QuantTradingStrategy_FinalProjectCode

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1 Pairs Trading with PCA and DBSCAN Clustering

Group member: Yicheng Wang, Guoyang Wan, Siyuan Liu

1.1 Assumptions

1. one pair from each cluster to diversify our portfolio, or pairs with lowest p-value among all the clusters
2. each pair is assigned $ $ $1,000,000 with a stop loss level to construct the portfolio
3. the portfolio is dollar neutral for each pair and for overall portfolio
4. 20100101-20150101 is the time period we construct our cluster and it is fixed throughout the testing
5. we pick the best pair from each cluster in a rolling basis for a training period, or optimal pairs among all the clsuters
6. we also tried different testing periods until 20180507
7. we also tried different lookback periods and band width for constructing Bollinger band for each pair trade
8. we also tried to apply kalman filter to generate dynamic hedge ratio

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
import datetime

import matplotlib.cm as cm
import statsmodels.formula.api as sm
import statsmodels.tsa.stattools as ts

from sklearn.cluster import KMeans, DBSCAN
from sklearn.decomposition import PCA
from sklearn.manifold import TSNE
from sklearn import preprocessing
from pykalman import KalmanFilter

from statsmodels.tsa.stattools import coint

from scipy import stats
```

```
import itertools
%matplotlib inline
import warnings
warnings.filterwarnings('ignore')
```

```
C:\ProgramData\Anaconda3\lib\site-packages\statsmodels\compat\pandas.py:56: FutureWarning: The
  from pandas.core import datetools
```

1.2 Data

Time: 2010-01-01 to 2018-05-07
Stocks: 3000 stocks from Russell3000, Russell3000 index as benchmark
Factors: Daily closing price, Industry(GICS Industry Group), Analyst Rating, Market to Book Value, Return on Assets, Debt to Assets, Earnings per Share, Market Capitalization

```
In [2]: # read Stock price and financial factors
        xls = pd.ExcelFile("ProjectData.xlsx")
        Price = pd.read_excel(xls,"Russell3000 Price") # stock price
        Industry = pd.read_excel(xls,"Industry")
        AR = pd.read_excel(xls,"AnalystRating")
        MTBV = pd.read_excel(xls,"MarketToBookValue")
        ROA = pd.read_excel(xls,"ROA" )
        DTA = pd.read_excel(xls,"DebtToAsset")
        EPS = pd.read_excel(xls,"EPS")
        MC = pd.read_excel(xls,"MarketCap")
```

```
In [3]: # read benchmark data
        Russell = pd.read_excel("RU3000PR.xlsx")
        Russell = Russell.dropna(axis=0, how='any')
        Russell = Russell.set_index("observation_date")
```

```
In [4]: # preprocess stock price and Industry classification
        Price = Price.set_index("Dates")
        # industry from 1-11
        Industry = (Industry['GICS_SECTOR'] - 5 )/ 5
        Industry = np.reshape(Industry, (1, np.size(Industry)))
        Price_Factor = Price.copy()
```

```
In [5]: # preprocess financial factors
        ContVal = [AR, MTBV, ROA, DTA, EPS, MC]
        for i in range(len(ContVal)):
            ContVal[i] = pd.DataFrame(ContVal[i])
            ContVal[i] = ContVal[i].set_index("Dates")
            ContVal[i] = ContVal[i].loc["2014-12-31", :]
            ContVal[i] = pd.qcut(ContVal[i], 5, labels=range(1,6,1))
            ContVal[i] = np.reshape(ContVal[i], (1, np.size(ContVal[i])))
```

```
In [6]: ContVal_name = ['AR', 'MTBV', 'ROA', 'DTA', 'EPS', 'MC']
        for i in range(len(ContVal)):
            Price_Factor.loc[ContVal_name[i],:] = ContVal[i][0]
        Price_Factor.loc["Industry",:] = Industry

        # Price_Factor: save stock price and finacial factors of each stock, drop NA
        Price_Factor = Price_Factor.dropna(axis=1, how='any')
        Price_Factor.shape # 1905 stocks

Out[6]: (2184, 1905)
```

1.3 PCA

To reduce dimension of daily price returns to 50 dimensions so that we can cluster from (50 + number of factors) dimensions

```
In [7]: # daily return dataframe
        Price = Price_Factor.iloc[:-7,:]
        Price.index = pd.to_datetime(Price.index)
        df_ret = Price.pct_change()
        df_ret.head()
        df_ret.dropna(axis=0, inplace=True) # drop first row (NA)

In [8]: # 5 year return data for clustering
        df_ret_train = df_ret.loc[:'20150101',:]
        df_ret_train.T.shape

Out[8]: (1905, 1304)

In [9]: # PCA
        N_PRIN_COMPONENTS = 50
        pca = PCA(n_components=N_PRIN_COMPONENTS)
        pca.fit(df_ret_train)

Out[9]: PCA(copy=True, iterated_power='auto', n_components=50, random_state=None,
          svd_solver='auto', tol=0.0, whiten=False)

In [10]: pca.components_.T.shape # reduce to 50 dimensions of 1905 stocks

Out[10]: (1905, 50)

In [11]: X = np.hstack(
              (pca.components_.T, # return data after PCA
               Price_Factor.iloc[-7:,:].T # factor data
               )
          )
          X.shape

Out[11]: (1905, 57)
```

1.4 DBSCAN Clustering

cluster 1905 stocks with 57 dimensions

In [12]:
```
# standardize data for clustering
X = preprocessing.StandardScaler().fit_transform(X)
print(X.shape) # 57 dimensions by adding 7 factors
```

```
(1905, 57)
```

In [13]:
```
clf = DBSCAN(eps=1.8, min_samples=3)
print(clf)

# labels is label values from -1 to x
# -1 represents noisy samples that are not in clusters
clf.fit(X)
labels = clf.labels_
n_clusters_ = len(set(labels)) - (1 if -1 in labels else 0) # eliminate noisy samples
print("\nClusters discovered: %d" % n_clusters_)

clustered = clf.labels_
```

```
DBSCAN(algorithm='auto', eps=1.8, leaf_size=30, metric='euclidean',
    metric_params=None, min_samples=3, n_jobs=1, p=None)

Clusters discovered: 10
```

In [14]:
```
# all stock with its cluster label (including -1)
clustered_series = pd.Series(index=df_ret_train.columns, data=clustered.flatten())
# clustered stock with its cluster label
clustered_series_all = pd.Series(index=df_ret_train.columns, data=clustered.flatten()
clustered_series = clustered_series[clustered_series != -1]
```

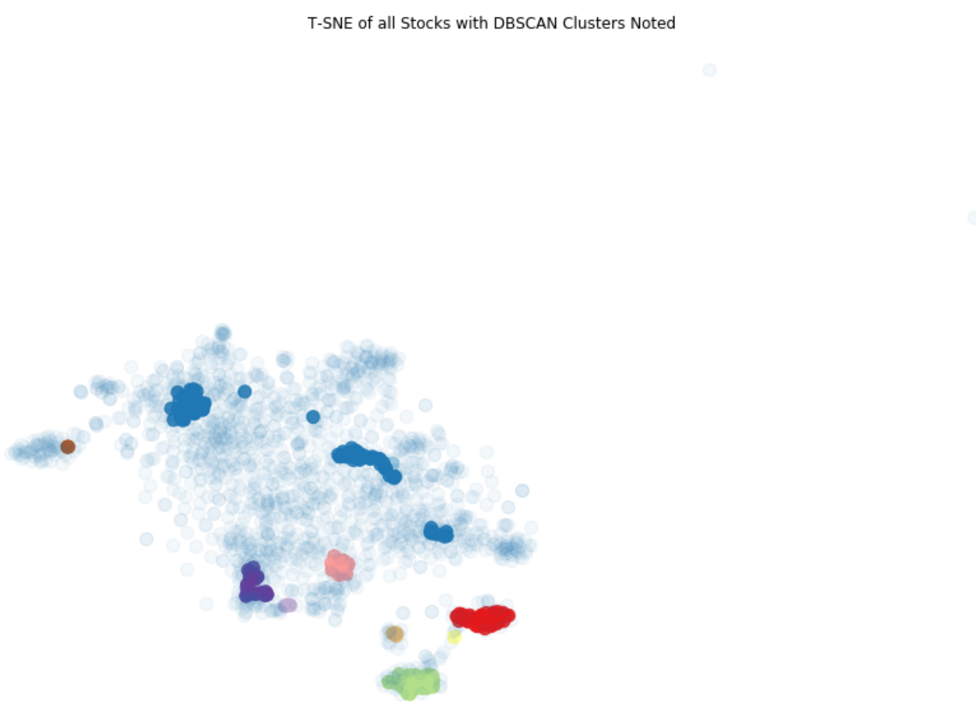
In [15]:
```
CLUSTER_SIZE_LIMIT = 9999
counts = clustered_series.value_counts()
ticker_count_reduced = counts[(counts>1) & (counts<=CLUSTER_SIZE_LIMIT)]
print("Clusters formed: %d" % len(ticker_count_reduced))
print("Pairs to evaluate: %d" % (ticker_count_reduced*(ticker_count_reduced-1)).sum()
```

