

THE IR STRUCTURE OF CELESTIAL GLUON AMPLITUDES

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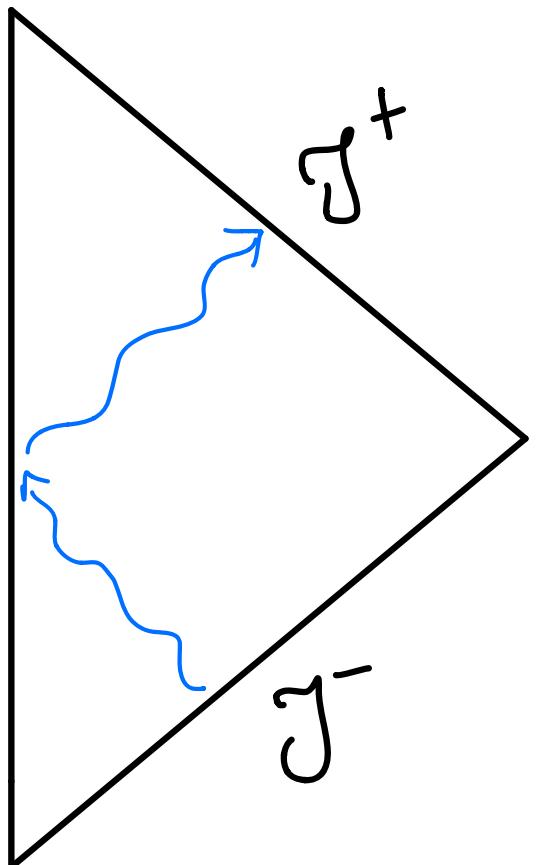
MOTIVATION

Flat Space Holography



SYMMETRIES

MOTIVATION

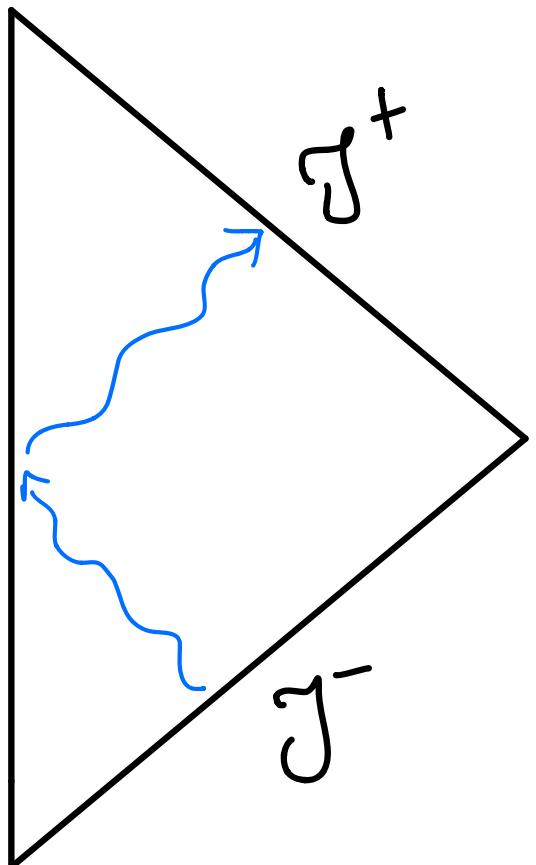


Flat Space Holography

↑
SYMMETRIES

- GRAVITY + BOUNDARY CONDITIONS AT $g^{+/-}$
∞-DIMENSIONAL
SYMMETRIES \supset POINCARÉ
(B.M.S.) [1962]

