Recall that after scaling, the line source solution was given by $T = -\log \rho$,(*) which vanishes on the unit circle $\rho = 1$. Here we use conformal mapping to obtain the line source solution which vanishes on the boundary of a regular polygon instead of a circle, and perform boundary tracing on that.

1. Schwarz-Christoffel mapping

Consider mapping the unit disk in ζ-space onto a polygon in z-space, sending the points $\zeta_1, \zeta_2, \ldots, \zeta_n$ on the unit circle unto the vertices $z_1, z_2, \ldots, z_n$ of the polygon with interior angles $\alpha_1\pi, \alpha_2\pi, \ldots, \alpha_n\pi$, respectively.

The conformal map which achieves this is given by the Schwarz-Christoffel formula

$$\frac{dz}{d\zeta} = c\left(1-\frac{\zeta}{\zeta_1}\right)^{\alpha_1-1}\left(1-\frac{\zeta}{\zeta_2}\right)^{\alpha_2-1}\cdots\left(1-\frac{\zeta}{\zeta_n}\right)^{\alpha_n-1}, \qquad (r4.1)$$

where c is a constant which ~~controls~~ determines dilation and rotation, and the constant of integration determines translation.

Note that

$$\alpha_1 + \alpha_2 + \cdots + \alpha_n = n-2, \qquad (r4.2)$$

since the vertices $z_1, z_2, \ldots, z_n$ form an n-gon.

(*) Here we use ρ rather than r, as ρ shall be the radius $\rho = |\zeta|$ for the complex coordinate ζ.

In particular, for a regular polygon in z-space, it follows by symmetry that the ζ_k must be the n-th roots of unity,

$$\zeta_k = e^{i2\pi k/n} \quad , \qquad k = 1, 2, \ldots, n. \qquad \text{(r4.3)}$$

Also the α_k must all be equal, so from (r4.2) we have

$$\alpha_k = 1 - \frac{2}{n} \quad , \qquad k = 1, 2, \ldots, n \qquad \text{(r4.4)}$$

whence the Schwarz-Christoffel formula (r4.1) reduces to

$$\frac{dz}{d\zeta} = c\left(1 - \frac{\zeta}{\zeta_1}\right)^{-2/n}\left(1 - \frac{\zeta}{\zeta_2}\right)^{-2/n} \cdots \left(1 - \frac{\zeta}{\zeta_n}\right)^{-2/n} \qquad \text{(r4.5)}$$

$$\boxed{\frac{dz}{d\zeta} = c\left(1 - \zeta^n\right)^{-2/n}} \qquad \text{(r4.6)}$$

Integrating, we obtain

$$\boxed{z = c\zeta \cdot H_n(\zeta)} \quad , \qquad \text{(r4.7)}$$

where

$$\boxed{H_n(\zeta) = {}_2F_1\left(\frac{1}{n}, \frac{2}{n}; 1 + \frac{1}{n}; \zeta^n\right)} \qquad \text{(r4.8)}$$

and we have taken $z = 0$ at $\zeta = 0$. To make the z_k lie on the unit circle in z-space (and ~~hence~~ be n-th roots of unity also), we take

$$c = \frac{1}{H_n(1)} \qquad \text{(r4.9)}$$

so that

$$\boxed{z = \zeta \cdot \frac{H_n(\zeta)}{H_n(1)}} \quad . \qquad \text{(r4.10)}$$

1.1 Summary

The mapping which takes the unit disk (ζ-space) onto the regular polygon of unit circumradius (z-space), sending $\zeta_k = e^{i2\pi k/n}$ to $z_k = e^{i2\pi k/n}$, is given by

$$\boxed{\frac{dz}{d\zeta} = \frac{1}{H_n(1)(1-\zeta^n)^{2/n}}} \qquad (r4.11)$$

or

$$\boxed{z = \zeta \cdot \frac{H_n(\zeta)}{H_n(1)}} \qquad (r4.12)$$

Now (r4.12) probably cannot be inverted analytically, but the inverse map $\zeta = \zeta(z)$ can be computed by either viewing (r4.12) as a trancendental equation in ζ and solving it numerically, or by inverting (r4.11) to obtain

$$\boxed{\frac{d\zeta}{dz} = H_n(1)(1-\zeta^n)^{2/n}} \qquad (r4.13)$$

and integrating forward from $\zeta(z=0) = 0$, along the straight line from 0 to the desired z.

2. Line source solution in a polygon

Observe that the line source solution $T = -\log \rho$ arises from taking the real part of $-\log \zeta$, and vanishes on the unit circle $\rho = |\zeta| = 1$.

We effect the inverse map $\zeta = \zeta(z)$, and define

$$\boxed{G(z) = -\log \zeta(z) = T + iV}, \qquad (r4.14)$$

with

$$\boxed{z = x + iy}; \qquad (r4.15)$$

it then follows that

$$\boxed{T = \mathrm{Re}\{G\}} \qquad (r4.16)$$

will be the solution to Laplace's equation (r7), which vanishes on the regular n-gon of circumradius unity, and corresponds to a line source at the origin.

3. Boundary tracing

For simplicity we shall ~~perform~~ consider boundary tracing in Cartesian coordinates (x, y). We have $h_u = h_v = 1$, and

$$du = dx \tag{r4.17}$$

$$dv = dy \tag{r4.18}$$

$$P = \frac{\partial T}{\partial x} = \mathrm{Re}\left\{\frac{\partial G}{\partial x}\right\} = \mathrm{Re}\left\{\frac{dG}{dz}\right\} \tag{r4.19}$$

$$\begin{aligned} Q = \frac{\partial T}{\partial y} &= \mathrm{Re}\left\{\frac{\partial G}{\partial y}\right\} = \mathrm{Re}\left\{\frac{dG}{dz}\cdot i\right\} \\ &= -\mathrm{Im}\left\{\frac{dG}{dz}\right\} \end{aligned} \tag{r4.20}$$

$$F = -\frac{T^4}{A} = -\frac{\mathrm{Re}^4\{G\}}{A} \tag{r4.21}$$

$$\Phi = P^2 + Q^2 - F^2$$

$$\boxed{\Phi = \left|\frac{dG}{dz}\right|^2 - \frac{\mathrm{Re}^8\{G\}}{A^2}}, \tag{r4.22}$$

where

$$\boxed{\frac{dG}{dz} = \frac{-1}{\zeta(z)}\frac{d\zeta}{dz} = -H_n(1)\frac{\left(1-\zeta^n(z)\right)^{2/n}}{\zeta(z)}}. \tag{r4.23}$$

Define

$$\boxed{\psi = \mathrm{Re}^4\{G\} \Big/ \left|\frac{dG}{dz}\right|}. \tag{r4.24}$$

Then

$$\Phi = \frac{1}{A^2}\left|\frac{dG}{dz}\right|^2\left(A^2 - \psi^2\right). \tag{r4.25}$$

Note that

$$(\vec{\nabla} T)^2 = P^2 + Q^2 = \left|\frac{dG}{dz}\right|^2. \tag{r4.26}$$

3.1 Viable domain

From (r4.25), the viable domain is the region

$$\psi = \mathrm{Re}^4\{G\}/\left|\frac{d\theta}{dz}\right| \le A, \qquad (r4.27)$$

or

$$\psi = \frac{\mathrm{Re}^4\{-\log\zeta\}\cdot|\zeta|}{\left|H_n(1)\,(1-\zeta^n)^{2/n}\right|} \le A\,.$$

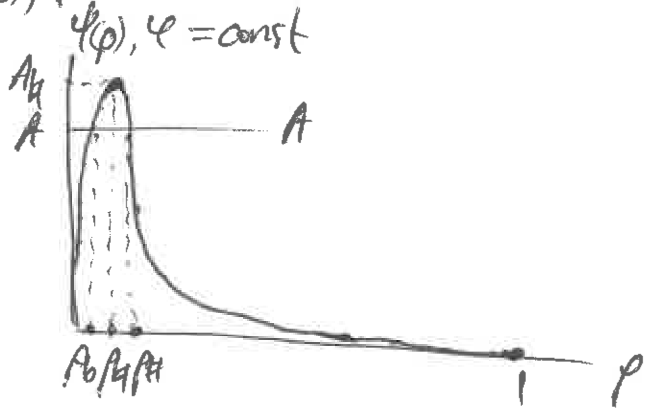
whilst it is expensive to visualise this in z-space, it is not so in ζ-space. Let

$$\boxed{\zeta = \rho e^{i\varphi}} \qquad (r4.28)$$

After some algebra, we obtain

$$\psi = \frac{4^{1/n}\sqrt{\pi}}{\Gamma\left(\frac{1}{2}-\frac{1}{n}\right)\Gamma\left(1+\frac{1}{n}\right)}\cdot\rho\log^4\rho\cdot\left[1+\rho^{2n}-2\rho^n\cos(n\varphi)\right]^{-1/n}, \qquad (r4.29)$$

and plotting these, we find that along any direction $\varphi = \text{const}$, $\psi = \psi(\rho)$ is qualitatively the same as its namesake in the line source case (see equation (r2.16)).



Writing A_H for max $\psi(\rho)$ at $\rho = \rho_H$, the entire ray $0<\rho<1$ is viable if $A \ge A_H$, but if $A < A_H$ then only the subintervals $0<\rho<\rho_0$ and $\rho_A < \rho < 1$ are viable.

Of course here A_H and ρ_H (and ρ_0 & ρ_A for any given $A \le A_H$) all depend on φ, but we find this dependence to be almost ~~negligible~~ unnoticeable.

Computing $\partial\psi/\partial\rho$ and setting it to zero, we find that $\rho_4 = \rho_4(\varphi)$ is given by the solution (on $0<\rho<1$) to

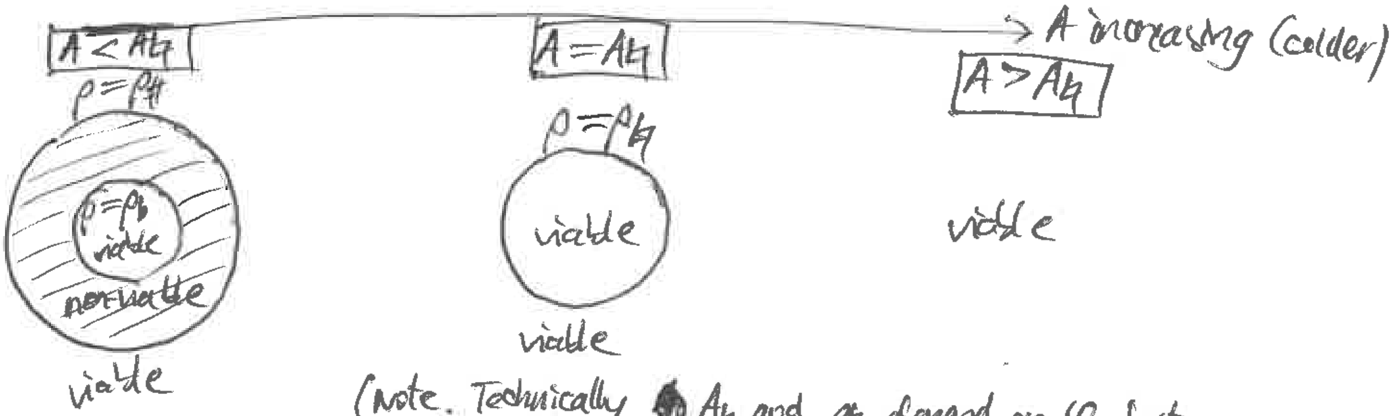
$$-4+8\rho^n\cos(n\varphi)+\rho^{2n}(-4+\log\rho)-\log\rho = 0. \qquad (r4.30)$$

Even for $n=3$, we find that

$$\frac{\max\rho_4}{\min\rho_4}-1 = \frac{\rho_4(0)}{\rho_4(\pi/3)} = 0.98\times 10^{-4},$$

and as n increases, this falls even lower. Thus, although ψ does depend upon φ, this dependence is negligible.

