Integrable Boundary States in Super-Chern-Simons Theory

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References

Holographic Wilson Loop One-point Functions in ABJM theory,
Xiao-Yi Zhang, Yunfeng Jiang, **JW**,
[arXiv: 2508.00281], JHEP11(2025)008.

Outline

- Introduction
- Wilson-loop one-point functions
- Holographic computations
- Comparison with results from localization
- Conclusion and outlook

Introduction

- Defects are everywhere in QFTs: line operators, boudanry, domain wall, relation to (higher-form) symmetry, $\cdots$
- Wilson(-'t Hooft) loop operators in gauge theory are related to confinement, precise definition of the gauge theories, $\cdots$.
- In supersymmetric gauge theory, it is natural to consider Wilson loops (WLs) preserving part of the supersymmetries.

BPS Wilson loops

Various tools are used to study the BPS WLs,

- Weak coupling: Feynman diagrams.
- Strong coupling: holographic dual, lattice(?).
- General coupling: supersymmetric localization, integrability, conformal bootstrap (defect CFT).

BPS Wilson loops in AdS_5/CFT_4

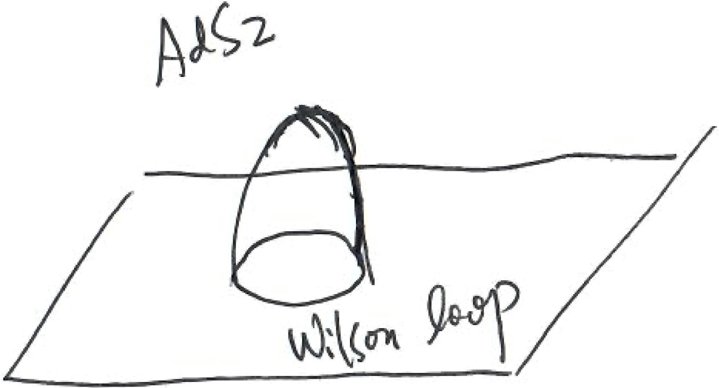
- The half-BPS circular WL in $\mathcal{N}=4$ SYM in the fundamental representation is dual to a half-BPS fundamental sting solution in $AdS_5 \times S^5$.
  [Berenstein, Corrado, Fischler, Maldacena, 98][Drukker, Gross, Ooguri, 99]
- String theory predicts that the vev of this WL is

$$\langle W[C] \rangle \sim e^{-\sqrt{\lambda}} \, ,$$

in the large N, large $\lambda := g_{\mathrm{YM}}^2 N$ limit.

BPS Wilson loops in $AdS_5 \times S^5$

The dual description is in term of a classical string solution in $AdS_5 \times S^5$. For the case at hand, its worldsheet is a minimal surface in AdS_5.



BPS Wilson loops in AdS_5/CFT_4

- Based on results from one-loop perturbation calculations in the field theory side, [Erickson, Semenoff, Zarembo, 00] conjectured that this vev can be computed using the Gaussian matrix model (Later confirmed by localization [Pestun, 07]).
- The field theory result in the large N limit is

$$\langle W[C] \rangle = \frac{2}{\sqrt{\lambda}} I_1(\lambda) \, .$$

- In the large λ limit, the above results gives

$$\langle W[C] \rangle \sim \frac{e^{\sqrt{\lambda}}}{\sqrt{\frac{\pi}{2}} \lambda^{3/4}} \, .$$

- This is one of the earliest precision checks of AdS/CFT duality beyond non-renormalization theorem.

ABJM theory

- ABJM theory is a three-dimensional $(3d)$ $\mathcal{N} = 6\, U(N) \times (N)$ Chern-Simons-matter theory.
- The fields in the theory include

Fields	representation of $U(N) \times U(N)$	representation of $SU(4)$ R-symmetry group
A_μ	$(\mathbf{adjoint}, \mathbf{1})$	$\mathbf{1}$
$\hat{A}_\mu$	$(\mathbf{1}, \mathbf{adjoint})$	$\mathbf{1}$
Y^I	$(\square, \bar{\square})$	$\mathbf{4}$
ψ_I	$(\square, \bar{\square})$	$\bar{\mathbf{4}}$

