x_i	y_i
x_1	y_1
x_2	y_2
$\vdots$	$\vdots$
x_m	y_m

$$g(x) = c_0 \varphi_0(x) + c_1 \varphi_1(x) + \dots + c_n \varphi_n(x)$$

$\varphi_i(x)$ – basis funcs

Example: $g(x) = c_0 + c_1 x + c_2 \sin(x)$

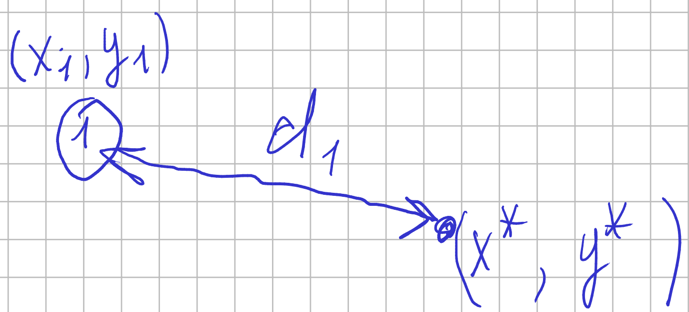
$$\begin{cases} g(x_1) = y_1 \\ g(x_2) = y_2 \\ \dots \\ g(x_m) = y_m \end{cases} \quad \begin{vmatrix} \underline{c_0} + \underline{c_1} x_1 + \underline{c_2} \sin(x_1) = y_1 \\ c_0 + c_1 x_2 + c_2 \sin(x_2) = y_2 \\ \dots \\ c_0 + c_1 x_m + c_2 \sin(x_m) = y_m \end{vmatrix}$$

So we have SLE $Ac = y$

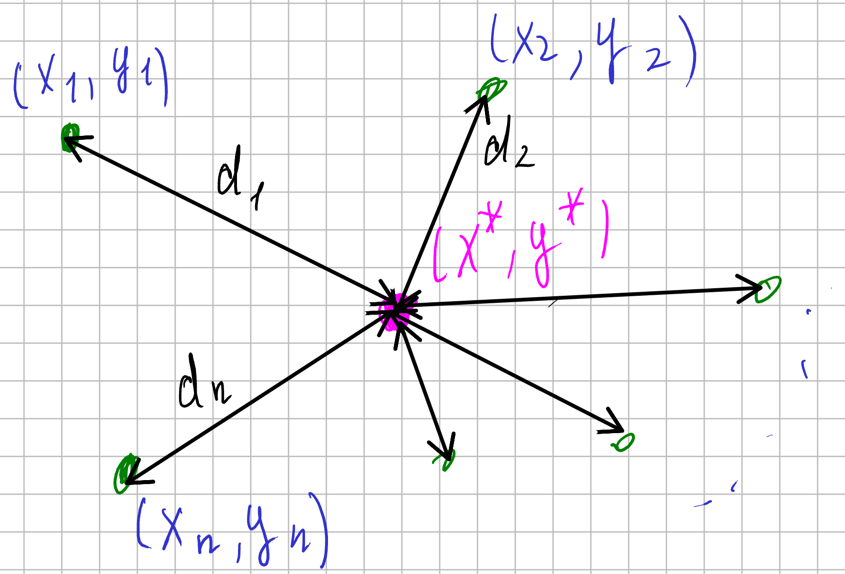
$$A = \begin{pmatrix} 1 & x_1 & \sin(x_1) \\ 1 & x_2 & \sin(x_2) \\ - & - & - \\ 1 & x_m & \sin(x_m) \end{pmatrix}, \quad c = \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix}, \quad y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix}$$

Let's find c: $\|Ac - y\| \longrightarrow \min$

Solving that we got $\underline{A^T A c = A^T y}$



$$\underbrace{d_1^2} = (\underline{x_1} - \underset{?}{x^*})^2 + (\underline{y_1} - \underset{?}{y^*})^2$$



$$\begin{cases} (x_1 - x^*)^2 + (y_1 - y^*)^2 = d_1^2 & (1) \\ (x_2 - x^*)^2 + (y_2 - y^*)^2 = d_2^2 & (2) \\ \dots \\ (x_n - x^*)^2 + (y_n - y^*)^2 = d_n^2 & (n) \end{cases}$$

Subtract every eq. from the first:

/* instead of x^*, use x */

$$(x_1 - x)^2 + (y_1 - y)^2 - (x_i - x)^2 - (y_i - y)^2 =$$
$$= d_1^2 - d_i^2$$

$$x_1^2 - 2xx_1 + \cancel{x^2} + y_1^2 - 2yy_1 + \cancel{y^2}$$
$$- x_i^2 + 2xx_i - \cancel{x^2} - y_i^2 + 2yy_i - \cancel{y^2} = d_1^2 - d_i^2$$

$$\underbrace{(2x_i - 2x_1)x + (2y_i - 2y_1)y = x_i^2 - x_1^2 + y_i^2 - y_1^2 + d_1^2 - d_i^2}_{\{i = 2, n\}}$$

$$A x = b$$

$$A = \begin{pmatrix} 2(x_2 - x_1) & 2(y_2 - y_1) \\ 2(x_3 - x_1) & 2(y_3 - y_1) \\ \cdots & \cdots \\ 2(x_n - x_1) & 2(y_n - y_1) \end{pmatrix}$$

$$b = \begin{pmatrix} x_2^2 - x_1^2 + y_2^2 - y_1^2 + d_1^2 - d_2^2 \\ x_3^2 - x_1^2 + y_3^2 - y_1^2 + d_1^2 - d_3^2 \\ \cdots \\ x_n^2 - \cdots \end{pmatrix}$$

$$\boxed{A^T A x = A^T b}$$