Therefore, in ζ-space, we ~~ag~~ have the following, very similar to the situation for the circular line source:



(Note. Technically A_4 and ρ_4 depend on φ, but for all practical purposes this can be ignored).

Note. The inner island $\rho\le\rho_b$ is so close to the origin, the traced boundaries there are effectively those for the line-source.

3.2 Tracing system of ODEs

We seek traced boundaries parametrised by arc length, $x=x(s)$, $y=y(s)$. From (48) & (49) (see 4.4 Arc length parametrisation) the tracing system of ODEs is

$$\frac{dx}{ds} = \frac{-QF \pm P\sqrt{\Phi}}{(\vec{\nabla}T)^2} \qquad (r4.31)$$

$$\frac{dy}{ds} = \frac{PF \pm Q\sqrt{\Phi}}{(\vec{\nabla}T)^2}, \qquad (r4.32)$$

but perhaps it would be advantageous to write this in terms of $z=x+iy$:

$$\frac{dz}{ds} = \frac{d(x+iy)}{ds}$$

$$= \frac{(-Q+iP)F \pm (P+iQ)\sqrt{\Phi}}{(\vec{\nabla}T)^2}. \qquad (r4.33)$$

Now

$$-Q+iP = \mathrm{Im}\left\{\frac{dG}{dz}\right\} + i\,\mathrm{Re}\left\{\frac{dG}{dz}\right\}$$

$$= i\left(\mathrm{Re}\left\{\frac{dG}{dz}\right\} - i\,\mathrm{Im}\left\{\frac{dG}{dz}\right\}\right)$$

$$= i\left(\frac{dG}{dz}\right)^*, \qquad (r4.34)$$

$$P+iQ = \mathrm{Re}\left\{\frac{dG}{dz}\right\} - i\,\mathrm{Im}\left\{\frac{dG}{dz}\right\}$$

$$= \left(\frac{dG}{dz}\right)^* \qquad (r4.35)$$

and

$$P^2+Q^2 = \left|\frac{dG}{dz}\right|^2$$

$$(\vec{\nabla}T)^2 = \left(\frac{dG}{dz}\right)\left(\frac{dG}{dz}\right)^*; \qquad (r4.36)$$

therefore (r4.33) becomes

$$\frac{dz}{ds} = \frac{i\left(\frac{dG}{dz}\right)^* F \pm \left(\frac{dG}{dz}\right)^* \sqrt{\Phi}}{\left(\frac{dG}{dz}\right)\left(\frac{dG}{dz}\right)^*}$$

$$\boxed{\frac{dz}{ds} = \frac{iF \pm \sqrt{\Phi}}{dG/dz}} \quad . \qquad (r4.37)$$

Now the right hand side is a known function of ζ, but since the inverse map $\zeta = \zeta(z)$ is very expensive to compute, (r4.37) is not quite suitable for boundary tracing as yet. But, multiplying both sides by $d\zeta/dz$ (see (r4.13)), we have

$$\frac{dz}{ds} \cdot \frac{d\zeta}{dz} = \frac{iF \pm \sqrt{\Phi}}{dG/dz} \cdot \frac{d\zeta}{dz}$$

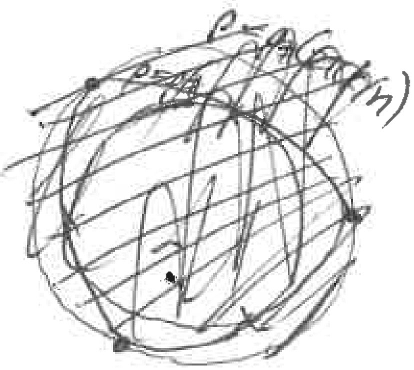
$$\boxed{\frac{d\zeta}{ds} = \frac{iF \pm \sqrt{\Phi}}{dG/d\zeta}} \quad , \qquad (r4.38)$$

