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HW2

Subject :- DSA 5400

1. An assembly consists of three mechanical components. Suppose that the probabilities that the first, second, and third components meet specifications are 0.8, 0.75 and 0.6, respectively. Assume that the components are independent. Determine the probability mass function of the Number of components in the assembly that meet specifications.

Sol: Given probability

P [first component meeting specification] = 0.8

P[second components meeting specification] = 0.75

P [Third component meeting specification] = 0.6

(1) $P(x=0) = P(A' \cap B' \cap C')$

$= P(A') \times P(B') \times P(C')$

$= 1 - P(A) \times 1 - P(B) \times 1 - P(C)$

$= 1 - 0.8 \times 1 - 0.75 \times 1 - 0.6$

$= 0.2 \times 0.25 \times 0.4$

$\boxed{P(X=0) = 0.02}$

② $P(X=1) = P(A' \cap B' \cap C) + P(A' \cap B \cap C') + P(A \cap B' \cap C')$

$= 0.2 \times 0.25 \times 0.6 + 0.2 \times 0.75 \times 0.4 + 0.8 \times 0.25 \times 0.4$

$= 0.03 + 0.06 + 0.08$

$\boxed{P(X=1) = 0.17}$

③ $P(X=2) = P(A' \cap B \cap C) + P(A \cap B' \cap C) + P(A \cap B \cap C')$

$= 0.2 \times 0.75 \times 0.6 + 0.8 \times 0.25 \times 0.6 + 0.8 \times 0.75 \times 0.4$

$= 0.09 + 0.12 + 0.24$

$\boxed{P(X=2) = 0.45}$

④ $P(X=3) = P(A \cap B \cap C)$

$= 0.8 \times 0.75 \times 0.6$

$\boxed{P(X=3) = 0.36}$

Probability of Mass function of components

$P(X=0) = 0.02$

$P(X=1) = 0.17$

$P(X=2) = 0.45$

$P(X=3) = 0.36.$

2. The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 15 calls are placed to the airline.

Sol: Given

Probability of occupied $P = 0.4$

Probability of not occupied $q = 0.6$

$n = 15$

a) what is probability that for exactly three calls, lines are occupied?

$$P(X=x) = nC_x p^x (1-p)^{n-x}$$

$$P(X=3) = 15C_3 (0.4)^3 (1-0.4)^{12}$$

$$= \frac{15!}{12!\,3!} \times (0.4)^3 (0.6)^{12}$$

$$= \frac{15 \times 14 \times 13 \times \cancel{12!}}{\cancel{12!}\,3!} \times (0.4)^3 \times (0.6)^{12}$$

$$\boxed{P(X=3) = 0.0633879}$$

R statement

→ dbinom(3,15,0.4)

→ 0.0633879

b) What is probability at least four calls, lines are not occupied?

$$P(x \geq 4) = 1 - P(x=0) - P(x=1) - P(x=2) - P(x=3)$$

$= 1 -$ dbinom (0, 15, 0.6) $-$ dbinom (1, 15, 0.6) $-$ dbinom(2, 15, 0.6)
$-$ dbinom (3, 15, 0.6)

$\boxed{P(x \geq 4) = 0.99\,80722}$

R statement

$\rightarrow$ 1 $-$ dbinom (0, 15, 0. 6) $-$ dbinom (1, 15, 0. 6) $-$ dbinom (2, 15, 0.6)
$-$ dbinom (3, 15, 0.6).

c) What is the expected Number of calls in which the lines are all occupied?

Mean $= n \times p$

$= 0.4 \times 15$

$= \underline{6}.$

3 Assume that each of your calls to apopular radio station has aprobability of 0.05 of connecting, thatis, of not obtaining a busy signal. Assume that your calls are independent.

sol: Given

Probability of connecting signal chances = 0.05

Probability of not connecting signals = 1-0.05

= 0.95

(a) What is the probability that your first connects is your 12th call?

$$P(X=12) = (1-P)^{x-1} P$$

$$= (0.95)^{11}(0.05)$$

$$= 0.028$$

R-statement

$$P(X=x) = dgeom\ (x-1, P)$$

$$P(X=12) = dgeom\ (11, 0.05)$$

$$= 0.28$$

(b) Whatis the probability that your third call that connects is your 12th call?

$$P(X=x) = (1-P)^{x-1} P$$

$\Rightarrow$ 3 trails before the first success

$$P(X=3) = (1-0.05)^2 (0.05)$$

$$= 0.045$$

The probability that 3rd call that connects is 12th call

$= 0.045 \text{ dgeom}(x-1, p)$

$= 0.45 \text{ dgeom}(11, 0.05)$

$= 0.0012.$

(c) What is the probability that requires more than 12 calls for you to get four connects.

$P(X \leq 12) = \text{pgeom}(x-1, p)$

$= \text{pgeom}(11, 0.05)$

$= 0.4596.$

4. A utility company might offer electrical rates based on time of day consumption to decrease the peak demand in a day. Enough customers need to accept the plan for it be successful. Suppose that among 100 major customers, 30 would accept the plan. The utility select 15 major customers randomly (without replacement) to contact and promote the plan.

Sol:- Given

Total No. of major customers $N = 100$

Acceptence of required No. of approvals $K = 30$

selected No. of major customers $n = 15$

selected major customer

(a) Probability exactly four of the selected major customer accept the plan?

$x = 4$

$$f(x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$f(4) = \frac{\binom{30}{4}\binom{70}{11}}{\binom{100}{15}}$$

$= 0.234$

R statement exactly four of the selected major customer accept = dhyper (4, 30, 70, 15)

(b) What is the probability that at least four of the selected major customer accepts the plan?

$P(x \geq 4) = 1 - \text{phyper}(4, 30, 70, 15)$

$= 0.512$

$P(x \geq 4) = 1 - P(x \leq 3)$

$= 1 - \text{Phyper}(3, 30, 70, 15)$

$= 0.722$

5. The Number of surface flaws in plastic panels used in the interior of automobiles has a poisson distribution with a mean of 0.06 flaw per square foot of plastic panel. Assume that an automobile interior contains 10 square feet of plastic panel.

sol:- Given

mean $\lambda T = 0.06$

length $T = 10$

(a) What is probability there are no surface flaws in auto interior?

$p(X=x) = dpois(x, \lambda T)$

$= dpois(0, 0.06)$

$= 0.941$

(b) Probability of more than two surface flaws in an auto interior?

$p(x \geq 2) = 1 - P(x \leq 2)$

$= 1 - ppois(2, 0.06)$

$\boxed{P(x \geq 2) = 0.0017}$