MOTIVATION

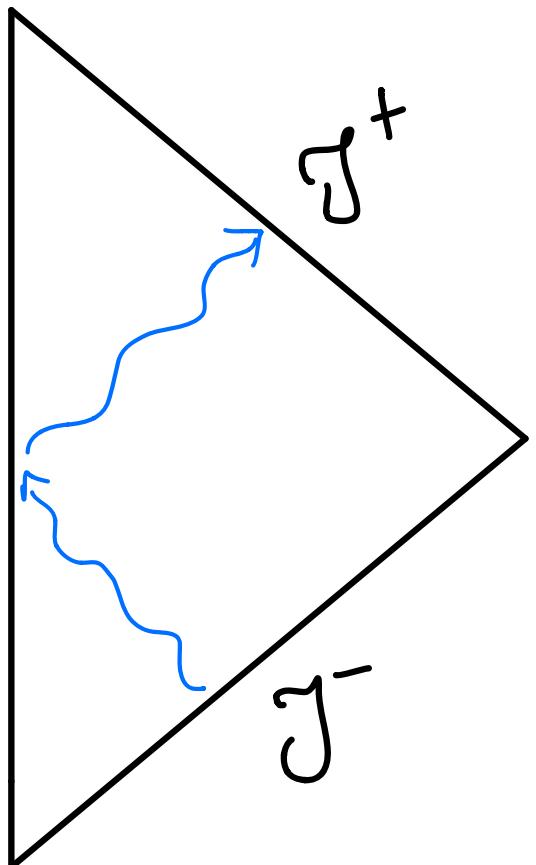


Flat Space Holography

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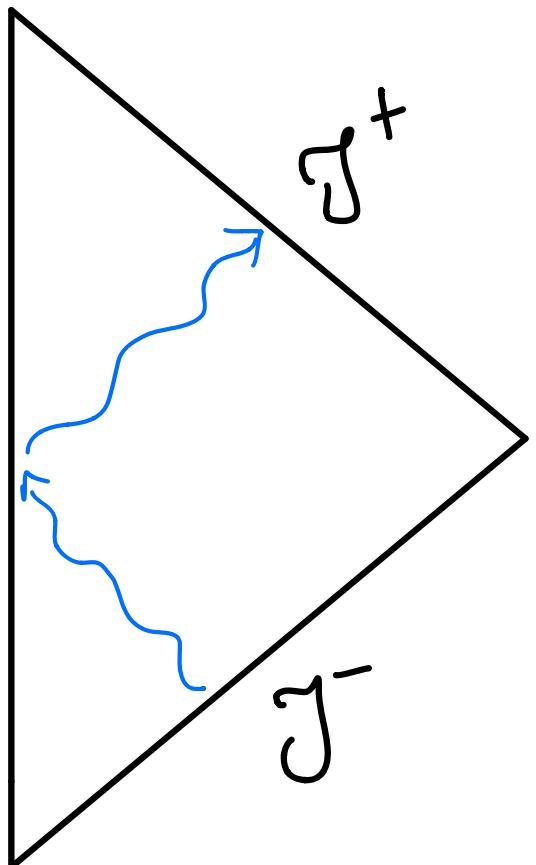
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[BARNICH-TROESSART (2010) CAMPIGLIA-LADDHA (2015)]

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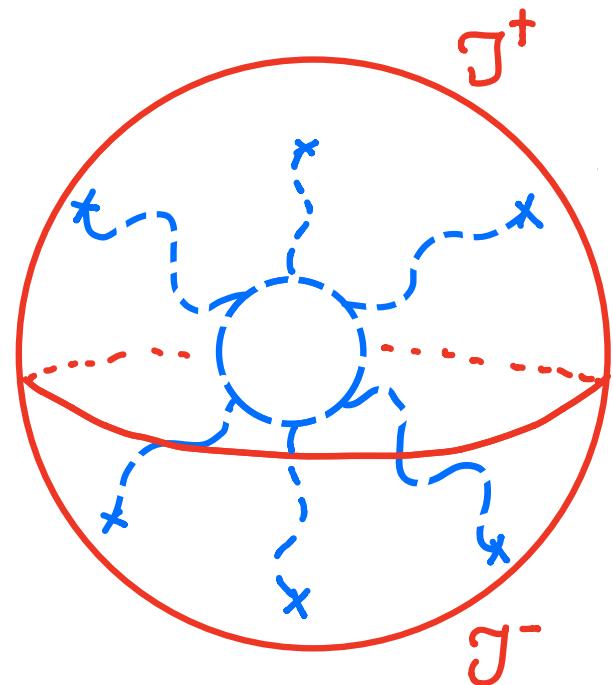
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WHAT ARE THE ROLE OF THESE
SYMMETRIES AT QUANTUM LEVEL?

S-MATRIX FOR MASSLESS PARTICLES

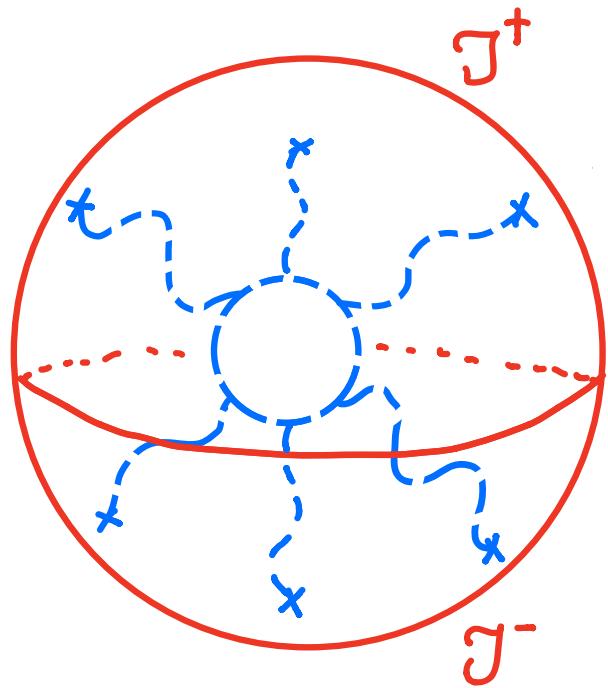


$$\langle \text{OUT} | S | \text{IN} \rangle$$

\downarrow \downarrow
 J^+ J^-

4D BULK / 2D CELESTIAL SPHERE

S-MATRIX FOR MASSLESS PARTICLES



$$\langle \text{OUT} | S | \text{IN} \rangle$$

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4D BULK / 2D CELESTIAL SPHERE

WHAT ARE THE PROPERTIES OF THIS 2D THEORY?

