ChatGPT

Critical Power (CP) and W′ in Endurance Running

Power-Time Hyperbolic Relationship and Critical Power (CP)

Endurance performance follows a **hyperbolic relationship** between power output and the time to exhaustion. If an athlete sustains a higher power, they fatigue faster, and conversely, they can sustain lower powers for much longer. When plotting power vs. time-to-fatigue from multiple exhaustive trials (e.g. efforts lasting ~2 to 15 minutes), the curve *levels off* toward an asymptote [1]. This asymptote is called **Critical Power (CP)** – the power output that can be sustained for a very long time without exhaustion [2] [1]. Mathematically, one form of this model is:

$$P = CP + \frac{W'}{t},$$

or equivalently $t = \frac{W'}{P-CP}$. This means the **time to exhaustion** t at any constant power P above CP is inversely proportional to how far P exceeds CP [3]. As P approaches CP from above, the sustainable duration t grows very large (approaching infinity in theory), illustrating that CP is an asymptotic "fatigue threshold." In simpler terms, **CP separates steady-state vs. non-steady-state intensities**: exercise at CP or below can reach a metabolic steady-state, whereas exercise above CP causes progressive fatigue until exhaustion [2].

In running, the same concept is often expressed as **critical speed (CS)** with a similar hyperbolic speed–time curve (and a corresponding **D′** in meters, analogous to W′) [4]. Whether using power (watts) or speed (m/s), CP/CS represents the boundary between the "heavy" exercise domain (sustainable, steady physiology) and the "severe" domain where fatigue inexorably accumulates [2] [5].

W′ (W Prime) – Finite Work Capacity Above CP

W′ (W prime) is the model parameter that quantifies the *finite amount of work* an athlete can perform above their CP. It is effectively the curvature constant of the power–duration hyperbola [6]. In practical terms, W′ is measured in energy units (joules or kilojoules) and represents a fixed "work capacity" that can be expended when exercising above CP [7] [1]. Once this extra work capacity is used up, fatigue is so high that the athlete must stop – this corresponds to exhaustion.

Importantly, **W′ is constant for a given individual** (at a given fitness level) and does not depend on how it's expended. Whether an athlete sprints briefly or maintains a moderately hard pace for longer, as long as the work done above CP equals their W′, they will reach the **limit of tolerance** [8]. For example, if an athlete's CP is 300 W and W′ is 20 kJ, then sustaining 400 W (which is 100 W above CP) would theoretically exhaust their W′ in about $20{,}000\ J/100\ J/s = 200\ s$ (since 100 J/s is the rate above CP). If they only ride at 330 W (30 W above CP), exhaustion would come later, around $20{,}000/30 \approx 667$ seconds, because W′ is burned more slowly. In other words, **the closer the exercise intensity is to CP, the longer it takes to use up W′** [7] [9]. W′ can thus be imagined as an energy reserve or "anaerobic capacity" (historically termed anaerobic work capacity) that fuels work above CP. However, it's not purely a fixed store of one substrate – it reflects a mix of physiological resources (phosphocreatine, anaerobic glycolysis, oxygen stores) and the tolerable accumulation of fatigue byproducts [10]. It remains remarkably constant across various high-intensity exercises; exhaustion in the severe domain consistently coincides with roughly the same W′ expended and similar levels of muscle metabolites, regardless of pacing strategy [8].

W′ Depletion in Intervals and Exponential Recovery (W′ Balance Model)

During **intermittent exercise** with work and rest intervals, the critical power model can be extended to track W′ usage and recovery in real time. The assumptions are: when power output **exceeds CP**, the athlete draws from their W′ store (depleting it), and when power falls **below CP**, the W′ can **reconstitute** (recover) partially [11]. Researchers formalized this with the concept of **W′ balance (W′_bal)** – essentially keeping a running tally of how much of W′ remains at any given moment [11].

W′ depletion: Above CP, W′ is expended in a roughly linear fashion. In fact, for a constant power > CP, the **rate** of W′ usage is approximately $P - CP$ (in J/s). This linear utilization is why a hard bout 50 W above CP will burn through W′ twice as fast as a bout 25 W above CP [7]. Complete exhaustion occurs when the cumulative work above CP equals the entire W′ (at which point W′_bal reaches zero). Notably, this linear usage holds during intermittent work bouts as well – each hard interval draws down the W′ reserve by the area under the curve above CP.