ABJM theory

- ABJM theory is the low energy effective theory of N $M2$-branes at the tip of $\mathbb{C}^4/\mathbb{Z}_k$ orbifold singularity.
- It admits two kinds of large N limits,

limit	holographic dual
$N, k \to \infty$, with $\lambda = N/k$ fixed	type IIA string theory on $AdS_4 \times \mathbb{CP}^3$
$N \to \infty$, with k fixed	M-theory on $AdS_4 \times S^7/\mathbb{Z}_k$

BPS Wilson loops in ABJM theory

WLs	gravity duals
Half-BPS WLs [Drukker, Trancanelli, 09]	$AdS_2(\subset AdS_4) \times \text{point}(\in \mathbb{CP}^3)$
Bosonic $1/6$-BPS WLs [Drukker, Plefka, Young][Chen,**JW**] [Rey, Suyama, Yamaguchi] 08	$F1$-string smearing on a $\mathbb{CP}^1 \subset \mathbb{CP}^3$
Fermionic $1/6$-BPS WLs [Ouyang, **JW**, Zhang, 15]	$F1$-string with a complicated boundary conditions in $\mathbb{CP}^3$ [Correa, Giraldo- Silva, 19]

Half-BPS Wilson loops

- We consider the half-BPS circular Wilson loop [Drukker, Trancanelli, 09] along the circle $C = \{x^\mu = (R\cos\tau, R\sin\tau, 0) | \tau \in [0, 2\pi]\}$,

$$W[C] = \mathrm{Tr}\mathcal{P}\,\exp\left(-\mathrm{i}\oint_C d\tau \mathcal{L}^C_{1/2}(\tau)\right) .$$

- Here the superconnection $\mathcal{L}^C_{1/2}$ is given by

$$\mathcal{L}^C_{1/2} = \begin{pmatrix} \mathcal{A}^C & \bar{f}^C_1 \\ f^C_2 & \hat{\mathcal{A}}^C \end{pmatrix} .$$

Half-BPS Wilson loops

- The components explicitly read:

$$
\begin{aligned}
\mathcal{A}^C &= A_\mu \dot{x}^\mu - \mathrm{i}\frac{2\pi}{k}(2\alpha_I \bar{\alpha}^J - \delta_I^J)|\dot{x}| Y^I \bar{Y}_J \,, \\
\hat{\mathcal{A}}^C &= \hat{A}_\mu \dot{x}^\mu - \mathrm{i}\frac{2\pi}{k}(2\alpha_I \bar{\alpha}^J - \delta_I^J)|\dot{x}| \bar{Y}_J Y^I \,, \\
\bar{f}_1^C &= -\sqrt{\frac{2\pi}{k}}\bar{\alpha}^I \bar{\zeta} \psi_I |\dot{x}| \,, \\
f_2^C &= \sqrt{\frac{2\pi}{k}}\bar{\psi}^I \eta \alpha_I |\dot{x}| \,,
\end{aligned}
$$

Half-BPS Wilson loops

- As usual
$$\dot{x}^\mu = dx^\mu/d\tau\,.$$
- α_I are complex parameters satisfying
$$\sum_{I=1}^{4} \alpha_I \bar{\alpha}^I = 1\,,$$
with $\bar{\alpha}^I = (\alpha_I)^*$.
- The spinors $\bar{\zeta}^\alpha$ and η_α are
$$\bar{\zeta}^\alpha = \left(e^{\mathrm{i}\tau/2}, e^{-\mathrm{i}\tau/2}\right)\,, \quad \eta_\alpha = \begin{pmatrix} e^{-\mathrm{i}\tau/2} \\ e^{\mathrm{i}\tau/2} \end{pmatrix}\,.$$

$1/3$-BPS chiral primary operators

- The $1/3$-BPS chiral primary operator (CPO)

$$\mathcal{O}^A = (C^A)^{J_1\cdots J_L}_{I_1\cdots I_L}\mathrm{Tr}(Y^{I_1}\bar{Y}_{J_1}\cdots Y^{I_L}\bar{Y}_{J_L})\,,$$

- Here C^A is a symmetric traceless tensor, satisfying

$$(C^A)^{J_1\cdots J_L}_{I_1\cdots I_L} = (C^A)^{J_1\cdots J_L}_{(I_1\cdots I_L)} = (C^A)^{(J_1\cdots J_L)}_{I_1\cdots I_L}$$

and

$$\delta^{I_1}_{J_1}(C^A)^{J_1\cdots J_L}_{I_1\cdots I_L} = 0\,.$$

Correlation functions

- We define the normalization factor $\mathcal{N}_{\mathcal{O}}$ using the two point function,

$$\langle \mathcal{O}^A(x)\mathcal{O}^{B\dagger}(y)\rangle = \frac{\delta^{AB}\mathcal{N}_{\mathcal{O}^A}}{|x-y|^{2\Delta_A}}\,,$$

