2018

Robust Flight Control

ASSIGNMENT

DR THEODOULIS SPILIOS & DR HYO-SANG SHIN

Introduction

The goal of this homework is to understand through a worked example the basic principles of robust control design using the tools of MATLAB/Simulink. This document should be consulted and used in parallel with the second part of the course and serves as a general guideline for the students.

The system considered in this homework is a high-fidelity linear model of an air-air missile. This model approximates the pitch axis flight dynamics of the missile for a fixed operating point of its flight envelope (i.e. fixed altitude, speed and angle-of attack). The missile model is accompanied by corresponding uncertainty levels as well as by a dynamic model of the actuators and sensors.

The work demanded from the students concerns three basic domains: modeling, control design and robustness analysis. Details for each task are given in the following paragraphs. The work must be presented as follows:

- A pdf document which analytically describes the answers and various plots/figures to each of the questions of the following paragraphs (i.e. modeling, control design, robustness analysis).
- A folder containing all MATLAB/Simulink code used to answer the questions named 'Main.m' divided into autonomous cells (which may call other functions) answering to each question. For example 'Cell II.1.A' answers question #1.A of control design etc. A 'Main_Simulink.slx' contains the final control loop.

The evaluation criteria for the final homework note are the following:

- Quality and structure of the results (25%)
- Quantity of results for each student per group (25%)
- MATLAB code quality and clarity (25%)
- Homework report quality (25%)

1. System Modeling (25%)

Introduction. An overview of the system to control is given in Figure 1 and is the same used throughout the course. The various variables involved are the angle of attack (AoA) α which is the angle between the airspeed V and the missile axis 1^b, the pitch rate q which is the second component of the missile angular velocity projected into the missile body coordinate system, the fin deflection δ_q which is the control input, and finally the aerodynamic acceleration along the vertical axis projected also into the missile coordinate system a_z.

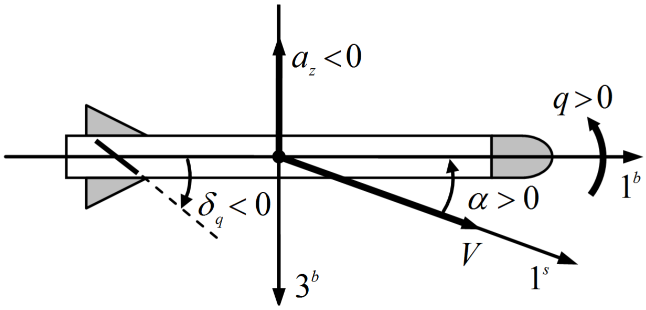

Figure 1 – Missile profile view

The linear state space model of the *missile airframe* pitch axis dynamics was developed during the course and is given by:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \frac{Z_\alpha}{V} & 1 \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} \frac{Z_\delta}{V} \\ M_\delta \end{bmatrix} \delta_q$$

$$a_z = \begin{bmatrix} \frac{A_\alpha}{g} & 0 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} \frac{A_\delta}{g} \end{bmatrix} \delta_q$$

The *actuator* model is a standard 2^{nd} order system given by:

$$\begin{bmatrix} \dot{\delta}_q \\ \ddot{\delta}_q \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_a^2 & -2\zeta_a\omega_a \end{bmatrix} \begin{bmatrix} \delta_q \\ \dot{\delta}_q \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_a^2 \end{bmatrix} \delta_{q,c}$$

Values for the various variables and uncertainty levels are given in Table 1 in the Appendix for a given flight point of the missile whereas the overall system open loop block diagram is given in Figure 2. Note in this figure that a sensor block was added on the block diagram which may correspond to accelerometers, gyros and potentiometers to measure the three output variables. For the sake of this study we may consider the sensor dynamics to be unitary. In addition we note that the fin deflection δ_q may or not be available for measurement and thus for feedback (hence the dashed line).

