MEPS
2023-2024

APPLIED MICROECONOMETRICS

Generalized Method of Moments (GMM)

Dynamic Panel Data Models

Yasin Tosun

Economic Policy

University of Siegen

Contents

A. Theoretical Coverage
B. Dynamic Panel Data Models Estimation Methods
B. 1. Pooled Least Squares Estimator
B. 2. Balestra and Nerlove Two-Stage Least Squares Method
B. 3. Random Effects Estimator
B. 4. Fixed Effects Estimator
B. 5. First Differences Estimator
B. 5. 1. Anderson & Hsiao
B. 5. 2. Arellano & Bond Generalized Method of Moments
B. 6. Arellano & Bover/Blundell & Bond System Generalized Method of Moments
C. GMM Applications: Arellano - Bond (1991)
C. 1. Anderson & Hsiao
C. 2. Arellano & Bond Generalized Method of Moments
C. 3. Arellano & Bover/Blundell & Bond System Generalized Method of Moments
D. Conclusion

A. Theoretical Coverage

- Dynamic panel data models differ from static panel data models by including lagged variables or variables.
- Dynamic panel data models can be examined under two groups:
  - Distributed lag panel data models
  - Autoregressive panel data models
- In autoregressive panel data models, the lagged values of the dependent variable are included as independent variables.
- An ***autoregressive panel data model*** with one lag can be represented as follows:

$$Y_{it} = \delta Y_{i,t-1} + \beta X'_{it} + \mu_i + u_{it} \tag{1}$$

- In distributed lag panel data models, the lagged values of the independent variables are included as independent variables. This can be represented as follows:

$$Y_{it} = \beta X'_{it} + \delta X'_{i,t-1} + \mu_i + u_{it} \tag{2}$$

A. Theoretical Coverage

- Typically, when discussing dynamic models, autoregressive models come to mind first due to the problems they cause.

- The most significant problem is the ***endogeneity*** issue caused by including the *lagged dependent variable* as an *independent variable* in the model.

- Generally, in dynamic models, it is known that $\boldsymbol{Y_{i,t-1}}$ is correlated with $\boldsymbol{u_{it}}$ it due to past shocks.

- Therefore, in model (1), $\boldsymbol{Y_{i,t-1}}$ is correlated with the error term, which also includes $\boldsymbol{\mu_i}$.

- In such cases, the estimators become ***biased*** and ***inconsistent***.

B. Dynamic Panel Data Models Estimation Methods

- We will examine Dynamic Panel Data models under the following headings.
- The main purpose is to explain a chronological solution process to problems in dynamic panel data models.
- STATA codes have also been added for coding basic notations.

1. Pooled Least Squares Estimator
2. Balestra and Nerlove Two-Stage Least Squares Method
3. Random Effects Estimator
4. Fixed Effects Estimator
    - 4. 1. Nickell Bias
5. First Differences Estimator
    - 5. 1. Anderson & Hsiao
    - 5. 2. Arellano & Bond Generalized Method of Moments
        - 5. 2. 1. Windmeijer (2005) : Small Sample Correction / Robust Standard Errors
6. Arellano & Bover/Blundell & Bond System Generalized Method of Moments
    - 6. 1. Roodman (2006): ‘xtabond2’ – Advanced Orthogonal Deviations

B. 1. Pooled Least Squares Estimator

- Dynamic panel data models can be estimated using the pooled ordinary least squares (POLS) method.

- However, due to the correlation between the lagged dependent variable $\boldsymbol{Y_{i,t-1}}$ and the error term, ***biased*** and ***inconsistent estimates*** are obtained with the POLS method.

- Additionally, as is well known, the classical model makes estimates by ignoring the presence of unit (and time) effects.

- If there are unit (and time) effects in the model, biased estimates are obtained with the POLS method.

$$\boldsymbol{Y_{it} = a + \beta_1 Y_{i,t-1} + \beta_2 X_{it} + u_{it}} \tag{3}$$

- Basic STATA codes to estimate the dynamic panel data model with the pooled least squares method,

xtreg Y L.Y X

B. 2. Balestra and Nerlove Two-Stage Least Squares Method

- A suitable instrumental variable is chosen and used in place of the lagged dependent variable that is correlated with the error term. Instrumental variables must meet the following conditions:
  - Instrumental variables must be uncorrelated with the error terms.
  - Instrumental variables must be highly correlated with the variables they are replacing.