OUTLINE

- Mellin Basis & Symmetries
- Celestial Factorization in IR Regulated Amplitudes
- CFT for IR divergences
- Conclusions

A NEW BASIS

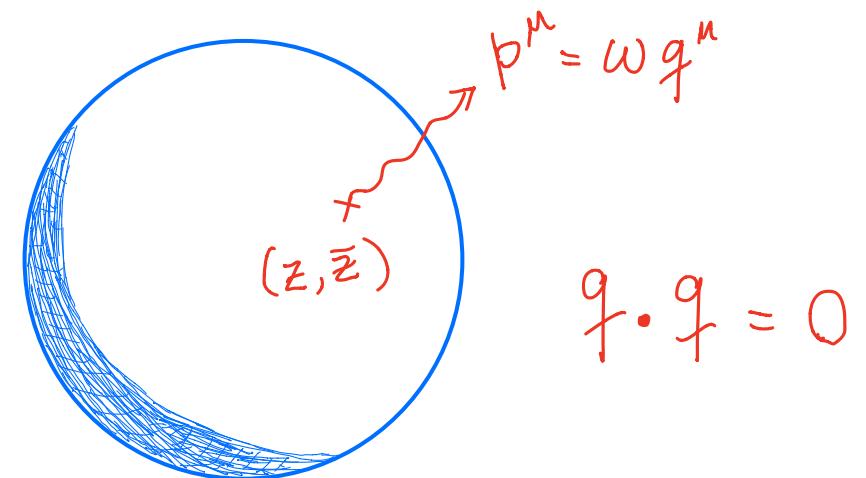
[PASTERSKI & SHAO (2017)]

- USUAL BASIS \leadsto PLANES WAVES $\psi_p(x) = e^{ip \cdot x}$

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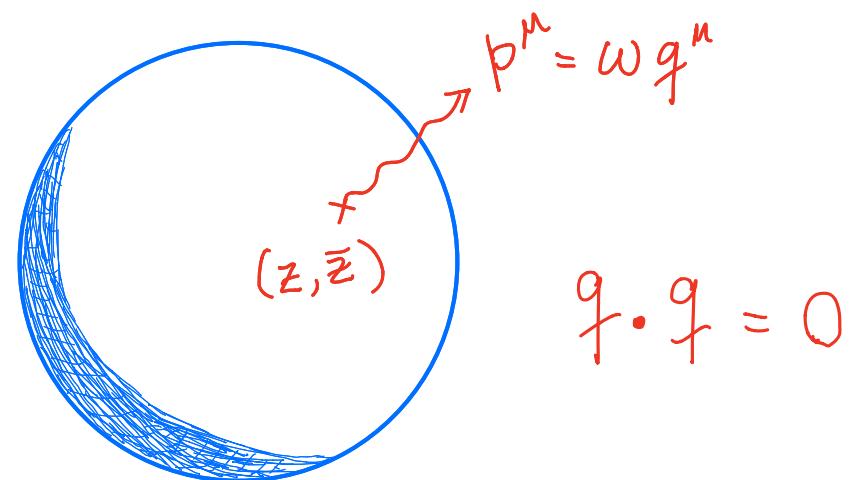


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$$P^\mu = \frac{\omega}{2} (1 + |z|^2, z + \bar{z}, -i(z - \bar{z}), 1 - |z|^2) = \omega q^\mu$$



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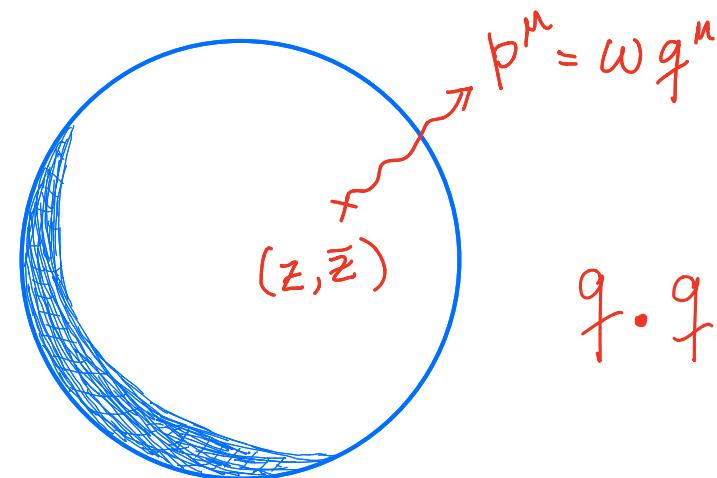
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$$z \rightarrow z' = \frac{az + b}{cz + d}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{C})$$