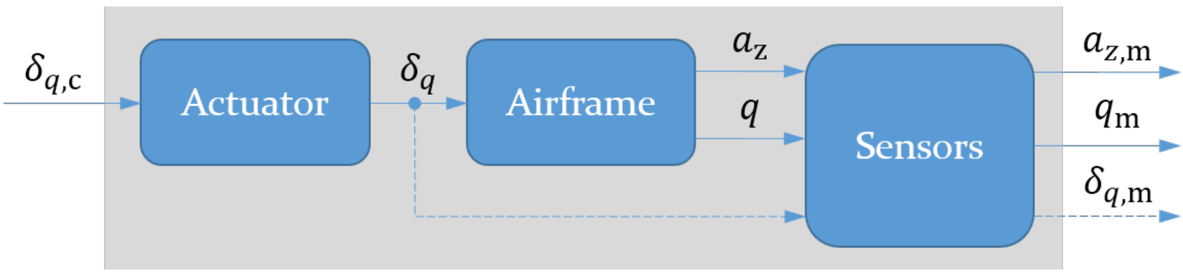

Figure 2 – Open loop block diagram

❖ ***Question 1.0:*** *Flight Dynamics. (5%)*

Having as a reference the lecture slides #1.1 provide a detailed but dense derivation of the nominal (no uncertainty) linear *missile airframe* state space model given in the previous page starting from the basic translational & attitude nonlinear equations. Explain intermediate steps (trimming, etc.) and discuss on the various hypotheses.

Hints: No MATLAB code needed. Additional references: Zipfel's book, Nichols1993 article.

❖ ***Question 1.1:*** *Parametric Uncertainty. (10%)*

A) Uncertain state space model. Use MATLAB to *form* the uncertain state space model $G_{\text{unc}}(s)$ of the complete open loop dynamics (actuator, airframe, sensors) of Figure 2 using the data and uncertainty levels given. Illustrate & comment the properties of the nominal & uncertain open loop system (poles & zeros, stability, etc.). Note: all three subsystems should have input, output & state names corresponding to the ones in Figure 2 for ease of verification.

B) LFT model. Starting from the uncertain state space model of the previous question, use MATLAB to obtain the airframe (only) $H - \Delta$ LFT form appearing at the end of lecture #2.1. Verify the computation with the theoretical derivation of the slides.

❖ ***Question 1.2:*** *Outer Loop Unstructured Uncertainty. (10%)*

A) *Uncertain inner loop state space model.* Compute a negative feedback inner loop gain K_q to place the transfer function $q_c \rightarrow a_{z,\text{m}}$ dominant poles at a damping of 0.707. Then scale the loop using a gain K_{sc} so that the transfer function $q_{\text{sc}} \rightarrow a_{\text{z,m}}$ has a unitary steady state gain. Form the resulting SISO inner closed loop uncertain state space model containing the same two uncertain variables as the open loop uncertain state space model of Question 1.1.A; document and comment the results. Note: put a zero feedback gain on $\delta_{q,\text{m}}$.

B) Uncertainty weight computation. Sample this uncertain system for a number (e.g. $n = 20$) of values for both uncertain variables and obtain an array of n^2 LTI systems. Using these, pick an unstructured uncertainty form (e.g. additive, multiplicative, inverse multiplicative, etc.) and then compute an approximation of the corresponding uncertainty weights W_i as in slide 15 of lecture 2.1. Play with the number of samples and with the order of the filter W_i.

Hints: Related MATLAB commands: 'ureal', 'uss', 'connect', 'usample', 'lftdata', etc. from RCT.