- Balestra and Nerlove (1966) proposed using the lagged values of the dependent variable as instrumental variables. This model is estimated using the pooled ordinary least squares (POLS) method.

- In this context, the instrumental variable matrix $\boldsymbol{Z}$ can be structured as follows:

$$\boldsymbol{Z} = \begin{bmatrix} Y_{i1} & 0 & 0 & \cdots & 0 \\ Y_{i1} & Y_{i2} & 0 & \cdots & 0 \\ Y_{i1} & Y_{i2} & Y_{i3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_{i1} & Y_{i2} & Y_{i3} & \cdots & Y_{i,t-1} \end{bmatrix}$$

B. 2. Balestra and Nerlove Two-Stage Least Squares Method

- When the model is transformed using the $\boldsymbol{Z}$ instrumental variable matrix:

$$\boldsymbol{ZY = \delta ZX + Zu} \qquad (4.1)$$

- In this equation:
- $\boldsymbol{ZY}$: The product of the $\boldsymbol{Z}$ matrix and the $\boldsymbol{Y}$ vector.
- $\boldsymbol{ZX}$: The product of the $\boldsymbol{Z}$ matrix and the $\boldsymbol{X}$ vector.
- $\boldsymbol{Zu}$: The product of the error term and the $\boldsymbol{Z}$ matrix.
- When estimation is done using the pooled ordinary least squares method, the estimator is obtained as follows:

$$\boldsymbol{\hat{\delta} = \left(X'Z(Z'Z)^{-1}Z'X\right)^{-1}X'Z(Z'Z)^{-1}Z'Y} \qquad (4.2)$$

- In this equation:
- $\boldsymbol{\hat{\delta}}$: The estimated coefficient vector.
- $\boldsymbol{X}$: The matrix of independent variables.
- $\boldsymbol{Z}$: The instrumental variable matrix.
- $\boldsymbol{Y}$: The dependent variable vector.
- $\boldsymbol{(Z'Z)^{-1}}$: The inverse of the $\boldsymbol{Z'Z}$ matrix.

B. 2. Balestra and Nerlove Two-Stage Least Squares Method

- To make this more concrete, let's go through these steps using a small example with 2 independent variables (X) and 3 observations:

$$Y = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 1 & 2 & 3 \end{bmatrix} \qquad X = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- Step 1: Calculate $\boldsymbol{Z'Z}$
- Step 2: Calculate $(\boldsymbol{Z'Z})^{-1}$
- Step 3: Calculate $\boldsymbol{X'Z}$
- Step 4: Calculate $\boldsymbol{X'Z(Z'Z)^{-1}Z'X}$
- Step 5: Calculate $\left(\boldsymbol{X'Z(Z'Z)^{-1}Z'X}\right)^{-1}$
- Step 6: Calculate $\boldsymbol{X'Z(Z'Z)^{-1}Z'Y}$

B. 2. Balestra and Nerlove Two-Stage Least Squares Method

- Through these calculations, the value of the estimator $\hat{\boldsymbol{\delta}}$ is obtained as follows:

$$\hat{\boldsymbol{\delta}} = \begin{bmatrix} \mathbf{1.00000000} \\ \mathbf{7.10542736 \times 10^{-15}} \end{bmatrix}$$

- These results show that the first component of the $\hat{\boldsymbol{\delta}}$ vector is 1, and the second component is very close to zero.

- The coefficient of 1 indicates that a one-unit increase in $\boldsymbol{X_1}$ causes a one-unit increase in the dependent variable $\boldsymbol{Y}$. The coefficient of $\mathbf{7.10542736 \times 10^{-15}}$, which is a very small value, indicates that the effect of $\boldsymbol{X_2}$ on $\boldsymbol{Y}$ is negligible.

- This <u>estimator</u> can <u>reduce bias </u>caused by <u>endogeneity</u> in dynamic models and can be used <u>if there are no unit effects</u>.

- However, <u>if unit (and time) effects are present</u>, it remains <u>biased because it ignores </u>their presence.