```
Clusters formed: 10
Pairs to evaluate: 13920
```

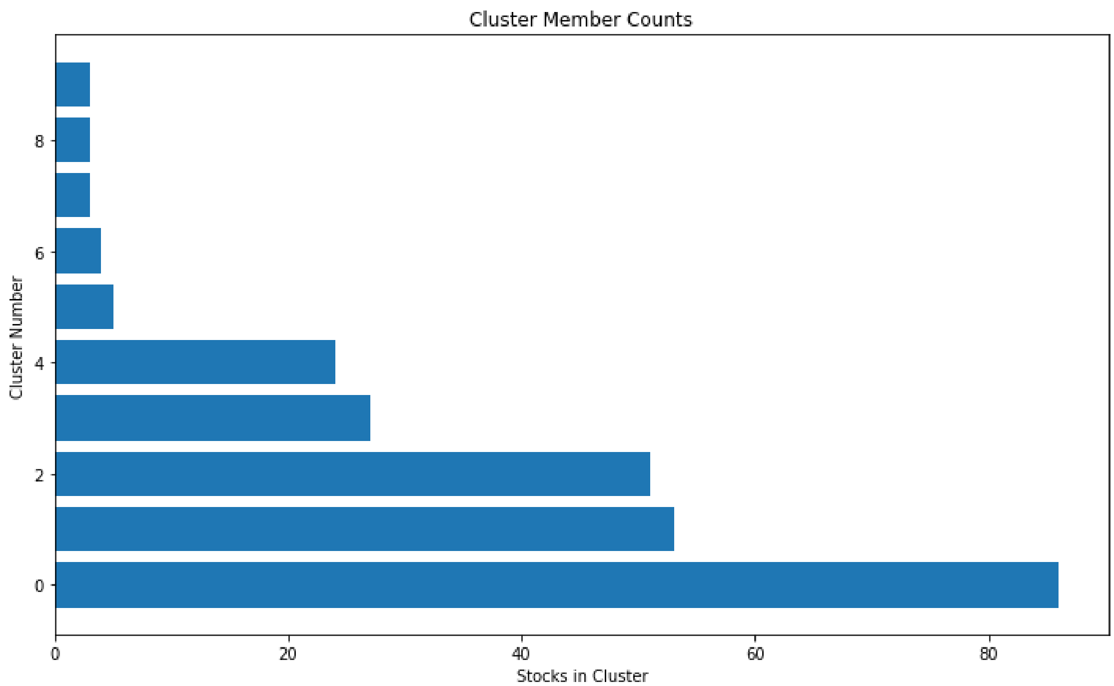
In [16]:
```
# to plot multidimension into 2D
X_tsne = TSNE(learning_rate=1000, perplexity=25, random_state=1337).fit_transform(X)
```

In [17]:
```
# PLOT
plt.figure(1, facecolor='white', figsize=(15,10))
```

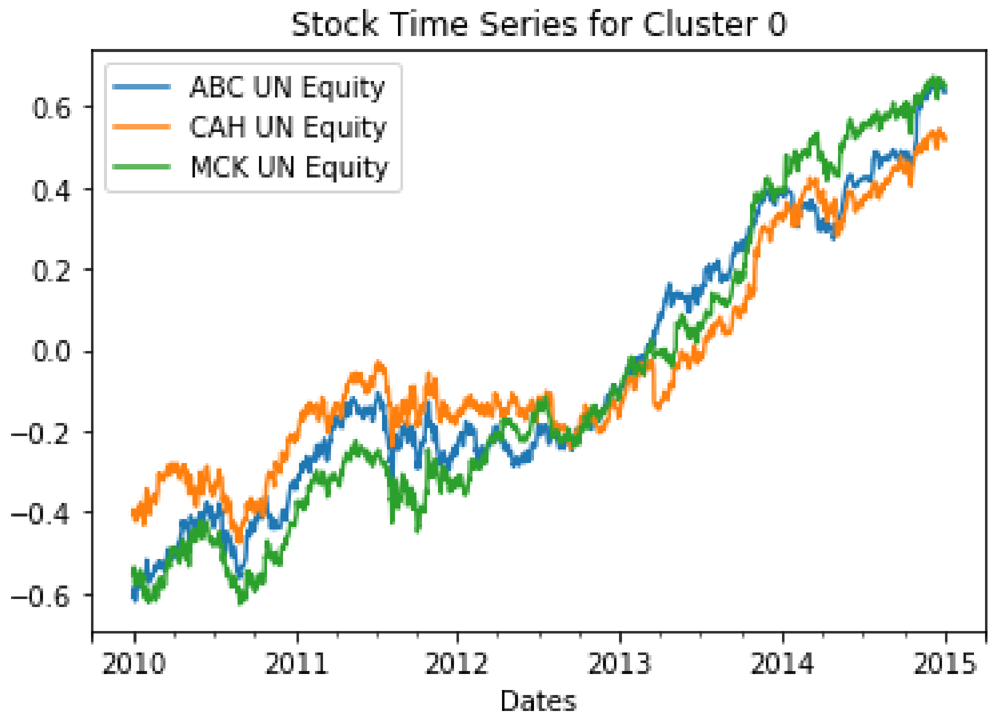
```
plt.clf()
plt.axis('off')

# clustered
plt.scatter(
    X_tsne[(labels!=-1), 0],
    X_tsne[(labels!=-1), 1],
    s=100,
    alpha=0.85,
    c=labels[labels!=-1],
    cmap=cm.Paired
)

# unclustered in the background
plt.scatter(
    X_tsne[(clustered_series_all==-1).values, 0],
    X_tsne[(clustered_series_all==-1).values, 1],
    s=100,
    alpha=0.05
)