GLOBAL
CONFORMAL
SYMMETRY
IN 2D



$$q \cdot q = 0$$

A NEW BASIS

[PASTERSKI & SHAO (2017)]

- CHANGE OF BASIS USING MELLIN TRANSFORM

PLANE WAVE :
(SCALAR)

$$\exp(i\omega q \cdot x)$$

ENERGY
↑
↓
POSITION
ON CS

NEW STATE : $O_\Delta(z; x)$

A NEW BASIS

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- SUMMING OVER ALL ENERGY SCALES
- EXCHANGING ENERGY (ω) BY SCALING DIMENSION (Δ)

CONFORMAL BASIS

- UNDER $SL(2, \mathbb{C})$: TRANSFORMS AS A PRIMARY FIELD

$$\mathcal{O}_\Delta (x^m, x^\nu, \frac{az+b}{cz+d}) = |cz+d|^\Delta \mathcal{O}_\Delta (x^m; z)$$

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- DELTA - NORMALIZABLE $(\Psi_p(x), \Psi_{p'}(x)) = (2\pi)^3 (2p^\circ) \delta^{(3)}(p-p')$

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SAME NORMALIZATION ON CONFORMAL BASIS $\rightarrow \Delta = 1 + i\lambda$

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{
DIFFERENT KIND OF CFT

CELESTIAL AMPLITUDES

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$$A_n(\{p_i, \epsilon_j\}) = i(2\pi)^4 \delta(p_1 + p_2 - \sum_{k=3}^n p_k) A(\{p_i, \epsilon_i\})$$

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INTEGRATION OVER EACH
EXTERNAL LEG

- \tilde{A} INVOLVES ARBITRARILY HIGH ENERGY PARTICLES (ω_k)

CELESTIAL = CORRELATION FUNCTION
AMPLITUDE 2D - CFT

$$\tilde{A}_{\{\Delta_k, \ell_k\}}(\{z_k, \bar{z}_k\}) = \langle O_{\Delta_1 \ell_1}(z_1, \bar{z}_1) \dots O_{\Delta_n \ell_n}(z_n, \bar{z}_n) \rangle$$

TRANSFORMATION PROPERTIES

- UNDER $SL(2, \mathbb{C})$ LORENTZ TRANSFORMATION

$$\tilde{\mathcal{A}}_{\{\Delta_k, l_k\}} \left(\begin{pmatrix} az_k + b \\ cz_k + d \end{pmatrix}, \begin{pmatrix} \bar{a}\bar{z}_k + \bar{b} \\ \bar{c}\bar{z}_k + \bar{d} \end{pmatrix} \right) = \prod_{k=1}^n ((cz_k + d)^{\Delta_k + l_k} (\bar{c}\bar{z}_k + \bar{d})^{\Delta_k - l_k}) \tilde{\mathcal{A}}_{\{\Delta_k, l_k\}} (z_k, \bar{z}_k)$$

$h_k = \frac{\Delta_k + l_k}{2}$ $\bar{h}_k = \frac{\Delta_k - l_k}{2}$ FOR EACH EXTERNAL PARTICLE

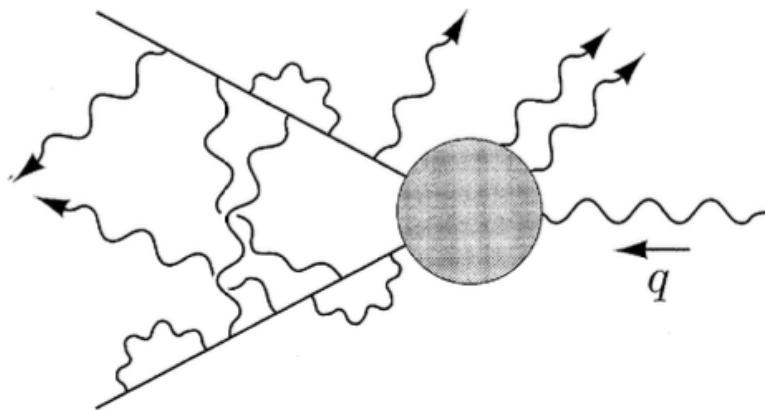
l_k : HELICITY

GLUON AMPLITUDES AT LOOP LEVEL

- MELLIN AMPLITUDES COMBINES HIGH AND LOW ENERGY STATES.
- THEORIES WITH MASSLESS EXCITATIONS (PHOTONS, GLUONS,...) SUFFERS FROM IR RADIATION AT LOOP LEVEL.

