

$$f(n) = O(g(n)) \Rightarrow \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$$

$$f(n) = \Omega(g(n)) \Rightarrow \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$$

$$f(n) = \Theta(g(n)) \Rightarrow 0 < \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} < \infty$$

a)  $f(n) = 2^n$   $g(n) = 2^{2n}$

$$\lim_{n \rightarrow \infty} \frac{2^n}{2^{2n}} = 2^{-1} = \frac{1}{2} \text{ so } f(n) \in \Theta(g(n))$$

b)  $f(n) = n^2$   $g(n) = n^3$

$$\lim_{n \rightarrow \infty} \frac{n^2}{n^3} = \frac{2n}{3n^2} = \frac{2}{3n} = \frac{1}{\infty} = 0 \Rightarrow f(n) \in O(g(n))$$

c)  $f(n) = 3n+1$   $g(n) = 2n-5$

$$\lim_{n \rightarrow \infty} \frac{3n+1}{2n-5} = \frac{3}{2} = f(n) \in \Theta(g(n))$$

d)  $f(n) = \ln n$   $g(n) = n^2$

$$\lim_{n \rightarrow \infty} \frac{\ln n}{n^2} = 4 = f(n) \in \Theta(g(n))$$

e)  $f(n) = \log_2 n$   $g(n) = \log_{10} n$

$$\lim_{n \rightarrow \infty} \frac{\log_2 n}{\log_{10} n} = \frac{1}{\frac{1}{\log_2 10}} = \log_2 10 \text{ so } f(n) \in \Theta(g(n))$$

f)  $f(n) = 2^n$   $g(n) = 3^n$

$$\lim_{n \rightarrow \infty} \frac{2^n}{3^n} = \frac{2}{3} \Rightarrow f(n) \in \Theta(g(n))$$

g)  $f(n) = n^3$   $g(n) = 1000n^2$

$$\lim_{n \rightarrow \infty} \frac{n^3}{1000n^2} = \frac{3n^2}{1000n} = \frac{3n}{1000} = \infty \text{ so } f(n) \in \Omega(g(n))$$

h)  $f(n) = 5n+4$   $g(n) = 2n+2$

$$\lim_{n \rightarrow \infty} \frac{5n+4}{2n+2} = \frac{5}{2} \text{ so } f(n) \in \Theta(g(n))$$

i)  $f(n) = \sqrt{n}$   $g(n) = \log_2 n$

$$\lim_{n \rightarrow \infty} \frac{\sqrt{n}}{\log_2 n} = \frac{1}{\frac{1}{n \log_2 e}} = n \log_2 e$$

j)  $f(n) = 2^n$   $g(n) = 2^{n+1}$

$$\lim_{n \rightarrow \infty} \frac{2^n}{2^{n+1}} = \frac{1}{2} \text{ so } f(n) \in \Theta(g(n))$$



2-  $\left( \frac{1}{2n} < \log n < \sqrt{n} < n+1 < n^2 / \log n < 2^n < 10^n < n! < n^{2n} \right)$

