Assignment-2

Course: SC-374

Computational and Numerical Methods

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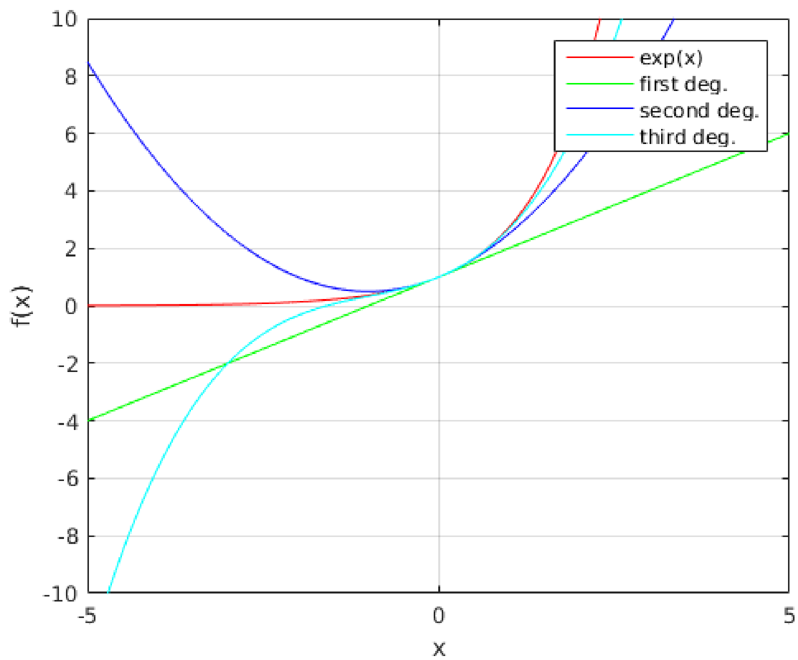
Problem: 1

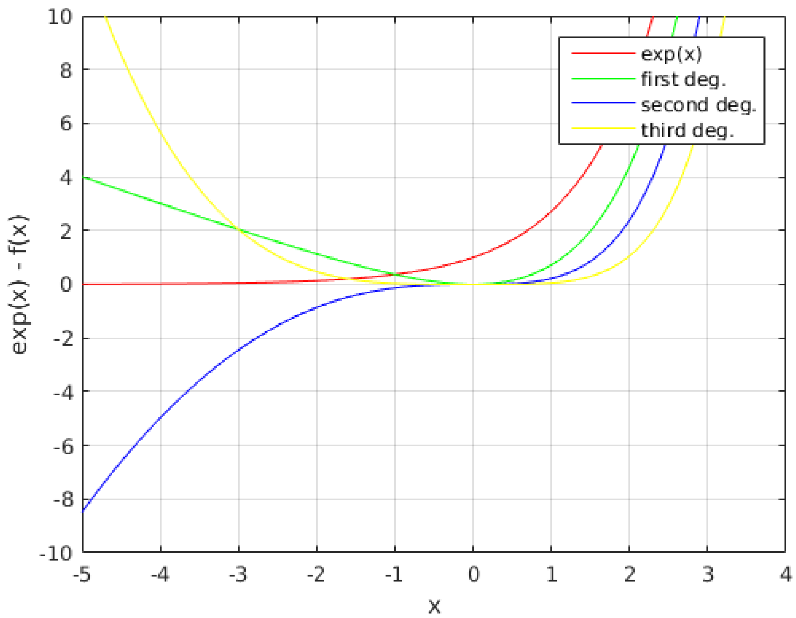
♦ **Statement:**

Consider the following functions $y = f(x)$, produce the first , the second and third-degree Taylor polynomials for each of the foregoing functions, using $a = 1$ as the point of approximation for $log x$ and $a = 0$ for the rest. In a suitably chosen neighbourhood of a , follow how the accuracy of a Taylor polynomial improves with the increasing degree. For this you will have to estimate the difference between $f(x)$ and its Taylor polynomials in a code. Present your results graphically for each function along with its Taylor polynomials of all three degrees.