plt.title('T-SNE of all Stocks with DBSCAN Clusters Noted')
plt.show()
```

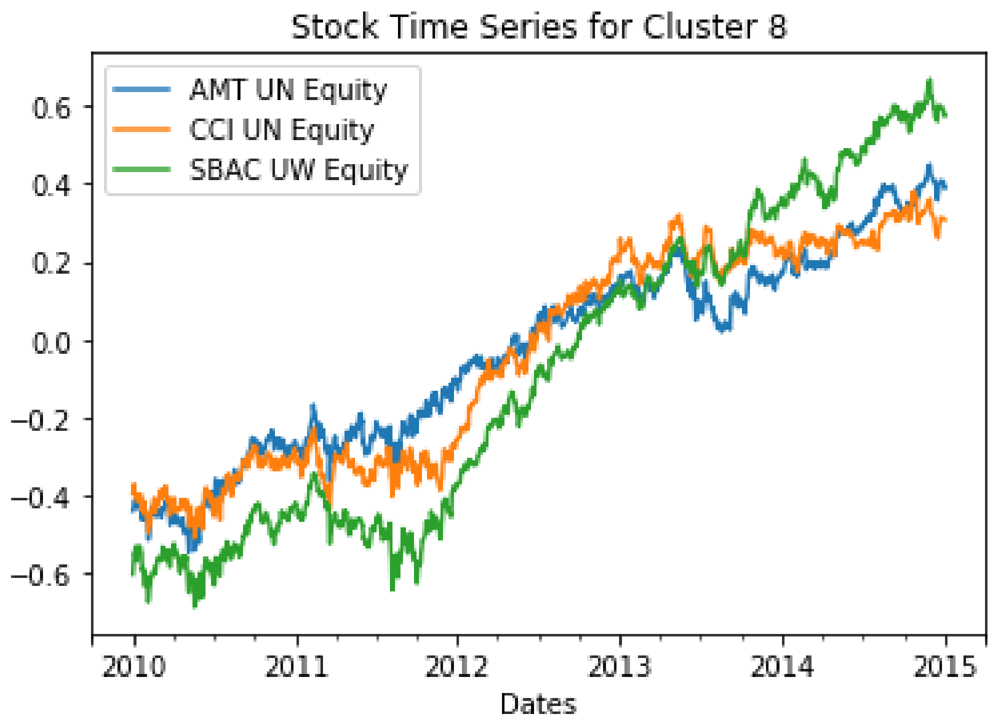


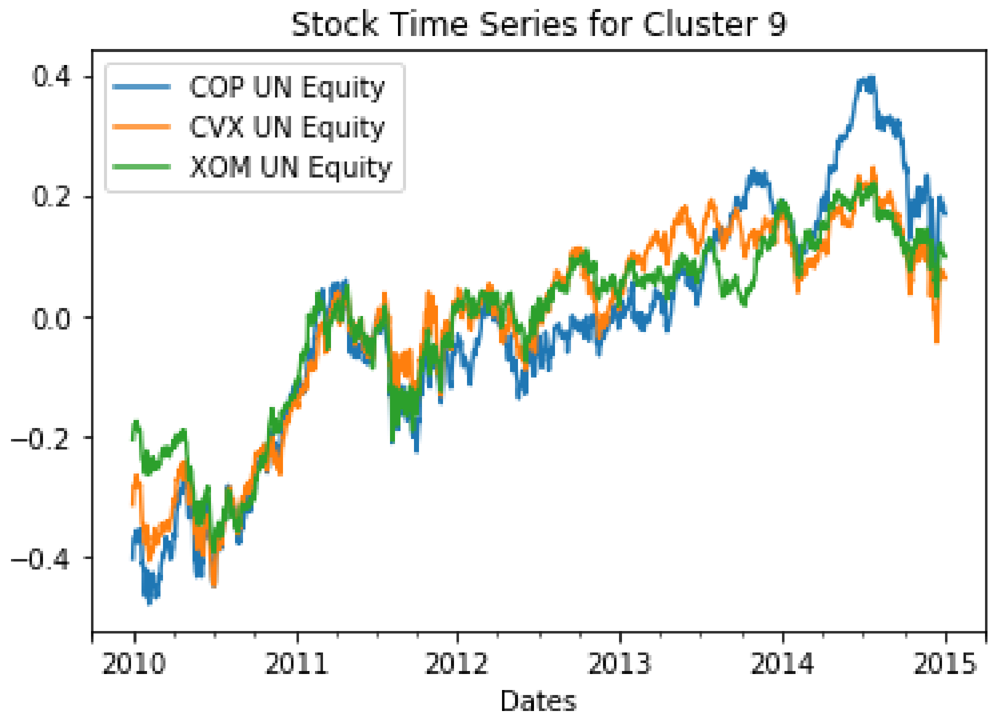
```
In [18]: # show number of clusters in each cluster
         plt.figure(figsize=(12,7))
         plt.barh(
             range(len(clustered_series.value_counts())), # cluster labels, y axis
             clustered_series.value_counts()
         )
         plt.title('Cluster Member Counts')
         plt.xlabel('Stocks in Cluster')
         plt.ylabel('Cluster Number')
         plt.show()
```

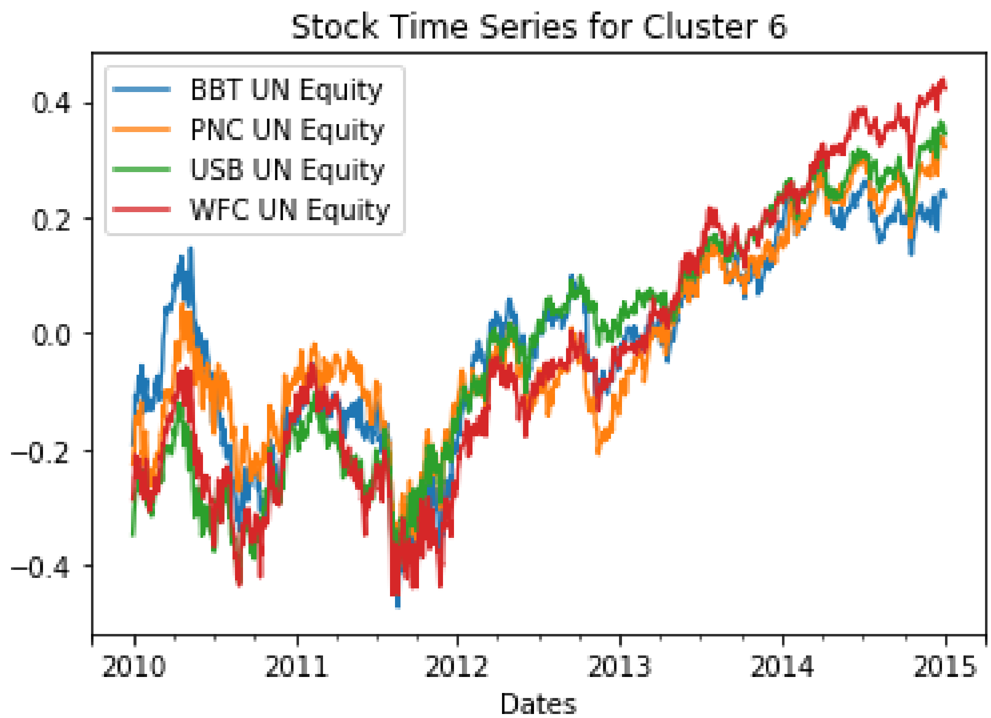



```
In [19]: # get the number of stocks in each cluster
         counts = clustered_series.value_counts()

         # let's visualize some clusters
         cluster_vis_list = list(counts[(counts<20) & (counts>1)].index)[::-1]

         # plot a handful of the smallest clusters
         for clust in cluster_vis_list[0:min(len(cluster_vis_list), 4)]:
             tickers = list(clustered_series[clustered_series==clust].index)
             means = np.log(Price.loc[:"2015-01-01", tickers].mean())
             data = np.log(Price.loc[:"2015-01-01", tickers]).sub(means)
             data.plot(title='Stock Time Series for Cluster %d' % clust)
         plt.show()
```


Stock Time Series for Cluster 0
ABC UN Equity
CAH UN Equity
MCK UN Equity
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
2010
2011
2012
2013
2014
2015
Dates


Stock Time Series for Cluster 8
AMT UN Equity
CCI UN Equity
SBAC UW Equity
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
2010
2011
2012
2013
2014
2015
Dates


Stock Time Series for Cluster 9
COP UN Equity
CVX UN Equity
XOM UN Equity
0.4
0.2
0.0
−0.2
−0.4
2010
2011
2012
2013
2014
2015
Dates


Stock Time Series for Cluster 6
BBT UN Equity
PNC UN Equity
USB UN Equity
WFC UN Equity
0.4
0.2
0.0
−0.2
−0.4
2010
2011
2012
2013
2014
2015
Dates

1.5 Pair Trading

1.5.1 *Cointegration* and *PairSelection* Function

```
In [20]: 'identify cointegrated pairs from clusters'

         def Cointegration(cluster, significance, start_day, end_day):
             pair_coin = []
             p_value = []
             n = cluster.shape[0]
             keys = cluster.keys()
             for i in range(n):
                 for j in range(i+1,n):
                     asset_1 = Price.loc[start_day:end_day, keys[i]]
                     asset_2 = Price.loc[start_day:end_day, keys[j]]
                     results = sm.OLS(asset_1, asset_2).fit()
                     predict = results.predict(asset_2)
                     error = asset_1 - predict
                     ADFtest = ts.adfuller(error)
                     if ADFtest[1] < significance:
                         pair_coin.append([keys[i], keys[j]])
                         p_value.append(ADFtest[1])
             return p_value, pair_coin
```

```
In [38]: "Pair selection method"
         "E_selection = True: select a pair with lowest p-value from each cluster"
         "E_selection = False: select pairs among all the cluster cluster"

         import heapq
         import operator
         def PairSelection(clustered_series, significance, start_day, end_day, E_selection):
             Opt_pairs = []   # to get best pair in cluster i

             if E_selection == True: # select one pair from each cluster
                 for i in range(len(ticker_count_reduced)):
                     cluster = clustered_series[clustered_series == i]
                     keys = cluster.keys()
                     result = Cointegration(cluster, significance, start_day, end_day)
                     if len(result[0]) > 0:
                         if np.min(result[0]) < significance:
                             index = np.where(result[0] == np.min(result[0]))[0][0]
                             Opt_pairs.append([result[1][index][0], result[1][index][1]])
             else:
                 p_value_contval = []
                 pairs_contval = []
                 for i in range(len(ticker_count_reduced)):
                     cluster = clustered_series[clustered_series == i]
                     keys = cluster.keys()
                     result = Cointegration(cluster, significance, start_day, end_day)
```