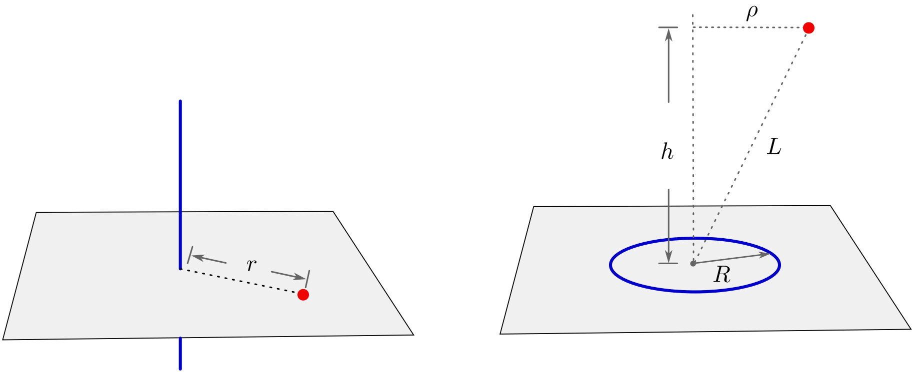
  with Δ_A being the conformal dimension of $\mathcal{O}^A$.

- For the circular Wilson loop with contour $x(\tau) = (R\cos\tau, R\sin\tau, 0)$ and a local operator at a generic point (y_1, y_2, y_3), conformal symmetry fixes the correlation function to be of the form [Berenstein et al., 98][Alday, Tseytlin, 11]

$$\frac{\langle W[C]\mathcal{O}(y)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[C]\rangle} = C_{\mathcal{O}}\frac{R^{\Delta}}{\left((h^2+\rho^2-R^2)^2+4R^2h^2\right)^{\Delta/2}}\,,$$

  where $\rho = \sqrt{y_1^2+y_2^2}$, $h = |y_3|$ and $C_{\mathcal{O}}$ is a function of k and N.

Correlation function



(a) Wilson line (b) Wilson loop

Figure: (a) Wilson line one-point function. (b) Wilson loop one-point function.

Correlation functions

- In the OPE limit $L := \sqrt{\rho^2 + h^2} \gg R$, we have

$$\frac{\langle W[C]\mathcal{O}(y)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[C]\rangle} = C_{\mathcal{O}} \frac{R^{\Delta}}{L^{2\Delta}} .$$

- For the Wilson line one-point function of a CPO, we have,

$$\frac{\langle W[L]\mathcal{O}(r)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \frac{B_{\mathcal{O}}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} \frac{1}{r^{\Delta}} .$$

  where r is the distance between the local operator.

- The coefficients $B_{\mathcal{O}}$ and $C_{\mathcal{O}}$ are related by [Lewkowycz, Maldacena, 13]

$$\frac{B_{\mathcal{O}}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} = \frac{C_{\mathcal{O}}}{2^{\Delta}} .$$

Holographic dual of ABJM theory

- ABJM with large N and finite k is dual to M-theory on on $AdS_4 \times S^7/\mathbb{Z}_k$.
- The eleven-dimensional supergravity background consists of the metric

$$ds^2 = g_{MN} dX^M dX^N = ds^2_{AdS_4} + 4 ds^2_{S^7/\mathbb{Z}_k} \, ,$$

and the four-form flux

$$F_4 = 3\Omega_{AdS_4} \, .$$

- we have set the radius of the AdS_4 to 1.
- From the AdS/CFT dictionary, the eleven-dimensional Planck length is given by

$$l_p = \left(\frac{2}{\pi^2 k N} \right)^{1/6} \, .$$

Holographic dual

- The metric on the unit (Euclidean) AdS_4 in Poincaré coordinates is
$$ds^2_{AdS_4} = \frac{1}{z^2}(dz^2 + \delta_{ij}dx^i dx^j)\,,$$
where $i, j = 1, 2, 3$.
- To describe the metric on $S^7/\mathbb{Z}_k$, we parameterize the space using four complex coordinates z^I, $(I = 1, \cdots, 4)$ subject to the constraint $\sum_{I=1}^4 |z^I|^2 = 1$. Explicitly,