W′ recovery: When power drops back **below CP** (during a recovery interval or a respite in pacing), the remaining W′ can refill, but **not linearly** – it follows a **curvilinear (exponential)** time course [12]. Early in recovery, W′ regenerates quickly, then the rate of recharge slows as it asymptotically approaches the original W′ level. In other words, W′ recovery behaves like a first-order **exponential process** with a certain time constant [13] [14]. Crucially, the *speed* of W′ reconstitution depends on how easy the recovery is: the farther *below CP* the recovery intensity is, the faster W′ recovers [15] [14]. For instance, one study found the recovery time constant τ was ~377 seconds when recovering at a very low 20 W, around 452 s at moderate-intensity recovery, and as slow as ~580 s when “recovering” just below CP in the heavy domain [16]. If the power never goes below CP at all, **W′ cannot recharge** (the model predicts an infinite τ when recovery power remains above CP, meaning the athlete is still net consuming W′ even in “recovery”) [17].

Modeling W′ balance: Using these principles, a continuous W′_bal model was developed (Skiba et al. 2012, 2015) to track the dynamic state of W′ during variable work rates. In mathematical form, it can be described piecewise by differential equations. When $P(t) > CP$:

- $\dfrac{dW\prime_{bal}}{dt} = -\big(P(t) - CP\big),$

meaning W′_bal decreases at the rate power exceeds CP (Joules per second). And when $P(t) < CP$:

- $\dfrac{dW\prime_{bal}}{dt} = \dfrac{W\prime_{\max} - W\prime_{bal}(t)}{\tau},$

meaning W′_bal replenishes toward its maximum value with an exponential time constant τ [14]. Here $W\prime_{\max}$ is the athlete’s full W′ capacity (recovered), and τ is a constant that may vary with recovery intensity as noted above. In plain language, after a hard effort, the *fraction* of W′ that regenerates per second depends on how much W′ is “missing” and how quickly the athlete physiologically recovers (fast at first, then slower). The **W′_balance** at any time is essentially: initial W′ minus what’s been used, plus what’s been regained via exponential recovery. Empirical models confirm that an exponential recharge function fits observed data well [13]. For example, Ferguson et al. (2010) showed W′ comes back more rapidly in the first minutes post-exercise than later, evidencing a curvilinear pattern [18]. Skiba and colleagues then validated an integral W′_bal model in cyclists, showing it could accurately predict exhaustion during varied interval protocols – when the model's W′_bal hit ~0, the athlete was indeed at failure [19] [20]. In summary, **W′ is utilized linearly when above CP, but its recovery below CP is exponential** (non-linear), a behavior that modern W′_bal models take into account [21].

CP vs. FTP: Physiological Accuracy and Differences

Functional Threshold Power (FTP) is a concept from cycling training that is often compared to CP. Both CP and FTP intend to pinpoint an athlete's threshold between sustainable and unsustainable effort, but they are defined differently. CP is derived from the power–duration *relationship* using multiple exhaustive tests, whereas **FTP is typically defined as the highest power one can sustain in a quasi-steady state for ~60 minutes** [22] (in practice, often estimated as ~95% of a best 20-minute power test). In essence, CP is a parameter from a physiological model, while FTP is a shorthand performance metric.

Magnitude: Studies on well-trained cyclists have found that CP and FTP are closely related but **not interchangeable**. In a 2021 study, for example, CP (determined by a 3-test model) averaged 282 W, while FTP (from a 60-min prediction via a 20-min test) averaged 266 W in the same athletes – CP was about 16 W (roughly 6%) higher than FTP on average [23]. This was a significant difference, meaning one cannot assume CP equals FTP for a given individual. Generally, CP tends to be a bit higher than the true one-hour power for many athletes [23]. Conceptually, CP often corresponds to the **upper limit of the heavy-intensity domain**, which some research suggests might be maintainable for ~30–40 minutes before slow fatigue processes (e.g. VO_2 slow component, lactate drift) intervene [22]. FTP, defined as a ~60-min power, might be slightly lower if an athlete cannot hold CP for a full hour. In other words, **CP is a more precise mathematically-defined threshold, whereas FTP is an empirical estimate of a one-hour sustainable power**.