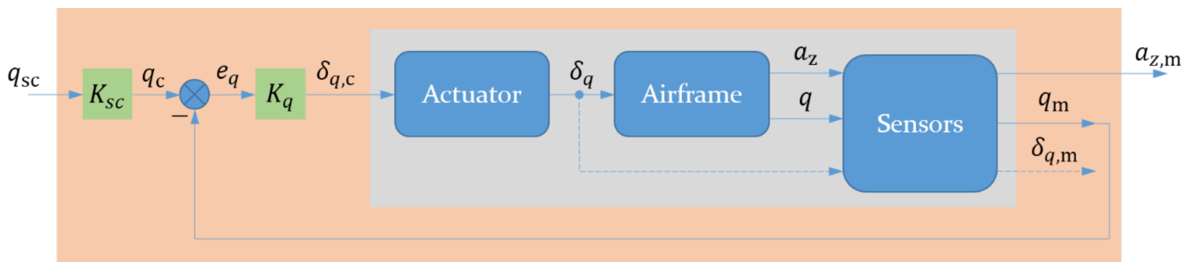

Figure 3 - Inner closed loop

2. Control Design (50%)

Introduction. In this section we are designing a 2DoF controller in order to control the normal acceleration $a_{\mathrm{z,m}}$ via the virtual control signal q_{sc} for our inner loop controlled system seen in Figure 3[1]. If we set the inner loop controlled system as $G_{\mathrm{in}}(s)$, then the controllers we want to design are the disturbance rejection controller $K_{\mathrm{dr}}(s)$ and the feedforward command following controller $K_{\mathrm{cf}}(s)$ illustrated in Figure 4.

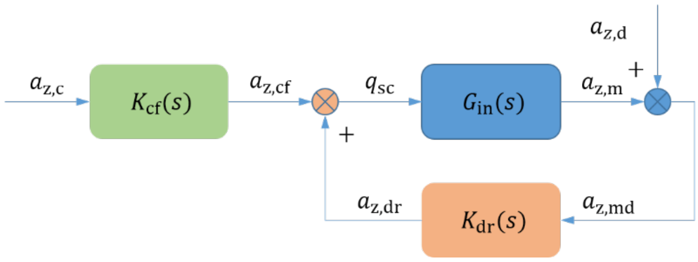

Figure 4 - Outer closed loop

Conformably to what we have seen during the course, the controller $K_{\mathrm{dr}}(s)$ is used to reject output an output disturbance $a_{\mathrm{z,d}}$ whereas the feedforward controller to make the closed loop transfer function $T_{\mathrm{CL}}(s)$ from the reference $a_{\mathrm{z,c}}$ to the measured output $a_{\mathrm{z,m}}$ follow a given reference model $T_{\mathrm{RM}}(s)$ which could be for example a 2nd order system with given damping and bandwidth.

❖ *Question 2.1: Mixed Sensitivity Design. (50%)*

A) Preliminary step (5%). Verify and write down using MATLAB the value of the obtained inner closed loop transfer function $G_{\mathrm{in}}(s)$ which should have four poles and two zeros as designed in Question 1.2.A. Compute now the step response of the inner closed loop. What are the 2% settling time and overshoot? Why is there an undershoot in the response? Why do we need an outer loop controller since the dynamic response appears to be correct?

B) Reference model computation (5%). As discussed in the introduction, we want that our final closed loop transfer function $T_{\mathrm{CL}}(s)$ behaves like a second order system of the following form:

$$T_{\mathrm{RM}}(s) = \frac{\omega_{\mathrm{RM}}^2\left(-\frac{s}{z_{\mathrm{NMP}}}+1\right)}{s^2 + 2\zeta_{\mathrm{RM}}\omega_{\mathrm{RM}}s + \omega_{\mathrm{RM}}^2}$$

The non-minimum phase zero z_{NMP} is of both the missile airframe dynamics and the inner closed loop and $\omega_{\mathrm{RM}}, \zeta_{\mathrm{RM}}$ are adjustable quantities. Find the values for these two variables so that $T_{\mathrm{RM}}(s)$ has a 2% settling time $t_s = 0.2s$ and a maximum overshoot $M_{\mathrm{p}} = 2\%$.