B. 2. Balestra and Nerlove Two-Stage Least Squares Method

- Let's estimate the dynamic model using the two-stage least squares (2SLS) estimator proposed by Balestra and Nerlove.
- Instead of using $\boldsymbol{Y_{i,t-1}}$ as an instrumental variable, we use $\boldsymbol{Y_{i,t-2}}$.
- To estimate the dynamic model using Balestra and Nerlove's approach, the following STATA command is used:

ivregress 2sls Y (L.Y=L2.Y) X

- The number of observations has decreased due to the lagged variable, resulting in a loss of two years of observations for each unit. As previously mentioned, this estimator is biased if there are unit effects since it does not account for them.
- To test for endogeneity in STATA, the following command is used:

estat endogenous

- The null and alternative hypotheses for the test are:

$\boldsymbol{H_0}$: The independent variables are exogenous.

$\boldsymbol{H_1}$: The independent variables are not exogenous.

- If the p-value ≤ 0.05, the null hypothesis is rejected, indicating that endogeneity is present.

B. 3. Random Effects Estimator

- The random effects generalized least squares estimators for dynamic panel data models are also **biased.**

- This is because the unit effect $\boldsymbol{\mu_i}$ included in the error term is correlated with one of the independent variables $(\boldsymbol{Y_{i,t-1}})$.

- This correlation violates one of the key assumptions of the random effects model (as it is known, in the random effects model, the assumption $\boldsymbol{E(X_{it}\mu_i) = 0}$ is made).

- Therefore, the estimation of the dynamic model with the random effects assumption is inconsistent.

$$\boldsymbol{Y_{it} = a + \beta_1 Y_{i,t-1} + \beta_2 X_{it} + u_{it}} \quad (3)$$

- To obtain the dynamic random effects estimator command on STATA,

xtreg Y L.Y X,re

B. 4. Fixed Effects Estimator

- In the estimation of dynamic panel data models, more attention has been given to fixed effects and first difference estimators, which take unit effects into account and **allow** for **correlation between unit effects and independent variables**.

- The within-group transformation under the fixed effects assumption eliminates the unit effect $(\boldsymbol{\mu_i})$.

- **Nickell (1981)** demonstrated that the instrumental variables least squares estimator in autoregressive panel data models is **inconsistent** when **N is large and T is small**. This phenomenon is known in the literature as "dynamic panel bias" or "Nickell bias."

- In their study, Judson and Owen (1999) found that even **when T=30**, the <u>bias in parameter estimates </u>was equal to <u>*20%*</u>.

B. 4. Fixed Effects Estimator

- Kiviet (1995, 1999), and later Bun and Kiviet (2003) and Bruno (2005a), proposed a **correction** for the dynamic panel **bias** when N is large and T is small.
- According to this method, the model is estimated in two stages:
  - 1) The model is estimated using the within-group estimation method,
  - 2) The parameters are then corrected.

- For the correction,
- **Nickell (1981):** As the length of the time series increases, the deviation decreases at a rate of $\boldsymbol{O}(\mathbf{1}/\boldsymbol{T})$.
- **Kiviet (1999):** As both the length of the time series and the number of units in the panel increase, the deviation decreases at a rate of $\boldsymbol{O}(\frac{\mathbf{1}}{\boldsymbol{NT}})$. It is the rate of convergence to " O" of the deviation.
- **Bun and Kiviet (2003):** As both the length of the time series and the number of units in the panel increase, the deviation decreases much more rapidly at a rate of $\boldsymbol{O}(\frac{\mathbf{1}}{\boldsymbol{NT^2}})$.
- Finally, consistent estimators by Anderson & Hsiao(1982), Arellano & Bond(1995), Blundell & Bond (1998) can be used.

B. 4. Fixed Effects Estimator

- To obtain the dynamic instrument variable least squares estimator in STATA, the command:

xtreg Y L.Y X, fe

- To estimate using bias-corrected instrument variable least squares method with these STATA commands, use:

xtlsdvc Y X, initial(ah) vcov(50) bias(1) lsdv

xtlsdvc Y X, initial(ab) vcov(50) bias(2)

xtlsdvc Y X, initial(bb) vcov(50) bias(3)

- Explanation of the command components:

❖ *xtlsdvc: Bias-corrected instrument variable least squares method*

❖ *initial(..estimator..): Estimator*

❖ *Estimator - ah: Anderson & Hsiao ; ab: Arellano & Bond ; bb: Blundell & Bond*

❖ *vcov(..robust..): Bootstrap variance-covariance matrix*

❖ *bias(..1 2 3..): Convergence rate - 1: Nickell, 2: Kiviet, 3: Bun & Kiviet*

❖ *lsdv: Least Squares Dummy Variable*

B. 5. First Differences Estimator

- Another method used when the individual (unit) effect is correlated with the independent variable is the first differences method.

