Subj :- DSA 5400

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HW3

1. Suppose that $f(x) = e^{-x}$ for $x \geq 0$ is a PDF of a distribution. Determine the following.

Sol :- Given that

$$f(x) = e^{-x}$$

For the PDF of distribution

$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad \Rightarrow \quad P(a \leq x \leq b) = \int_a^b f(x)\,dx$$

(a) $P(x>1)$

The range of x is greater than 1

$$P(x>1) = \int_1^{\infty} e^{-x}\,dx$$

$$= \left[-e^{-x}\right]_1^{\infty}$$

$$= \left[-e^{-\infty} + e^{-1}\right] = \left[0 + \frac{1}{e}\right]$$

$$P(x>1) = e^{-1}$$

$$\boxed{P(x>1) = 0.3679}$$

(b) $P(1 < x < 2.5)$

The range of x is $[1, 2.5]$

$$P(1 < x < 2.5) = \int_{1}^{2.5} e^{-x} dx$$

$$= \left[-e^{-x}\right]_{1}^{2.5}$$

$$= \left[-e^{-2.5} + e^{-1}\right]$$

$$\boxed{P(1 < x < 2.5) = 0.2858}$$

(c) $P(x = 3)$

For continuous distribution, the probability at a specific point is zero.

$$P(x = 3) = 0.$$

(d) $P(x < 4)$

Given $x \geq 0$

$$P(x < 4) = \int_{0}^{4} e^{-x} dx$$

$$= \left[-e^{-x}\right]_{0}^{4}$$

$$= \left[-e^{-4} + e^{-0}\right]$$

$$= \left[-e^{-4} + 1\right]$$

$$\boxed{P(x < 4) = 0.9817}$$

(e) $P(X \geq a) = 0.10$

$$P(X \geq a) = \int_a^\infty e^{-x} dx = 0.10$$

$$\Rightarrow \left[-e^{-x}\right]_a^\infty = 0.10$$

$$e^{-\infty} + e^{-a} = 0.10$$

$$0 + e^{-a} = 0.10$$

$$e^{-a} = 0.10$$

$$-a = \ln(0.10)$$

$$\boxed{a = 2.3026}$$

2] The demand for water use in phoenix in 2003 hit a high of about 440 million gallons per day on june 27. Water use in the summer is normally distributed with a mean of 320 million gallons per day and a standard deviation of 43 million gallons per day. City reservoirs have a combined storage capacity of nearly 350 million gallons.

Sol: Given that

Mean $\mu = 320$

Standard deviation $(\sigma) = 43$

Total capacity of city reservoir $X = 350$

(a) Probability that a day requires more water than is stored in city reservoirs.

$$P(X > 350)$$

$$P(X > 350) = 1 - P(X \leq 350)$$

∴ using R-program : pnorm (x, mean, sd)

$$= 1 - \text{pnorm}(350, 320, 43)$$

$$\boxed{P(X > 350) = 0.2426}$$

(b) ~~Rese~~ Reservoir capacity is needed so that the probability that exceeded is 1%

$$P(X > x) = 0.01$$

∴ R-program $x = \text{qnorm}(1 - P, \mu, \sigma)$

$$= \text{qnorm}(0.99, 320, 43)$$

$\boxed{x = 420.033}$ million gallons

(c) Amount of water use is exceeded with 95%

$$P(X > x) = 0.95$$

$$= \text{qnorm}(1 - 0.95, 320, 43)$$

$= \underline{\underline{249.2713}}$ million gallons.

3] The Number of views on a web-site follows a poisson distribution with an average of 1.6 view per minute.

Sol: Given

Average no. of views on website $= \lambda = 1.6$ per minute

For poisson distribution

$$p(x=k) = \frac{e^{-\lambda} \lambda_T^k}{k!}$$

where, λ is avg rate

k is no. of events.

(a) Probability of no views in a minute.

$k=0$

$\lambda_T = 1.6 \times 1 = 1.6$

$$p(x=0) = \frac{e^{-1.6} (1.6)^0}{0!}$$

$\therefore$ R-program $\Rightarrow$ $p(x=0) =$ dpois(0, 1.6)

$$\boxed{P(x=0) = 0.2018}$$

(b) Probability of two or fewer views in 10 minutes.

$$P(x \leq 2) = \sum_{k=0}^{2} \frac{e^{-\lambda_T} \lambda_T^k}{k!}$$

$$\boxed{\lambda_T = 1.6 \times 10 = 16}$$

$$= \left[\frac{e^{-16} (16)^0}{0!} + \frac{e^{-16} (16)^1}{1!} + \frac{e^{-16} (16)^2}{2!} \right]$$

R program $\Rightarrow P(x \leq 2) = \text{dpois}(0,16) + \text{dpois}(1,16) + \text{dpois}(2,16)$

$$\boxed{P(x \leq 2) = 1.63176\, e^{-05}}$$

(c) Length of a time interval such that probability of no views in an interval of the length is 0.001.

$$P(x=0) = 0.001$$

$$\frac{e^{-\lambda T} (\lambda T)^0}{0!} = 0.001$$

$$e^{-1.6t} = 0.001$$

$$-1.6t = \ln(0.001)$$

$$-1.6\, t = -6.9077$$

$$\boxed{t = 4.317}$$

4]. The distance between major cracks in a highway follows exponential distribution with a mean of 6 miles.

Sol:
Given

mean $\mu = 6$ miles.

$$\mu = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{\mu} = \frac{1}{6}$$

(a) Probability there no major cracks in 12 mile strech of the highway.

$P(x > 12)$

$P(x > 10) = 1 - P(x \leq 12)$

$\therefore$ R program $= 1 - \text{pexp}(x, \lambda)$

$= 1 - \text{pexp}(12, 1/6)$

$\boxed{P(x > 10) = 0.1353}$

(b) Probability that there are two major cracks in a 12 mile streeth of the highway.

$T = 12$ mile , $\lambda = 1/6$

using poisson distribution

$P(X = x) = \text{dpois}(x, \lambda T)$

$P(X = 2) = \text{dpois}(2, (1/6)\,12)$

$\boxed{P(X = 2) = 0.270}$

(c) Probability the first major crack occurs bt 10 and 12 miles of the start of inspection.

miles $= P(10 < x < 12)$

R program $=$ pexp (12, 1/6) - pexp(10, 1/6)

miles $= \underline{0.0632}$

(d) Probability there are no major cracks in two seperate 6 mile streches of the highway.

$= P(x>6) \times P(x>6)$

$P(x>6)^2 = (1 - P(x \leq 6))^2$

$\therefore$ R-program $= (1-$ pexp $(6, 1/6))^2$

$P(x>6)^2 = 0.1352.$

(e) No cracks in the first 6 miles inspected, probability there No major cracks in next 12 miles inspected

$P(x > 12) = 1 - P(x \leq 12)$

$= 1 -$ pexp $(12, 1/6)$

$\boxed{P(x>12) = 0.1353}$