1

a) $T(n) = 3 * T(n-1) - 2 * T(n-2)$

$r^2 = 3r - 2 \rightarrow r^2 - 3r + 2 \quad (r-2)(r-1)$

$r \quad -2$
$r \quad -1 \quad r = 2, 1 \quad = c_1 2^n + c_2 1^n$

b) $T(n) = T(n/2) + 1$, $\quad n^{\log_b a} = 1 \quad n^0 = 1$

$\log_b a = \log_2 1 = 0 \quad = n^{\log_b a}, \log(n) = \log n$

c) $r^2 = 4r - 4 \quad = c_1 2^n + c_2 2^n n$

$r^2 - 4r + 4 \quad r = 2$
$r \quad -2 \quad r_2 = 2$
$r \quad -2$

d) $T(n) = 4T(n/2) + n^2 \quad 2 f(n)$

$n^{\log_b a} = n^2 = f(n) \quad n^{\log_b a}, (\log n)$

$= n^2 . \log(n)$

e) $T(n) = 2T(n/2) + \Theta(n)$

$n^{\log_b a} = n$

$\log_b a = \log_2 2 = 1 \quad n = n \quad n^{\log_b a}, \log(n)$

$\Theta(n \log n)$

f) $r^2 = r + 1 \quad r^2 - r - 1 \quad \frac{1 \pm \sqrt{1+4}}{2} \quad c_1 \frac{\sqrt{5}-1}{2} + c_2 \frac{-\sqrt{5}-1}{2}$

g) $T(n) = T(n/2) + n \quad T(1) = 1$ forward substitution

$T(2) = 3$

$T(2) = T(1) + 2 \quad T(2) = 3$

$T(4) = T(2) + 4 \quad T(4) = 7 \quad T(n) = 2n - 1$

$T(8) = T(4) + 8 \quad T(8) = 15 \quad n + 1 =$

$2n + 1 = O(n)$

$2n - 1 = 2(\frac{n}{2}) - 1 + n \quad T(n) = 2n - 1$

$2n - 1 = 2n - 1 \quad T(n) = \Theta(n)$

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h) $T(n) = 2T(\sqrt{n}) + 1 \qquad T(1) = 1 \qquad T(4) = 3$

$n^{\frac{1}{2}}$

$T \quad 2(2\,T(n^{\frac{1}{4}}) + 1) + 1$

$2(2(2\,T(n^{1/8}) + 1) + 1) + 1$

$2^i . T(n^{\frac{1}{2^i}}) + \sum_{k=0}^{i-1} 2^k . 1 \qquad 2^i - 1$

$n^{\frac{1}{2^i}} = 1 \qquad T(1) = 1 \qquad 2^{i+1} - 1$

$2^i + 2^i - 1$

$\frac{1}{2^i} . \log n = \log_2 1 \qquad = 2\log(n) - 1$

$\log n = 2^i$

2-) a)

$T(n) = 2T(n/2) + n$

By using master teorem

$n^{\log_2 2} = n$ 2. $\Rightarrow n^{\log_b a} \cdot \log n$

$= \Theta(n \log n)$

b-)

$T(n) = 2T(n/2) + n$

By using master theorem

$n^{\log_b a} = n^{\log_2 2} = n$ 2. $\Rightarrow n^{\log_b a} \cdot \log n$

$= \Theta(n \log n)$

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3\. a) $T(n) = 5 T(n/2) + n^3$

b) $T(n) = 2T(n-2) + n$

c) $T(n) = 3T(n/2) + n^2$

a) $n^{\log_2 5} = n^{\log_2 5} < n^3$ 3

$a \cdot f(n/b) \le c f(n)$ $c=5$ holds

$\Theta(f(n)) = \Theta(n^3)$

b) $T(n) = 2T(n-2) + n$

$2(2T(n-4) + n) + n$

$2(2(2T(n-6) + n) + n) + n$

$2^{\frac{n}{2}} T(1)$

$2^i . T(n-2i) + \sum_{k=0}^{i} 2^k . n$

$n - 2i = 1$

$n = 2i + 1$

$\frac{n-1}{2} = i$

$\frac{(n \cdot 2^{\frac{n-1}{2}} + n) n}{2}$

$= O(n^2)$

$2^{\frac{n-1}{2}} . T(1) + O(n^2) \Rightarrow O(n^2)$

c) $3T(n/2) + n^2$

$f(n) = n^2$

$n^{\log_2 3} < n^2$ so $\Theta(f(n))$

$\Rightarrow O(n^2)$

a) $O(n^3)$

$\boxed{\begin{array}{l} b)\ O(n^2) \\ c)\ O(n^2) \end{array}}$

a is complexier than the others, Choosing b and c is more convenient.

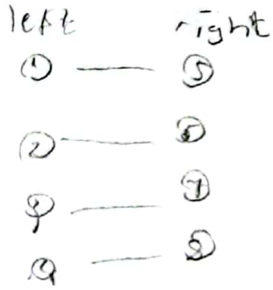
5-) $T(n) = 2(n/2)+n$

4-) For a bipartite graph there must be two list holds right and the other holds left the algorithm must iterate through left list than takes current vertices in iteration and search its adjented vertices by using bfs algorithm, If the found adjacented vertices in right list remove it from the list and continue that for the other items in left list until no one items left as paired.

n: number of vertices

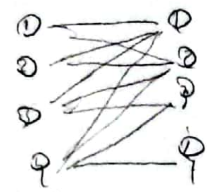
Best Case

list [1,2,3,4]



$$T(n) = \sum_{i=1}^{n/2} 1 = O(n)$$

Worst case



$$T(n) = \sum_{i=1}^{n/2} \sum_{j=1}^{i-1} j = 1+2+3+\dots+\frac{n}{2}-1$$

$$\frac{\left(\frac{n}{2}-1+1\right)\cdot\frac{n}{2}}{2} = \frac{n^2}{8} = O(n^2)$$

Average Case:

$$\sum_{i=1}^{n} = \frac{p}{n} O(n) + \frac{p}{n} O(n+1) \dots + \frac{p}{n} \cdot O(n^2)$$

$$\frac{(n-1)\cdot(n+(n-2))+n}{2} \cdot \frac{p}{n} + (n-1) \cdot n^2 \cdot \frac{p}{n}$$

$$= \frac{2n^2-4n+2+n}{2} \cdot \frac{p}{n}(n-1)$$

$$= O(n^2) + O(n^3)$$

$$= O(n^3)$$

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