$$\begin{aligned}
z^1 &= \cos\xi \cos\frac{\theta_1}{2} \exp\left[\mathrm{i}(\zeta + \frac{\psi + \varphi_1}{2})\right]\,, \\
z^2 &= \cos\xi \sin\frac{\theta_1}{2} \exp\left[\mathrm{i}(\zeta + \frac{\psi - \varphi_1}{2})\right]\,, \\
z^3 &= \sin\xi \cos\frac{\theta_2}{2} \exp\left[\mathrm{i}(\zeta + \frac{-\psi + \varphi_2}{2})\right]\,, \\
z^4 &= \sin\xi \sin\frac{\theta_2}{2} \exp\left[\mathrm{i}(\zeta + \frac{-\psi - \varphi_2}{2})\right]\,,
\end{aligned}$$

Holographic dual

- We should impose the angular identification $\zeta \sim \zeta + \frac{2\pi}{k}$ due to the $\mathbb{Z}_k$ quotient.
- The metric on $S^7/\mathbb{Z}_k$, induced from the flat metric $ds_{\mathbb{C}^4} = \sum_{i=1}^{4} dz^I d\bar{z}_I$, takes the form

$$ds_{S^7/\mathbb{Z}_k} = (d\zeta + A)^2 + ds_{\mathbb{CP}^3} .$$

- The one-form A is given by

$$A = \frac{1}{2}\cos(2\xi)d\psi + \frac{1}{2}\cos^2\xi\cos\theta_1 d\varphi_1 + \frac{1}{2}\sin^2\xi\cos\theta_2 d\varphi_2 ,$$

- the Fubini-Study metric on $\mathbb{CP}^3$ is

$$\begin{aligned} ds_{\mathbb{CP}^3} &= d\xi^2 + \frac{1}{4}\cos^2\xi(d\theta_1^2 + \sin^2\theta_1 d\varphi_1^2) \\ &+ \frac{1}{4}\sin^2\xi(d\theta_2^2 + \sin^2\theta_2 d\varphi_2^2) \\ &+ \cos^2\xi\sin^2\xi(d\psi + \frac{1}{2}\cos\theta_1 d\varphi_1 - \frac{1}{2}\cos\theta_2 d\varphi_2) . \end{aligned}$$

Holographic dual

In Poincaré coordinates, the four-form flux F_4 and its corresponding three-form potential c_3 are

$$F_4 = \frac{3}{z^4} dz \wedge dx^1 \wedge dx^2 \wedge dx^3 \,,$$
$$c_3 = -\frac{1}{z^3} dx^1 \wedge dx^2 \wedge dx^3 \,.$$

M-theory dual of the WL

- The WL in the fundamental representation (of $U(N|N)$) is dual to a probe membrane in $AdS_4 \times S^7/\mathbb{Z}_k$.
- The membrane action is

$$S_{\text{M2}} = T_{\text{M2}} \left(\int d^3\sigma \sqrt{\det \tilde{g}} - P[c_3] \right)$$

- $\tilde{g}_{ab} = \partial_a X^M \partial_b X^N g_{MN}$ is the induced metric on the M2-brane worldvolume and $P[c_3]$ is the pullback of c_3 to the membrane worldvolume.
- T_{M2} is the tension of the M2-brane

$$T_{\text{M2}} = \frac{1}{4\pi^2 l_p^3} .$$

M-theory dual of the WL

- Denoting the worldvolume coordinates of this M2-brane as $(\sigma^0, \sigma^1, \sigma^2)$, the embedding of this brane is

$$z = R\sin\sigma^1\,, \quad x_1 = R\cos\sigma^1\cos\sigma_0\,, \quad x_2 = R\cos\sigma^1\sin\sigma^0\,,$$
$$x_3 = 0\,, \zeta = \sigma^2 + \zeta^0\,, \xi = \xi^0\,, \theta_1 = \theta_1^0\,, \varphi_1 = \varphi_1^0\,, \theta_2 = \theta_2^0\,,$$
$$\varphi_2 = \varphi_2^0\,, \psi = \psi^0\,,$$

  where $\zeta^0, \xi^0, \theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \psi^0$ are constants and the worldvolume coordinate ranges are

$$0 \le \sigma^0 \le 2\pi, \quad 0 \le \sigma^1 \le \frac{\pi}{2}, \quad 0 \le \sigma^2 \le \frac{2\pi}{k}\,,$$

- The worldvolume of this M2-brane has the the topology $AdS2 \times S^1$ with AdS_2 in AdS_4 and S^1 along the ζ-cycle which is the M-theory cycle.