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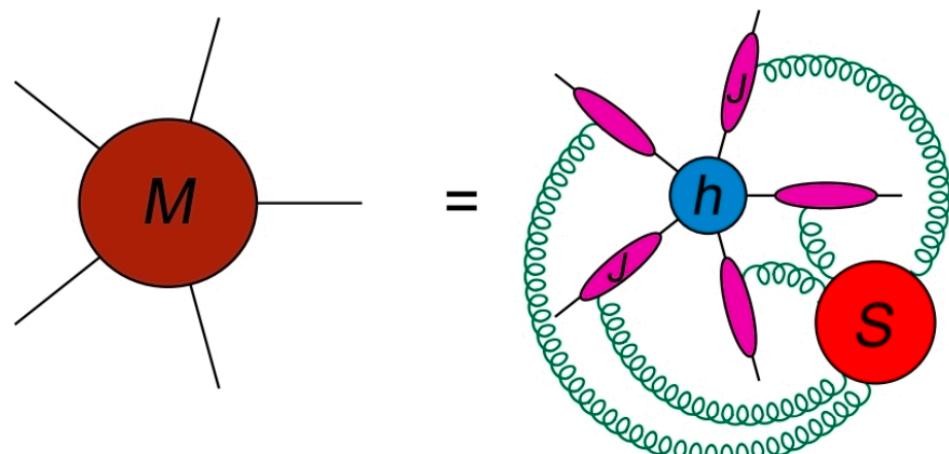
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IR PROBLEM : ∞ INTERNAL EXCHANGES
WITH VERY LOW ENERGY

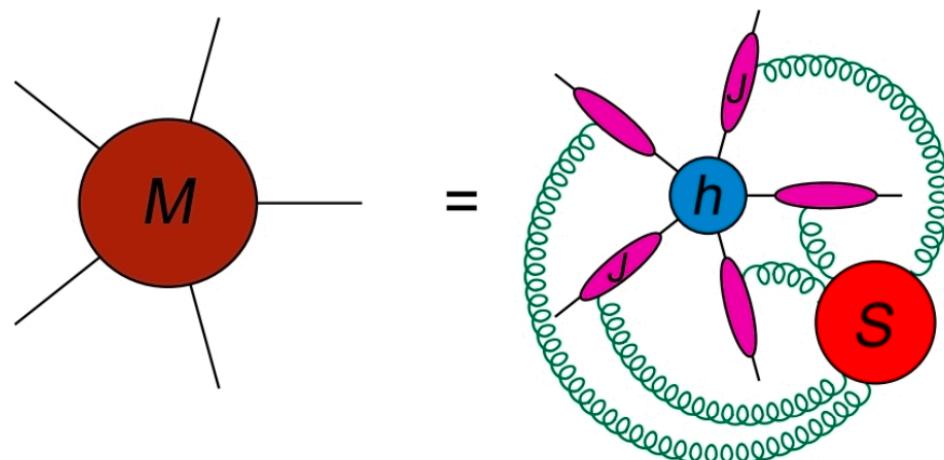
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$$M = \underbrace{\left(\prod_{i=1}^n J_i \right)}_{\text{IR RESSUMATION}} S \mathcal{H}$$

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The diagram illustrates the decomposition of a gluon amplitude M into a hard piece h and an infrared resummation piece S . On the left, a red circle labeled M has four external lines. An equals sign follows. To the right is a more complex diagram: a blue circle labeled h with three external lines, connected by a green wavy line to a red circle labeled S , which also has three external lines. The green wavy line is labeled J at both ends.

$$M = \underbrace{\left(\prod_{i=1}^n J_i \right) S}_{\text{IR RESSUMATION}} J$$

HARD PIECE

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

- WE CONSIDER A PROCESS INVOLVING n EXTERNAL GLUONS WITH HELICITIES $\ell_i = \pm$ AND SU(N) INTERNAL SYMMETRY

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- DIMENSIONAL REGULARIZATION $D=4-2\epsilon$ $\epsilon < 0$ FOR INTERNAL INTERACTIONS

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

n-POINT AMPLITUDES $M_n(p_1, \dots, p_n) = \underbrace{Z_n}_{\substack{\text{DIVERGENT} \\ \text{FOR } \Lambda \rightarrow 0}} \times \underbrace{A_n}_{\text{FINITE}}$