```
                if len(result[0]) > 0:
                    p_value_contval += result[0]
                    pairs_contval += result[1]

            Opt_pair_index = heapq.nsmallest(7, range(len(p_value_contval)), key=p_value_c
            Opt_pairs = operator.itemgetter(*Opt_pair_index)(pairs_contval)
        return Opt_pairs
```

1.5.2 Preview of selected pairs

```
In [ ]: # Pairs with lowest p-value among all the clusters
        significance = 0.05
        start_day = "2010-01-01"
        end_day = "2015-01-01"
        E_selection = False

        Opt_pairs = PairSelection(clustered_series, significance, start_day, end_day, E_selecti
```

```
In [24]: print("Number of clusters: ",len(ticker_count_reduced))
         print("Number of cointegrated pairs: ",len(Opt_pairs))
         print("Pairs with lowest p-value among all the clusters:")
         Opt_pairs
```

```
Number of clusters:  10
Number of cointegrated pairs:  7
Pairs with lowest p-value among all the clusters:
```

```
Out[24]: (['AVA UN Equity', 'UTL UN Equity'],
          ['O UN Equity', 'VTR UN Equity'],
          ['EPR UN Equity', 'HIW UN Equity'],
          ['FISV UW Equity', 'SNA UN Equity'],
          ['DDR UN Equity', 'PLD UN Equity'],
          ['DDR UN Equity', 'WRI UN Equity'],
          ['ITW UN Equity', 'ZBH UN Equity'])
```

```
In [26]: # Pairs with lowest p-value of each cluster
         significance = 0.05
         start_day = "2010-01-01"
         end_day = "2015-01-01"
         E_selection = True # True: select a pair from each cluster

         Opt_pairs = PairSelection(clustered_series, significance, start_day, end_day, E_select

         print("Number of clusters: ",len(ticker_count_reduced))
         print("Number of cointegrated pairs: ",len(Opt_pairs))
         print("Pairs with lowest p-value of each cluster:")
         Opt_pairs
```

```
Number of clusters:  10
Number of cointegrated pairs:  6
Pairs with lowest p-value of each cluster:
```

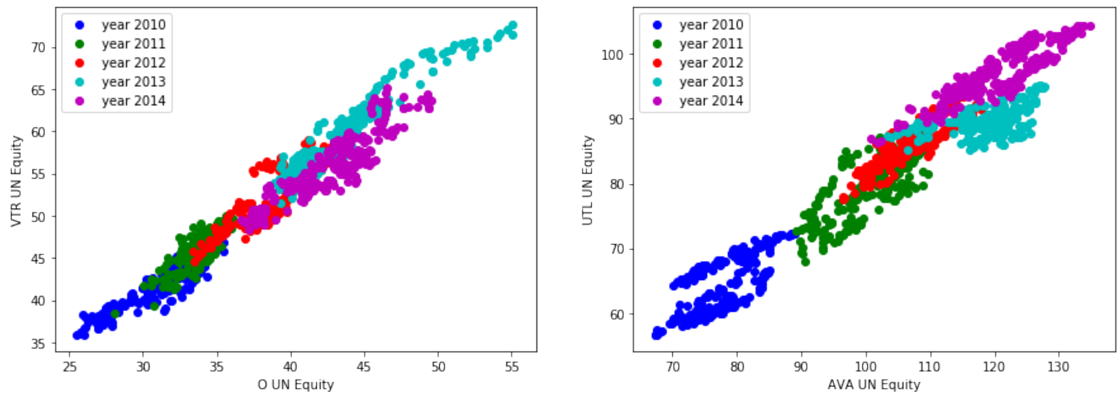
```
Out[26]: [['FISV UW Equity', 'SNA UN Equity'],
          ['O UN Equity', 'VTR UN Equity'],
          ['BRO UN Equity', 'VR UN Equity'],
          ['AVA UN Equity', 'UTL UN Equity'],
          ['TRST UW Equity', 'UBSI UW Equity'],
          ['CVX UN Equity', 'XOM UN Equity']]
```

1.6 Kalman Filter

1.6.1 Preview of Cointegrated Patterns

```
In [29]: 'plot one cointegrated pair in different time interval'
         stock_O = Price.loc["2010-01-01":"2015-01-01", Opt_pairs[1][0]]
         stock_VTR = Price.loc["2010-01-01":"2015-01-01", Opt_pairs[1][1]]

         stock_CVX = Price.loc["2010-01-01":"2015-01-01", Opt_pairs[5][0]]
         stock_XOM = Price.loc["2010-01-01":"2015-01-01", Opt_pairs[5][1]]

         fig, (ax1, ax2) = plt.subplots(1,2, sharey=False, figsize=(15,5))

         ax1.set_xlabel("O UN Equity")
         ax1.set_ylabel("VTR UN Equity")
         ax1.scatter(stock_O[0:252],stock_VTR[0:252], color='b',
                     label = 'year 2010')
         ax1.scatter(stock_O[252:504],stock_VTR[252:504], color='g',
                     label = 'year 2011')
         ax1.scatter(stock_O[504:756],stock_VTR[504:756], color='r',
                     label = 'year 2012')
         ax1.scatter(stock_O[756:1008],stock_VTR[756:1008], color='c',
                     label = 'year 2013')
         ax1.scatter(stock_O[1008:],stock_VTR[1008:], color='m',
                     label = 'year 2014')
         ax1.legend()

         ax2.set_xlabel("AVA UN Equity")
         ax2.set_ylabel("UTL UN Equity")
         ax2.scatter(stock_CVX[0:252],stock_XOM[0:252], color='b',
                     label = 'year 2010')
         ax2.scatter(stock_CVX[252:504],stock_XOM[252:504], color='g',
                     label = 'year 2011')
         ax2.scatter(stock_CVX[504:756],stock_XOM[504:756], color='r',
                     label = 'year 2012')
         ax2.scatter(stock_CVX[756:1008],stock_XOM[756:1008], color='c',
                     label = 'year 2013')
```

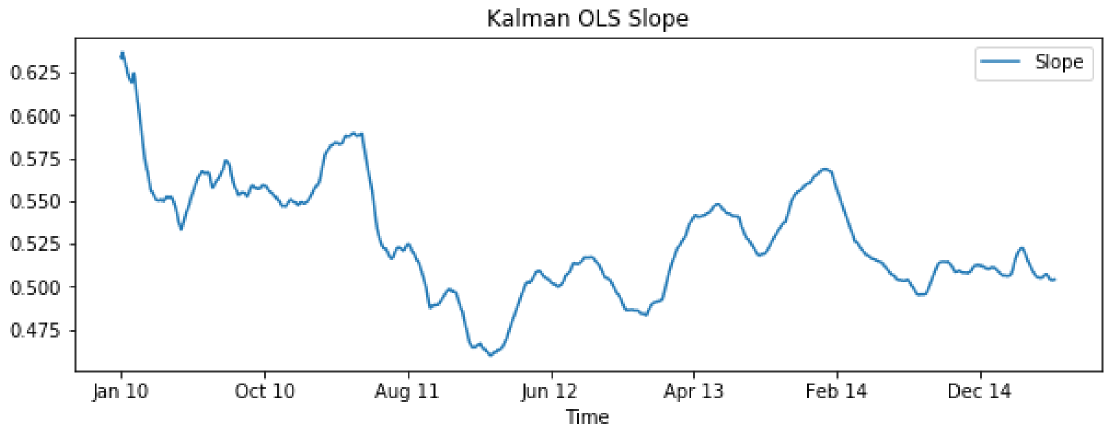
```
ax2.scatter(stock_CVX[1008:],stock_XOM[1008:], color='m',
            label = 'year 2014')
ax2.legend()

