1) $r_{ii} = \|v_i\|$

n multiplications
n-1 additions
1 square root

$q_i = v_i / r_{ii}$

n divisions

$r_{ij} = q_i^* v_j$

n multiplications
n-1 additions

$v_j = v_j - r_{ij} q_i$

n multiplications
n subtractions

The formula:

$$f(n) = \sum_{i=1}^{n} \left[2n + n + \sum_{j=i+1}^{n} 2n-1 + 2n \right]$$

which sums to:

$$2n^3 + \frac{n^2}{2} + \frac{n}{2}$$

L8_notebook

September 16, 2022

[ ]:
```
import numpy as np
```

[ ]:
```
# Modified Gram-Schmidt
def mgs(A):
    m,n = A.shape
    Q = np.zeros((m,n))
    R = np.zeros((n,n))
    for i in range(n):
        v = A[:,i]
        R[i,i] = np.linalg.norm(v)
        Q[:,i] = v/R[i,i]
        for j in range(i+1,n):
            R[i,j] = Q[:,i].T@A[:,j]
            A[:,j] = A[:,j] - R[i,j]*Q[:,i]
    return Q,R
```

[ ]:
```
# Random 6x4 matrix
A = np.random.rand(6,4)

# QR decomposition
Q,R = mgs(A)
```

```
[[5.55111512e-17 2.68623866e-01 5.96988586e-01 7.01159114e-01]
 [0.00000000e+00 3.91902760e-01 4.75176175e-01 4.27199678e-01]
 [0.00000000e+00 3.18473686e-02 1.07988672e-01 4.14615338e-01]
 [0.00000000e+00 4.38898211e-01 5.64577221e-01 8.02240538e-01]
 [0.00000000e+00 6.47511457e-02 6.59832365e-01 5.69488549e-01]
 [0.00000000e+00 5.69886577e-01 1.93222718e-01 5.24007008e-01]]
1.056238235584049
```

[ ]:
```
print(Q,R,A)
```

```
[[ 0.31052067  0.41189449  0.11696298 -0.35711114]
 [ 0.45302716  0.10320336 -0.52007671 -0.38959615]
 [ 0.03681455  0.09562804  0.81075306 -0.16591619]
 [ 0.50735241  0.15634851  0.20485227 -0.17763019]
 [ 0.07485027  0.74809227 -0.07073306  0.63335466]
 [ 0.6587708  -0.47587852  0.10747901  0.51036035]] [[1.41781068 0.86507564
```

```
0.86773829 1.22136689]
 [0.         0.92325975 0.79519859 0.67463628]
 [0.         0.         0.87760253 0.3763626 ]
 [0.         0.         0.         0.38956737]] [[ 0.44025952  0.3802856
0.10264701 -0.13911885]
 [ 0.64230674  0.09528351 -0.45642064 -0.15177395]
 [ 0.05219606  0.08828952  0.71151894 -0.06463554]
 [ 0.71932966  0.14435028  0.17977887 -0.06919893]
 [ 0.10612351  0.69068349 -0.06207552  0.24673431]
 [ 0.93401228 -0.43935949  0.09432386  0.19881974]]
```

[ ]:
```
print(Q@R-A)
print(np.linalg.norm(Q@R-A)/np.linalg.norm(A))
```

```
[[5.55111512e-17 2.68623866e-01 5.96988586e-01 7.01159114e-01]
 [0.00000000e+00 3.91902760e-01 4.75176175e-01 4.27199678e-01]
 [0.00000000e+00 3.18473686e-02 1.07988672e-01 4.14615338e-01]
 [0.00000000e+00 4.38898211e-01 5.64577221e-01 8.02240538e-01]
 [0.00000000e+00 6.47511457e-02 6.59832365e-01 5.69488549e-01]
 [0.00000000e+00 5.69886577e-01 1.93222718e-01 5.24007008e-01]]
1.056238235584049
```

[ ]: