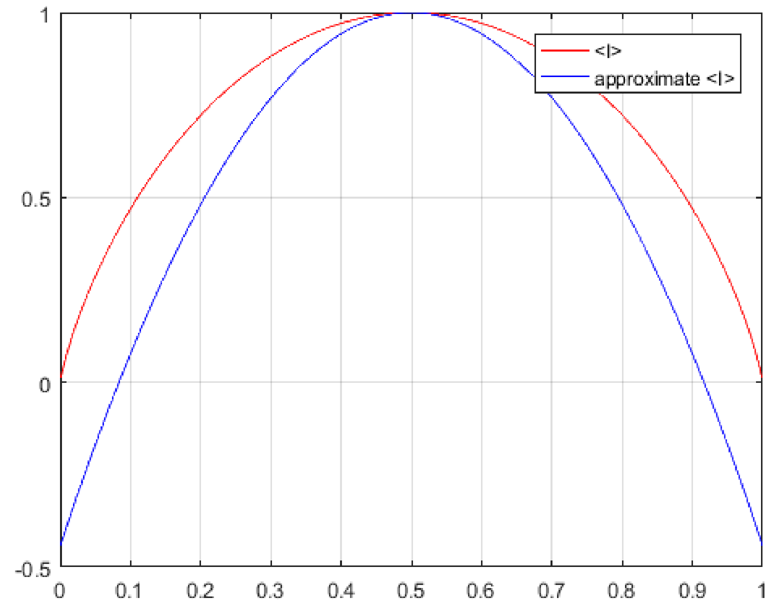
$$<I> = -k(\varepsilon+122\varepsilon\ln 2-1+12-\,\varepsilon-2\varepsilon\ln 2-1)$$

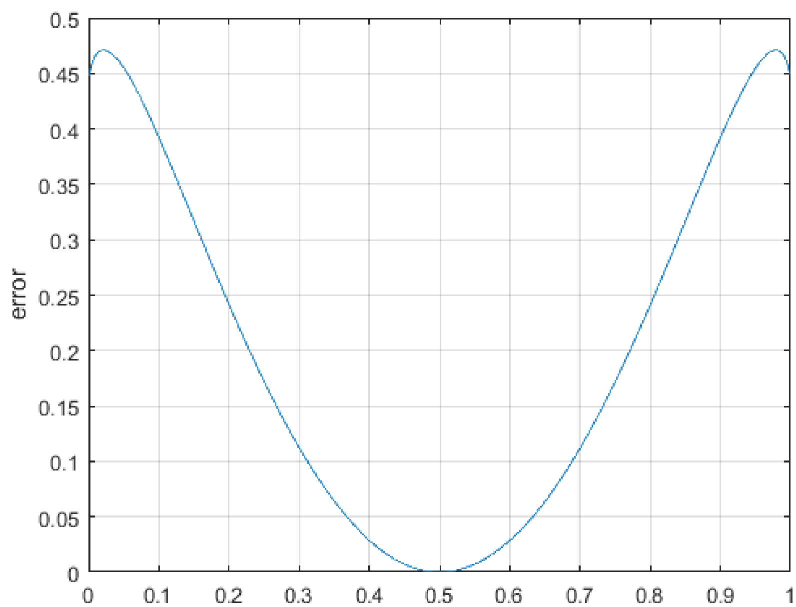
$$<I> = -k4\varepsilon 2\ln 2-1$$

$$<I> =k-4k\ln 2\varepsilon 2$$

$$a=k \text{ and } b=4k\ln 2$$

(c) plot for both the actual function and the approximate function together and then compare the graph for closeness on the line. For plotting choose k=1.





Assignment - 2

An Astrophysical Inflow

In the problem of spherically symmetric astrophysical accretion, interstellar fluid matter (a very thin gas) travels a great distance (almost from infinity) along radial lines and falls on to a massive star (or a neutron star or even a black hole) located at the origin of coordinates. The star can be treated as a point-like particle, and the rate of the fluid flow (matter flowing in unit time) on to it is given as