plt.show()
```

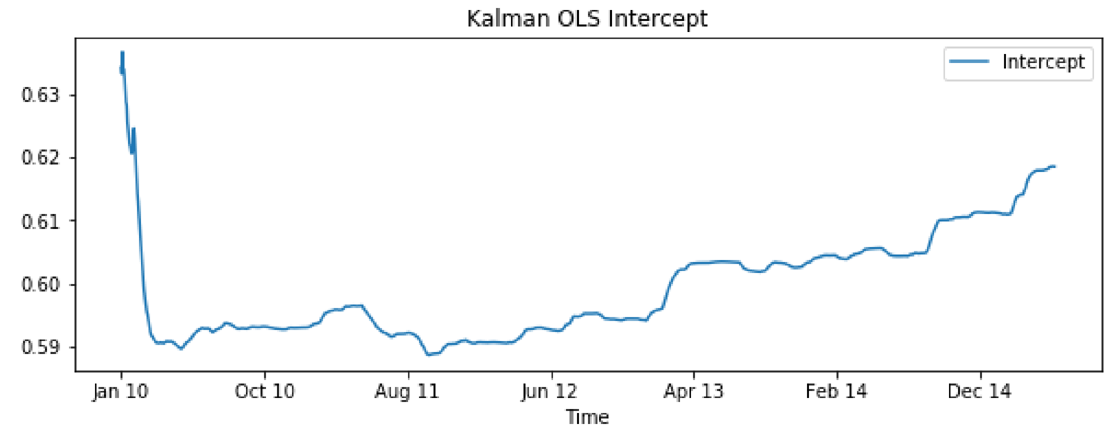


1.6.2 Kalman Filter Regression

```
In [204]: 'kalman filter regression'
          # observation matrix

          obs_mat = np.vstack([stock_FFBC,
                               np.ones(stock_FFBC.shape)]).T[:, np.newaxis]

          delta = 1e-5
          trans_cov = delta / (1 - delta) * np.eye(2)

          kf = KalmanFilter(n_dim_obs=1, n_dim_state=2,
                            initial_state_mean=np.zeros(2),
                            initial_state_covariance=np.ones((2, 2)),
                            transition_matrices=np.eye(2),
                            observation_matrices=obs_mat,
                            observation_covariance=1.0,
                            transition_covariance=trans_cov)

          state_means, state_covs = kf.filter(stock_BRKL)

In [208]: 'plot kalman OLS coefficients'

          fig, (ax1, ax2) = plt.subplots(2,1, sharex=False,figsize=(10,8))
          'plot kalman OLS coefficients: slope'
          ax1.set_title('Kalman OLS Slope')
          ax1.set_xlabel('Time')
```

```
ax1.plot(state_means[:,0], label = 'Slope')
ax1.legend()
# change x label
ax1.set_xticks([0,200,400,600,800,1000,1200])
ax1.set_xticklabels(['Jan 10','Oct 10','Aug 11','Jun 12','Apr 13','Feb 14','Dec 14'])

'plot kalman OLS coefficients: intercept'
ax2.set_title('Kalman OLS Intercept')
ax2.set_xlabel('Time')
ax2.plot(state_means[:,1], label = 'Intercept')
ax2.legend()
# change x label
ax2.set_xticks([0,200,400,600,800,1000,1200])
ax2.set_xticklabels(['Jan 10','Oct 10','Aug 11','Jun 12','Apr 13','Feb 14','Dec 14'])

plt.subplots_adjust(wspace = 1, hspace=0.4)
plt.show()
```





1.7 Bollinger Band Trading Strategy

1.7.1 *MovingAverage*, *BBand*, *BBTrading* and *PortfolioTrading* Function

In [30]:
```
'define moving average and rolling std of data series with time period t'
def MVaverage(a, t):
    # a is an array of a vector of data
    # t is the looking back period
    i = np.size(a)
    j = 0
    mv_t = np.zeros(i-t+1)  # moving average of data interval with period t
    std_t = np.zeros(i-t+1) # std of data interval with period t
    while i>(t-1):
        mv_t[j] = np.mean(a[j:(j+t)])
        std_t[j] = np.std(a[j:(j+t)])
        j += 1
        i -= 1
    return mv_t, std_t
```

In [31]:
```
'define bollinger band function to calculate Bollinger band width'
def BBand(a, t, const):
    # a is an array of a vector of data
    # t is the looking back period
    # const is the number of std away from the moving average
    BollingerBand = []
    mv, std = MVaverage(a, t)
    LowerBand = np.subtract(mv, const*std)
    UpperBand = np.add(mv, const*std)
    BollingerBand.append([LowerBand, UpperBand])
    return mv, BollingerBand
```

In [32]:
```
'define function to calculate maximum drawdown'
def MaxDrawdown(Ret_Cum):
    # ret_cum also can be portfolio position series
    ContVal = np.zeros(np.size(Ret_Cum))
    MaxDD = np.zeros(np.size(Ret_Cum))
    for i in range(np.size(Ret_Cum)):
        if i == 0:
            if Ret_Cum[i] < 0:
                ContVal[i] = Ret_Cum[i]
            else:
                ContVal[i] = 0
        else:
            ContVal[i] = Ret_Cum[i] - np.max(Ret_Cum[0:(i+1)])
        MaxDD[i] = np.min(ContVal[0:(i+1)])
    return MaxDD
```

In [33]:
```
'design the black box of moving average trading strategy'
'with bollinger band'
```

```
def BBTrading(capital, stock_1, stock_2, t, const, stop_loss, KF):
    # a is an array of a vector of data
    # t is the looking back period
    # const is the number of std away from the moving average
    # stop_loss: amount of loss we take (eg. 0.2*capital)
    # KalmanFilter: True or False, whether use Kalman Filter, default is False

    if KF==True:
        obs_mat = np.vstack([stock_2,
                             np.ones(stock_2.shape)]).T[:, np.newaxis]
        delta = 1e-5
        trans_cov = delta / (1 - delta) * np.eye(2)
        kf = KalmanFilter(n_dim_obs=1, n_dim_state=2,
                          initial_state_mean=np.zeros(2),
                          initial_state_covariance=np.ones((2, 2)),
                          transition_matrices=np.eye(2),
                          observation_matrices=obs_mat,
                          observation_covariance=1.0,
                          transition_covariance=trans_cov)
        state_means, state_covs = kf.filter(stock_1)
        'generate spread based on kalman OLS'
        spread_k = np.zeros(np.size(stock_1))
        for i in range(np.size(spread_k)):
            spread_k[i] = stock_1[i] - state_means[i,0]*stock_2[i] - state_means[i,1]
        a = pd.Series(spread_k)

    else:
        results = sm.OLS(stock_1, stock_2).fit()
        predict = results.predict(stock_2)
        a = np.subtract(stock_1, predict)


    mv, BB = BBand(a, t, const) # moving average and bollinger band


    m = np.size(mv)
    Entry = np.zeros(m-1)  # entry signal
    Holding = np.zeros(m)  # holding state
    PL_Holding = np.zeros(m-1) # holding return
    PL_Cum = np.zeros(m-1) # cumulative return of trading strategy

    'produce entry signal'
    for i in range(m-1):
        if i == 0:
            if a[i+t]<BB[0][0][i+1] and a[i+t-1]>BB[0][0][i]:
                Entry[i] = 1
            else:
                Entry[i] = 0
```

```
    else:
        if a[i+t]<BB[0][0][i+1] and a[i+t-1]>BB[0][0][i]:
            Entry[i] = 1
        else:
            if a[i+t]>BB[0][1][i+1] and a[i+t-1]<BB[0][1][i]:
                Entry[i] = -1
            else:
                Entry[i] = 0

'holding decision'
for i in range(1,m,1):
    if Entry[i-1]==0:
        Holding[i] = Holding[i-1]
    else:
        if Entry[i-1] + Holding[i-1] == 0:
            Holding[i]=0
        else:
            if np.abs(Entry[i-1]+Holding[i-1])==1:
                Holding[i] = Entry[i-1] + Holding[i-1]
            else:
                Holding[i] = Holding[i-1]

PL_Holding = np.multiply(Holding[1:], a[(t):])*(-1) * (capital/stock_1[t])
PL_Holding.index = range(np.size(PL_Holding))

commission = Entry * (4.95 + 0.0001 * capital)*2
PL_Holding = PL_Holding - commission

'Calculate cumulative P/L'
for i in range(np.size(PL_Holding)):
    if i ==0:
        PL_Cum[i] = PL_Holding[0]
    else:
        PL_Cum[i] = PL_Cum[i-1]  + PL_Holding[i]


if np.min(PL_Cum) < (-stop_loss * capital):
    po = np.min(np.where(PL_Cum < (-stop_loss*capital)))
    PL_Cum[po:] = PL_Cum[po]

po = np.where(PL_Cum < (stop_loss*capital))