EVOLUTION EQUATIONS

$$\frac{d}{d \log \mu^2} Z_n = - \Gamma_n Z_n$$

ENERGY SCALE

$$Z(\mu^2, p_i \cdot p_j) = P \exp \left\{ -\frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \Gamma_n(p_i \cdot p_j, \lambda^2) \right\}$$

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

MAIN OBJECTIVE :

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

MAIN OBJECTIVE :

CELESTIAL CFT BEHIND CORRELATOR

$$Z_n(\mu^2, p_i \cdot p_j)$$

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MAIN OBJECTIVE :

CELESTIAL CFT BEHIND CORRELATOR

$$Z_n(\mu^2, p_i \cdot p_j)$$

1) $N \gg 1$

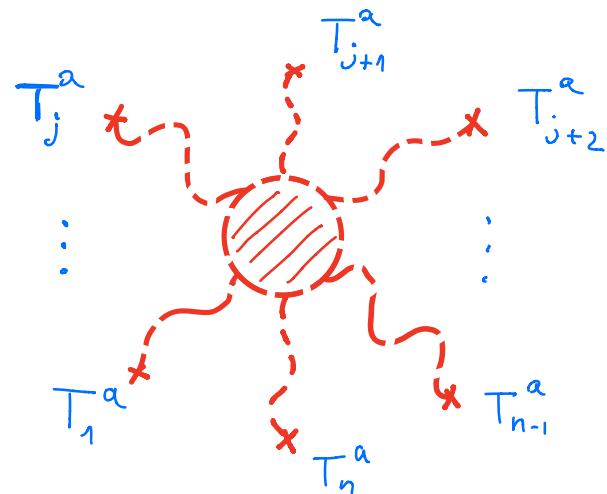
2) FINITE N
TRUNCATED in α_s

[MAGNEA 2021]



COLORED
FREE SCALAR

○ SU(N) GLUON AMPLITUDES (COLOR SPACE FORMALISM) CATANI [1998]



T_i^a : COLOR CHARGE OPERATOR
IN ADJOINT REPRESENTATION

$\sum_{i=1}^n T_i^a = 0$: COLOR CHARGE CONSERVATION

GARDI-MAGNEA
BECHER-NEUBERT
[2009]

$$\sum_{j \neq i} \frac{\partial}{\partial \log p_{ij}} \Gamma_n^{(\text{SOFT})} = \gamma_{\text{cusp}}^i(\alpha_s) \mathbb{1}$$

p_{ij} invariant $p_i \rightarrow \lambda_i p_i$
under

cusp

$$\langle \text{Tr } P e^{ig_s f_A} \rangle$$

CELESTIAL n-POINT GLUON AMPLITUDE (ALL LOOPS)

- MELLIN AMPLITUDE FACTORIZES AS WELL:

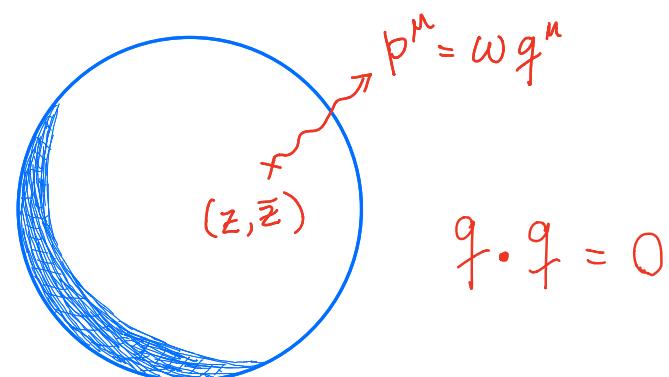
$$\tilde{A}_n^{\text{ALL-loops}} = \mathcal{Z}_n(\mu^2, |z_{ij}|^2) \tilde{A}_{\text{finite}}(h'_i = h_i - \frac{1}{4}K)$$

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RECALL:

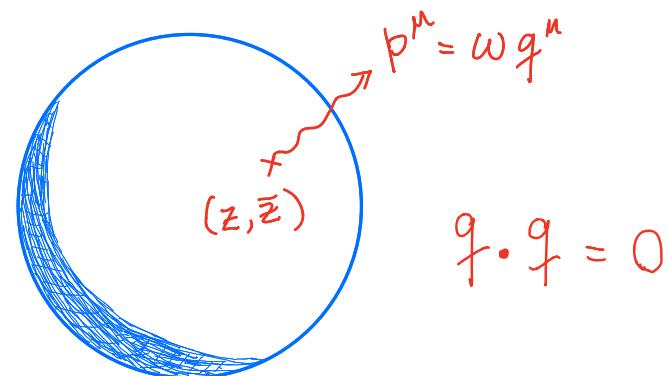


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- INTERESTING PHYSICS ENCODED IN Z_n SPECIFIC THEORY?