$$m.=\pi G2M2\rho\infty cs3\infty \ 25-3\gamma5-3\gamma2\gamma-2$$

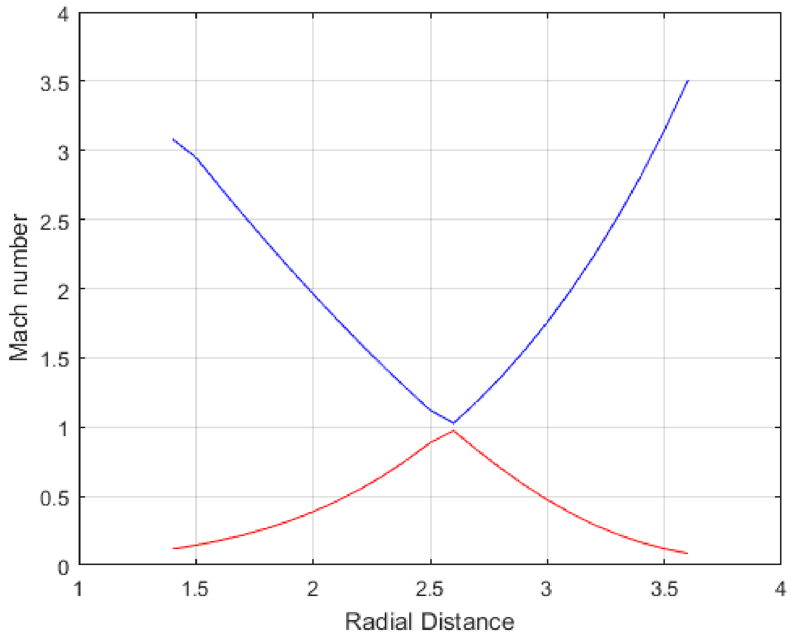
in which G is Newton's universal gravitational constant, M is the mass of the central astrophysical object, ρ∞ is the constant density of the gas at infinity, cs(∞) is the speed of sound at infinity, and γ is a dimensionless number called the polytropic exponent ($1 \leq \gamma \leq 5/3$). The velocity of the fluid flow v, as a function of the radial distance from the centre r, is given by the equation

$$fv,r=v22+n\mu.vr21n \ -GMr-ncs2\infty=0$$

$$\mu.=m.4\pi\rho\infty cs2n$$

$$n=1\gamma-1$$

Solve Eq.(1) by the bisection method to find v(r), using the values M = 2 × 1030 kg, cs(∞) = 10 km s−1, ρ∞ = 10−21 kg m−3 and n = 2.5. These values are typical of accretion of the interstellar medium on to a star. Each value of r in Eq.(1) will give a set of two real and physical roots of v. The plot of v(r) is shown in Fig. 1, in which v is scaled as the Mach number, v(r)/cs(r). Obtain a similar plot.



Assignment - 3

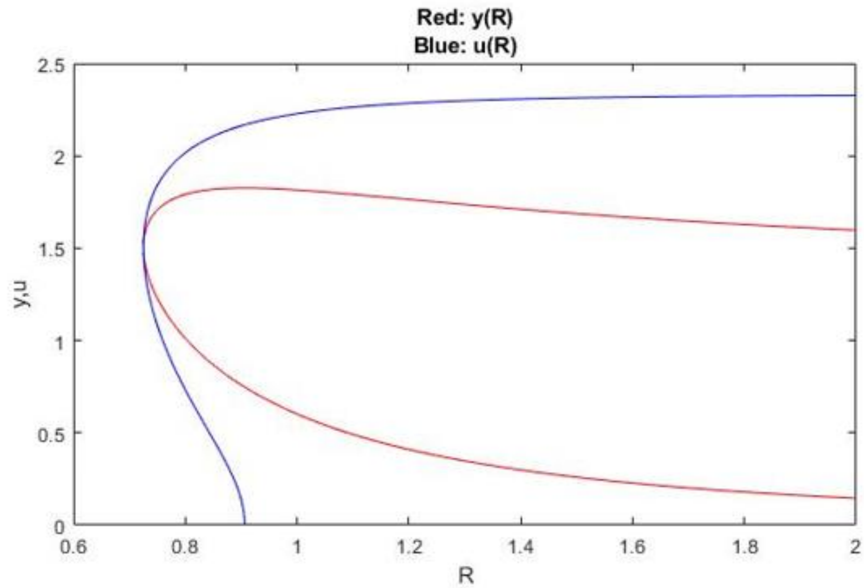
A Nuclear Outflow

High-energy impacts and collisions among elementary particles can result in an outflow of nuclear fluid. The rescaled equations of the steady outflow are,

$$xyR^2 = 1 \,,$$

$$y^2 + 3x^2 - 4x = B,$$

the velocity of an acoustic wave in the nuclear matter is $u^2 = x(3x - 2)$. On the same graph now plot R versus u.