- With the first difference transformation in this method, the individual (unit) effect $\boldsymbol{\mu_i}$ drops out of the model in model (1). However, the lagged dependent variable is endogenous, meaning that estimates are obtained.

- Therefore, after the first difference transformation, it is recommended to control for the correlation between the lagged dependent variable and the error term using instrumental variables.

B. 5. 1. Anderson & Hsiao

- In this method, the first differences of the dynamic panel data model are taken, and then the variable $\boldsymbol{Y_{i,t-2}}$ or $\Delta\boldsymbol{Y_{i,t-2}}(=\boldsymbol{Y_{i,t-2}}-\boldsymbol{Y_{i,t-3}})$ is used as an instrumental variable instead of the independent variable $\Delta\boldsymbol{Y_{i,t-1}}(=\boldsymbol{Y_{i,t-1}}-\boldsymbol{Y_{i,t-2}})$, which is correlated with the error term.

- When the dynamic panel data model is expressed as follows:

$$\boldsymbol{Y_{it} = \gamma Y_{i,t-1} + \beta X'_{it} + \mu_i + u_{it}} \qquad (5)$$

- The first difference of this model is:

$$\boldsymbol{Y_{it} - Y_{i,t-1} = \gamma\left(Y_{i,t-1} - Y_{i,t-2}\right) + \beta\left(X'_{it} - X'_{i,t-1}\right) + \left(u_{it} - u_{i,t-1}\right)}$$

$$\boldsymbol{\Delta Y_{it} = \delta \Delta X_{it} + \Delta u_{it}}$$

$$\boldsymbol{\Delta X = \left[\left(Y_{i,t-1} - Y_{i,t-2}\right), \left(X'_{it} - X'_{i,t-1}\right)\right] = \left[\Delta Y_{i,t-1}, \Delta X'_{it}\right]} \qquad (6)$$

- By taking the first difference, the unit effect and fixed parameter are eliminated from the model.
- Here, the lagged dependent variable, which is an independent variable, is correlated with the error term.

B. 5. 1. Anderson & Hsiao

- To represent the independent variables $\Delta \boldsymbol{X} = \left[(\boldsymbol{Y}_{i,t-1} - \boldsymbol{Y}_{i,t-2}), (\boldsymbol{X}'_{it} - \boldsymbol{X}'_{i,t-1})\right] = \left[\Delta \boldsymbol{Y}_{i,t-1}, \Delta \boldsymbol{X}'_{it}\right]$, the model is estimated using pooled least squares method with one of the following instrumental variables:

$$\boldsymbol{Z} = \left[\boldsymbol{Y}_{i,t-2}, (\boldsymbol{X}'_{it} - \boldsymbol{X}'_{i,t-1})\right] = \left[\boldsymbol{Y}_{i,t-2}, \Delta \boldsymbol{X}'_{it}\right]$$

- It is proposed to use $\boldsymbol{Y}_{i,t-2}$ or $\Delta \boldsymbol{Y}_{i,t-2}$ as instruments instead of the variable $\Delta \boldsymbol{Y}_{i,t-1}$.

- In his work on dynamic panel data models, Arellano (1989) proved that $\boldsymbol{Y}_{i,t-2}$ is a more suitable instrument than $\Delta \boldsymbol{Y}_{i,t-2}$.
- Additionally, if $\boldsymbol{Y}_{i,t-2}$ is used as an instrumental variable, there will be a loss of two periods; if $\Delta \boldsymbol{Y}_{i,t-2}$ is used, there will be a loss of three periods.

- The pooled least squares estimator for the first-differenced instrumental variable model can be expressed as follows:

$$\widehat{\boldsymbol{\delta}} = \left(\Delta \boldsymbol{X}'\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'\Delta \boldsymbol{X}\right)^{-1}\Delta \boldsymbol{X}'\boldsymbol{Z}(\boldsymbol{Z}'\boldsymbol{Z})^{-1}\boldsymbol{Z}'\Delta \boldsymbol{Y}$$

- Anderson and Hsiao's (1982) estimator produces <u>consistent</u> estimators, but is <u>ineffective</u> because the <u>autocorrelation problem is not taken into account.</u>

B. 5. 1. Anderson & Hsiao

- To estimate the dynamic panel data model using Anderson and Hsiao's method with $\Delta \boldsymbol{Y}_{i,t-2}$ as an instrumental variable, the following command is used in Stata:

ivregress 2sls D.Y (LD.Y=L2D.Y) D.(X), noconstant first

- To estimate using the dynamic panel data model of Anderson and Hsiao with $\boldsymbol{Y}_{i,t-2}$ as an instrumental variable, the following command is used:

ivregress 2sls D.(Y X) (LD.Y=L2.Y), noconstant

B. 5. 2. Arellano & Bond Generalized Method of Moments

- If the error terms of the first difference model have constant variance and no autocorrelation, it is appropriate to use the estimator of Anderson and Hsiao for estimation.