$O(\sqrt{n})$   $O(\log n)$   $O(n)$   $O(n^2 \log n)$   $O(n^2)$   $O(n^3)$   $O(n^4)$   $O(n^5)$   $O(n^6)$   $O(n^7)$   $O(n^8)$   $O(n^9)$   $O(n^{10})$   $O(n^{11})$   $O(n^{12})$   $O(n^{13})$   $O(n^{14})$   $O(n^{15})$   $O(n^{16})$   $O(n^{17})$   $O(n^{18})$   $O(n^{19})$   $O(n^{20})$   $O(n^{21})$   $O(n^{22})$   $O(n^{23})$   $O(n^{24})$   $O(n^{25})$   $O(n^{26})$   $O(n^{27})$   $O(n^{28})$   $O(n^{29})$   $O(n^{30})$   $O(n^{31})$   $O(n^{32})$   $O(n^{33})$   $O(n^{34})$   $O(n^{35})$   $O(n^{36})$   $O(n^{37})$   $O(n^{38})$   $O(n^{39})$   $O(n^{40})$   $O(n^{41})$   $O(n^{42})$   $O(n^{43})$   $O(n^{44})$   $O(n^{45})$   $O(n^{46})$   $O(n^{47})$   $O(n^{48})$   $O(n^{49})$   $O(n^{50})$   $O(n^{51})$   $O(n^{52})$   $O(n^{53})$   $O(n^{54})$   $O(n^{55})$   $O(n^{56})$   $O(n^{57})$   $O(n^{58})$   $O(n^{59})$   $O(n^{60})$   $O(n^{61})$   $O(n^{62})$   $O(n^{63})$   $O(n^{64})$   $O(n^{65})$   $O(n^{66})$   $O(n^{67})$   $O(n^{68})$   $O(n^{69})$   $O(n^{70})$   $O(n^{71})$   $O(n^{72})$   $O(n^{73})$   $O(n^{74})$   $O(n^{75})$   $O(n^{76})$   $O(n^{77})$   $O(n^{78})$   $O(n^{79})$   $O(n^{80})$   $O(n^{81})$   $O(n^{82})$   $O(n^{83})$   $O(n^{84})$   $O(n^{85})$   $O(n^{86})$   $O(n^{87})$   $O(n^{88})$   $O(n^{89})$   $O(n^{90})$   $O(n^{91})$   $O(n^{92})$   $O(n^{93})$   $O(n^{94})$   $O(n^{95})$   $O(n^{96})$   $O(n^{97})$   $O(n^{98})$   $O(n^{99})$   $O(n^{100})$   $O(n^{101})$   $O(n^{102})$   $O(n^{103})$   $O(n^{104})$   $O(n^{105})$   $O(n^{106})$   $O(n^{107})$   $O(n^{108})$   $O(n^{109})$   $O(n^{110})$   $O(n^{111})$   $O(n^{112})$   $O(n^{113})$   $O(n^{114})$   $O(n^{115})$   $O(n^{116})$   $O(n^{117})$   $O(n^{118})$   $O(n^{119})$   $O(n^{120})$   $O(n^{121})$   $O(n^{122})$   $O(n^{123})$   $O(n^{124})$   $O(n^{125})$   $O(n^{126})$   $O(n^{127})$   $O(n^{128})$   $O(n^{129})$   $O(n^{130})$   $O(n^{131})$   $O(n^{132})$   $O(n^{133})$   $O(n^{134})$   $O(n^{135})$   $O(n^{136})$   $O(n^{137})$   $O(n^{138})$   $O(n^{139})$   $O(n^{140})$   $O(n^{141})$   $O(n^{142})$   $O(n^{143})$   $O(n^{144})$   $O(n^{145})$   $O(n^{146})$   $O(n^{147})$   $O(n^{148})$   $O(n^{149})$   $O(n^{150})$   $O(n^{151})$   $O(n^{152})$   $O(n^{153})$   $O(n^{154})$   $O(n^{155})$   $O(n^{156})$   $O(n^{157})$   $O(n^{158})$   $O(n^{159})$   $O(n^{160})$   $O(n^{161})$   $O(n^{162})$   $O(n^{163})$   $O(n^{164})$   $O(n^{165})$   $O(n^{166})$   $O(n^{167})$   $O(n^{168})$   $O(n^{169})$   $O(n^{170})$   $O(n^{171})$   $O(n^{172})$   $O(n^{173})$   $O(n^{174})$   $O(n^{175})$   $O(n^{176})$   $O(n^{177})$   $O(n^{178})$   $O(n^{179})$   $O(n^{180})$   $O(n^{181})$   $O(n^{182})$   $O(n^{183})$   $O(n^{184})$   $O(n^{185})$   $O(n^{186})$   $O(n^{187})$   $O(n^{188})$   $O(n^{189})$   $O(n^{190})$   $O(n^{191})$   $O(n^{192})$   $O(n^{193})$   $O(n^{194})$   $O(n^{195})$   $O(n^{196})$   $O(n^{197})$   $O(n^{198})$   $O(n^{199})$   $O(n^{200})$   $O(n^{201})$   $O(n^{202})$   $O(n^{203})$   $O(n^{204})$   $O(n^{205})$   $O(n^{206})$   $O(n^{207})$   $O(n^{208})$   $O(n^{209})$   $O(n^{210})$   $O(n^{211})$   $O(n^{212})$   $O(n^{213})$   $O(n^{214})$   $O(n^{215})$   $O(n^{216})$   $O(n^{217})$   $O(n^{218})$   $O(n^{219})$   $O(n^{220})$   $O(n^{221})$   $O(n^{222})$   $O(n^{223})$   $O(n^{224})$   $O(n^{225})$   $O(n^{226})$   $O(n^{227})$   $O(n^{228})$   $O(n^{229})$   $O(n^{230})$   $O(n^{231})$   $O(n^{232})$   $O(n^{233})$   $O(n^{234})$   $O(n^{235})$   $O(n^{236})$   $O(n^{237})$   $O(n^{238})$   $O(n^{239})$   $O(n^{240})$   $O(n^{241})$   $O(n^{242})$   $O(n^{243})$   $O(n^{244})$   $O(n^{245})$   $O(n^{246})$   $O(n^{247})$   $O(n^{248})$   $O(n^{249})$   $O(n^{250})$   $O(n^{251})$   $O(n^{252})$   $O(n^{253})$   $O(n^{254})$   $O(n^{255})$   $O(n^{256})$   $O(n^{257})$   $O(n^{258})$   $O(n^{259})$   $O(n^{260})$   $O(n^{261})$   $O(n^{262})$   $O(n^{263})$   $O(n^{264})$   $O(n^{265})$   $O(n^{266})$   $O(n^{267})$   $O(n^{268})$   $O(n^{269})$   $O(n^{270})$   $O(n^{271})$   $O(n^{272})$   $O(n^{273})$   $O(n^{274})$   $O(n^{275})$   $O(n^{276})$   $O(n^{277})$   $O(n^{278})$   $O(n^{279})$   $O(n^{280})$   $O(n^{281})$   $O(n^{282})$   $O(n^{283})$   $O(n^{284})$   $O(n^{285})$   $O(n^{286})$   $O(n^{287})$   $O(n^{288})$   $O(n^{289})$   $O(n^{290})$   $O(n^{291})$   $O(n^{292})$   $O(n^{293})$   $O(n^{294})$   $O(n^{295})$   $O(n^{296})$   $O(n^{297})$   $O(n^{298})$   $O(n^{299})$   $O(n^{300})$   $O(n^{301})$   $O(n^{302})$   $O(n^{303})$   $O(n^{304})$   $O(n^{305})$   $O(n^{306})$   $O(n^{307})$   $O(n^{308})$   $O(n^{309})$   $O(n^{310})$   $O(n^{311})$   $O(n^{312})$   $O(n^{313})$   $O(n^{314})$   $O(n^{315})$   $O(n^{316})$   $O(n^{317})$   $O(n^{318})$   $O(n^{319})$   $O(n^{320})$   $O(n^{321})$   $O(n^{322})$   $O(n^{323})$   $O(n^{324})$   $O(n^{325})$   $O(n^{326})$   $O(n^{327})$   $O(n^{328})$   $O(n^{329})$   $O(n^{330})$   $O(n^{331})$   $O(n^{332})$   $O(n^{333})$   $O(n^{334})$   $O(n^{335})$   $O(n^{336})$   $O(n^{337})$   $O(n^{338})$   $O(n^{339})$   $O(n^{340})$   $O(n^{341})$   $O(n^{342})$   $O(n^{343})$   $O(n^{344})$   $O(n^{345})$   $O(n^{346})$   $O(n^{347})$   $O(n^{348})$   $O(n^{34$