'calculate maximum drawdown'
MaxDD = MaxDrawdown(PL_Cum)

results = []
results.append([a, Entry, Holding, PL_Holding, PL_Cum, MaxDD])
```

```
            return results
```

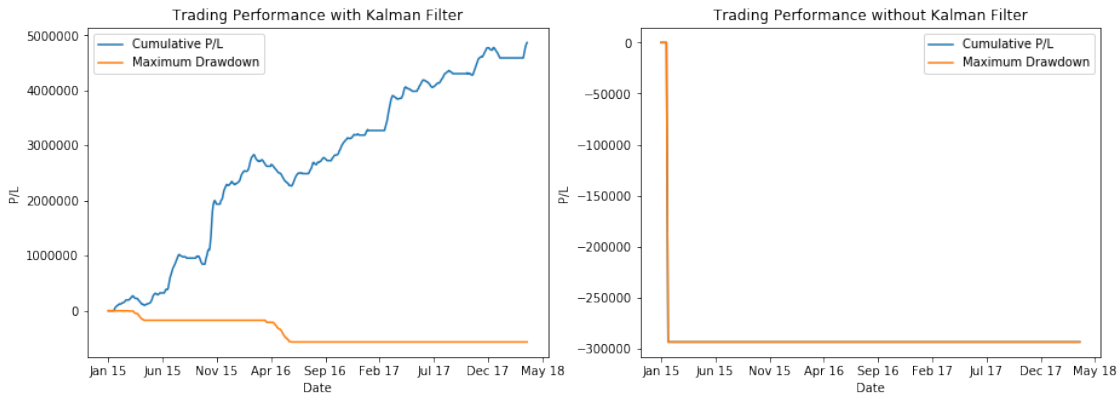
```
In [34]: def Portfolio(capital, clustered_series, significance, coint_start_day,
                   coint_end_day, t, const, stop_loss, rolling_days, KF, E_selection):

             end_day_index = Price.index.get_loc(coint_end_day)
             num = np.size(Price.index)

             PL_daily_portfolio = []
             PL_cum_portfolio = []

             while(end_day_index < num):
                 Opt_pairs = PairSelection(clustered_series, significance,
                                           coint_start_day, coint_end_day, E_selection)
                 num_pairs = len(Opt_pairs) # count pairs

                 # start date of testing period
                 start_day = Price.index[end_day_index -t]
                 if (end_day_index + rolling_days) < num:
                     end_day = Price.index[end_day_index + rolling_days] # rolling day is 126
                 else:
                     end_day = Price.index[num-1]

                 PL_daily_ContVal = [] # save each pair's daily P/L

                 for i in range(num_pairs):
                     stock_1 = Price.loc[start_day:end_day, Opt_pairs[i][0]]
                     stock_2 = Price.loc[start_day:end_day:, Opt_pairs[i][1]]
                     each_pair = BBTrading(capital, stock_1, stock_2, t, const, stop_loss, KF)
                     PL_daily_ContVal.append(each_pair[0][3])


                 PL_daily_portfolio.append(np.sum(PL_daily_ContVal, axis=0))

                 coint_start_day = Price.index[Price.index.get_loc(coint_start_day) + rolling_
                 coint_end_day_index = Price.index.get_loc(coint_end_day) +rolling_days
                 if coint_end_day_index < num:
                     coint_end_day = Price.index[coint_end_day_index]
                     end_day_index = coint_end_day_index
                 else:
                     end_day_index = coint_end_day_index


             PL_daily_portfolio = list(itertools.chain.from_iterable(PL_daily_portfolio))
             PL_cum_portfolio = np.zeros(np.size(PL_daily_portfolio))
             for i in range(np.size(PL_daily_portfolio)):
                 if i==0:
                     PL_cum_portfolio[i] = PL_daily_portfolio[i]
```

```
            else:
                PL_cum_portfolio[i] = PL_cum_portfolio[i-1] + PL_daily_portfolio[i]

        maxdd_portfolio = MaxDrawdown(PL_cum_portfolio)

        return PL_daily_portfolio, PL_cum_portfolio, maxdd_portfolio, num_pairs
```

1.7.2 Visualize performance for one pair with or without Kalman Filter

```
In [141]: # one pair, trading performance
          stock_1 = Price.loc["2015-01-01":, Opt_pairs[1][0]]
          stock_2 = Price.loc["2015-01-01":, Opt_pairs[1][1]]
          t = 100
          const = 1.5
          stop_loss = 0.2
          capital = 1000000

          test_pair_T = BBTrading(capital, stock_1, stock_2, t, const, stop_loss, KF=True)
          test_pair_F = BBTrading(capital, stock_1, stock_2, t, const, stop_loss, KF=False)
          # return: spread, Entry, Holding, PL_Holding, PL_Cum, MaxDD
```

```
In [146]: # a pair trading performance with/without Kalman Filter
          fig, (ax1, ax2) = plt.subplots(1,2, sharey=False, figsize=(15,5))

          ax1.set_title("Trading Performance with Kalman Filter")
          ax1.set_xlabel("Date")
          ax1.set_ylabel("P/L")
          ax1.plot(test_pair_T[0][4], label="Cumulative P/L")
          ax1.plot(test_pair_T[0][5], label="Maximum Drawdown")
          ax1.legend()
          ax1.set_xticks(range(0,900,100))
          ax1.set_xticklabels(['Jan 15','Jun 15','Nov 15',
                               'Apr 16','Sep 16','Feb 17','Jul 17','Dec 17','May 18'])


          ax2.set_title("Trading Performance without Kalman Filter")
          ax2.set_xlabel("Date")
          ax2.set_ylabel("P/L")
          ax2.plot(test_pair_F[0][4], label="Cumulative P/L")
          ax2.plot(test_pair_F[0][5], label="Maximum Drawdown")
          ax2.legend()
          ax2.set_xticks(range(0,900,100))
          ax2.set_xticklabels(['Jan 15','Jun 15','Nov 15',
                               'Apr 16','Sep 16','Feb 17','Jul 17','Dec 17','May 18'])
          plt.show()
```



1.8 Portfolio Compared with Benchmark

```
In [148]: capital = 1000000 # capital for each pair
          total_capital = len(Opt_pairs)*capital
          print('Capital for each pair: '+str(capital))
          print('Total capital invested in portfolio: ' + str(total_capital))
```

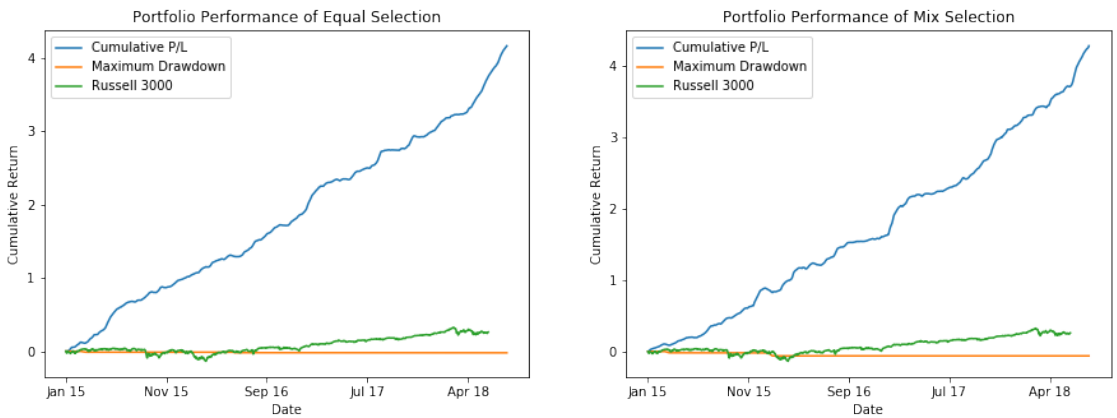
```
Capital for each pair: 1000000
Total capital invested in portfolio: 7000000
```

```
In [35]: # benchmark cumulative return
         Russell = Russell.loc["2014-12-31":, :]
         ret_daily_russell = np.log(np.divide(Russell.iloc[1:,0], Russell.iloc[:-1,0]))
         ret_cum_russell = np.zeros(np.size(ret_daily_russell))
         for i in range(np.size(ret_daily_russell)):
             if i == 0:
                 ret_cum_russell[i] = ret_daily_russell[i]
             else:
                 ret_cum_russell[i] = ret_cum_russell[i-1] + ret_daily_russell[i]
```

