Homework 5

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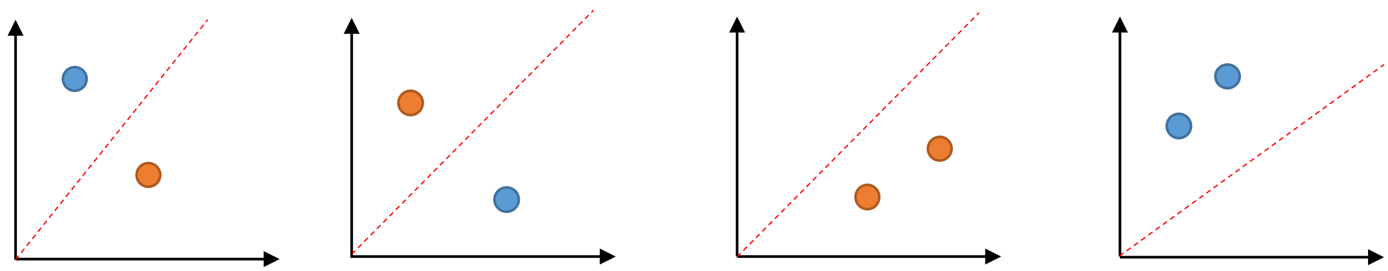
1.
$\epsilon = 0.15,\ \delta = 0.05,\ |H| = C^3_{100\times100} = 166616670000$
$\because m \geq \frac{1}{\epsilon}(\ln|H| + ln\frac{1}{\delta})$
$\therefore m \geq 193$

2.
Define:

$$h(\boldsymbol{x}) = \begin{cases} +1 & if\ a \leq \arctan\left(\frac{x_j}{x_i}\right) \leq b \\ -1 & otherwise \end{cases}$$

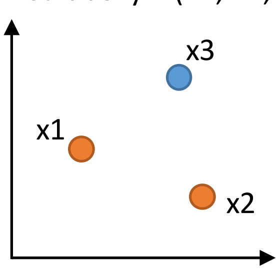
Here: $\boldsymbol{x}$ are 2D points ($\boldsymbol{i} = \mathbf{1}, \boldsymbol{j} = \mathbf{2}$), and $a = 0°\ \boldsymbol{or}\ b = 90°$ acting as lower bound or upper bound, another bound is $\mathbf{arctan}(\boldsymbol{k})$, k is the slope of the linear hypoplane ($\boldsymbol{y} = \boldsymbol{kx}$).
(1) lower bound: $VCdim(\boldsymbol{H}) \geq 2$



(2) upper bound: $VCdim(\boldsymbol{H}) < 3$
Let $\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}$ be arbitrary points s.t.

$$\frac{x_{12}}{x_{11}} > \frac{x_{32}}{x_{31}} > \frac{x_{22}}{x_{21}}$$

as shown in the picture below.
Let label y = (+1, +1, -1) and assume some h = [a, b] works.



since $y_1 = +1$, must have $a \leq \arctan\left(\frac{x_{12}}{x_{11}}\right) \leq b$
since $y_2 = +1$, must have $a \leq acrtan\left(\frac{x_{22}}{x_{21}}\right) \leq b$
As $\boldsymbol{y} = \mathbf{arctan}\ (\boldsymbol{x})$ is an incresing function, then according to the statement, we should have:

$$a \leq acrtan\left(\frac{x_{22}}{x_{21}}\right) < acrtan\left(\frac{x_{32}}{x_{31}}\right) < \arctan\left(\frac{x_{12}}{x_{11}}\right) \leq b$$

But $y_3 = -1$ which is a contradiction.

Therefore, $VCdim(\boldsymbol{H}) = 2$

3.

Technical solution:

Main idea: $\sin(x) = -\sin(x+\pi)$, in this way, if we want to change positive $\sin(x)$ to negative one, we just need to add a π

My solution:

(1) set the initial $w = \pi$

(2) sort $\boldsymbol{k}$**:** $\{k_1, k_2, k_3 \dots k_m\}$ **in an increasing order**, $k_1 < k_2 < k_3 \dots < k_m$

(3) loop from k_1 to k_m:

$$h_w(x) = \begin{cases} True & if \sin(wx) \ge 0 \\ False & if \sin(wx) < 0 \end{cases}$$

if $h_w(x_i)$ is equal to $label(x_i)$, which means the classification is right, then, we do not change w
else if $h_w(x_i)$ is not equal to $label(x_i)$, which means the classification is wrong, then, we will **update** $\boldsymbol{w}$

$\because wx_i + \pi = w_{update}x_i$ and $x_i = 2^{-k_i}$
$\therefore \boldsymbol{w_{update} = w + 2^{k_i}\pi}$

in this way, we can make sure that with $\boldsymbol{w_{update}}$, $h_w(x_i)$ is correctly classified.

(4) we should check, if we update $\boldsymbol{w}$, will the samples previous being checked stay correct?
The answer is **Yes**.
Previous checked samples $\{x_1, x_2 \dots x_{i-1}\}$, as we have sorted $\boldsymbol{k}$ in an increasing order, that means:
$k_1 < \dots < k_{i-1} < k_i$
in this way, if we update $\boldsymbol{w_{update} = w + 2^{k_i}\pi}$
they will have $\boldsymbol{sin(w_{update}2^{-k_j}) = sin(w2^{-k_j} + 2^{k_i - k_j}\pi)}$
since $k_j < k_i$, then $\boldsymbol{2^{k_i - k_j}\pi = 2n\pi}$, n is integer
thus, $\boldsymbol{sin(w_{update}2^{-k_j}) = sin(w2^{-k_j})}$, indicating that the classfication of $\{x_1, x_2 \dots x_{i-1}\}$ will not change

(5) after finishing the loop, we finish updating $\boldsymbol{w}$.
at last, only **the negative values** get w updated, which means:

$$\boldsymbol{w = \pi(1 + \sum_{i=negative\_1}^{negative\_m} 2^{k_i})}$$

$\boldsymbol{negative\_1}$ means $\boldsymbol{k_i}$ corresponding to the first negative value with $label = False$
$\boldsymbol{negative\_m}$ means $\boldsymbol{k_i}$ corresponding to the last negative value with $label = False$

(6) Note: if we change the initial value of $\boldsymbol{w}$ between 0 and π, we will get a range of $\boldsymbol{w}$

$$\boldsymbol{w \in (\sum_{i=negative_1}^{negative_m} 2^{k_i}\pi, \pi(1 + \sum_{i=negative\_1}^{negative\_m} 2^{k_i})]}$$