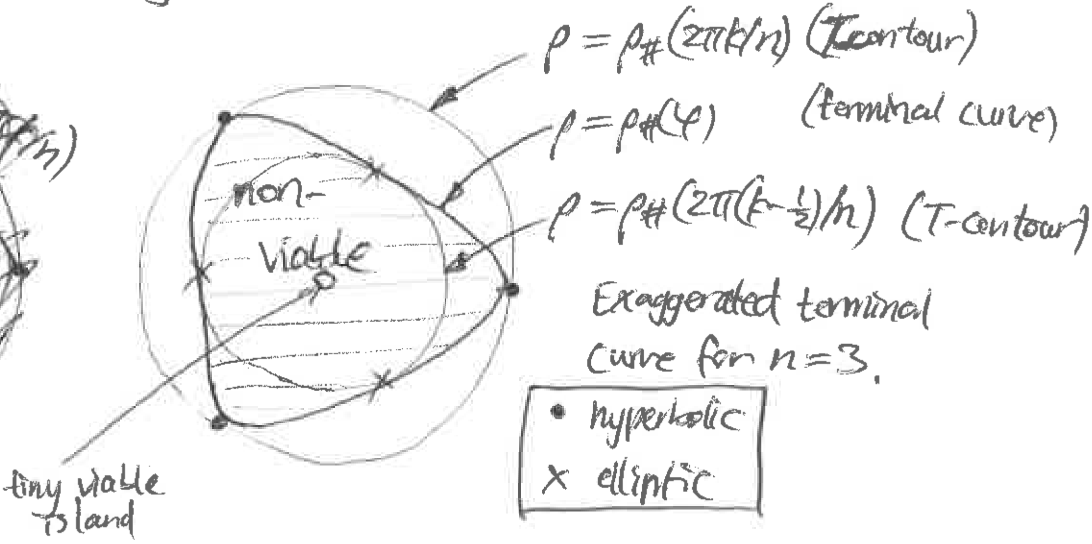
and still the right hand side is a known function of ζ, but now we have an ODE for traced boundaries $\zeta = \zeta(s)$ in ζ-space. Thus we avoid ever having to compute ~~bound~~ the inverse map $\zeta = \zeta(z)$, since boundary tracing may now be done without leaving ζ-space, by using (r4.38); and when we are done, we only need to use the forward map $z = z(\zeta)$, or (r4.12), which is easy to compute, to bring the traced boundaries into physical z-space.

3.3 Constructing domains

Although the outer terminal curve $\rho = \rho_\#$ is practically indistinguishable from a circle in ζ-space, the slight φ-dependence has theoretical consequences. In fact $\rho_\#(\varphi)$ is maximal ~~along~~ at $\varphi = 2\pi k/n$ (k integer) and minimal at $\varphi = 2\pi(k-1/2)/n$; since the T-contours in ζ-space are simply circles $|\zeta| = \rho = \text{const}$, it follows that along the ~~the~~ outer terminal curve $\rho = \rho_\#$:

- $\varphi = 2\pi k/n$ are hyperbolic critical terminal points
- $\varphi = 2\pi(k-1/2)/n$ are elliptic critical terminal points
- All other φ are ordinary terminal points.





Exaggerated terminal curve for $n=3$.

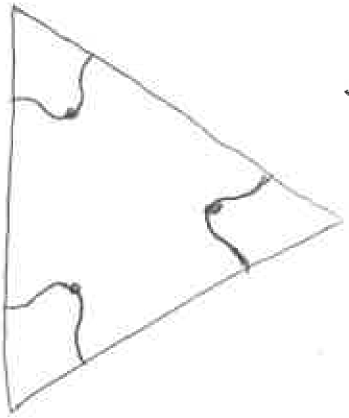
Using similar arguments to those before, the only possibly convex domain enclosing $\zeta = 0$ constructible from ~~[illegible]~~ patching together traced boundaries must pass through the n hyperbolic critical terminal points along $\rho = \rho_\#(\varphi)$ at $\varphi = 2\pi k/n$, k integer.