- However, the first difference error terms are often negatively autocorrelated, and in this case, it is more appropriate to use the generalized method of moments (GMM) estimator by Arellano and Bond (1991).

- In this method, the first difference model is first transformed using the instrumental variable matrix, and then this transformed model is estimated using the generalized least squares method.

- Therefore, the generalized method of moments estimator is also known as the "two-stage instrumental variables estimator."

B. 5. 2. Arellano & Bond Generalized Method of Moments

- When the dynamic panel data model has no explanatory variable other than the lagged value of the dependent variable, it can be represented as follows:

$$\boldsymbol{Y_{it} = \gamma Y_{i,t-1} + \mu_i + u_{it}} \qquad \mathbf{(8)}$$

- The first difference of this model can be written as:

$$\boldsymbol{Y_{it} - Y_{i,t-1} = \gamma\left(Y_{i,t-1} - Y_{i,t-2}\right) + \left(u_{i,t} - u_{i,t-1}\right)} \qquad \mathbf{(9)}$$

- As seen, the unit effect is eliminated from the model.

- $\boldsymbol{Y_{i,t-1}}$ is correlated with $\boldsymbol{u_{i,t-1}}$ and thus the first difference estimator is biased downwards.

- In this model, <u>suitable instrumental variables</u> for $\boldsymbol{(Y_{i,t-1} - Y_{i,t-2})}$ are the lagged values because each lagged variable has zero correlation with the preceding difference error term, $\boldsymbol{\left(for\ example\ \ E\left(Y_{i,t-2}(u_{i,t} - u_{i,t-1})\right) = 0\right)}$

B. 5. 2. Arellano & Bond Generalized Method of Moments

- The instrumental variables matrix is :

$$Z_i = \begin{bmatrix} Y_{i1} & 0 & 0 & 0 & 0 & 0 & \vdots \\ 0 & Y_{i2} & Y_{i1} & 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & Y_{i3} & Y_{i2} & Y_{i1} & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{10}$$

- The first difference model with instrumental variables, including independent variables other than the lagged value, can be generally represented with matrices as:

$$Z'\Delta Y = Z'\Delta Y_{-1}\gamma + Z'\Delta X\beta + Z'\Delta u \tag{11}$$

- The generalized method of moments (GMM) estimator can be represented with matrices as follows:

$$\hat{\delta}_{GMM} = \left(\Delta X'Z\left(Z'\hat{\Omega}Z\right)Z'\Delta X\right)^{-1}\left(\Delta X'Z\left(Z'\hat{\Omega}Z\right)Z'\Delta Y\right) \tag{12}$$

- Here, $\hat{\Omega}$ is the variance-covariance matrix of the error terms.
- **In situations where N is small and T is large, tests tend to be biased. As a general rule, the number of instrumental variables used should be equal to or less than N.**

B. 5. 2. Arellano & Bond Generalized Method of Moments

- To make estimates using Arellano and Bond's generalized method of moments (GMM) estimator, the command:

  xtabond Y X, lag(1) noconstant

- Having the number of instrumental variables exceed the size of the unit can negatively affect the accuracy and reliability of the model. The number of instrumental variables can be reduced by defining a maximum lag using the code maxldep(#) added to the command:

  xtabond Y X, lag(1) noconstant maxldep(2)

- To make estimates using Arellano and Bond's two-step generalized method of moments estimator, the command:

  xtabond Y X, lag(1) noconstant twostep maxldep(2)

- **This command adds a lagged value of the dependent variable as an independent variable in the model and performs a two-step estimation.**

B. 5. 2. Arellano & Bond Generalized Method of Moments

- Additionally, using the two-step GMM corrects one-step GMM estimators for autocorrelation and heteroskedasticity (White, 1980).

- However, the standard errors obtained from the two-step GMM tend to be seriously biased downwards.

- Therefore, as indicated in the output, it is necessary to correct the standard errors.

- To make estimates using Arellano and Bond's two-step generalized method of moments estimator with Windmeijer's robust standard errors, the command:

xtabond Y X, lag(1) noconstant twostep vce(robust) maxldep(2)

B. 6. Arellano & Bover/Blundell & Bond System Generalized Method of Moments

- The **Arellano and Bond estimator becomes weak when the number of autoregressive parameters is too high or the variance of the unit effect is very large.**

- Additionally, when working with unbalanced panel data or when T is small, the first difference transformation also becomes weak.

- Therefore, instead of the first difference transformation, another proposed transformation is the "forward orthogonal deviations" or "orthogonal deviations" method.

- **Arellano and Bover (1995) suggest that instead of taking the difference of the previous period in the first differences method, the difference of the average of all possible future values of a variable is taken.**

B. 6. Arellano & Bover/Blundell & Bond System Generalized Method of Moments

- In summary, a two-system equation (original and transformed equations) is established and estimated together as a system.
- Therefore, the estimator is known as "system GMM".
- **Blundell and Bond (1998) emphasized the importance of the extra moment condition used to obtain an efficient estimator of the dynamic panel data model when T is small (N>T).**

- Let's use forward orthogonal deviations instead of first differences.
- This command allows for more test results to be obtained and provides options for obtaining consistent estimators in the presence of heteroskedasticity and autocorrelation (no correlation between units).

xtabond2 Y L.Y X, noconstant gmm(L.Y, lag(1 12) collapse) iv(X) orthogonal

- Estimate the model using the two-step system generalized method of moments with Windmeijer's standard errors.
- The command used is:

xtabond2 Y L.Y X, noconstant gmm(L.Y lag(1 12) collapse) iv(X) orthogonal robust twostep

C. GMM Applications: Arellano - Bond (1991)

- It is used to model labor demand for United Kingdom firms between 1976 and 1984.
- The aim is to find the lagged effect of wages, capital stock, and industrial production on labor.

$$n_{it} = \beta_0 + \beta_1 n_{it-1} + \beta_2 n_{it-2} + \beta_3 w_{it} + \beta_4 w_{it-1} + \beta_5 ys_{it} + \beta_6 ys_{it-1} + \beta_7 k_{it} + u_{it}$$

- n : Logarithm of labor
- w : Logarithm of wages
- k : Logarithm of capital stock
- ys : Logarithm of the industry's output where the firm is located
- i : id (firms) 140 firms
- t : year (1976 - 1984)

C. 1. Anderson & Hsiao

```
use http://www.stata-press.com/data/r11/abdata.dta
xtset id year
```

* The first one (lsdv) is the instrumental variable least squares estimator.

```
quietly xtreg l(0/1).(n w ys) k, fe robust
estimates store lsdv
```

* The second (lsdvb1), third (lsdvb2), and fourth (lsdvb3) provide the Anderson and Hsiao estimator.

```
help xtlsdvc
quietly xtlsdvc n w wL1 ys ysL1 k, initial(ah) vcov(50) bias(1)
estimates store lsdvb1
quietly xtlsdvc n w wL1 ys ysL1 k, initial(ah) vcov(50) bias(2)
estimates store lsdvb2
quietly xtlsdvc n w wL1 ys ysL1 k, initial(ah) vcov(50) bias(3)
estimates store lsdvb3
```

* The fifth (ahl2) (instrumental variable n_{it-2})

```
quietly ivregress 2sls D.n (LD.n=L2.n) D.(l(0/1).(w ys) k), noconstant vce(robust)
estimates store ahl2
```

* The sixth (ahl2d) (instrumental variable Δn_{it-2})

```
quietly ivregress 2sls D.n (LD.n=L2D.n) D.(l(0/1).(w ys) k), noconstant vce(robust)
estimates store ahl2d
```

*All results

```
estimates table lsdv lsdvb1 lsdvb2 lsdvb3 ahl2 ahl2d, b(%7.3g) star stat(N r2)
```

C. 1. Anderson & Hsiao

Figure 1

Variable	lsdv	lsdvb1	lsdvb2	lsdvb3	ahl2	ahl2d
n						
L1.	.571***	.776***	.775***	.798***		
LD.					.873*	.179
w						
--.	-.598***	-.613***	-.613***	-.614***		
L1.	.245*					
D1.					-.628***	-.573***
LD.					.474*	.0924
ys						
--.	.657***	.711***	.711***	.717***		
L1.	-.47***					
D1.					.787***	.567***
LD.					-.689*	-.116
k	.316***	.247***	.247***	.239***		
wL1		.327***	.326***	.335***		
ysL1		-.633***	-.632***	-.65***		
k						
D1.					.254**	.416***
_cons	.812					
N	891	891	891	891	751	611
r2	.771					

legend: * p<0.05; ** p<0.01; *** p<0.001

C. 1. Anderson & Hsiao

- When choosing among these six estimators to estimate the dynamic model, selections can be made from the four estimators **excluding** the first (lsdv) and the sixth (ahl2d) estimators.