(A) $\boldsymbol{y = e^x}$

♦ **Graphs:**





♦ Observations :

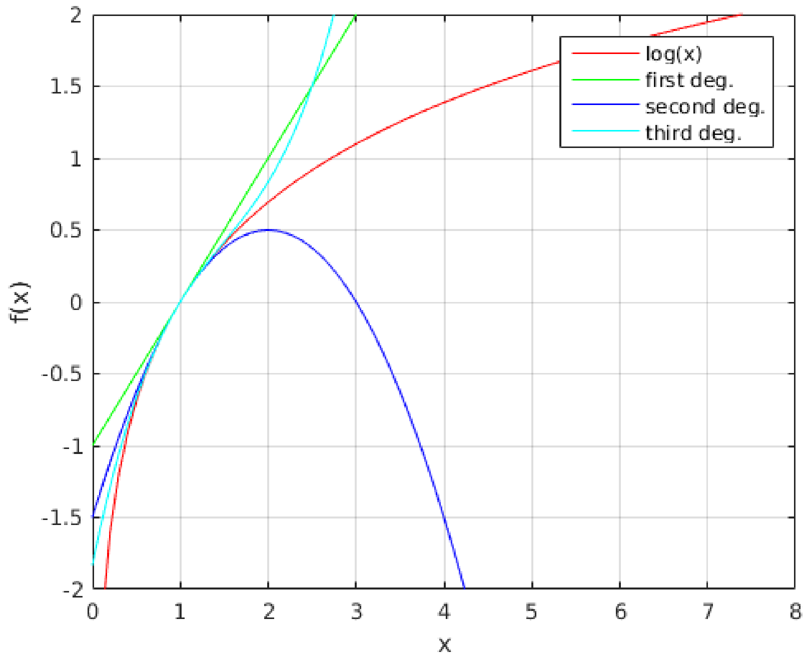
We have plotted the graph for first, second and third degree of function y.

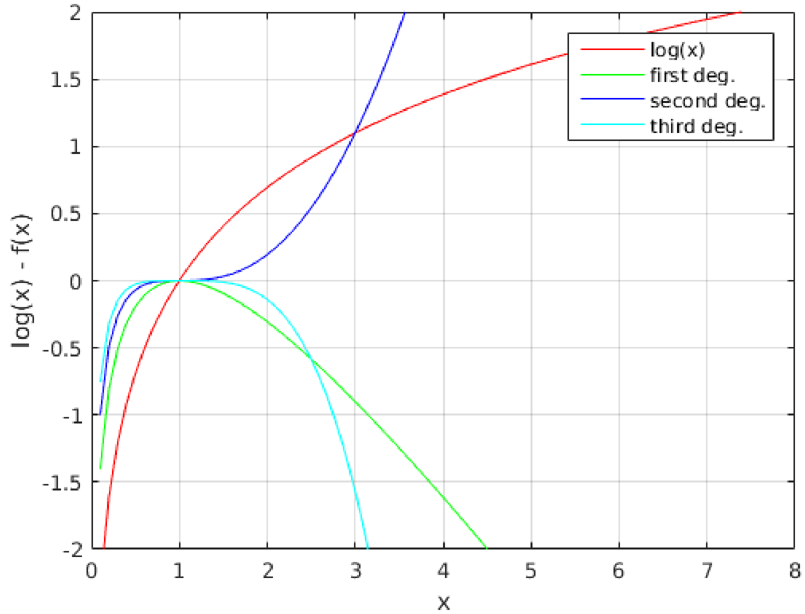
Where $f1(x) = 1 + x$, $f2(x) = f1(x) + \frac{(x^2)}{2}$, $f3(x) = f2(x) + \frac{x^3}{6}$. We can say that as the degree of polynomial increases the function comes nearer and nearer to e^x. If we compare the second-degree polynomial with first degree polynomial, we can see that second-degree polynomial is more nearer to y as compared to first degree polynomial. And if we compare third degree polynomial with second degree polynomial, we can see that third-degree polynomial is more nearer to y as compared to second degree polynomial. In the second-degree polynomial, the highest degree term is even, so for $x < 0$ the graph of second degree polynomial is above the function $y = e^x$ and first degree and third-degree polynomials are below the function $y = e^x$. for $x > 0$, $y = e^x$ increases very rapidly as compared to first, second and third-degree polynomials, because it contains much higher degree terms as compared to first, second and third-degree terms.

As we go away from $x = 0$, the difference of the function $y - f(x)$ increases. Where difference of first degree polynomial increasing rate is more than second degree polynomial. And difference of second degree polynomial increasing rate is more than third degree polynomial.