in general rule

$$C(1/2n) = O(1/n)$$

for  $n \geq 1$

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\frac{1000}{1000} \frac{10^7}{2^n} = 5^n = 1000$$

$\hookrightarrow z^n \in m^n$

$$O(\sqrt{n+5}) = O(\sqrt{n})$$

$$\sqrt{a} \cdot a^{\frac{1}{2}}$$

$\frac{1}{7}$  C9

$$\lim_{n \rightarrow \infty} \frac{n \log n}{n^2 \log n} = \frac{1}{n} = 0$$

else

else

5

4-7)  $i = 2$   
while  $i \leq n$ :  
if  $i \% 2 == 0$  then  $O(1)$   
 $i = i - 1$   $O(1)$

else  
 $i = i * i$   $O(1)$   
 $i = i + 1$   $O(1)$  }  $O(1000)$

Print(i)

Step: 1

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$$p_2, x_2$$

$$g = i + 1$$

$j=5$

else

step 3:  $1 - 2 = 0$

since  $i = 4$

$$i, 5i, i^*$$

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$p = 37$

Step 2  $i \neq 0$   
since  $i = 5$

5.8  $f = \frac{1}{2}$

$$j = 4$$

15  
step 4

$$i/o \neq 0$$

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$$j \in \{1, \dots, n\}$$

$j=10$

1050 percentage of steps

Inside the loop get into

if statement and 10 to 50 percentage  
set into else so,

$$ATC = \frac{P_1}{1} T_1 + P_2 T_2$$

$$O\left(\frac{1}{2}, \log n\right) + \frac{1}{2}, O(n)$$

$$O\left(\frac{\log n + O(n)}{2}\right) = O(n)$$



5) the algorithm can work like this. Search the array one by one then if its find the even element it returns otherwise continues.

$$T(n) = \sum_{i=1}^3 P_i \times T(n) \Rightarrow P_1 O(1) + P_2 O(1) + P_3 O(n)$$

$$P_1 = 0.2 \times O(1)$$

$$P_3 = (0.8)^n$$

$$P_2 = 1 - P_1 - P_3$$

$$\lim_{n \rightarrow \infty} (0.8)^n = 0$$

$$\approx 0.8$$

$$O, O(n)$$

$$T(n) = 0.2 O(1) + 0.8 O(n) + 0 O(n)$$

$$\approx O(n)$$

3-) Time complexity analysis given both in paper and in the PG code.

a-) adding items to the list by using inorder traversal takes  $O(n)$  time.

Merging the two bst list takes  $O(n+m) \Rightarrow O(n)$  time

Creating a node from the list items takes  $O(n)$  time

$$O(n) + O(n) + O(n+m) + O(n) \Rightarrow O(n)$$

b-) Used inorder search algorithm to achieve smallest element on binary search tree

visits all the nodes after even it finds the correct data, so it takes  $O(n)$  time.

d-) Search all the nodes from bst and compare if its between low and high variable.