M-theory dual of the WL

- The parameters α_I defining the Wilson line in the boundary theory are related to the brane embedding via [Lietti, Mauri, Penati, Zhang, 17]

$$\alpha_I = \bar{z}_I(\xi^0, \theta_1^0, \varphi_1^0, \theta_2^0, \varphi_2^0, \zeta^0)\,.$$

- This provided important ingredients of the precise holographic dual of the WL, and was not mentioned in [Drukker, Trancanelli, 09].

M-theory dual of the CPO

- The chiral primary operators $\mathcal{O}^A$ are dual to fluctuations of both the metric and the three-form potential in the eleven-dimensional background:

$$\begin{aligned} G_{MN} &= g_{MN} + \delta g^A_{MN} \mathbf{Y}^A \,, \\ C_{MNP} &= c_{MNP} + \delta c^A_{MNP} \mathbf{Y}^A \,. \end{aligned}$$

- Here $\mathbf{Y}^A = (C^A)^{J_1 \cdots J_L}_{I_1 \cdots I_L} \alpha_{J_1} \cdots \alpha_{J_L} \bar{\alpha}^{I_1} \cdots \bar{\alpha}^{I_L}$ are spherical harmonics of $S^7/\mathbb{Z}_k$ with radius 1.

M-theory dual of the CPO

- The explicit forms of the fluctuations are [Biran, et al., 84][Castellani, et al. 84][Bastianelli, Zucchini, 99]:

$$
\begin{aligned}
\delta g^A_{mn} &= \frac{4}{J+2}\left[\nabla_m \nabla_n + \frac{J(J+6)}{8} g_{mn}\right] s^A - \frac{7J}{6} g_{mn} s^A \,, \\
\delta g^A_{\alpha\beta} &= \frac{1}{3} J g_{\alpha\beta} s^A \,, \\
\delta c^A_{mnp} &= 2\epsilon_{mnpq} \nabla^q s^A \,,
\end{aligned}
$$

  where $J = 2\Delta = 2L$.
- The scalar field $s^A(x,z)$ depends only on the coordinates (x,z) of AdS_4 and is determined by the boundary source $s^A_0(y)$

$$
s^A(x,z) = \int d^3y G_\Delta(y;x,z) s^A_0(y) \,,
$$

M-theory dual of the CPO

- $G_\Delta(y; x, z)$ is the boundary-to-bulk propagator

$$G_\Delta(y; x, z) = c \left(\frac{z}{z^2 + |x-y|^2} \right)^\Delta ,$$

with the constant c being

$$c = 2^{\Delta-1} \pi l_p^4 \sqrt{k} \frac{\Delta+1}{\Delta} \sqrt{2\Delta+1} \,.$$

The OPE limit

- In the OPE limit $L \gg R$, we can approximate

$$
\begin{aligned}
G_\Delta(y; x, z) &\approx c\frac{z^\Delta}{\tilde{L}^{2\Delta}}\,, \\
\partial_z s^A &\approx \frac{\Delta}{z} s^A\,, \\
\partial_\mu \partial_\nu s^A &\approx \delta^z_\mu \delta^z_\nu \frac{\Delta(\Delta-1)}{z^2} s^A,
\end{aligned}
$$

- The metric fluctuation simplifies to:

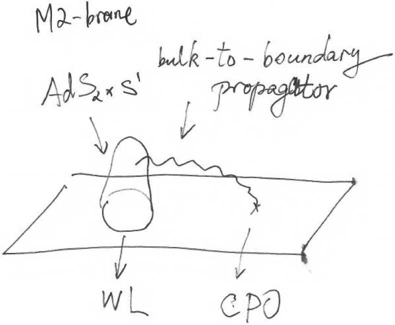
$$
\delta g^A_{mn} = -\frac{2}{3} J g_{mn} s^A + \frac{J}{z^2} \delta^z_m \delta^z_n s^A\,.
$$

Holographic WL 1-pt function

- The WL one-point correlation function

$$\frac{\langle W[C]\mathcal{O}^A(y)\rangle}{\langle W[C]\rangle\sqrt{\mathcal{N}_{\mathcal{O}}}}$$

is computed from the variation of the M2-brane action due to background field fluctuations.



