Assignment-9, 10, 11

Course: SC-374

Computational and Numerical Methods

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SET - 9

The Gaussian Elimination Method

Problem - 1

♦ **Statement:**

Numerically solve the following system:

$$x_1 + 2x_2 + x_3 = 0$$

$$2x_1 + 2x_2 + 3x_3 = 3$$

$$-x_1 - 3x_2 = 2$$

Roots are :-

$$x_1 = 1\,, x_2 = -1 \text{ and } x_3 = 1$$

Problem - 2

♦ **Statement:**

Numerically solve the following system:

$$4x_1 + 3x_2 + 2x_3 + x_4 = 1$$

$$3x_1 + 4x_2 + 3x_3 + 2x_4 = 1$$

$$2x_1 + 3x_2 + 4x_3 + 3x_4 = -1$$

$$x_1 + 2x_2 + 3x_3 + 4x_4 = -1$$

Roots are :-

$$x_1 = 0\,, x_2 = 1\,, x_3 = -1 \; and \; x_4 = 0$$

Problem - 3

♦ **Statement:**

Numerically Find the inverse of the following matrix:

$$\begin{matrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{matrix}$$

Inverse Matrix is:-

$$\begin{matrix} 2 & -1 & 0 \\ 1.5 & -0.5 & 0.5 \\ 2.5 & -1.5 & 0.5 \end{matrix}$$

SET - 10

The Jacobi Iteration and The Gauss-Seidel Methods

Problem - 1

♦ **Statement:**

Numerically solve the following system of equations by both the Jacobi iteration and the Gauss-Seidel methods. Compare the efficiency of both methods.

$$9x_1 + x_2 + x_3 = 10$$

$$2x_1 + 10x_2 + 3x_3 = 19$$

$$3x_1 + 4x_2 + 11x_3 = 0$$

Take initial guess values of $x_1^0 = x_2^0 = x_3^0 = 0$. Try two more sets of initial guess values of your choice, and check for convergence to the actual solution (which you can easily compute by hand from the three foregoing equations).

Roots are:-

$$x_1 = 1 , x_2 = 2 \ and \ x_3 = -1$$

For, $x_1 = 0 , x_2 = 0 \ and \ x_3 = 0$,

Jacobi Method:-

no	x1	x2	x3	error
1	0	0	0	1
2	1.1111	1.9	0	3.0111
3	0.9	1.6778	-0.99394	1.4273
4	1.0351	2.0182	-0.85556	0.61392
5	0.98193	1.9496	-1.0162	0.28238
6	1.0074	2.0085	-0.97676	0.12373
7	0.99648	1.9915	-1.0051	0.056179
8	1.0015	2.0022	-0.99597	0.024845
9	0.9993	1.9985	-1.0012	0.011204
10	1.0003	2.0005	-0.99926	0.0049801
11	0.99986	1.9997	-1.0003	0.0022374
12	1.0001	2.0001	-0.99986	0.00099725
13	0.99997	1.9999	-1.0001	0.00044711
14	1	2	-0.99997	0.00019959
15	0.99999	2	-1	8.9385e-05
16	1	2	-0.99999	3.9935e-05
17	1	2	-1	1.7873e-05
18	1	2	-1	7.989e-06
19	1	2	-1	3.5744e-06
20	1	2	-1	1.5981e-06
21	1	2	-1	7.1486e-07

Gauss-Seidel Method:-

no	x1	x2	x3	error
1	0	0	0	1
2	0.9	1.9	-1.5943e-15	2.8
3	0.99283	2.0009	-0.93636	1.1301
4	0.99971	2.0009	-0.99837	0.068935
5	1	2.0001	-1.0003	0.0030015
6	1	2	-1.0001	0.00034624
7	1	2	-1	6.3979e-05
8	1	2	-1	6.9605e-06
9	1	2	-1	5.4465e-07

For, $x_1 = 1, x_2 = 2$ *and* $x_3 = 3$

Jacobi Method:-

no	x1	x2	x3	error
1	1	2	3	1
2	0.55556	0.8	-1	5.6444
3	1.1333	2.0889	-0.44242	2.4242
4	0.92817	1.8061	-1.0687	1.1143
5	1.0292	2.035	-0.90989	0.48872
6	0.9861	1.9671	-1.0207	0.22171
7	1.0059	2.009	-0.98426	0.098119
8	0.99725	1.9941	-1.0049	0.044224
9	1.0012	2.002	-0.9971	0.019665
10	0.99945	1.9989	-1.0011	0.0088322
11	1.0002	2.0004	-0.99945	0.0039375
12	0.99989	1.9998	-1.0002	0.0017651
13	1	2.0001	-0.99989	0.00078803
14	0.99998	2	-1	0.00035288
15	1	2	-0.99998	0.00015767
16	1	2	-1	7.0564e-05
17	1	2	-1	3.1541e-05
18	1	2	-1	1.4112e-05
19	1	2	-1	6.3093e-06
20	1	2	-1	2.8223e-06
21	1	2	-1	1.262e-06
22	1	2	-1	5.6447e-07

