

# Estimating error rates for bullet comparisons in forensic science

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## Abstract

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It consists of two paragraphs.

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## Introduction and literature review

The firearm examiners are focusing on the same source and different source problems of bullets and cartridge cases which serve as important forensic evidence in the court/to the jurors. The subjectivity in the traditional forensic process is called to be reduced or complemented by more objective methods [The paper]. Some automatic matching algorithms are developed which usually return a similarity score (0-1) to quantify the similarity or the probability to be an actual match for a certain comparison. However, this raises questions about how to interpret the reported probabilities and how these probabilities are distributed. Thus, it is not all clear how to conduct inferences based on the similarity scores. [The paper] proposed binomial and beta-binomial for the number of matched cells of the CMC method. Thus, it provides a way to quantify the theoretical error rate of the algorithm. However, for the bullets LEA comparison, we haven't established similar distributional results. In this paper, we will evaluate the possible models/distributions for the LEA comparisons scores generated by the random forest proposed by Hare etc. And then, we will also evaluate the error rates based on the estimated distribution for the automatic matching algorithm.

In section 2, we will discuss proper distributional forms of the RF scores. In section 3, we will introduce the LAPD and the estimated distributions. In section 4, the theoretical error rates based on the distributions are discussed. In section 5, we evaluate the performance of the estimation, stability of the distribution within a changing sample size context. In section 6, we will conclude the discussion.

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## The distributional forms of the similarity scores

The quantitative methods used to objectively measure the similarity between LEAs reports various quantities, such as counts, correlations, distances, probabilities and more generally similarity scores references. To understand how those quantities reflect the strength of evidence and to study the underlying the error rates in making decisions based on those quantities, distributional forms are usually set up references. Particularly, we are focusing on the similarity scores which range from 0 to 1 and the probabilities reported as the likelihood of an actual match. The similarity scores reported in the forensic researches are usually categorized into two categories. One is that the compared LEAs or other forensic evidences are actual matches, the other is that the compared LEAs or other forensic evidences are not matches. We name the former as known matches (KM) and the latter as known non-matches (KNM), and the corresponding distributions are named as KM distributions and KNM distributions [figures]. However, this kind of classifications can be investigated only in the lab environment where the ground truth is known. When we make any decisions based on any quantitative measurement, we are actually making a distinguish between those two potential distributions. The strength of any identification process is also measured by the disparities of those distributions. However, in practice, we can hardly discriminate those two distributions totally, thus, we are never 100% sure which distribution the observed score comes from. This is where the identification error raises.

For the purpose of illustration, we have a look at the examples (could use Hamby and other sets, not necessarily LAPD) [figure(histogram?)]. Those are RF scores generated from the automatic matching algorithm references. The scores from the RF can be explained as probabilities that calculated through the algorithm based on the trained model to quantify the likelihood that a pair of LEAs are actual a match. Or we can think of the RF scores as general similarity scores that quantify the similarities. As the name indicated, the higher RF scores imply higher chances to be a match. For different combination of ammunition and firearms, the scores are distributed differently. It is expected that systematic differences exist there for different cases. So, it is necessary to study the scores under controlled conditions, thus, a threshold for the scores to do classifications varies based on the changes of the underlying distributions.

We can see from the figure [figure], which is a typical one we usually have for the scores, that the distributions are apart for the majorities. In the bullet LEA comparison, we usually have a well separated bullet scores but for the land scores, there are usually some overlaps [Susan]. We propose beta distributions for those scores. Because the beta distribution is a well-used one in statistics to describe a quantity from 0 to 1 which usually a probability for another distribution in Bayesian analysis. And it is very flexible to capture any unimodal shape in 0 to 1. However, it may not be adequate explain a very heavy tail or even a second mode. Thus, we also consider the beta mixture distribution which is a more complex distribution than the beta distribution. For the beta mixture distribution, we introduce a hierarchical structure with a prior probability to

combine two beta distributions as one. The distributions are stated as follows:  
(formal definition with notations, assumptions, conditions for beta and beta-mix distribution)

### LAPD data set and the estimated distributions

For the following sections of the paper, we will base our analysis on the LAPD data sets. This is a large data set of ... It is the first time such a large data set available to the researcher, which makes it possible for a statistical analysis for the distribution of the RF scores. (More!)

The estimation was done using EM algorithm implemented in “betareg” package in R [reference]. This is a numeric method for calculating maximum likelihood estimators (MLE) when direct computation is not available or hard. Instead of a simpler beta model, we start with the more complex beta mixture model, see how well it fits the data and test if some components are necessary. We estimate two-component beta mixture distribution for both KNM and KM distributions as follows:

(table: report alpha, beta, mean, variance)

(figure: the estimated curve with observations drawn as rugs or boxplots in the bottom)

The estimated distributions are also shown in the figure [figure] with rugs in the bottom shown as the actual observations. As expected, the estimated distributions show the properties we desire for the similarity scores. The majorities of the estimated distributions are apart from each other, while the minority part between the two distributions has some overlap. It’s worth to note that (how the actual observations are distributed compared with the curves). The distributions usually have longer tails than points where the observations show up but with small probabilities. And the shapes of the distributions are ideal and potentially flexible enough to capture the empirical distributional information of the observations.

It’s also helpful to see how the individual components look like in the beta mixture distributions [figure]. The KNM and KM distributions seem to share a common component while keep the other components far apart from each other. The separated components represent the ideal cases where the bullet land engraved areas preserved the information of source well and result clear distinct separation. The shared components represent the cases where the KM results lower scores because of some degree of tank rash, pitting, breakoff or other damages on the bullets and the KNM results higher score because of the random identification effect. These properties together well explained the observed empirical distribution of similarity scores in our cases. Particularly, the heavier tail of the KM comparisons is explicitly included the form of the model by one of the components.

We need to do formal statistical tests to verify some of the points mentioned before. We would like to know if simpler beta distributions are sufficient to model the data. We would also quantify the variability of estimations of the distribution through parametric bootstrap.

(test, bootstrap inference, and report the model for the full data)

### **Evaluate the error rates**

(report theoretical error rates)

### **Estimations with changing sample size**

(quantify the variation, reproduce some results in the previous sections with changing sample size)

### **Conclusion**

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Here are two sample references: Feynman and Vernon Jr. (1963; Dirac, 1953).

## References

Dirac, P.A.M., 1953. The lorentz transformation and absolute time. *Physica* 19, 888–896. doi:10.1016/S0031-8914(53)80099-6

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