Holographic WL 1-pt function

The Wess-Zumino term does not contribute, so the action variation is

$$\delta S_{\mathsf{M2}} = T_{\mathsf{M2}} \int d^3\sigma \sqrt{\det \tilde{g}_{ab}} \frac{1}{2} \tilde{g}^{ab} \partial_a X^M \partial_b X^N \delta g^A_{MN} \mathbf{Y}^A \,,$$

where $\tilde{g}_{ab} = \partial_a X^M \partial_b X^N g_{MN}$ is the induced metric on the worldvolume.

Holographic WL 1-pt function

- The action variation is

$$\begin{aligned}\delta S_{\mathsf{M2}} &= T_{\mathsf{M2}} \int d^3\sigma \left(2\frac{\cos\sigma_1}{\sin^2\sigma_1} \right) \frac{1}{2}(-J \sin^2\sigma) s^A \mathbf{Y}^A \\ &= -J T_{\mathsf{M2}} \int d^3\sigma \cos\sigma_1 s^A \mathbf{Y}^A \,.\end{aligned}$$

- Using the AdS/CFT dictionary, the correlation function reads

$$\begin{aligned}\frac{\langle W[C] \mathcal{O}^A(y) \rangle}{\langle W[C] \rangle \sqrt{\mathcal{N}_\mathcal{O}}} &= -\frac{\delta S_{\mathsf{DBI}}}{\delta s_0^A(y)} = J T_{\mathsf{M2}} \int d^3\sigma \cos\sigma_1 G_\Delta(y; x, z) \mathbf{Y}^A \\ &= \frac{2^{\Delta + \frac{1}{4}} \sqrt{\pi(2\Delta+1)}}{k \lambda^{\frac{1}{4}}} \frac{R^\Delta}{L^{2\Delta}} \mathbf{Y}^A \,,\end{aligned}$$

Holographic WL 1-pt function

- We define the OPE coefficients $C_{\mathcal{O}^A}$ as

$$\frac{\langle W[C]\mathcal{O}^A(y)\rangle}{\langle W[C]\rangle\sqrt{\mathcal{N}_{\mathcal{O}}}} = C_{\mathcal{O}^A}\frac{R^\Delta}{L^{2\Delta}}\mathbf{Y}^A\,.$$

- The holographic prediction is

$$C_{\mathcal{O}^A} = \frac{2^{\Delta+\frac{1}{4}}\sqrt{\pi(2\Delta+1)}}{k\lambda^{\frac{1}{4}}}\,.$$

Localization

- The correlation function of a half-BPS Wilson line and a CPO with $\Delta = 1$ can be computed using the supersymmetric localization. [Guerrini, 23]
- The half-BPS Wilson line is placed along the x^3-axis at $(x^1, x^2, x^3) = (0, 0, s)$.
- The polarization vector α_I in the definition of the Wilson loop is chosen to be $\alpha_I = (1, 0, 0, 0)$.

Localization

- The $1/3$-BPS operator is inserted at $(r\cos\tau, r\sin\tau, 0)$, and its polarization vector are chosen as

$$n_I = \frac{1}{\sqrt{2}}\left(e^{-\frac{\mathrm{i}}{2}\tau}, 0, e^{\frac{\mathrm{i}}{2}\tau}, 0\right)\,, \qquad \bar{n}^I = \frac{1}{\sqrt{2}}\left(e^{\frac{\mathrm{i}}{2}\tau}, 0, -e^{-\frac{\mathrm{i}}{2}\tau}, 0\right)\,.$$

- Then $1/3$-BPS operator with $\Delta = 1$ in the along the circle is

$$\mathcal{O}(\tau) = \mathrm{Tr}(n_I(\tau) Y^I(\tau) \bar{n}^J(\tau) \bar{Y}_J(\tau))\,.$$

- This special choice of polarization vector makes the local operator in the twisted-translated frame. [Drukker, Plefka, 09][Basso, Komatsu, Vieira, 15][Pereira, 17][Yang, Jiang, Komatsu, **JW**, 21]

Two point functions

- The conformal symmetry and R-symmetry give strong constraints on the two-point function of $\mathcal{O}$,

$$\langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle = \mathcal{N}_{\mathcal{O}} d_{12} d_{21} \,.$$