- The results of the other four estimators are quite similar to each other, and the parameter signs are in the economically expected direction.

- Labor is positively affected by its own lag to a significant degree.

- Wages negatively affect labor demand, and their lag also has a negative effect.

- The production of the industry in which the firm operates positively affects labor demand, while its lag has a negative effect.

- The capital stock positively affects labor demand.

C. 2. Arellano & Bond Generalized Method of Moments

* The first (abond) is the one-step estimator by Arellano and Bond. (Windmeijer)

```
quietly xtabond n l(0/1).(w ys) k, lag(1) noconstant robust
estimates store abond
```

* The second (abond-two) is the two-step estimator. (Windmeijer)

```
quietly xtabond n l(0/1).(w ys) k, lag(1) twostep noconstant robust
estimates store abondtwo
```

* The subsequent estimators (abb1, abb2, and abb3) are bias-corrected instrumental variable least squares

```
quietly xtlsdvc n w wL1 ys ysL1 k, initial(ab) vcov(50) bias(1)
estimates store abb1
```

```
quietly xtlsdvc n w wL1 ys ysL1 k, initial(ab) vcov(50) bias(2)
estimates store abb2
```

```
quietly xtlsdvc n w wL1 ys ysL1 k, initial(ab) vcov(50) bias(3)
estimates store abb3
```

* All results

```
estimates table abond abondtwo abb1 abb2 abb3, b(%7.3g) star stat(N arm1 arm2)
```

C. 2. Arellano & Bond Generalized Method of Moments

Figure 2

Variable	abond	abondtwo	abb1	abb2	abb3
n					
L1.	.474***	.395**	.707***	.707***	.718***
w					
--.	-.581***	-.51***	-.606***	-.606***	-.607***
L1.	.235*	.19			
ys					
--.	.702***	.681***	.687***	.688***	.69***
L1.	-.362*	-.19			
k	.308***	.281***	.266***	.266***	.262***
wL1			.288***	.288***	.292***
ysL1			-.57***	-.57***	-.578***
N	751	751	891	891	891
arm1	-2.58	-1.49			
arm2	-1.17	-.949			

legend: * p<0.05; ** p<0.01; *** p<0.001

C. 2. Arellano & Bond Generalized Method of Moments

- Among these five estimators, the last three can be preferred.

- According to the **autocorrelation test results of Arellano and Bond**, there is a first-order negative autocorrelation in the one-step estimation, but no second-order autocorrelation; (z-statistics > 2.0 : Autocorrelation to %5)

- According to the results of the two-step estimation, there is neither first-order nor second-order autocorrelation.

- Additionally, the number of instrumental variables (33) is far below the number of units (140).

C. 3. Arellano & Bover / Blundell & Bond

```
* The first (bb) and the second (bbtwo) are one-step and two-step estimators
quietly xtdpdsys n l(0/1).(w ys) k, lag(1) noconstant vce(robust)
estimates store bb

quietly xtdpdsys n l(0/1).(w ys) k, lag(1) noconstant twostep vce(robust)
estimates store bbtwo

* The third (abond2) and the fourth (abond2two) are orthogonal estimators
help xtabond2
quietly xtabond2 n l(0/1).(w ys) k, noconstant gmm(L.n) iv(l(0/1).(w ys) k) orthogonal robust
estimates store abond2

quietly xtabond2 n l(0/1).(w ys) k, noconstant gmm(L.n) iv(l(0/1).(w ys) k) orthogonal twostep robust
estimates store abond2two

* Least squares dummy variable method using three different methods
quietly xtlsdvc n w wL1 ys ysL1 k, initial(bb) vcov(50) bias(1)
estimates store bb1

quietly xtlsdvc n w wL1 ys ysL1 k, initial(bb) vcov(50) bias(2)
estimates store bb2

quietly xtlsdvc n w wL1 ys ysL1 k, initial(bb) vcov(50) bias(3)
estimates store bb3
* All results
estimates table bb bbtwo abond2 abond2two bb1 bb2 bb3, b(%7.3g) star stat(N chi2 arm1 arm2 ar1 ar1p ar2 ar2p hansen hansenp)
```

