4) a) J : clockwise rotation

F : Reflects accross the $\theta/2$ line, measuring clockwise from the positive y-axis

b) Note - lots of credit to several online sets of notes on QR w/ givens rotations.

We need c, s such that

$$\begin{bmatrix} c & -s \\ s & c \end{bmatrix}^T \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} r \\ 0 \end{bmatrix}$$

with $r = \sqrt{a^2+b^2}$

Then

$$c = \frac{a}{\sqrt{a^2+B^2}} \qquad s = \frac{b}{\sqrt{a+2+b^2}}$$

Then for each i,j you want

to zero out you use the matrix G, which is the identity matrix but with: $G_{ii} = G_{jj} = c$, $G_{ij} = -G_{ji} = s$ and $i = j-1$, so that you always operate on adjacent rows, giving G a 2x2 sub-matrix differing from I.

Then for each element to be eliminated you apply:

$$A = G(c,s,i,j)^T A$$

which can be computationally simplified.

c) Because we know the structure of G, we can simplify the $G^T A$ calculation to only the small matrix calculation

$$\begin{bmatrix} c & -s \\ s & c \end{bmatrix}^T \begin{bmatrix} a \\ b \end{bmatrix}$$

for 2 rows (supplying a & b) of each column of A. The other rows are unchanged.

This calculation has 4 multiplications and 2 additions per element, yielding

6ℓ flops where ℓ is the length of the row vector.

This is in comparison to 4ℓ via

Householder Transformations.