IT CAN BE WRITTEN IN TERMS OF n-POINT CORRELATORS OF
VERTEX OPERATORS $\mathcal{V}_i(z, \bar{z})$ WITH CONFORMAL DIMENSION $\frac{1}{4}K$

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VERTEX OPERATORS $V_i(z, \bar{z})$ WITH CONFORMAL DIMENSION $\frac{1}{4}K$

$$Z_n = \langle V_{T_1} \circledast V_{T_2} \circledast \cdots \circledast V_{T_n} \rangle$$

2D - CFT CORRELATOR

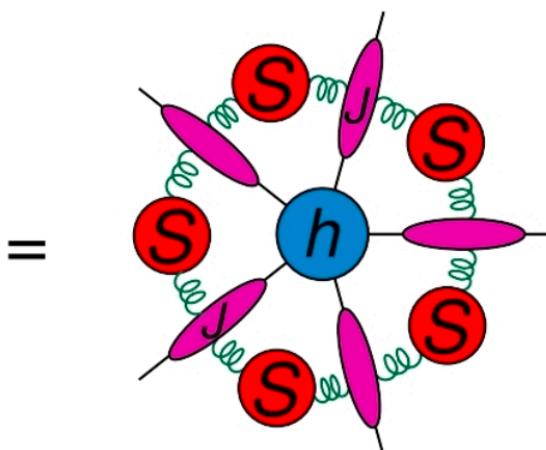
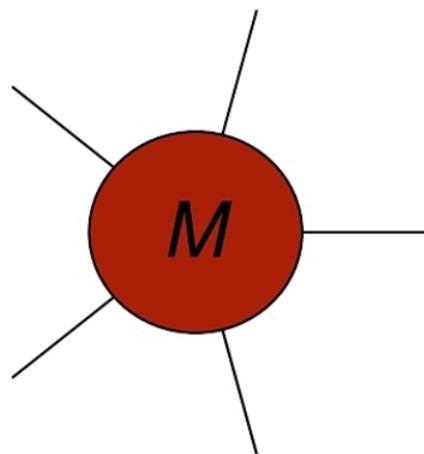
GARDI, SMILLIE [2011, 2013]
WHITE

$$K = \frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \gamma_{\text{cusp}}(\alpha_s(\lambda^2))$$

DIVERGES WITH
THE REGULATOR ϵ

LARGE N

$$Z_n = \exp \left(-\frac{NK}{4} \sum_{i < j} \underbrace{(\delta_{i,j+1} + \delta_{j,i+1})}_{\text{NEAREST NEIGHBOUR INTERACTIONS}} \log (|z_{ij}|^2) \right)$$



- ONLY PLANAR DIAGRAMS CONTRIBUTE
- INTERACTIONS BETWEEN NEIGHBOURING LEGS

LARGE N

$$Z_n = \exp \left(-\kappa \sum_{i < j} (\delta_{i,j+1} + \delta_{j,i+1}) \log (|z_{ij}|^2) \right)$$

$4\chi = NK = \text{fixed}$
 $N \rightarrow \infty$

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COULOMB GAS DESCRIPTION :

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COULOMB GAS DESCRIPTION : $\langle \phi^a(z) \phi^b(0) \rangle = -\kappa \delta^{ab} \log (|z_{ij}|^2)$

↑
FREE BOSON WITH A COLOR INDEX

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$$Z_n = \langle :e^{iT_1 \cdot \phi}: :e^{iT_2 \cdot \phi}: \dots :e^{iT_n \cdot \phi}: \rangle = \exp \left[- \sum_{i < j} \langle \phi \cdot T_i, \phi \cdot T_j \rangle \right]$$

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NO LONGER
AN OPERATOR $\rightarrow T_i^a$

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POSITION i POSITION i+1

NO LONGER
AN OPERATOR → $T_i^a = \left(\underbrace{0, \dots, 1}_{N^2 \rightarrow \infty \text{ COMPTES}}, \underbrace{-1}_{\downarrow}, 0, \dots, 0 \right) \rightsquigarrow \sum_{i=1}^n T_i^a = 0$

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$$V_{T_i} = e^{iT_i \cdot \phi}$$

VERTEX OPERATOR

LARGE N

$$Z_n = \langle :e^{iT_1 \cdot \phi} : :e^{iT_2 \cdot \phi} : \dots :e^{iT_n \cdot \phi} : \rangle = \exp \left[- \sum_{i < j} \langle \phi \cdot T_i, \phi \cdot T_j \rangle \right]$$

$$V_{T_i} = e^{iT_i \cdot \phi}$$



$$I(\phi^a) = \frac{1}{8\pi K} \int d^2z \bar{\partial}\phi^a \partial\phi^a$$

VERTEX OPERATOR

EFFECTIVE ACTION

FINITE N

Γ_n AT FINITE N AND TWO-LOOP TRUNCATION

$$Z_n = \exp \left(\frac{K}{2} \sum_{i < j} T_i \cdot T_j \log(|z_{ij}|^2) \right)$$

ALSO ADMITS A COLORED FREE SCALAR REP.