~~This yields~~

Thus we have n curves in ζ-space, which, using the forward map $z = z(\zeta)$, may be easily brought into z-space, whereafter they are referred to as the candidate curves.

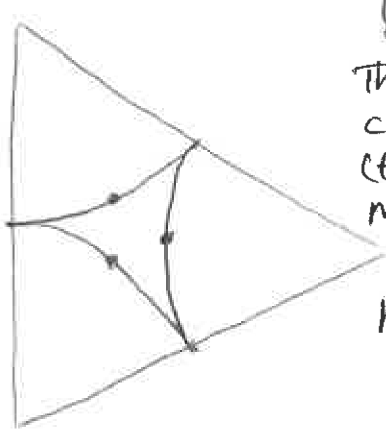
We find that there exist A_m and A_i such that:

$\boxed{0 < A < A_m}$



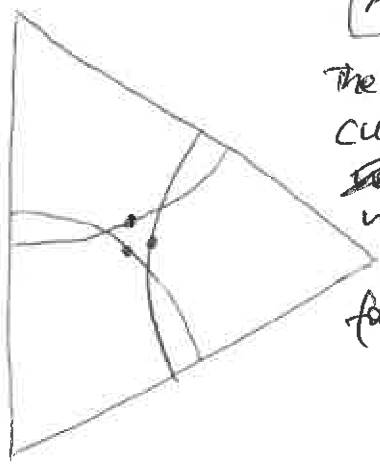
The n candidate curves never intersect with each other

$\boxed{A = A_m}$



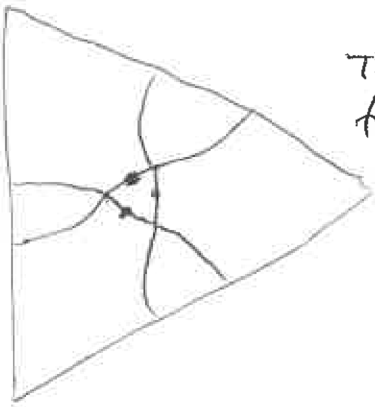
The n candidate curves meet (touch) at the midpoints of the edges of the polygon (i.e. $\varphi = \pi/n$)

$\boxed{A_m < A < A_i}$



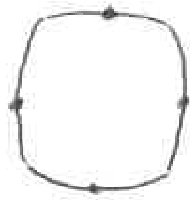
The n candidate curves intersect ~~at~~ ~~[illegible]~~ strictly within the n-gon, but the n-gon-like domain formed is not convex

$\boxed{A = A_i}$



The n-gon-like domain formed becomes convex.

Thus for $A_i \leq A < A_h$ we can produce n-gon-like domains:



To determine A_i, we seek the A at which the ~~vertices~~ corners (at $\varphi = 2\pi(k-1/2)/n$) of the n-gon-like domain are points of inflection. A bit of effort is required to compute the curvature, since F, Φ and dG/dz in the right hand side of (r4.37) involve Re and $|\cdot|^2$ which are not complex differentiable.

Now recall that a curve $z = z(s) = x(s) + iy(s)$ in the complex plane corresponds to the physical curve $\vec{r}(s) = x(s)\,\vec{a}_x + y(s)\,\vec{a}_y$. With primes denoting s differentiation, the signed curvature is

$$
\begin{aligned}
&\vec{a}_z \cdot \vec{r}\,' \times \vec{r}\,'' \\
&= \vec{a}_z \cdot (x'\vec{a}_x + y'\vec{a}_y) \times (x''\vec{a}_x + y''\vec{a}_y) \\
&= x'y'' - y'x'', \qquad \text{(r4.39a)}
\end{aligned}
$$

where $(x', y') = (\mathrm{Re}, \mathrm{Im})(z')$ and $(x'', y'') = (\mathrm{Re}, \mathrm{Im})(z'')$.
Given a traced boundary $\zeta = \zeta(s)$ obtained by numerical integration of (r4.38), z' is known from (r4.37),