- Here d_{ij} is defined as

$$d_{ij} := \frac{n_i \cdot \bar{n}_j}{|x_i - x_j|} \,.$$

- For the operators in the twisted-translated frame, we have

$$d_{12} = \frac{n(\tau_1)\cdot \bar{n}(\tau_2)}{|x(\tau_1) - x(\tau_2)|} = \frac{\mathrm{i}\sin\frac{\tau_1-\tau_2}{2}}{2r|\sin\frac{\tau_1-\tau_2}{2}|} \,,$$

leading to

$$d_{12} d_{21} = \frac{1}{4r^2} \,,$$

Two point functions

- $\mathcal{N}_{\mathcal{O}}$ is the normalization constant.
- In ABJM, it is a non-trivial function of the coupling constant $1/k$!
- For arbitrary finite N and k, the integrated two-point function is given by [Agmon, et al., 17][Binder, et al., 19]

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = -\frac{1}{16\pi^2 r^4} \frac{1}{Z} \frac{\partial^2 Z[m]}{\partial m^2}\bigg|_{m=0},$$

where $Z[m]$ is the partition function of the mass-deformed ABJM theory where m is the mass-deformation parameter.

Localization

- Using localization [Kapustin, Willett, Yaakov, 09][Jafferis, 10][Hama, Hosomichi, Lee, 10], $Z[m]$ can be expressed as

$$
\begin{aligned}
Z[m] &= \frac{1}{(N!)^2(2\pi)^{2N}} \int d\lambda_i d\mu_i \exp\left[\frac{\mathrm{i}k}{4\pi}(\sum \lambda_i^2 - \sum \mu_i^2)\right] \\
&\times \frac{\prod_{i\neq j}^N 2\sinh\frac{\lambda_i-\lambda_j}{2} \prod_{i\neq j}^N 2\sinh\frac{\mu_i-\mu_j}{2}}{\prod_{i,j} 2\cosh\frac{\lambda_i-\mu_j}{2} 2\cosh(\frac{\mu_j-\lambda_i}{2} - \pi m r)}.
\end{aligned}
$$

Localization

- By using the Fermi gas approach, all order perturbatrive $1/N$ corrections to $Z[m]$ can be resumed. The result is given in terms of the Airy function [Marino, Putrov, 11][Guerrini, 23]

$$Z^{\text{pert.}}[m] = e^{A} C^{-1/3} \mathrm{Ai}[C^{-1/3}(N-B)]\,,$$

where A, B, C are functions of k, r, m. Here we do not need the explicit expressions for A and B. As for C, we have

$$C = \frac{2}{\pi^2 k(1+4m^2r^2)}\,.$$

Localization

- the large N limit of the integrated two-point function reads

$$\begin{aligned}\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle &= -\frac{1}{16\pi^2 r^4} \frac{1}{Z[m]} \frac{\partial^2 Z[m]}{\partial m^2}\bigg|_{m=0} \\ &= \frac{\sqrt{2k} N^{3/2}}{12\pi r^2} \,.\end{aligned}$$

- Here we have used the asymptotic expansion of the Airy function

$$\mathrm{Ai}(x) \sim \exp\left(-\frac{2}{3} x^{3/2}\right) \,,$$

where an irrelevant factor $(1/x)^{1/4}/(2\sqrt{\pi})$ has been omitted.

Localization

- Let us recall that,

$$\langle \mathcal{O}(\tau_1)\mathcal{O}(\tau_2)\rangle = \mathcal{N}_{\mathcal{O}} d_{12} d_{21} \, .$$

- Performing the τ-integrals on both sides of this equation yields

$$\left\langle \int d\tau_1 \mathcal{O}(\tau_1) \int d\tau_2 \mathcal{O}(\tau_2) \right\rangle = \frac{\pi^2 \mathcal{N}_{\mathcal{O}}}{r^2}.$$

Comparing this with the localization results fixes the normalization

$$\mathcal{N}_{\mathcal{O}} = \frac{\sqrt{2k} N^{3/2}}{12\pi^3} \, .$$

in the large N limit.