C) Weighting filter selection (5%). We want to impose weights on the sensitivity function S_{o} and on the control times sensitivity function KS_{o} as discussed in lecture #3.5. A general weight template that may be used for the i'th transfer function is:

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[1] For clarity purposes we name the signal q_{sc} as *virtual* since it is not the *actual* control signal $\delta_{q,c}$ given to the actuators.

$$W_i^{-1}(s) = \frac{M_{h,i}s + \omega_i' M_{l,i}}{s + \omega_i'}$$

The parameters $M_{l,i}, M_{h,i}$ are the low and high frequency gains and ω_i' is an adjustable pole so that the filter inverse (which corresponds to the corresponding target transfer function) has a given gain k_i at a certain frequency ω_i (this gain may be unitary and hence ω_i is the gain crossover frequency or 0.707 (corresponding to $-3d\mathrm{B}$) and hence ω_i is the bandwidth frequency). This pole can be readily computed as:

$$\omega_i' = \sqrt{\frac{M_{h,i}^2 - k_i^2}{k_i^2 - M_{l,i}^2}} \cdot \omega_i$$

Form the two weighting filters W_{S_o}, W_{KS_o} using the above formulas. Take appropriate gains for both following the discussion of lecture #3.5. Illustrate the corresponding target transfer functions and discuss on your selection.

Hints: For the low/high frequency gain parameters use low/high values (respectively high/low values) for the sensitivity and complementary sensitivity functions. For the gain k_i use either selection and for the bandwidths use a value smaller than the one of the reference model for the sensitivity function, and a value corresponding to the actuator dynamics for the control times sensitivity function.

D) Synthesis block diagram (10%). The standard form for the controller computation is shown in Figure 5. We find the desired reference model transfer function $T_{\mathrm{RM}}(s)$, the inner closed loop $G_{\mathrm{in}}(s)$, two interface gain matrices[2] with $K_z = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $K_v = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and finally the weighting filter matrix $W(s) = \begin{bmatrix} W_{S_o} & 0 \\ 0 & W_{KS_o} \end{bmatrix}$. We can see that we have two exogenous generalized disturbances r, d (reference plus disturbance), one control input u, two performance signals $\tilde{z}_1, \tilde{z}_2$, two measure signals v_1, v_2 and finally the weighted performance signals z_1, z_2. Use MATLAB and/or Simulink to compute the transfer matrix $\tilde{P}(s)$ (orange box) and the weighted transfer matrix $P(s)$ (orange plus green box). Write down each of the transfer functions:

$$\begin{bmatrix} z_1 \\ z_2 \\ v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \\ * & * & * \end{bmatrix} \begin{bmatrix} r \\ d \\ u \end{bmatrix}$$

Finally comment on the choice of the control layout of Figure 5.

[2] The interface matrices are used mainly to avoid naming problems related to the connect function of RCT.

