$egin{aligned} & m{u}, m{vu} \cdot m{v??u} \cdot m{v} = |m{u}| |m{v}| \cos heta m{u}, m{v} heta 0 \leq heta \leq \pi \ & m{u} = (-1, 0, 7), m{v} = (-4, 5, 3) \ & m{u} = (-1, 1, 0), m{v} = (1, -2, 2) \ & m{u} 
eq m{0}, m{v} 
eq m{0} m{u} m{v} & \iff m{u} \cdot m{v} = 0 \ & m{u}, m{vu} \times m{v} \ & m{u} = (-2, 3, 7), m{v} = (-4, 2, 3) \ & m{u} = (4, 1, 1), m{v} = (0, -2, 2) \ & 3 m{u} m{v} m{u} \times m{v} m{u} m{v} \ & \mathbf{u} \times m{v} \ & \mathbf{u} = (0, -2, 2) \ & \mathbf{u} \cdot m{v} \cdot m{v} = (0, -2, 2) \ & \mathbf{u} \cdot m{v} \cdot m{v} \cdot m{v} \cdot m{v} = (-2, 2, 2) \ & \mathbf{u} \cdot m{v} \cdot$ 

$$\mathbf{u} \cdot \mathbf{v} = (-1) \times (-4) + 0 \times 5 + 7 \times 3 = 25$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \mathbf{v}} = \frac{25}{\sqrt{50}\sqrt{50}} = \frac{1}{2}$$

$$\mathbf{u} \cdot \mathbf{v} = (-1) \times 1 + 1 \times (-2) + 0 \times 2 = -3$$

$$\cos \theta = \frac{\mathbf{u} \cdot \mathbf{v}}{\mathbf{u} \mathbf{v}} = \frac{-3}{\sqrt{2}\sqrt{9}} = -\frac{1}{\sqrt{2}}$$

 $\begin{aligned} & \boldsymbol{u} \neq \boldsymbol{0}, \boldsymbol{v} \neq \boldsymbol{0} | \boldsymbol{u} | \neq 0, | \boldsymbol{v} | \neq 0 \boldsymbol{u} \cdot \boldsymbol{v} = | \boldsymbol{u} | | \boldsymbol{v} | \cos \theta \boldsymbol{u} \boldsymbol{v} \Leftrightarrow \cos \theta = 0 \Leftrightarrow \boldsymbol{u} \cdot \boldsymbol{v} = | \boldsymbol{u} | | \boldsymbol{v} | \cos \theta = 0 \\ & \boldsymbol{u} \times \boldsymbol{v} = (-5, -22, 8) \\ & \boldsymbol{u} \times \boldsymbol{v} = (4, -8, -8) \\ & \boldsymbol{u} = (a, b, c) \boldsymbol{v} = (x, y, z) \boldsymbol{u} \boldsymbol{v} \theta \\ & \boldsymbol{u} \times \boldsymbol{v} = (bz - cy, cx - az, ay - bx) \end{aligned}$ 

$$\mathbf{u} \cdot (\mathbf{u} \times \mathbf{v}) = a(bz - cy) + b(xz - az) + c(ay - bx) = 0$$
$$\mathbf{v} \cdot (\mathbf{u} \times \mathbf{v}) = x(bz - cy) + y(xz - az) + z(ay - bx) = 0$$

 $u \times vuvuv$ 

$$|\mathbf{u} \times \mathbf{v}|^{2} = (bz - cy)^{2} + (cx - az)^{2} + (ay - bx)^{2}$$

$$= (a^{2} + b^{2} + c^{2})(x^{2} + y^{2} + z^{2}) - (ax + by + cz)^{2}$$

$$= |\mathbf{u}|^{2}|\mathbf{v}|^{2} - (\mathbf{u} \cdot \mathbf{v})^{2}$$

$$= |\mathbf{u}|^{2}|\mathbf{v}|^{2} - |\mathbf{u}|^{2}|\mathbf{v}|^{2}\cos^{2}\theta$$

$$= |\mathbf{u}|^{2}|\mathbf{v}|^{2}(1 - \cos^{2}\theta)$$

$$= |\mathbf{u}|^{2}|\mathbf{v}|^{2}\sin^{2}\theta$$

 $\begin{aligned} |\boldsymbol{u}\times\boldsymbol{v}| &= |\boldsymbol{u}||\boldsymbol{v}||\sin\theta|\boldsymbol{u}\times\boldsymbol{v}\boldsymbol{u}\boldsymbol{v}\\ |\boldsymbol{u},\boldsymbol{v},\boldsymbol{u}\times\boldsymbol{v}||\boldsymbol{u}\times\boldsymbol{v}\boldsymbol{u}\boldsymbol{v}\end{aligned}$ 

$$\begin{vmatrix} a & b & c \\ bz - cycx - azay - bx \end{vmatrix} = a^2y^2 + a^2z^2 - 2abxy - 2acxz + b^2x^2 + b^2z^2 - 2bcyz + c^2x^2 + c^2y^2 = (bz - cy)^2 + (cx - az)^2 + (ay - bx)^2 > 0$$