Localization

- The correlation function of the Wilson line $W[L]$ and $\mathcal{O}$ is given by [Guerrini, 23]
$$\frac{\langle W[L]\mathcal{O}(0)\rangle}{\langle W[L]\rangle} = \frac{\mathcal{B}}{r},$$
where $\mathcal{B}$ is the Bremsstranhlung function.
- Its large N expansion (with finite k) takes the form [Beccaria, Tseytlin, 25]
$$\mathcal{B}(k,N)\big|_{N\gg 1} = \frac{1}{2\pi}\sqrt{\frac{N}{2k}} - \frac{1}{2\pi k}\cot\frac{2\pi}{k} + \mathcal{O}(N^{-1/2}).$$

Localization

- Including the normalization factor, we have

$$\frac{\langle W[L]\mathcal{O}(0)\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \frac{\mathcal{B}}{\sqrt{\mathcal{N}_{O}}r} \,,$$

The coefficient of $1/r$ is

$$\frac{\mathcal{B}}{\sqrt{\mathcal{N}_{\mathcal{O}}}} = \frac{\sqrt{3\pi}}{2^{3/4}k\lambda^{1/4}} \,,$$

Comparison

- The holographic computation of the Wilson loop OPE coefficient for the CPO gives

$$\mathcal{C}_{\mathcal{O}} = \frac{2^{5/4}\sqrt{3\pi}}{k\lambda^{1/4}} \, ,$$

by setting $\Delta = 1$. The holographic correlation function for a normalized CPO of dimension $\Delta = 1$ with the Wilson line takes the form

$$\frac{\langle W[L]\mathcal{O}\rangle}{\sqrt{\mathcal{N}_{\mathcal{O}}}\langle W[L]\rangle} = \mathcal{C}_{\mathcal{O}}\mathbf{Y}^{A}\frac{1}{2r},$$

where r is the distance from the CPO to the line, and $\mathbf{Y}^{A}$ is the spherical harmonic factor.

Comparison

- For the Wilson line considered here, the polarization vector is $\alpha_I = (1,0,0,0)$, and the spherical harmonic evaluates to

$$\mathbf{Y}^A = \bar{n}^I n_J \alpha_I \bar{\alpha}^J = 1/2\,.$$

- Thus, the holographic prediction for $\Delta = 1$ is

$$\frac{\langle W[L]\mathcal{O}\rangle}{\sqrt{\mathcal{N}_\mathcal{O}}\langle W[L]\rangle} = \frac{\mathcal{C}_\mathcal{O}\mathbf{Y}^A}{2r} = \frac{\sqrt{3\pi}}{2^{3/4}k\lambda^{1/4}r}\,,$$

which exactly matches the localization result.

Conclusion

- Correlation functions of BPS Wilson loops and local operators are important observables in supersymmetric gauge theories.
- We computed $\langle W[C]\mathcal{O}\rangle$ for a half-BPS Wilson loop and a CPO by using AdS/CFT duality.
- The results coincide with localization prediction for $\Delta = 1$.
- We also computed $\langle W[L]T_{\mu\nu}\rangle$ for a half-BPS Wilson line holographically.
- The results involving the Bremsstrahlung function $\mathcal{B}(k, N)$ and confirmed by the previous field theory results.

Outlook: holography

- It is valuable to compute holographical WL one-point functions for WLs in higher-dimensional respresentation and/or with less supersymmetries.
- For the $F1$-string smearing in a $\mathbb{CP}^1 \subset \mathbb{CP}^3$, I am wonder that whether the correct prescription is simply to average over this $\mathbb{CP}^1$.
- How to treat the $F1$-string complicated boundary condition in $\mathbb{CP}^3$.

Outlook: localization and bootstrap

- A localization formula for $\langle W[C]\mathcal{O}\rangle$ with generic $\mathcal{O}$ a generic CPO would be highly desirable.
- It is interesting to study more ABJM (integrable) correlation functions of WLs, single-trace BPS operator or giant gravitons from localization results in mass-deformed ABJM theory.
- WL two-point functions encode defect operator spectra, We should try to compute them using superconformal bootstrap.

Outlook: Integrablity

- Integrability can be applied to compute the WL one-point functions for non-BPS operators in the planar limit.
- For WLs that correspond to integrable boundary states, the OPE coefficients can be computed using the worldsheet g-function approach [Jiang, Komastu, Vescovi, 19].
- Steps have been taken at weak coupling and the result showed that half-BPS and bosonic $1/6$-BPS WLs give integrable boundary states. [Jiang, **Wu**, Yang, 23]
- A finite-coupling analysis remains an open challenge.

Thanks for your Attention !