ψ
$$\frac{4^{\frac{1}{n}}\, \rho\, \sqrt{\pi}\, \left(1+\rho^{2n}-2\,\rho^n\, \text{Cos}[n\,\varphi]\right)^{-1/n}\, \text{Log}[\rho]^4}{\Gamma\left[\frac{1}{2}-\frac{1}{n}\right]\, \Gamma\left[1+\frac{1}{n}\right]}$$

$\partial\psi/\partial r$
$$-\frac{4^{\frac{1}{n}}\, \sqrt{\pi}\, \left(1+\rho^{2n}-2\,\rho^n\, \text{Cos}[n\,\varphi]\right)^{-\frac{1+n}{n}}\, \left(-4+8\,\rho^n\, \text{Cos}[n\,\varphi]+\rho^{2n}\, \left(-4+\text{Log}[\rho]\right)-\text{Log}[\rho]\right)\, \text{Log}[\rho]^3}{\Gamma\left[\frac{1}{2}-\frac{1}{n}\right]\, \Gamma\left[1+\frac{1}{n}\right]}$$