(B) $y = lnx$

♦ **Graphs :**





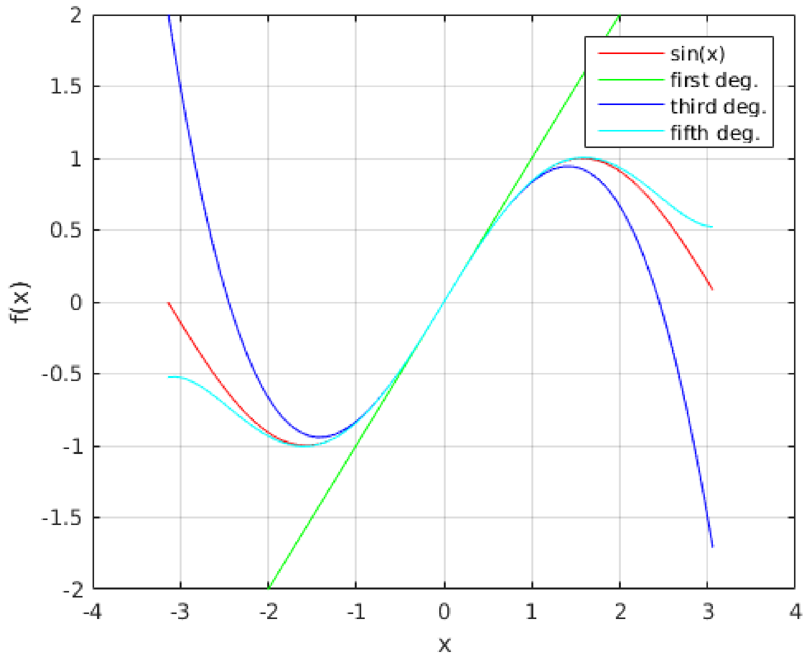
♦ Observations:

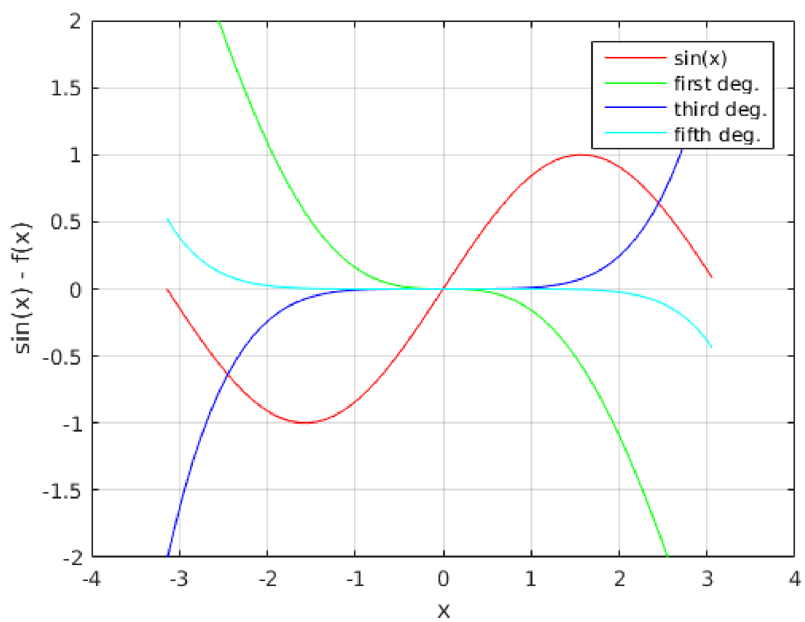
We have plotted the graph for first, second and third-degree polynomials of function y. Where $f1(x) = x - 1$, $f2(x) = f1(x) - \frac{(x-1)^2}{2}$, $f3(x) = f2(x) + \frac{(x-1)^3}{3}$. We can see that third-degree polynomial is the closest to function y as compared to first and second-degree polynomials.

When $x > 1$, the value of the function will increase for odd degree polynomials and will decrease for even degree polynomials. This can be seen in the third figure when the second-degree curve is below y and the first and third-degree curves are above y. The difference between y and the given degrees of polynomials can be seen in the figure. The discussion stated above justifies it.

(C) $\boldsymbol{y = sinx}$

♦ Graphs:





♦ Observations:

We have plotted the graph for first, second and third-degree polynomials of function y. Where $f1(x) = x$, $f3(x) = f1(x) - \frac{(x)^3}{6}$, $f5(x) = f3(x) +$

