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Subject: DSA 5400 Stat found

1. Given the following joint probability distribution. Please evaluate

X	Y	fXY (x, y)
-1	-2	1/4
-0.5	-1	1/8
0.5	1	1/2
1	2	1/8

Sol:

(a) $P(X < 0.5, Y < 1.5) = P(X=-1,Y=-2) + P(X=-0.5,Y=-1)$

$= 1/4 + 1/8$

$= 3/8$

$= 0.375$

(b) $P(X < 0.5) = P(X=-0.5, Y=-1) + P(X=-1, Y=-2)$

$= 1/8 + 1/4 = 3/8$

$= 0.375$

(c) $P(Y < 1.5) = P(Y=1, X=0.5) + P(Y=-1,X=-0.5) + P(Y=-2,X=-1)$

$= 1/2 + 1/8 + 1/4 = 7/8$

$= 0.875$

(d) $P(X > 0.25, Y < 4.5) = P(X=0.5, Y=1)+P(X=1, Y=2)$

$= 1/2 + 1/8 = 5/8$

$= 0.625$

(e) $E(X) = (-1)(1/4) + (-0.5)(1/8) + (0.5)(1/2) +(1)(1/8)$

$=-1/4 - 1/16 +1/4 +1/8$

$= 0.0625$

(f) V(X) = $(-1 - 0.0625)^2 * \left(\frac{1}{4}\right) + (-0.5 - 0.0625)^2 * \left(\frac{1}{8}\right) + (0.5 - 0.0625)^2 * \left(\frac{1}{2}\right) +$
$(1 - 0.0625)^2 * \left(\frac{1}{8}\right)$

= 0.2822 + 0.0395 + 0.0957 + 0.1098

= 0.5272

2. Determine the covariance and correlation for the following joint probability distribution.

X	Y	fXY (x, y)
1	3	1/4
1	4	1/2
2	5	1/8
4	6	1/8

Sol:

E(X) = (1)*(3/4) + (2)*(1/8) + (4)*(1/8) = 3/2 = 1.5

V(X) = $(1 - 1.5)^2 * \left(\frac{3}{4}\right) + (2 - 1.5)^2 * \left(\frac{1}{8}\right) + (4 - 1.5)^2 * \left(\frac{1}{8}\right)$

= 0.1875 + 0.0312 + 0.7812 = 0.999

E(Y) = (3)*(1/4) + (4)*(1/2) + (5)*(1/8) + (6)*(1/8) = 4.125

V(Y) = $(3 - 4.125)^2 * (\frac{1}{4}) + (4 - 4.125)^2 * \left(\frac{1}{2}\right) + (5 - 4.125)^2 * \left(\frac{1}{8}\right) + (6 - 4.125)^2 * (\frac{1}{8})$

=0.3164 + 0.0078 + 0.0957 + 0.4394

V(Y) = 0.8593

Covariance:

$C(X,Y)$ = (1)(3)(1/4) + (1)(4)(1/2) + (2)(5)(1/8) + (4)(6)(1/8) –(1.5)(4.125)

= 7 – 6.18 = 0.82

Correlation :

$$\rho_{xy} \;=\; = \frac{0.82}{\sqrt{0.999*0.8593}} = 0.885$$

3. Assume that the weights of individuals are independent and normally distributed with a mean of 165 pounds and a standard deviation of 25 pounds.
Suppose that 25 people squeeze into an elevator that is designed to hold 4300 pounds.
Sol:

Given that Normal distribution

Individual , mean = 165 , Standard deviation = 25

For 25 people ,

E(Y) = (25)* (165) = 4125

V(Y) = $(25)^2 * 25 = 15625$

S.D = $\sqrt{15625}$ = 125

(a) What is the probability that the load (total weight) exceeds the design limit?

Design limit = 4300 pounds

Using R statements for normal distribution ,

P(X>4300) = 1 – pnorm(4300 , 4125 , 125)

= 0.0807

(b) What design limit is exceeded by 25 occupants with probability 0.001?

Using R statements for normal distribution ,

P(X>x) = 0.001

x = qnorm(0.999 , 4125 , 125)

= 4511.279

4. The weight of a small candy is normally distributed
with a mean of 0.2 ounce and a standard deviation of 0.01 ounce. Suppose that 20 candies are placed in a package and that the weights are independent.

Sol:

Given is a Normal distribution,

For Individual , mean = 0.2 ounce , Standard deviation = 0.01 ounce

For 20 candies ,

E(Y) = E(X1) + E(X2) +.....+ E(X20)

= (20)* (0.2) = 4

V(Y) = $(0.01)(0.01)(20) = 0.002$

S.D = $\sqrt{\mathbf{0.002}}$ = 0.0447

(a) What is the probability that the net weight of a package is less than 3.5 ounces?

Using R statements for normal distribution ,

P(X< 3.5) = pnorm(3.5 , 4 , 0.0447)

= 2.396026e-29

(b) What value will the mean weight exceed with probability 0.98?,

Using R statements for normal distribution ,

= qnorm(0.02, 4 , 0.0447)

= 3.908197