DCIT105
Mathematics for IT Professionals

Session 1 – Number Systems I: Radix

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Session Outline

The key topics to be covered in the session are as follows:

- Analog vs. Digital signals
- Electronic Translators
- Decimal Number System
- Binary Number System
- Advantages of Binary Number System
- Decimal to Binary Conversion
- Binary to Decimal Conversion
- Hexadecimal to Binary Conversion
- Application Example
- Binary Coded Decimal

Topic One

NUMBER SYSTEMS I: RADIX

Introduction

- Humans use base ten (or decimal)
- The computer uses base-two (binary)
  - Because it only understands two states
  - High and Low voltage
  - 1 (+5V) and 0 (0V)
  - ON and OFF
- Need for converting numbers between these two number systems

Analog versus Digital

- Two ways of representing numerical values
  - Analog
  - Digital
- Analog system
  - The physical quantities or signals may vary continuously over a specified range.
  - Gives a continuous output
- Digital system
  - The physical quantities or signals can assume only discrete values.
  - Gives a discrete output.
  - Greater accuracy

Electronic Translators

- Devices that convert from decimal to binary numbers and from binary to decimal numbers.
- Encoders -

  translates from decimal to binary
- Decoders -

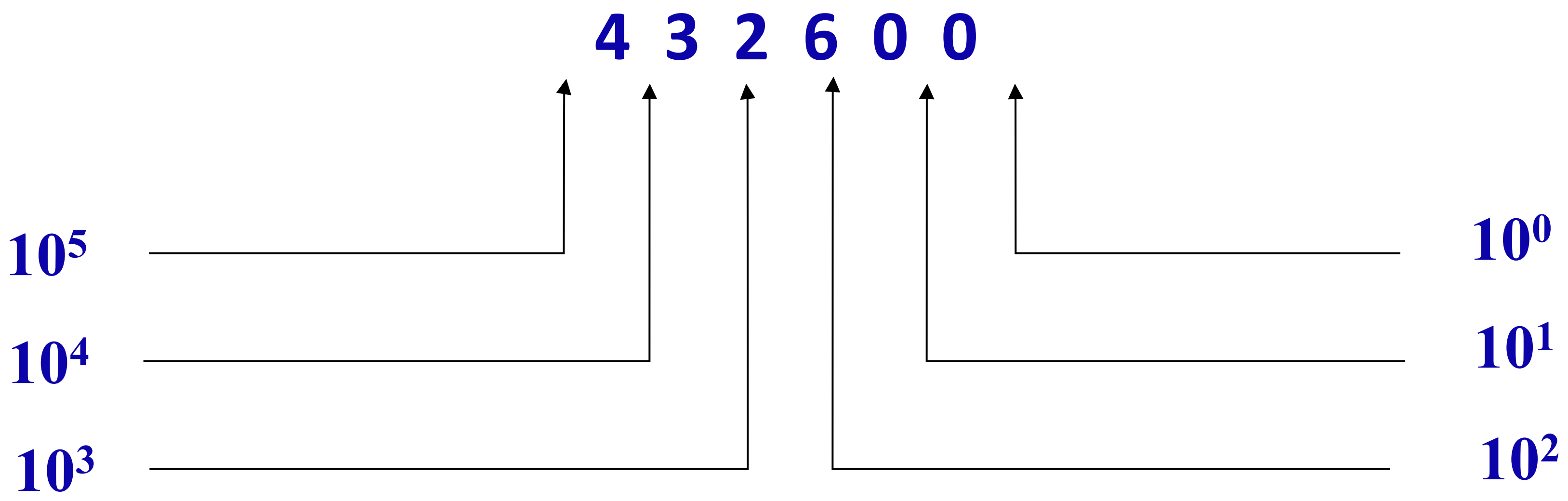
  translates from binary to decimal

Decimal {0 1 2 3 4 5 6 7 8 9}

- Decimal (base 10) {0 1 2 3 4 5 6 7 8 9}
  - Place value gives a logarithmic representation of the number
  - Eg. 4378 means
    - $4 \times 10^3 = 4000$
    - $3 \times 10^2 = 300$
    - $7 \times 10^1 = 70$
    - $8 \times 10^0 = 8$
  - The place also gives the exponent of the base

Example

- **432,600**



Powers of ten:

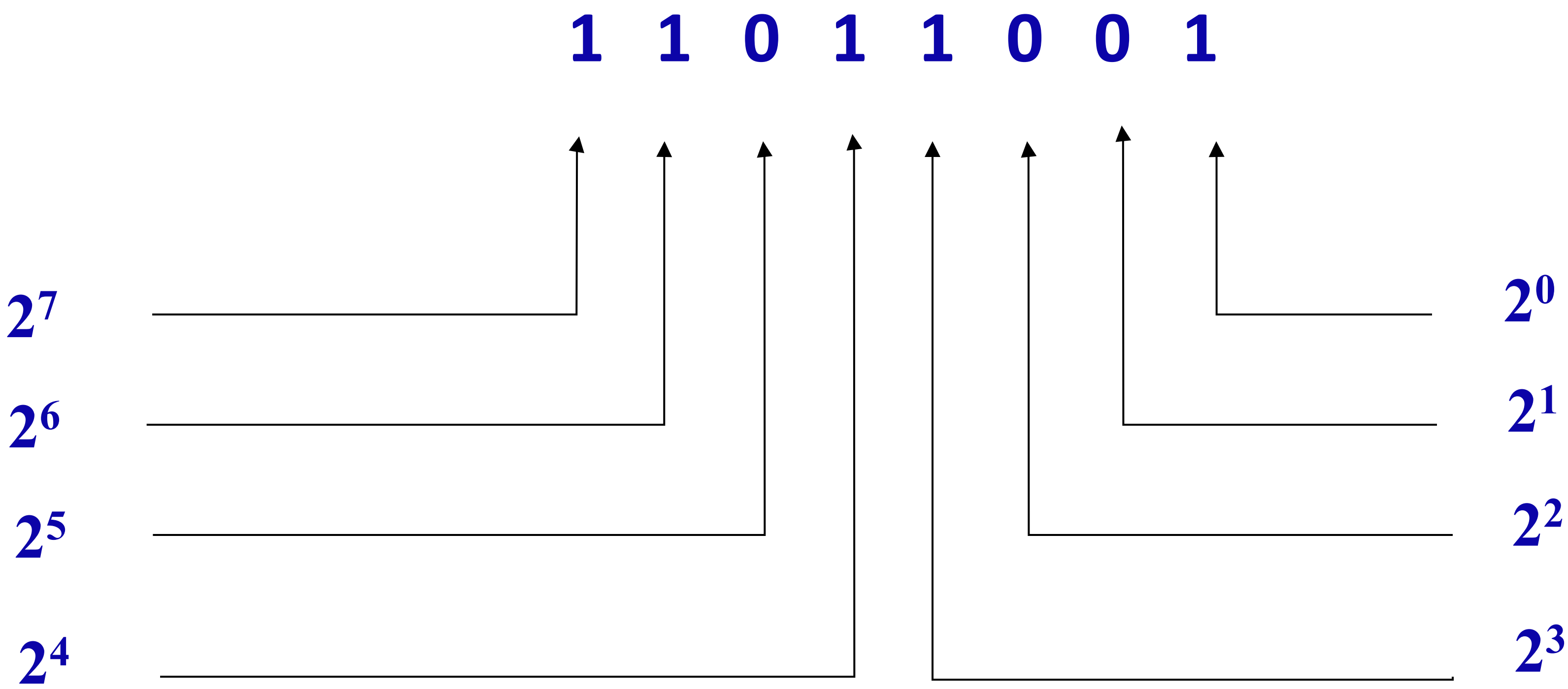
$10^0 = 1$ $\quad$ $10^4 = 10000$

$10^1 = 10$ $\quad$ $10^3 = 1000$ $\quad$ $10^5 = 100000$

Binary (base 2) {0 1}

Binary	Decimal
0	0
1	1
10	2
11	3
100	4
101	5
110	6
111	7
1000	8
1001	9
1010	10

Example



Decimal Equivalent

- **1101 1001**

$$
\begin{array}{rrcr}
 & 1 \times 2^7 & = & 128 \\
+ & 1 \times 2^6 & = & 64 \\
+ & 0 \times 2^5 & = & 0 \\
+ & 1 \times 2^4 & = & 16 \\
+ & 1 \times 2^3 & = & 8 \\
+ & 0 \times 2^2 & = & 0 \\
+ & 0 \times 2^1 & = & 0 \\
+ & 1 \times 2^0 & = & 1 \\
 & & & 217
\end{array}
$$

Notice how powers of two stand out:

$2^0 = 1$

$2^1 = 10$

$2^2 = 100$

$2^3 = 1000$

Example

- Write the first 16 numbers in the binary number system?
- Consider an arbitrary number system with the independent digits as 0, 1 and X. What is the radix of this number system? List the first 10 numbers in this number system.
- Solution

Will be solved in class

Advantages of Binary Number System

- Reduction of logic mathematics to binary notation.
- All kinds of data can be represented in 0s and 1s.
- Electronic devices operate in 2 distinct different modes.
- Performing arithmetic operations is simplified.

