$\boldsymbol{u}, \boldsymbol{vu} \cdot \boldsymbol{v??u} \cdot \boldsymbol{v} = |\boldsymbol{u}||\boldsymbol{v}| \cos \theta \boldsymbol{u}, \boldsymbol{v} \theta 0 \leq \theta \leq \pi$
$\boldsymbol{u} = (-1, 0, 7), \boldsymbol{v} = (-4, 5, 3)$
$\boldsymbol{u} = (-1, 1, 0), \boldsymbol{v} = (1, -2, 2)$
$\boldsymbol{u} \neq \mathbf{0}, \boldsymbol{v} \neq \mathbf{0} \boldsymbol{uv} \iff \boldsymbol{u} \cdot \boldsymbol{v} = 0$
$\boldsymbol{u}, \boldsymbol{vu} \times \boldsymbol{v}$
$\boldsymbol{u} = (-2, 3, 7), \boldsymbol{v} = (-4, 2, 3)$
$\boldsymbol{u} = (4, 1, 1), \boldsymbol{v} = (0, -2, 2)$
$3 \boldsymbol{uvu} \times \boldsymbol{v} 3 \boldsymbol{uvuv}$

$$\boldsymbol{u}\cdot\boldsymbol{v}=(-1)\times(-4)+0\times 5+7\times 3=25$$

$$\cos\theta=\frac{\boldsymbol{u}\cdot\boldsymbol{v}}{\boldsymbol{u}\boldsymbol{v}}=\frac{25}{\sqrt{50}\sqrt{50}}=\frac{1}{2}$$

$$\boldsymbol{u}\cdot\boldsymbol{v}=(-1)\times 1+1\times(-2)+0\times 2=-3$$

$$\cos\theta=\frac{\boldsymbol{u}\cdot\boldsymbol{v}}{\boldsymbol{u}\boldsymbol{v}}=\frac{-3}{\sqrt{2}\sqrt{9}}=-\frac{1}{\sqrt{2}}$$

$\boldsymbol{u}\neq\boldsymbol{0},\boldsymbol{v}\neq\boldsymbol{0}|\boldsymbol{u}|\neq 0,|\boldsymbol{v}|\neq 0\boldsymbol{u}\cdot\boldsymbol{v}=|\boldsymbol{u}||\boldsymbol{v}|\cos\theta\boldsymbol{u}\boldsymbol{v}\Leftrightarrow\cos\theta=0\Leftrightarrow\boldsymbol{u}\cdot\boldsymbol{v}=|\boldsymbol{u}||\boldsymbol{v}|\cos\theta=0$
$\boldsymbol{u}\times\boldsymbol{v}=(-5,-22,8)$
$\boldsymbol{u}\times\boldsymbol{v}=(4,-8,-8)$
$\boldsymbol{u}=(a,b,c)\boldsymbol{v}=(x,y,z)\boldsymbol{u}\boldsymbol{v}\theta$
$\boldsymbol{u}\times\boldsymbol{v}=(bz-cy,cx-az,ay-bx)$

$$\boldsymbol{u}\cdot(\boldsymbol{u}\times\boldsymbol{v})=a(bz-cy)+b(xz-az)+c(ay-bx)=0$$

$$\boldsymbol{v}\cdot(\boldsymbol{u}\times\boldsymbol{v})=x(bz-cy)+y(xz-az)+z(ay-bx)=0$$

$\boldsymbol{u}\times\boldsymbol{v}\boldsymbol{u}\boldsymbol{v}\boldsymbol{u}\boldsymbol{v}$

$$\begin{aligned}|\boldsymbol{u}\times\boldsymbol{v}|^2=&(bz-cy)^2+(cx-az)^2+(ay-bx)^2\\=&(a^2+b^2+c^2)(x^2+y^2+z^2)-(ax+by+cz)^2\\=&|\boldsymbol{u}|^2|\boldsymbol{v}|^2-(\boldsymbol{u}\cdot\boldsymbol{v})^2\\=&|\boldsymbol{u}|^2|\boldsymbol{v}|^2-|\boldsymbol{u}|^2|\boldsymbol{v}|^2\cos^2\theta\\=&|\boldsymbol{u}|^2|\boldsymbol{v}|^2(1-\cos^2\theta)\\=&|\boldsymbol{u}|^2|\boldsymbol{v}|^2\sin^2\theta\end{aligned}$$

$|\boldsymbol{u}\times\boldsymbol{v}|=|\boldsymbol{u}||\boldsymbol{v}||\sin\theta|\boldsymbol{u}\times\boldsymbol{v}\boldsymbol{u}\boldsymbol{v}$
$[\boldsymbol{u},\boldsymbol{v},\boldsymbol{u}\times\boldsymbol{v}]\boldsymbol{u}\times\boldsymbol{v}\boldsymbol{u}\boldsymbol{v}$

$$\begin{vmatrix}a & b & c\\ x & y & z\\ bz-cy & cx-az & ay-bx\end{vmatrix}=a^2y^2+a^2z^2-2abxy-2acxz+b^2x^2+b^2z^2-2bcyz+c^2x^2+c^2y^2$$
$$=(bz-cy)^2+(cx-az)^2+(ay-bx)^2>0$$