$$
z' = \frac{iF \pm \sqrt{\Phi}}{dG/dz} \qquad \text{(r4.39)}
$$

which is a function of ζ (see (r4.14), (r4.21), (r4.22), (r4.23)). But this function of ζ is not an analytic one, since it involves Re and $|\cdot|^2$, so z'' cannot be ~~directly~~ obtained by applying directly applying d/ds unto the whole expression in the form $(d\zeta/ds)\cdot d/d\zeta$. Only the denominator dG/dz is analytic in ζ, and we must proceed carefully for the numerator:

$$
\begin{aligned}
z'' &= (iF \pm \sqrt{\Phi})\left(\frac{1}{dG/dz}\right)' + \frac{1}{dG/dz}(iF \pm \sqrt{\Phi})' \\
&= (iF \pm \sqrt{\Phi})\left(\frac{d\zeta}{ds}\right)\frac{d}{d\zeta}\left(\frac{1}{dG/dz}\right) + \frac{iF' \pm (\sqrt{\Phi})'}{dG/dz} \qquad \text{(r4.40)}
\end{aligned}
$$

Now

$$
\begin{aligned}
F' &= \frac{dF}{ds} \\
&= \frac{d}{ds}\left(-\frac{1}{A}\,\mathrm{Re}^4\{G\}\right) \\
&= -\frac{1}{A}\cdot 4\,\mathrm{Re}^3\{G\}\,\frac{d}{ds}\mathrm{Re}\{G\} \\
&= -\frac{1}{A}\cdot 4\,\mathrm{Re}^3\{G\}\,\mathrm{Re}\left\{\frac{dG}{ds}\right\}
\end{aligned}
$$

or
$$F' = -\frac{4\,\mathrm{Re}^3\{G\}}{A}\,\mathrm{Re}\left\{\frac{d\varsigma}{ds}\frac{dG}{d\varsigma}\right\}, \qquad (r4.41)$$
and
$$\left(\sqrt{\Phi}\right)' = \frac{1}{2\sqrt{\Phi}}\cdot\Phi' \qquad (r4.42)$$
where
$$\begin{aligned}
\Phi' &= \frac{d}{ds}\left(\left|\frac{dG}{dz}\right|^2 - \frac{\mathrm{Re}^8\{G\}}{A^2}\right)\\
&= \frac{d}{ds}\left(\mathrm{Re}^2\left\{\frac{dG}{dz}\right\} + \mathrm{Im}^2\left\{\frac{dG}{dz}\right\} - \frac{1}{A^2}\mathrm{Re}^8\{G\}\right)\\
&= 2\,\mathrm{Re}\left\{\frac{dG}{dz}\right\}\frac{d}{ds}\mathrm{Re}\left\{\frac{dG}{dz}\right\} + 2\,\mathrm{Im}\left\{\frac{dG}{dz}\right\}\frac{d}{ds}\mathrm{Im}\left\{\frac{dG}{dz}\right\} - \frac{8\,\mathrm{Re}^7\{G\}}{A^2}\frac{d}{ds}\mathrm{Re}\{G\}\\
&= 2\,\mathrm{Re}\left\{\frac{dG}{dz}\right\}\mathrm{Re}\left\{\frac{d}{ds}\left(\frac{dG}{dz}\right)\right\} + 2\,\mathrm{Im}\left\{\frac{dG}{dz}\right\}\mathrm{Im}\left\{\frac{d}{ds}\left(\frac{dG}{dz}\right)\right\} - \frac{8\,\mathrm{Re}^7\{G\}}{A^2}\mathrm{Re}\left\{\frac{dG}{ds}\right\}\\
&= 2\,\mathrm{Re}\left\{\frac{dG}{dz}\right\}\mathrm{Re}\left\{\frac{d\varsigma}{ds}\frac{d}{d\varsigma}\left(\frac{dG}{dz}\right)\right\} + 2\,\mathrm{Im}\left\{\frac{dG}{dz}\right\}\mathrm{Im}\left\{\frac{d\varsigma}{ds}\frac{d}{d\varsigma}\left(\frac{dG}{dz}\right)\right\} - \frac{8\,\mathrm{Re}^7\{G\}}{A^2}\mathrm{Re}\left\{\frac{d\varsigma}{ds}\frac{dG}{d\varsigma}\right\}.
\end{aligned} \qquad (r4.43)$$

Thus may we evaluate (r4.40).