C. 3. Arellano & Bover / Blundell & Bond

Figure 3

Variable	bb	bbtwo	abond2	abond2two	bb1	bb2	bb3
n							
L1.	.667***	.603***			.748***	.748***	.765***
w							
--.	-.575***	-.54***	-.281*	-.325**	-.604***	-.604***	-.605***
L1.	.316**	.29**	-.0867	.0302			
ys							
--.	.782***	.692***	-.102	.164	.675***	.675***	.677***
L1.	-.508***	-.408***	.65**	.343			
k	.271***	.271***	.797***	.788***	.252***	.252***	.247***
wL1					.293***	.293***	.298***
ysL1					-.564***	-.564***	-.573***
N	891	891	891	891	891	891	891
chi2	3091.68	2247.40	1373.83	1191.69			
arm1	-4.39	-2.50					
arm2	-1.11	-1.11					
ar1			-0.73	-0.58			
ar1p			0.46	0.56			
ar2			-1.45	-1.97			
ar2p			0.15	0.05			
hansen			45.64	45.64			
hansenp			0.11	0.11			

legend: * p<0.05; ** p<0.01; *** p<0.001

C. 3. Arellano & Bover / Blundell & Bond

- According to all estimators, all independent variables are significant, and the parameter signs are as expected.

- In general, it can be seen that there are no significant differences in the estimation results.

- The autocorrelation results, there is 1st-order negative autocorrelation, while there is no 2nd-order autocorrelation.

- The Hansen test shows that the over-identification restrictions are valid.
- (H0): The instruments are valid. If the p-value > 0.05, the instruments are valid.

- The number of instruments (40) is much lower than the number of units (140), which makes it reliable.

- When all the application results are evaluated in general, although there are no significant differences, it is more appropriate to use the estimators of Arellano and Bover / Blundell and Bond for our sample, where the unit dimension is much larger than the time dimension.

D. Conclusion

- When dynamic panel data models are mentioned, autoregressive panel data models are generally understood.
- The most important problem in autoregressive panel data models is the issue of endogeneity.
- The estimation of autoregressive panel data models using the pooled least squares method is biased due to the endogeneity problem.
- The two-stage least squares estimator by Balestra and Nerlove controls for endogeneity using instrumental variables but is not preferred because it does not account for unit effects.
- Due to the correlation between the lagged independent variable and the unit effect, the random effects estimator is also not suitable for use.
- The fixed effects estimator can be preferred because it considers unit effects and allows for the correlation between the unit effect and the independent variable.

D. Conclusion

- Consistent estimates can be obtained using the bias-corrected instrumental variables least squares method.

- The first differences estimator can also be preferred, like the fixed effects estimator, because it considers unit effects and allows for the correlation between the unit effect and the independent variable.

- However, the lagged value of the dependent variable obtained through the first-difference transformation is correlated with the error term, and this correlation can be controlled by using instrumental variables.

- When instrumental variables are used in the first-differenced dynamic model and estimated with the pooled least squares method, the Anderson and Hsiao estimator is obtained.

- The Anderson and Hsiao estimator is inefficient because it ignores the autocorrelation in the first-difference error term.

D. Conclusion

- To address autocorrelation, it is suggested to transform the first-differenced dynamic model with instrumental variables and then estimate it using the generalized least squares method. This results in the Arellano and Bond generalized method of moments (GMM) estimator.

- Arellano and Bond/Blundell and Bover proposed forward orthogonal deviations instead of first differences. This estimator allows for the use of multiple instrumental variables, increasing efficiency. A two-system equation is established: the original equation and the transformed equation. Therefore, this estimator is known as the "System GMM.«

- When estimating dynamic panel data models using instrumental variables, three criteria determine which method to prefer:

➢ i. whether the error term is autocorrelated,

➢ ii. whether the independent variables are strictly exogenous, and

➢ iii. the size of N and T.

MEPS
2023-2024

APPLIED MICROECONOMETRICS

Generalized Method of Moments (GMM)

Dynamic Panel Data Models

Yasin Tosun

Economic Policy

University of Siegen