Midterm Exam: Topics and Preparation

February 10, 2026

Topics brief

- Root-finding algorithms (Bisection, Newton, Fixed-Point).
- Systems of linear equations (Gaussian elimination, Jacobi, and Gauss–Seidel).
- Interpolation (Polynomial interpolation, Lagrange formula, Cubic Spline).
- Basic polynomial approximation (least squares).

Details below

1. Root Finding Algorithms

(a) **Bisection Algorithm:**

- Given a function $f(x)$ and a segment $[a, b]$ check, if we can guarantee root existence on the given interval.
- Determine how many iterations of bisection are required to achieve a certain precision ε.
- Do several iterations of the algorithm.

(b) **Newton's Method:**

- Given $f(x)$, choose a starting point x_0 and perform several iterations of Newton's method.

(c) **Fixed-Point Iteration (relaxation):**

- Given $f(x)$ and interval $[a, b]$, transform $f(x) = 0$ into $x = g(x)$ using $g(x) = x + \alpha f(x)$.

- Check, if with α you have choosen, $|g'| \leq q < 1$ on the interval.
- Select x_0 and do several (2-3) iterations of the algorithm.
- Determine how many iterations of FP algorithm are required to achieve a certain precision ε.

2. Systems of Linear Equations (SLE)

(a) **Gaussian Elimination:**

- Given a system of linear equations, solve it with Gaussian elimination.
- Show all operations (steps).
- Check that the solution really fits the SLE.

(b) **Jacobi Iteration:**

- Given a system of linear equations, perform two iterations of the Jacobi method.
- Show all operations.
- Estimate how many iterations we need to achieve precision ε (make an error less than ε).
- Check if the sufficient condition of convergence holds.

(c) **Gauss–Seidel Iteration:**

- Given a system of linear equations, perform two iterations of Gauss–Seidel method.
- Show all operations.
- Check if sufficient condition of convergence holds.

3. Interpolation

(a) **Polynomial Interpolation (System Approach):**

- Given 3 or 4 points (x_i, y_i), set up and solve the corresponding system of linear equations to find a polynomial of appropriate degree.
- Write down the resulting polynomial explicitly. Check that it goes through the points!

(b) **Lagrange Polynomial:**

- Given 3 or 4 points, construct the interpolation polynomial using the Lagrange formula.
- Show each Lagrange basis polynomial $L_i(x)$.
- Check that the polynomial goes through the points.

(c) **Cubic spline:**

- Given 3 points, construct the interpolation cubic spline (natural).
- Check that the spline goes through the points and the derivative is continuous.

4. Approximation

Least squares:

- Given 4 or 5 points (x_i, y_i), set up and solve the corresponding SLE to find an approximation polynomial $p(x)$ of degree 1 or 2.
- Write down the resulting polynomial explicitly. Calculate error - distance between the sequences y_i and $p(x_i)$

Important: For each problem, you are expected to show *all the relevant steps* (not just the final answer).