```
In [151]: "Portfolio trading performance"
          significance = 0.05
          capital = 1000000
          coint_start_day = "2010-01-01" # cointegration test start day
          coint_end_day = "2015-01-01"   # cointegration test end day

          t = 100 # lookback days
          const = 1.5 # band width
          stop_loss = 0.2
          rolling_days = 126 # testing period
          KF = True # whether use kalman Filter, True or False
```

```
    # one pair from each cluster
    test_port_T = Portfolio(capital, clustered_series, significance, coint_start_day,
                            coint_end_day, t, const, stop_loss, rolling_days, KF, E_sele
    # best pairs from all clusters
    test_port_F = Portfolio(capital, clustered_series, significance, coint_start_day,
                            coint_end_day, t, const, stop_loss, rolling_days, KF, E_sele
```

In [187]:
```
# a pair trading performance with Kalman Filter, and equal/mix selection
fig, (ax1, ax2) = plt.subplots(1,2, sharey=False, figsize=(15,5))

ax1.set_title("Portfolio Performance of Equal Selection")
ax1.set_xlabel("Date")
ax1.set_ylabel("Cumulative Return")
ax1.plot(test_port_T[1]/(capital*7), label="Cumulative P/L")
ax1.plot(test_port_T[2]/(capital*7), label="Maximum Drawdown")
ax1.plot(ret_cum_russell, label = "Russell 3000")
ax1.legend()
ax1.set_xticks([0,200,400,600,800])
ax1.set_xticklabels(['Jan 15','Nov 15','Sep 16','Jul 17','Apr 18'])

ax2.set_title("Portfolio Performance of Mix Selection")
ax2.set_xlabel("Date")
ax2.set_ylabel("Cumulative Return")
ax2.plot(test_port_F[1]/(capital*7), label="Cumulative P/L")
ax2.plot(test_port_F[2]/(capital*7), label="Maximum Drawdown")
ax2.plot(ret_cum_russell, label = "Russell 3000")
ax2.legend()
ax2.set_xticks([0,200,400,600,800])
ax2.set_xticklabels(['Jan 15','Nov 15','Sep 16','Jul 17','Apr 18'])

plt.show()
```



In [186]:
```
# total return and sharp ratio
print("Trading period: Jan 1, 2015 - May 7, 2018")
```

```
print("Russell 3000, total return: ",ret_cum_russell[-1])
print("Russell 3000, shape ratio: ",np.nanmean(ret_daily_russell)/np.nanstd(ret_daily
print("Equal selection, total return: ", test_port_T[1][-1]/total_capital)
print("Equal selection, sharp ratio:", np.nanmean(test_port_T[0])/np.nanstd(test_por
print("Mix selection, total return: ", test_port_F[1][-1]/total_capital)
print("Mix selection, sharp ratio:", np.nanmean(test_port_F[0])/np.nanstd(test_port_
```

```
Trading period: Jan 1, 2015 - May 7, 2018
Russell 3000, total return:  0.2599044812785878
Russell 3000, shape ratio:  0.03706069642439636
Equal selection, total return:  4.160061172655936
Equal selection, sharp ratio: 0.973699138500516
Mix selection, total return:  4.277556105551584
Mix selection, sharp ratio: 0.7929686919732171
```

One pair from each cluster performs better because of low correlation between pairs.

1.9 Portfolio Optimization

by changing parameters

```
In [154]: port_compare = pd.DataFrame(columns=['port','train_months','test_days','lookback_days
                                               'stop_loss','Kalman_Filter','Sharpe'])
```

1.9.1 Optimize for Bollinger Band Strategy for Each Pair Trade

fix training period 5-year, stop loss level, testing period 0.5-year

```
In [157]: port_compare_1 = pd.DataFrame(columns=['port','train_months','test_days','lookback_da
                                               'stop_loss','Kalman_Filter','Sharpe'])
```

```
In [158]: significance = 0.05
          capital = 1000000
          coint_start_day = "2010-01-01" # cointegration test start day
          coint_end_day = "2015-01-01"   # cointegration test end day
          train_month = 60
          lookback_days = [20, 50, 100, 200]
          band_width = [1.5, 2, 2.5]
          stop_loss = 0.2
          rolling_days = 126
          KF = True
          E_selection = False

          n = 0
          for t in lookback_days:
              for const in band_width:
                  n += 1
                  port = Portfolio(capital, clustered_series, significance, coint_start_day,
```

```
                  coint_end_day, t, const, stop_loss, rolling_days, KF, E_sele
        sharpe = np.nanmean(port[0])/np.nanstd(port[0])
        port_compare_1.loc[n, :] = [port, train_month, rolling_days, t, const, stop_l
```

In [159]:
```
save1 = port_compare_1.copy()
save1
```

Out[159]:
```
                                                 port train_months test_days  \
1   ([0.0, 0.0, 51376.205580549926, 43016.74917161...           60       126
2   ([0.0, 0.0, 51376.205580549926, 32619.98521171...           60       126
3   ([0.0, 0.0, 51376.205580549926, 32619.98521171...           60       126
4   ([0.0, 0.0, 41675.17989623978, 42150.573654585...           60       126
5   ([0.0, 0.0, 41675.17989623978, 32422.896102090...           60       126
6   ([0.0, 0.0, 41675.17989623978, 32422.896102090...           60       126
7   ([0.0, 0.0, 41663.77518432925, 42054.266838365...           60       126
8   ([0.0, 0.0, 41663.77518432925, 32418.306420600...           60       126
9   ([0.0, 0.0, 41663.77518432925, 32418.306420600...           60       126
10  ([0.0, 0.0, 41668.79580473811, 32420.322995752...           60       126
11  ([0.0, 0.0, 41668.79580473811, 32420.322995752...           60       126
12  ([0.0, 0.0, 41668.79580473811, 32420.322995752...           60       126

   lookback_days band_width stop_loss Kalman_Filter    Sharpe
1             20        1.5       0.2          True   1.10832
2             20          2       0.2          True  0.920283
3             20        2.5       0.2          True  0.629173
4             50        1.5       0.2          True  0.980591
5             50          2       0.2          True  0.636623
6             50        2.5       0.2          True  0.575766
7            100        1.5       0.2          True  0.792969
8            100          2       0.2          True  0.661294
9            100        2.5       0.2          True  0.558647
10           200        1.5       0.2          True  0.813391
11           200          2       0.2          True  0.582521
12           200        2.5       0.2          True   0.44463
```

In [161]:
```
# without Kalman Filter
port_compare_2 = pd.DataFrame(columns=['port','train_months','test_days','lookback_da
                                       'stop_loss','Kalman_Filter','Sharpe'])
```

In [162]:
```
significance = 0.05
capital = 1000000
coint_start_day = "2010-01-01" # cointegration test start day
coint_end_day = "2015-01-01"   # cointegration test end day
train_month = 60
lookback_days = [20, 50, 100, 200]
band_width = [1.5, 2, 2.5]
stop_loss = 0.2
rolling_days = 126
KF = False
```

```
E_selection = True

n = 0
for t in lookback_days:
    for const in band_width:
        n += 1
        port = Portfolio(capital, clustered_series, significance, coint_start_day,
                         coint_end_day, t, const, stop_loss, rolling_days, KF, E_sele
        sharpe = np.nanmean(port[0])/np.nanstd(port[0])
        port_compare_2.loc[n, :] = [port, train_month, rolling_days, t, const, stop_l
```

```
In [163]: port_compare_2
```

```
Out[163]:
                                                   port train_months test_days  \
   1   ([0.0, 0.0, 0.0, -44268.916779650615, -10279.7...           60       126
   2   ([0.0, 0.0, 0.0, 0.0, -3500.9965207924306, -19...           60       126
   3   ([0.0, 0.0, 0.0, 0.0, 0.0, -13057.247699164353...           60       126
   4   ([0.0, 0.0, 0.0, -32200.2805991863, -29752.611...           60       126
   5   ([0.0, 0.0, 0.0, 0.0, -28802.075036284084, -41...           60       126
   6   ([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 5726.599938589...           60       126
   7   ([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 42462.39046307...           60       126
   8   ([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,...           60       126
   9   ([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,...           60       126
   10  ([0.0, 0.0, 0.0, 0.0, 35650.48687635291, 30454...           60       126
   11  ([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 9184...           60       126
   12  ([0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,...           60       126

      lookback_days band_width stop_loss Kalman_Filter    Sharpe
   1             20        1.5       0.2         False  0.948962
   2             20          2       0.2         False  0.971121
   3             20        2.5       0.2         False  0.836479
   4             50        1.5       0.2         False   1.08567
   5             50          2       0.2         False  0.923465
   6             50        2.5       0.2         False  0.825383
   7            100        1.5       0.2         False  0.988879
   8            100          2       0.2         False     1.021
   9            100        2.5       0.2         False  0.696258
   10           200        1.5       0.2         False   1.31589
   11           200          2       0.2         False  0.970149
   12           200        2.5       0.2         False  0.980413
```

1.9.2 Optimize for testing and training periods