Physiological basis: CP has a strong grounding in physiology – it reflects the maximal steady-state oxidative energy supply and corresponds to the critical intensity above which fatigue-related metabolites (lactate, H^+, P_i, etc.) continually accumulate [5] [24]. FTP, on the other hand, was not originally derived from lab measures but has been correlated to the maximal lactate steady state (MLSS) or critical power in many cases. Both CP and FTP assume a "threshold" for metabolic steady-state, but the **accuracy differs**. Because CP is obtained via multiple exhaustive efforts and modeling, it accounts for both aerobic capacity and anaerobic work capacity (W′), giving insight into an individual's profile. FTP is easier to test (requiring just a time trial effort) but can be influenced by testing protocol and doesn't explicitly yield a W′-like parameter. Researchers note that calling FTP and CP equivalent "maximal metabolic steady-states" can be misleading – in the aforementioned study, **CP was significantly higher than FTP while purportedly representing a similar concept** [25]. This suggests that athletes might reach a true physiological steady state at a power a bit below CP (closer to FTP), and CP itself might be slightly above the sustainable lactate threshold for some. In practice, coaches often treat CP and FTP as analogous benchmarks (both delineate the boundary of tolerable sustainable intensity), but **CP is considered more precise and individualized**, whereas FTP is a convenient approximation for training zones [22] [23].

Using W′ Balance in Coaching and Real-Time Training Prescription

One of the powerful applications of the CP/W′ model is in **designing and adjusting interval workouts** to optimally challenge an athlete. A coaching algorithm that knows an athlete's CP and W′ can use the **W′ balance** (W′_bal) to prescribe interval intensities and durations in real time for maximal effectiveness [26]. For example, consider an athlete with a W′ of 20 kJ. How many intervals at a given intensity can they handle before exhaustion? W′_bal modeling provides the answer by tracking the remaining W′. A coach or software can simulate a planned workout and ensure that by the end of the last interval, the athlete's W′_bal is nearly drained (e.g. reaches zero or a small reserve), indicating a full utilization of high-intensity capacity without failing too early [27] [28]. This takes the guesswork out of crafting intervals like *4×2 minutes vs. 3×3 minutes*: since **each athlete's W′ is unique**, the ideal interval count and duration should match their capacity [29]. If intervals are too long or too many, the model will predict W′_bal hitting zero *before* the workout is done – a warning that the athlete would likely not complete the session. If intervals are too short or recovery too easy, W′_bal might not be sufficiently challenged by the end.

In real time, such an algorithm (or device feedback) can adjust on the fly. **During a session**, if the athlete's W′_bal is dropping faster than expected (meaning they are working harder relative to CP than planned), the system could recommend extending the next recovery interval or reducing the next interval's power slightly to avoid premature exhaustion. Conversely, if W′_bal is still high after an interval (meaning they didn't go as hard as prescribed), the next interval could be made more intense or longer. Modern training software and bike computers have started incorporating live W′_bal charts for this purpose [30] [31]. Seeing a "W′ remaining" metric (sometimes visualized like a battery of matches left to burn) gives athletes and coaches actionable insight – for instance, a cyclist in a race might decide whether it's *safe to attack* based on how much W′ they have left in the tank [32] [33]. From a coaching perspective, the **precision in interval prescription** is greatly improved. Instead of prescribing generic interval lengths, a coach can specify, say, repeats at 120% of CP until W′_bal is 20% (nearly exhausted), then recover until W′_bal is back to 90%, and repeat. This ensures each athlete is stressed appropriately relative to their individual capacities [26]. As one sports science article summarizes: with knowledge of an athlete's CP, W′, and W′ recovery kinetics, a coach can prescribe work and recovery intervals "with greater precision to achieve specific physiological goals" [26].

In summary, the CP and W′ model not only explains the fundamental power–duration relationship (hyperbolic curve) and defines W′ as a finite reserve of work above threshold, but it also provides practical tools. W′ balance modeling allows real-time monitoring of an athlete's high-intensity energy reserves, enabling smarter interval training design and pacing strategies. By avoiding excessive depletion of W′ too early (or ensuring it's fully utilized by the end of key workouts), athletes can train and compete more effectively [34] [35]. This integration of physiology and training prescription exemplifies how the CP/W′ concept bridges lab theory with on-the-road (or on-the-track) coaching decisions.

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