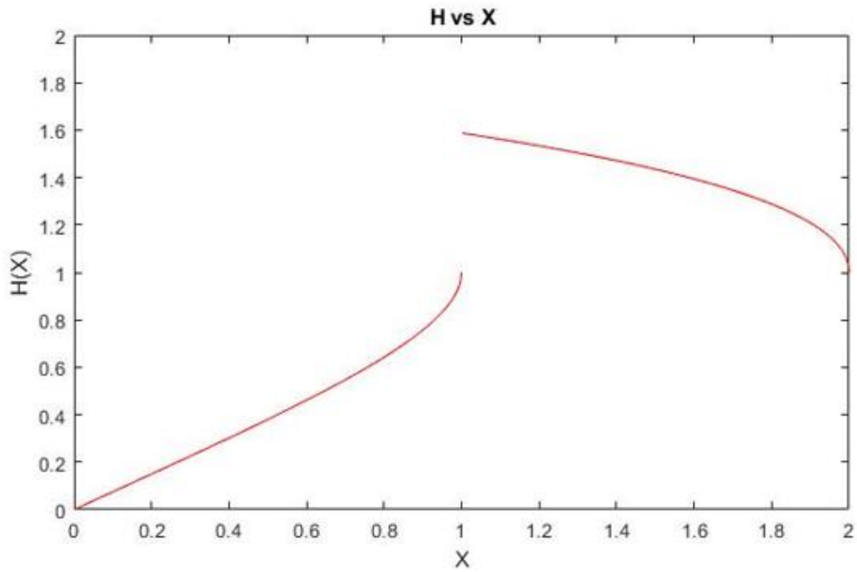
Assignment - 4

The Hydraulic Jump

In a hydraulic jump, the height of a flowing liquid increases abruptly, without any pumping action. In a steady one-dimensional liquid flow, the rescaled equation of the flow is $4H - H4 = 3\ (X - D)$, in which H is the flow height, X is the distance, and D is a constant.

(a) Restrict your study to the range $X \geq 0$, and first analyse all the implications of dH/dX.

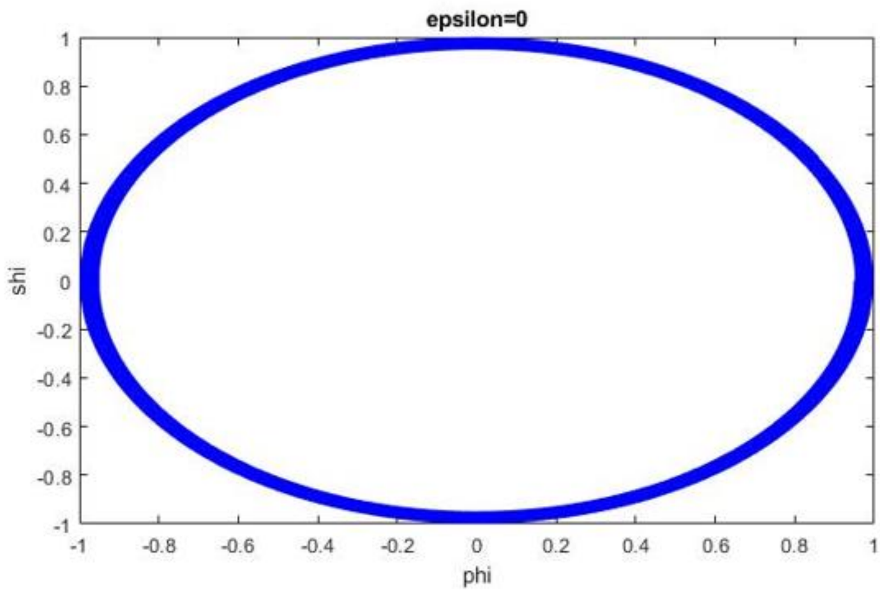
(b) Analytically solve the quartic equation $H \equiv H(X)$. For the condition $X = H = 0$, plot X along the horizontal axis and H along the vertical axis of a graph. On the same graph, repeat the plotting exercise for $H = 1$ when $X = 2$