Decimal to Binary Conversion

- Example: 575

  Step 1. Find the largest power of two less than the number
  - $2^9 = 512$

  Step 2. Subtract that power of two from the number
  - $575 – 512 = 63$

  Step 3. Repeat steps 1 and 2 for the new result until you reach zero.
  - $2^5 = 32$ $63 – 32 = 31$
  - $2^4 = 16$ $31 – 16 = 15$
  - $2^3 = 8$ $15 – 8 = 7$
  - $2^2 = 4$ $7 – 4 = 3$
  - $2^1 = 2$ $3 – 2 = 1$
  - $2^0 = 1$ $1 – 1 = 0$

  Step 4. Construct the number
  - 1000111111

Decimal to Binary Conversion

- Example: 144
  - $2^7 = 128$ 144 – 128 = 16
  - $2^4 = 16$ 16 – 16 = 0
- Result 10010000

Decimal to Binary Conversion

- 53

  $= 32 + 16 + 4 + 1$

  $= 2^5 + 2^4 + 2^2 + 2^0$

  = 110101 in binary

  = 0011 0101 as a full byte in binary

- 211

  $= 128 + 64 + 16 + 2 + 1$

  $= 2^7 + 2^6 + 2^4 + 2^1 + 2^0$

  = 1101 0011 in binary

Decimal (fraction) to Binary Conversion

- **Multiply the number by the 'Base' (=2)**
- **Take the integer (either 0 or 1) as a coefficient**
- **Take the resultant fraction and repeat the division**

Example: $(0.625)_{10}$

	Integer		Fraction	Coefficient
0.625 * 2 =	1	.	25	$a_{-1} = 1$
0.25 * 2 =	0	.	5	$a_{-2} = 0$
0.5 * 2 =	1	.	0	$a_{-3} = 1$

Answer: $(0.625)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3})_2 = (0.101)_2$

MSB LSB

Binary to Decimal Conversion

- What is 10011010 in decimal?

  10011010 $= 1*2^7 + 0*2^6 + 0*2^5 + 1*2^4 + 1*2^3 + 0*2^2 + 1*2^1 + 0*2^0$

  $= 2^7 + 2^4 + 2^3 + 2^1$

  $= 128 + 16 + 8 + 2$

  $= 154$

- What is 00101001 in decimal?

  00101001 $= 0*2^7 + 0*2^6 + 1*2^5 + 0*2^4 + 1*2^3 + 0*2^2 + 0*2^1 + 1*2^0$

  $= 2^5 + 2^3 + 2^0$

  $= 32 + 8 + 1$

  $= 41$

Octal Number System

- Has a radix of 8 and therefore 8 distinct digits.
- The independent digits are

  0, 1, 2, 3, 4, 5, 6 and 7

- The place values for the different digits in the octal number system are
  - 8^0, 8^1 and so on (for the integer part) and
  - 8^{-1}, 8^{-2}, and so on (for the fractional part).
- Exercise:
  - Write the next 10 numbers that follow '7'

Decimal to Octal Conversion

Example: $(175)_{10}$

	Quotient	Remainder	Coefficient
175 / 8 =	21	7	$a_0 = 7$
21 / 8 =	2	5	$a_1 = 5$
2 / 8 =	0	2	$a_2 = 2$

Answer: $(175)_{10} = (a_2\ a_1\ a_0)_8 = (257)_8$

Example: $(0.3125)_{10}$

	Integer		Fraction	Coefficient
0.3125 * 8 =	2	.	5	$a_{-1} = 2$
0.5 * 8 =	4	.	0	$a_{-2} = 4$

Answer: $(0.3125)_{10} = (0.a_{-1}\ a_{-2}\ a_{-3})_8 = (0.24)_8$

Binary – Octal Conversion

- $8 = 2^3$
- Each group of 3 bits represents an octal digit

Example:

Assume Zeros

$$(\; 1\,0\,1\,1\,0\,.\,0\,1 \;)_2$$

$$(\; 2 \quad 6 \; . \; 2 \;)_8$$

Octal	Binary
0	0 0 0
1	0 0 1
2	0 1 0
3	0 1 1
4	1 0 0
5	1 0 1
6	1 1 0
7	1 1 1

Works both ways (Binary to Octal & Octal to Binary)

Hexadecimal (base 16)

- **{0 1 2 3 4 5 6 7 8 9 A B C D E F}**
- **Decimal to Hexadecimal equivalents shown in Table**

Dec	Hex	Dec	Hex
0	0	8	8
1	1	9	9
2	2	10	A
3	3	11	B
4	4	12	C
5	5	13	D
6	6	14	E
7	7	15	F

Example: Hexadecimal to Decimal

3 B 6 E

16^3 16^2 16^1 16^0

$3 \times 16^3 = 12288$

$11 \times 16^2 = 2816$

$6 \times 16^1 = 96$

$14 \times 16^0 = 14$

? **15214**

Hexadecimal to Binary Conversion

$16 = 2^4$

Each hexadecimal digit will be represented by 4 bits (nibble)