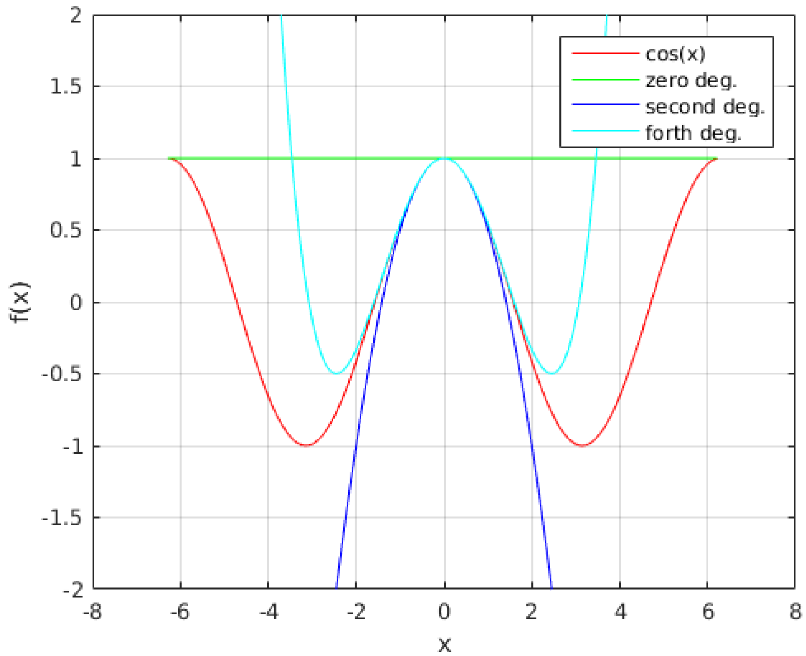
$\frac{(x)^5}{120}$. We can see that fifth-degree polynomial is the closest to function y as compared to first and third-degree polynomials.

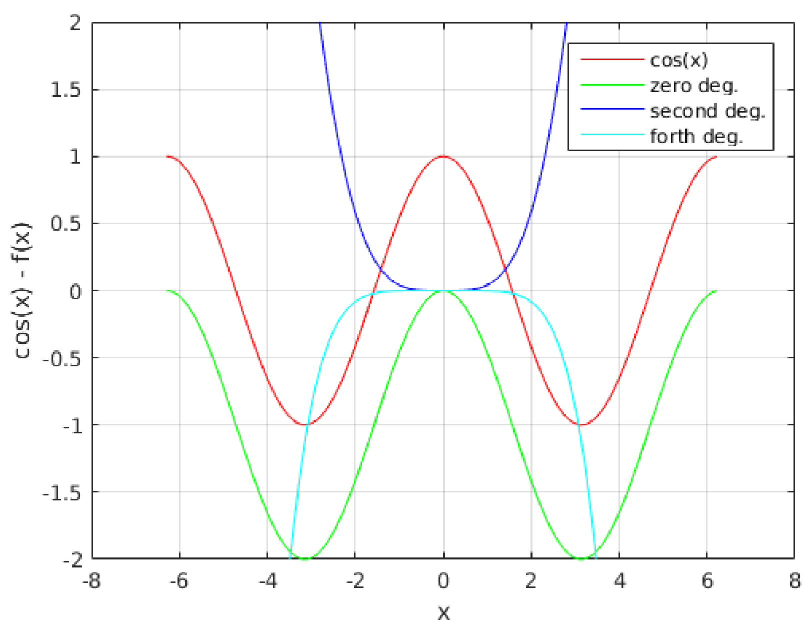
If we compare the third-degree polynomial with first degree polynomial, we can see that third-degree polynomial is more nearer to y as compared to first degree polynomial. And if we compare fifth degree polynomial with third degree polynomial, we can see that fifth-degree polynomial is more nearer to y as compared to third degree polynomial.

For the even degrees, we can say that they lie above $sin(x)$ for the negative x And the odd ones are below but in the case of the increases values of x in the positive side, the odd degree polynomials lie above, and the even ones are found Below y.

(D) y = cos(x)

♦ Graphs:





♦ Observations:

We have plotted the graph for first, second and third-degree polynomials of function y. Where $f0(x) = 1$, $f2(x) = f0(x) - \frac{(x)^2}{2}$, $f4(x) = f2(x) + \frac{(x)^4}{24}$. We can see that fourth-degree polynomial is the closest to function y as compared to zeroth and second-degree polynomials.

If we compare the second-degree polynomial with zeroth degree polynomial, we can see that second-degree polynomial is more nearer to y as compared to zeroth degree polynomial. And if we compare forth degree polynomial with second degree polynomial, we can see that forth-degree polynomial is more nearer to y as compared to second degree polynomial.

We also observe that the zeroth order approximation and the second order approximation lie above the equation y = cos(x) while the forth order equation lies below the given equation.