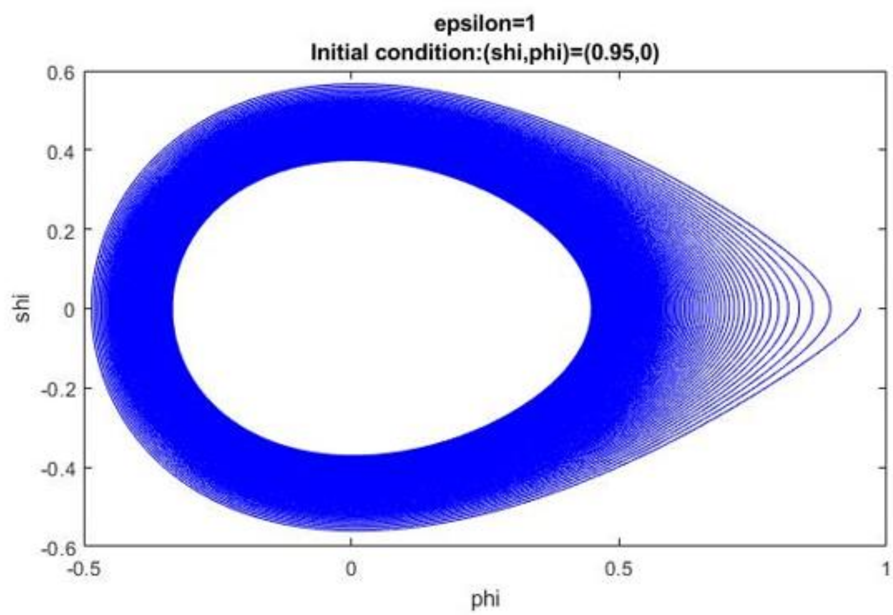


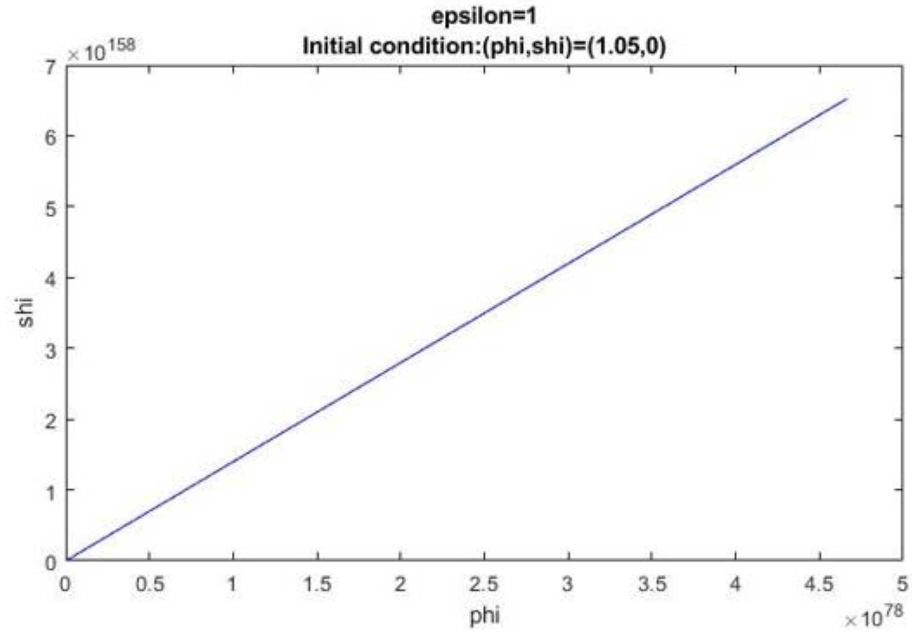
Assignment - 5

The Lienard system

Apply the fourth-order Runge-Kutta method on this system. Solve separately for the two initial conditions (φ, ψ) = (0.95, 0),(1.05, 0). Plot φ versus ψ.


epsilon=0
shi
phi
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1


epsilon=1
Initial condition:(shi,phi)=(0.95,0)
shi
phi
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.5
0
0.5
1


epsilon=1
Initial condition:(phi,shi)=(1.05,0)
×10^158
7
6
5
4
3
2
1
0
shi
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
phi
×10^78