Gauss-Seidel Method:-

no	x1	x2	x3	error
1	1	2	3	1
2	1	2	-1	4
3	1	2	-1	4.4409e-16

For, $x_1 = 1, x_2 = 2$ and $x_3 = 3$

Jacobi Method:-

no	x1	x2	x3	error
1	1	-2	3	1
2	1	0.8	0.45455	5.3455
3	0.97172	1.5636	-0.56364	1.8101
4	1	1.8747	-0.83361	0.60937
5	0.99543	1.9501	-0.95445	0.20075
6	1.0005	1.9873	-0.9806	0.068372
7	0.99926	1.9941	-0.9955	0.022952
8	1.0002	1.9988	-0.99765	0.0077598
9	0.99987	1.9993	-0.99961	0.002709
10	1	1.9999	-0.9997	0.00090203
11	0.99998	1.9999	-0.99998	0.00034654
12	1	2	-0.99996	0.00014661
13	1	2	-1	6.8342e-05
14	1	2	-0.99999	2.9645e-05
15	1	2	-1	1.3563e-05
16	1	2	-1	5.9647e-06
17	1	2	-1	2.701e-06
18	1	2	-1	1.1969e-06
19	1	2	-1	5.3895e-07

Gauss-Seidel Method:-

no	x1	x2	x3	error
1	1	-2	3	1
2	0.88687	1.5636	0.45455	6.2222
3	0.98274	1.9658	-0.81047	1.763
4	0.99828	1.9983	-0.98285	0.22045
5	0.99988	2	-0.99892	0.019378
6	1	2	-0.99997	0.0011774
7	1	2	-1	5.0685e-05
8	1	2	-1	6.0559e-06
9	1	2	-1	1.1055e-06
10	1	2	-1	1.1969e-07

Initial Values			Total Steps for Jacobi Method	Total Steps for Gauss-Seidel method
x_1	x_2	x_3		
0	0	0	20	8
1	2	3	21	2
1	-2	3	18	9

So, Gauss-Seidel Method is more efficient then Jacobi Method in terms of faster convergence.

SET - 11

Non-linear system and Newton Method

Problem - 1

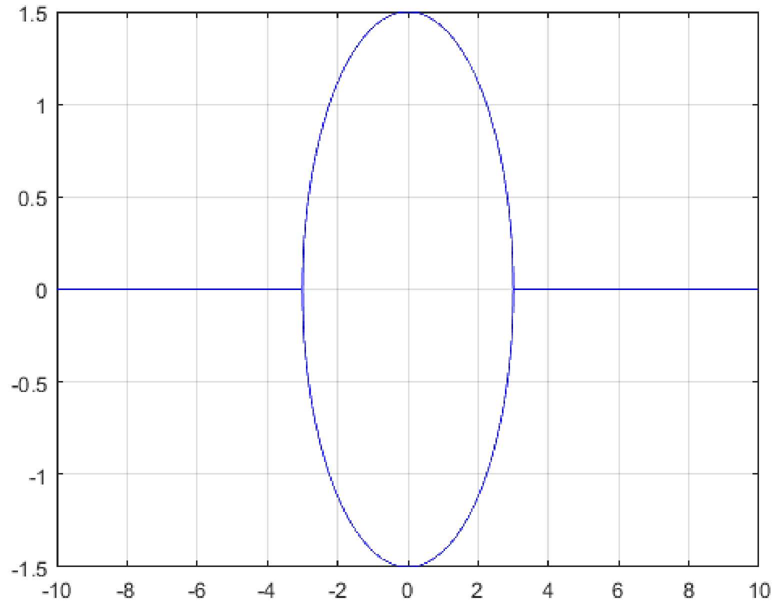
♦ **Statement:**

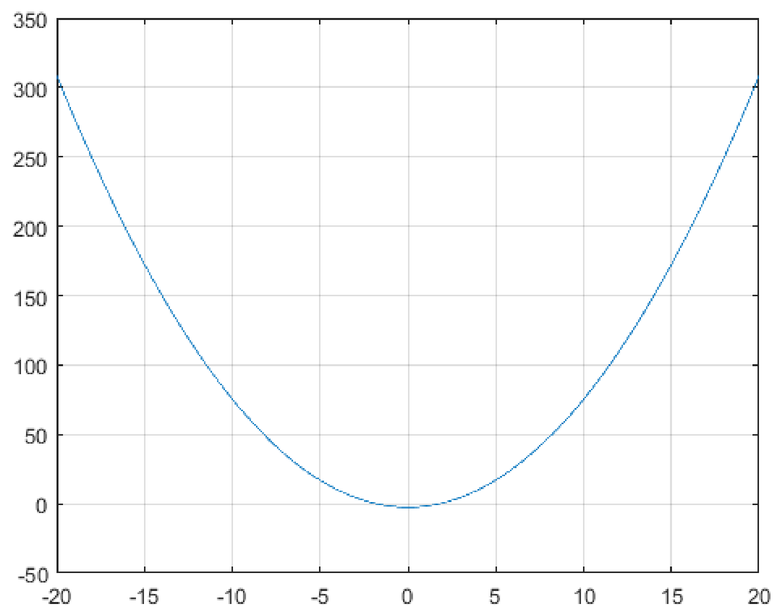
Consider the following system of nonlinear equations.

$$f(x, y) = x^2 + 4y^2 - 9 = 0$$

$$g(x, y) = 18y - 14x^2 + 45 = 0$$

(a) Plot the foregoing functions on the x-y plane.





(b) Obtain all the roots (where f and g intersect) by the general Newton method.

For $x = 1 \; and \; y = 2$

Roots are:

$$x = 2.1372 \; and \; y = 1.0527$$