Homework 11

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1 The text shows that $A^\dagger$ has the SVD $A^\dagger = V\Sigma_A^{-1}U^*$. Compare this to $A_1^{-1} = V\Sigma_{A_1}^{-1}U^*$. The unitary matrices all have 2-norm of 1, and the 2-norm is equivalent to the largest singular value, so we have

$$||A^\dagger||_2 = \frac{1}{\sigma_{\min}(A)}$$

$$||A_1^{-1}||_2 = \frac{1}{\sigma_{\min}(A_1)}$$

Then we only need show that $\sigma_{\min}(A) \geq \sigma_{\min}(A_1)$. This follows from the fact that the singular values for A are the square roots of

$$A^*A = \begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} = A_1^*A_1 + A_2^*A_2 \geq A_1^*A_1$$

which is greater than the singular values for A_1, the square roots of the eigenvalues of

$$A_1^*A_1$$

.

This shows that A has larger singular values, and thus $A^\dagger$ has a smaller 2-norm than A_1^{-1}.