Binary	Hex	Binary	Hex
0000	0	1001	9
0001	1	1010	A
0010	**2**	1011	B
0011	3	1100	C
0100	**4**	1101	D
0101	5	1110	E
0110	6	1111	F
0111	7		
1000	**8**	⍰ Nibble	

Binary to Hex Conversion

- Group binary number by fours (nibbles)
  - 1101 1001 0110
- Convert each nibble into hex equivalent
  - 1101 1001 0110

    D 9 6

Binary to Hex Conversion

- $16 = 2^4$
- **Each group of 4 bits represents a hexadecimal digit**

Example:

Assume Zeros

$(\ 1\ 0\ 1\ 1\ 0\ .\ 0\ 1\ )_2$

$(\ 1\quad 6\ .\ 4\ )_{16}$

Hex	Binary
0	0 0 0 0
1	0 0 0 1
2	0 0 1 0
3	0 0 1 1
4	0 1 0 0
5	0 1 0 1
6	0 1 1 0
7	0 1 1 1
8	1 0 0 0
9	1 0 0 1
A	1 0 1 0
B	1 0 1 1
C	1 1 0 0
D	1 1 0 1
E	1 1 1 0
F	1 1 1 1

Works both ways (Binary to Hex & Hex to Binary)

Octal to Hex Conversion

- **Convert to Binary as an intermediate step**

Example:

$$(2\quad 6\quad .\quad 2)_8$$

Assume Zeros

$$(0\,1\,0\,1\,1\,0\,.\,0\,1\,0)_2$$

Assume Zeros

$$(1\quad 6\quad .\quad 4)_{16}$$

Works both ways (Octal to Hex & Hex to Octal)

Decimal to Hex Conversion

- Eg. 284
  - $16^2 = 256$ $284 - 256 = 28$
  - $16^1 = 16$ $28 - 16 = 12$ (Hex C)
  - Result: 1 1 C
  - **Repeated Division Approach:**

	Quotient	Remainder	Coefficient
284/ 16 =	17	12	$a_0 = C$
17/ 16 =	1	1	$a_1 = 1$
1/ 16 =	0	1	$a_2 = 1$

Answer: $(284)_{10} = (a_2\, a_1\, a_0)_{16} = (11C)_{16}$

  - Exercise: Try 1054
  - Result: 4 1 E

Hexadecimal to Decimal Conversion

- Convert hexadecimal number 2DB to a decimal number

Place Value	16^2	16^1	16^0
Hexadecimal	**2**	**D**	**B**
	$(16^2 \times 2)$	$(16^1 \times 13)$	$(16^0 \times 11)$

Decimal $\quad 512 + 208 + 11 \boxed{=} 731$

Decimal, Binary, Octal and Hexadecimal

Decimal	Binary	Octal	Hex
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Application Example

- ***ASCII*** (American Standard Code for Information Interchange)
  - Most widely used character code.
  - The eighth bit is often used for error detection (parity bit)
  - ***Example***: ASCII code representation of the word ***Digital***

Character	Binary Code	Hexadecimal Code
D	1000100	44
i	1101001	69
g	1100111	67
i	1101001	69
t	1110100	74
a	1100001	61
l	1101100	6C

Binary Coded Decimal (BCD) System

- The BCD system is used to represent each of the 10 decimal digits (0-9) as a 4-bit binary code.
  - 0 1 2 3 4 5 6 7 8 9
- To form a BCD number, simply convert each decimal digit to its 4-bit binary code.
  - Used to encode numbers for output to numerical displays
  - Used in processors that perform decimal arithmetic.
- Eg1. Convert 469_{ten} to BCD
- Answer:

$$(\,4 \quad 6 \quad 9\,)_{ten}$$

$$(\,0100 \quad 0110 \quad 1001\,)_{BCD}$$

Advantage of BCD to Binary Number System

- There is no limit to the size of a number in BCD.
  - To add another digit, you just need to add a new 4-bit sequence.
- In contrast, numbers represented in binary format are generally limited to the largest number that can be represented by 8, 16, 32 or 64 bits.
- Assignment:
  - What will be the disadvantage of using BCD system?

Example of BCD Representation

- Eg2. Convert 0110 0100 1011$_{BCD}$ to decimal
- Answer:

$$(\,0110 \quad 0100 \quad 1011\,)_{BCD}$$

$$(\,6 \quad 4 \quad \#\,)_{ten}$$

- # - not possible because 1011 is not a valid BCD
- Assignment:
  – Convert $(9750)_{10}$ to BCD

Summary

- Digital electronics use base-two (binary)
  - Because it only understands two states
  - ON and OFF
- Digital Systems operate on discrete digits – ON and OFF states
- Analog Systems operate on continuously varying electrical/physical magnitudes – temperature, pressure, velocity, etc
- Number conversions
  - Binary, Octal, Hexadecimal, BCD

Thank you