[MAGNEA 2021]

FINITE N : BEYOND DIPOLEAR APPROXIMATION

$$Z_n = P_{\text{exp}} \left\{ -\frac{1}{2} \int_0^{\mu^2} \frac{d\lambda^2}{\lambda^2} \Pi_n(|z_{ij}|^2, \lambda^2) \right\}$$

Γ_n ANOMALOUS DIMENSION CONTRIBUTIONS

Kinematic Dependence	Tensor Structure	Loop order	Reference
$\frac{z_{ij} z_{kl}}{z_{il} z_{jk}}$ SL(2,C) INVARIANT RATIOS	$f_{abc} f^e{}_{cd} \{T_i^a, T_j^b\} T_j^b T_k^c$	3	ALMELID, DUHR, GARDI, MCLEOD WHITE [2016] [2017]
$\log(z_{ij} ^2)$	$d_{abcd} \text{tr}(T_i^a T_j^b T_k^c T_l^d)$	4	BECHER NEUBERT [2019]

FINITE N : BEYOND DIPOLEAR APPROXIMATION

- CCFT CONTAINING NON-ABELIAN STRUCTURE

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↳ BOUNDARY ACTIONS FROM
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- 3 AND 4 LOOPS STRUCTURES ?

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- 3 AND 4 LOOPS STRUCTURES ?

- IR SAFE CELESTIAL AMPLITUDES $\mathcal{O}_{(\Delta, l)}^{\text{BARE}} = \mathcal{V}_i \mathcal{O}_{(\Delta, l)}^{\text{DRESSED}}$

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↳ FADDEEV - KULISH CONSTRUCTION

CONCLUSION

- NEW KIND OF HOLOGRAPHY:

CELESTIAL AMPLITUDES \longleftrightarrow 2D - CFT

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- $SU(N)$ GAUGE THEORY, CELESTIAL AMPLITUDE SPLITS INTO HARD AND INFRARED DIVERGENT PIECES.

CONCLUSION

- NEW KIND OF HOLOGRAFY:

CELESTIAL AMPLITUDES \longleftrightarrow 2D - CFT

- SU(N) GAUGE THEORY, CELESTIAL AMPLITUDE SPLITS INTO HARD AND INFRARED DIVERGENT PIECES.
- CFT IDENTIFICATION FOR INFRARED DIVERGENCES

$$Z_n = \langle V_{T_1} \otimes V_{T_2} \otimes \dots \otimes V_{T_n} \rangle$$

- LARGE N
- FINITE N
- TRUNCATED

$$V_i = e^{iT_i \cdot \phi} \text{ PRIMARIES} \quad \phi^a \rightsquigarrow \begin{matrix} \text{COLORED} \\ \text{SCALAR} \end{matrix}$$

THANKS!

FINITE N

WZW SATISFIES KNIZHNİK-ZAMOLODCHIKOV EQUATIONS

$$\left(\frac{\partial}{\partial z_i} - \frac{k}{2} \sum_{i \neq j} \frac{T_i \cdot T_j}{z_i - z_j} \right) Z_n = 0$$

IN OUR CASE THERE IS AN EXTRA TERM ON THE RIGHT HAND SIDE

Z_n NOT COMPATIBLE WITH WZW

CONFORMAL BASIS

[STIEBERGER & TAYLOR]
2019

- UNDER SPACE-TIME TRANSLATION : $\hat{P}^\mu = \hat{\omega} q^\mu(z, \bar{z})$

$$\delta_{\hat{P}^\mu} O_\Delta(z; x) = q^\mu \int_0^\infty \frac{d\omega}{\omega} \omega^{(\Delta+1)} e^{i\omega q \cdot x}$$

$$\propto O_{\Delta+1}(z; X)$$

IN PARTICULAR $\hat{P}^+ = \hat{P}^0 + \hat{P}^3$

$$\delta_{P^+} O_\Delta = O_{\Delta+1}$$

TRANSLATION IS A SHIFT
IN CONFORMAL DIMENSION

CONFORMAL BASIS: SUMMARY

$\mathcal{O}_\Delta(z; x) \rightarrow$ PRIMARY STATE UNDER
LORENTZ TRANSFORMATIONS

↓
SCALING DIMENSION : $\Delta = 1 + i\lambda \rightsquigarrow SL(2, \mathbb{C})$ PRINCIPAL
SERIES?

NOT SO CLEAR AS TRANSLATIONS RELATE \mathcal{O}_Δ WITH $\mathcal{O}_{\Delta+1}$