```
In [ ]: port_compare_3 = pd.DataFrame(columns=['port','train_months','test_days','lookback_days
                                               'stop_loss','Kalman_Filter','Sharpe'])
```

```
In [ ]: significance = 0.05
        capital = 1000000
        train_month = range(48,1,-12)
```

```
lookback_days = [20, 50, 100]
band_width = [1.5, 2] # [1, 1.5, 2]
stop_loss = 0.2 # only useful when KF is False
test_days = [21, 63, 126, 252]
Kalman = [True, False] # whether use kalman Filter, True or False

n = 0
for tm in range(len(train_month)):
    train_period = train_date[tm]
    coint_start_day = train_period[0]
    coint_end_day = train_period[1]

    for t in lookback_days:
        for const in band_width:
#             for stop_loss in stoploss:
            for rolling_days in test_days:
                n += 1
                port = Portfolio(capital, clustered_series, significance, coint_start_c
                                  coint_end_day, t, const, stop_loss, rolling_days, KF)
                sharpe = np.nanmean(port[0])/np.nanstd(port[0])
                port_compare_3.loc[n, :] = [port, train_month[tm], rolling_days, t, cor
```

```
In [151]: port_compare_3
```

```
Out[151]:                                                    port train_months test_days  \
          1   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        21
          2   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        63
          3   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       126
          4   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       252
          5   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        21
          6   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        63
          7   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       126
          8   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       252
          9   ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        21
          10  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        63
          11  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       126
          12  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       252
          13  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        21
          14  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        63
          15  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       126
          16  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       252
          17  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        21
          18  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60        63
          19  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       126
          20  ([0.0, 0.0, 46528.14028730437, 43146.518784630...           60       252

             lookback_days band_width stop_loss Kalman_Filter   Sharpe
          1             20          1       0.1          True  1.24564
```

2	20	1	0.1	True	1.40082
3	20	1	0.1	True	1.46476
4	20	1	0.1	True	1.33455
5	20	1	0.2	True	1.24564
6	20	1	0.2	True	1.40082
7	20	1	0.2	True	1.46476
8	20	1	0.2	True	1.33455
9	20	1	0.5	True	1.24564
10	20	1	0.5	True	1.40082
11	20	1	0.5	True	1.46476
12	20	1	0.5	True	1.33455
13	20	1	0.8	True	1.24564
14	20	1	0.8	True	1.40082
15	20	1	0.8	True	1.46476
16	20	1	0.8	True	1.33455
17	20	1	1	True	1.24564
18	20	1	1	True	1.40082
19	20	1	1	True	1.46476
20	20	1	1	True	1.33455

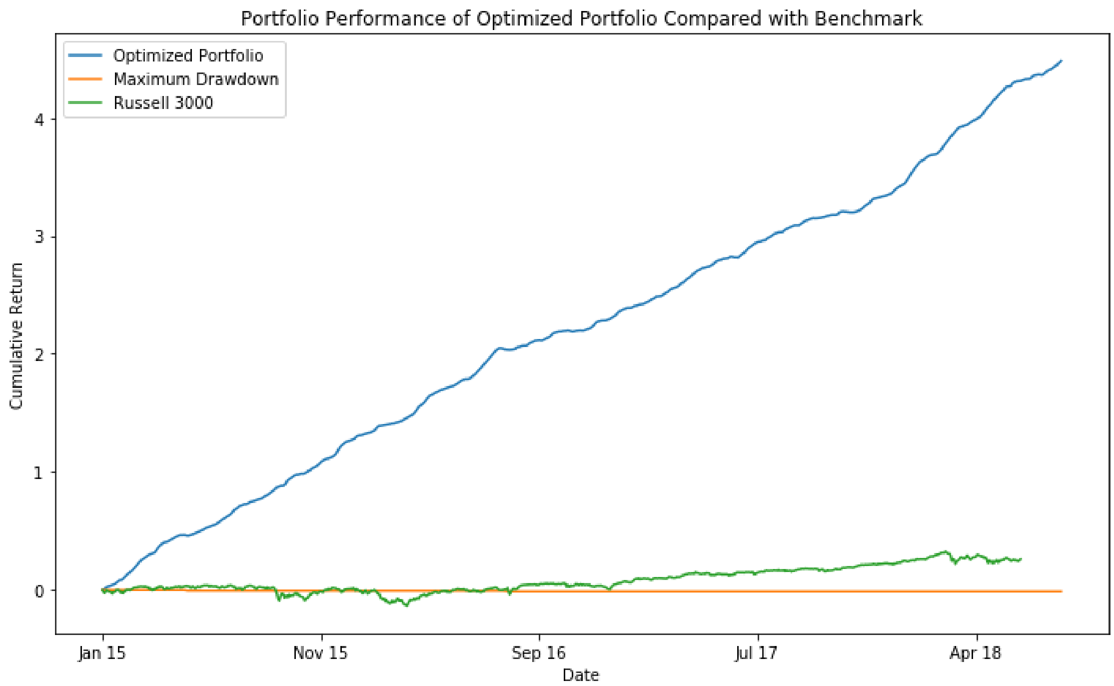
1.9.3 Optimized Portfolio compared with Benchmark

```
In [39]: "Portfolio trading performance"
         significance = 0.05
         capital = 1000000
         coint_start_day = "2014-01-02" # cointegration test start day
         coint_end_day = "2015-01-01"   # cointegration test end day

         t = 100 # lookback days
         const = 1 # band width
         stop_loss = 0.2
         rolling_days = 126 # testing period

         Optimized_port = Portfolio(capital, clustered_series, significance, coint_start_day,
                                    coint_end_day, t, const, stop_loss, rolling_days, KF=True, E
```

```
In [41]: total_capital = capital * Optimized_port[-1]
```

```
In [49]: fig, ax1 = plt.subplots(figsize=(12,7))
         ax1.set_title("Portfolio Performance of Optimized Portfolio Compared with Benchmark")
         ax1.set_xlabel("Date")
         ax1.set_ylabel("Cumulative Return")
         ax1.plot(Optimized_port[1]/total_capital, label="Optimized Portfolio")
         ax1.plot(Optimized_port[2]/total_capital, label="Maximum Drawdown")
         ax1.plot(ret_cum_russell, label = "Russell 3000")
         ax1.legend()
         ax1.set_xticks([0,200,400,600,800])
         ax1.set_xticklabels(['Jan 15','Nov 15','Sep 16','Jul 17','Apr 18'])
```

```
Out[49]: [Text(0,0,'Jan 15'),
          Text(0,0,'Nov 15'),
          Text(0,0,'Sep 16'),
          Text(0,0,'Jul 17'),
          Text(0,0,'Apr 18')]
```




```
In [52]: # total return and sharp ratio
         print("Trading period: Jan 1 2015 - May 7, 2018")
         print("Russell 3000, total return: ",ret_cum_russell[-1])
         print("Russell 3000, shape ratio: ",np.nanmean(ret_daily_russell)/np.nanstd(ret_daily_
         print("Optimized Portfolio, total return: ", Optimized_port[1][-1]/total_capital)
         print("Optimized Portfolio, sharp ratio:", np.nanmean(Optimized_port[0])/np.nanstd(Op
         print("Optimized Portfolio Total profit: ", Optimized_port[1][-1])
```

```
Trading period: Jan 1 2015 - May 7, 2018
Russell 3000, total return:  0.2599044812785878
Russell 3000, shape ratio:  0.03706069642439636
Optimized Portfolio, total return:  4.481015884979954
Optimized Portfolio, sharp ratio: 1.3341860596845443
Optimized Portfolio Total profit:  31367111.19485968
```