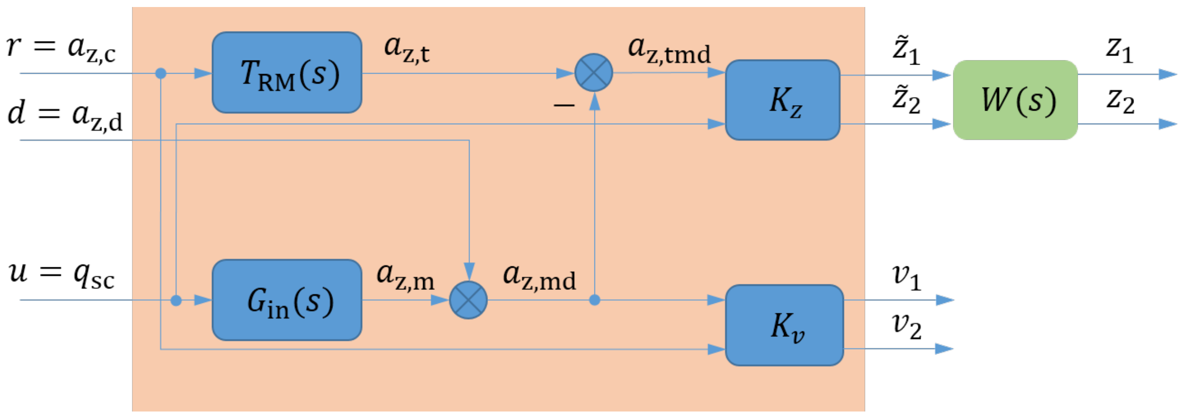

Figure 5 - Standard form

E) Controller Synthesis (10%). Use MATLAB hinfsyn to design a full order $\mathcal{H}_\infty$ controller $K_{\mathrm{FO}}(s)$ for the standard form of Figure 5. How are $K_{\mathrm{dr}}(s)$ and $K_{\mathrm{cf}}(s)$ of Figure 4 obtained from $K_{\mathrm{FO}}(s)$? What is the order of the (minimal) controllers? Write the weighted closed loop transfer matrix components:

$$z(s) = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} T_{r\to z_1} & T_{d\to z_1} \\ T_{r\to z_2} & T_{d\to z_2} \end{bmatrix} \begin{bmatrix} r \\ d \end{bmatrix} = T_{w\to z}(s) w(s)$$

Compute $\|T_{w\to z}\|_\infty$ and compare with the performance level γ obtained from hinfsyn. Plot the singular values (using a 2x2 figure) of the four transfer functions of the unweighted closed loop $T_{w\to\tilde{z}}(s)$ and superimpose for each one the target template (inverse of the corresponding weighting function). Finally play around with the disturbance rejection weighting filter bandwidth ω_{S_o} supposing that the high frequency gain M_{h,S_o} corresponds to $\|1 - T_{\mathrm{CL}}(s)\|_\infty$ (roughly 1.51), so that $\gamma \simeq 1$.

F) Controller Implementation (10%). We want now to convert the control implementation scheme of Figure 4 into the one of Figure 6. Compute first the controllers $K'_{\mathrm{cf}}, K'_{\mathrm{dr}}$ as a function of $K_{\mathrm{cf}}, K_{\mathrm{dr}}$ taking care of the now negative feedback. What is the controller order of the controllers obtained? Proceed to a more detailed examination of the controller poles and zeros and verify the high frequency dynamics of the feedback controller; a characteristic proper to $\mathcal{H}_\infty$ control. Reduce the order of the controller using model order reduction techniques as much as possible; you should be able to reach 3rd order; compare the singular values of the full and reduced order controllers.

Hints: MATLAB functions for minimal realization is 'minreal', model order reduction is 'reduce' etc.

G) Controller Simulation (5%). Use Simulink to simulate the complete control system (inner and outer loop). Provide a reference step command $\mathrm{a_{z,c}}$ of $1\mathrm{g}$ and observe the output normal acceleration response $\mathrm{a_{z,m}}$ the pitch rate q as well as the control signal $\delta_{\mathrm{q,c}}$ (in degrees) and its derivative. How does the output follows the reference model? Comment on the results. Check the control system disturbance rejection performance for a step disturbance $\mathrm{a_{z,d}} = 0.1\mathrm{g}$ and plot the same variables.

Hints: The overall control system must be a combination of Figure 3 and Figure 6 for the inner and output control loops respectively.

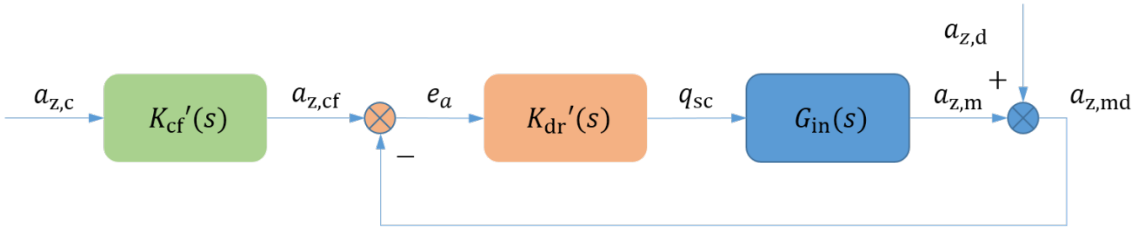

Figure 6 - Controller Implementation

H) Fixed Order Controller Synthesis (25%)[3]. We saw in question E) that it is possible to à posteriori reduce the controller order using model order reduction. Design controllers of the same order as before using structured $\mathcal{H}_\infty$ control design. Play with the various orders of the disturbance rejection and command following controller; simulate and compare both full and fixed order design methods.

