A new result for the Degree/Diameter Problem

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Abstract

The Degree/Diameter Problem is one of the most famous problem in graph theory. I found a new $(306, 2)$-graph with 88723 vertices.

1 Prerequisite

A *graph* $G = (V, E)$ consists of a set V called *vertices* and a set $E \subset V^2$ called *edges*. If (v, w) is in E, it is said that v and w are *adjacent*. If the vertex v is adjacent to itself, the edge (v, v) is called *loop*. The graph G without any loops is *simple*. The *order* N of the graph is the size of the set of vertices. The *degree* of the vertex $\delta(v)$ is a number of vertices which are adjacent to v. The *degree* of the graph Δ is the maximal degree of the vertex. The *distance* for each pair (v, w) of vertices is the shortest path length between v and w. The *diameter* D of the graph is the maximum distance for all pairs of vertices. The *Degree/Diameter problem* is finiding the graph with the maximum vertices for given degree Δ and diameter D. The order of a graph with degree Δ $(\Delta > 2)$ of diameter D is easily seen to be bounded by

$$1 + \Delta \sum_{k=1}^{D-1} (\Delta - 1)^k$$

which is called *Moore bound*. The genaral constructions for small degree and small diameter are known. Especially for $D = 2$ there exists the general construction called Brown's construction. Given the finite field F_q where q is a power of prime, we construct Brown's one $B(F_q)$ whose vertices are lines in F_q^3 and two lines are adjacent if and only if they are orthogonal. The order of $B(F_q)$ is $q^2 + q + 1$ and the degree of it is $q + 1$. The diameter of it is 2 because $B(F_q)$ includes many triangles.