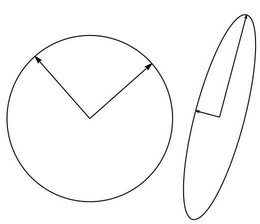
In[112]:=
```
plotVectors[A_] := Module[{u, v1, s, v},
  {u, s, v} = SingularValueDecomposition[A];
  v1 = v[[All, 1]];
  Graphics[
    {
      Circle[{0, 0}],
      Arrow[{{0, 0}, u[[All, 1]]}], Arrow[{{0, 0}, u[[All, 2]]}]
    }
   ] ×
   Graphics[
    {
      GeometricTransformation[Circle[{0, 0}, {s[[1, 1]], s[[2, 2]]}],
       RotationTransform[ArcTan[v1[[2]] / v1[[1]]], {0, 0}]],
      Arrow[{{0, 0}, s[[1, 1]] * v[[All, 1]]}],
      Arrow[{{0, 0}, s[[2, 2]] * v[[All, 2]]}]
    }
   ]
 ]
```

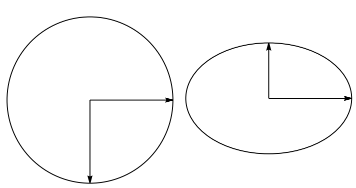
In[115]:=
```
plotVectors[{{1, 2}, {0, 2}}]
```

Out[115]=



In[116]:=
```
plotVectors[{{3, 0}, {0, -2}}]
```

Out[116]=



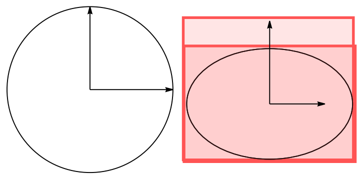
In[117]:=
```
plotVectors[{{2, 0}, {0, 3}}]
```

··· Power : Infinite expression $\frac{1}{0}$ encountered .

··· GeometricTransformation :

TransformationFunction [{{Indeterminate , Indeterminate , Indeterminate }, {≪1≫}, {Indeterminate , ≪13≫, Indeterminate }}] is not an affine transformation function .

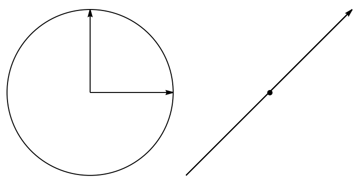
Out[117]=



This one doesn't work because ArcTan won't work for it

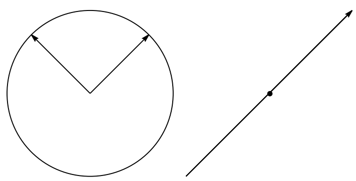
In[120]:=
```
plotVectors[{{1, 1}, {0, 0}}]
```

Out[120]=



In[121]:=
```
plotVectors[{{1, 1}, {1, 1}}]
```

Out[121]=



4) $AQ = QB$

$$U_A \Sigma_A (V_A^* Q) = (Q U_B) \Sigma_B V_B^*$$

$$U \Sigma V = (Q U') \Sigma (V' Q^*)$$

$$\Sigma = \Sigma$$

Scratch Work

$$U \Sigma V = U \Sigma V$$

$$(U' U^*) \Sigma (V^* V')$$

$$(U' U^*) A (U^* V') = B$$

$$A = Q$$

---

* True

$\Rightarrow$ let $A = QBQ^*$

$$U_A \Sigma_A V_A = Q U_B \Sigma_B V_B Q^*$$

$$\Sigma_A = (U_A^* Q U_B) \Sigma_B (V_B Q^* V_A^*)$$

And unitarily equivalent diagonal matrices must be equivalent up to a permutation, so

$$\Sigma_A = \Sigma_B$$

$\Leftarrow$ let $\quad \Sigma_A = \Sigma_A$

Then

$$U_B \Sigma_A V_B = U_B \Sigma_B V_B = B$$

$$U_A U_B^* U_B \Sigma_A V_B V_B^* V_A = U_A U_B^* B V_B^* V_A$$

$$A = (U_A U_B^*) B (V_B^* V_A)$$

$$= Q B Q^*$$

3) If A is real, then

$$(U \Sigma V^*)^* = (U \Sigma V^*)^T$$

$$V \Sigma U^* = \overline{V} \Sigma U^T$$

Which means

$$V = \overline{V}$$

and

$$U^* = U^T \Rightarrow \overline{U} = U$$

So both must be real