Hints: Related MATLAB functions are hinfstruct, hinfstructOptions, ltiblock.tf, etc.

❖ ***Question 2.2: Loop Shaping Design.***

For future use.

3. Robustness Analysis (25%)

A) Basic Robustness Analysis (10%). Use Simulink to verify the gain and phase margins of the resulting SISO loops by opening the loop at the input of the actuator and at the output of each sensor.

Hint: An easy way to do that is to use the capabilities of Simulink Control Design. Right click on the place of the loop that you want to open and then on the menu choose Linear Analysis Point → Loop Transfer. Then launch the wizard: Analysis → Control Design → Linear Analysis and click on the pole-zero map. Finally export the resulting model linsys1 to the MATLAB workspace by dragging and dropping and plot its Bode (take care to inverse the sign of the plant before).

B) Advanced Robustness Analysis (15%). We want to use mu-analysis techniques to verify the robust stability of the closed loop system with respect to the modeled uncertainty of Question 1.1. To do this we will use the so called $M - \Delta$ structure for robustness analysis. The departure point is the uncertain open loop system in its $H - \Delta$ form in feedback with all the controllers K_q, K_{sc}, K_{dr}' but not the feedforward controller (why?) and no external references present as seen in Figure 7. Compute the transfer function $M(s)$ and use mu-analysis to conclude on robust stability. What is the robustness margin of your closed loop system? Plot the minimum and maximum bounds of mu as a function of frequency and comment on the selected frequency grid.

[3] The student may choose this question or Part 3 on robustness analysis.

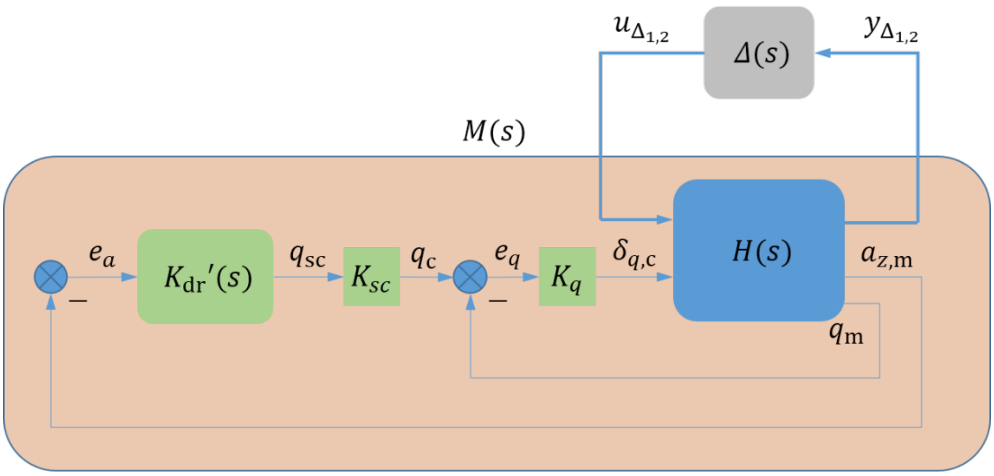

Figure 7 - Robustness Analysis

Hints: Related MATLAB functions are lftdata, frd and mussv for mu analysis.

4. Appendix

The values for the computation of the open loop uncertain state space model are shown in the following table.

Z_α	M_α	M_q	Z_δ	M_δ	A_a	A_δ	V	g
-1231.914	-299.26	$\simeq 0$	-107.676	-130.866	-1429.131	-114.59	947.684	9.81

ω_a	ζ_a	r_{M_α}	r_{M_δ}
150	0.7	57.813%	32.716%

Table 1 - Data values