Using the bisection algorithm where appropriate, we ~~find that~~ obtain the following values of A:

n	A_m	A_i	A_h
3	1.18154	1.74256	2.65409
4	1.03501	2.08796	3.57643
5	0.770825	2.17300	3.99234

4. Summary

With scaling as per the line source case, consider the complex function

$$G = -\log \zeta = T + iV \qquad (r4.44)$$

in $\zeta = \rho e^{i\theta}$-space, whose real part

$$T = \mathrm{Re}\{G\} = -\log \rho \qquad (r4.45)$$

corresponds to the real line source solution which vanishes on the unit circle $|\zeta| = \rho = 1$ in ζ-space. ~~A Stw~~ The Schwarz-Christoffel mapping

$$\frac{dz}{d\zeta} = \frac{1}{H_n(1)(1-\zeta^n)^{2/n}} \qquad (r4.46)$$

sends the unit disk (ζ-space) onto the regular ~~polygon~~ n-gon of unit circumradius ($z = x+iy$-space), and effecting the inverse map $\zeta = \zeta(z)$ we obtain the solution

$$T = T(x,y) \qquad (r4.47)$$

to Laplace's equation (r7) which vanishes on the boundary of the polygon in z-space. Boundary tracing says that ~~using~~ with (r4.47) as the known temperature profile, convex n-gon-like ~~radiation~~ domains ~~are~~ corresponding to radiating boundaries may be constructed for $A_i \le A < A_H$ (see Section 3, Page r4-4) where

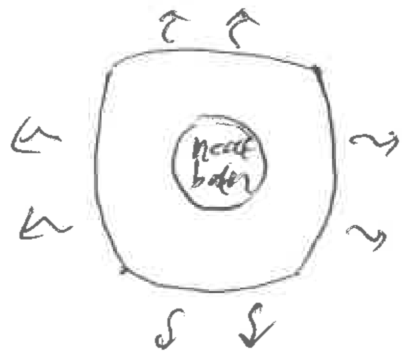
$$A = \frac{1}{c r \sigma T_0^3} \qquad (r4.48)$$

is dimensionless.

4.1 A hypothetical situation

The domains constructed will admit the solution (r4.47) to the boundary value problem (r7) & (r8). The heat may be generated by heat bath for any ~~any~~ curve in z-space which is the image under (r4.46) of a circle in ζ-space of radius ρ_* at temperature T_* (both in scaled terms) satisfying

$$T_* = -\log \rho_* . \qquad (r4.49)$$



5. Temperature offset

While Laplace's equation (r7) is invariant with respect to uniform temperature offset, the radiation condition (r8) is affected non-trivially. Consider, instead of (r4.44),

$$G = \gamma - \log \zeta = T + iV, \tag{r4.50}$$

where γ is real and dimensionless, whose real part is

$$T = \mathrm{Re}\{G\} = \gamma - \log \rho. \tag{r4.51}$$

In dimensional terms this amounts to beginning with the line source solution to Laplace's equation with $T = \gamma T_0$ (rather than zero) on $\rho = \rho_0$, so $T = 0$ at the larger radius $\rho = e^{\gamma}\rho_0$. After applying the Schwarz-Christoffel transformation as before, this means that the known solution has ~~temperature~~ dimensional temperature γT_0 (and scaled temperature γ) on the boundary of the regular n-gon, rather than zero.

5.1 Viable domain

Proceeding as before, the viable domain is the region

$$\psi = \mathrm{Re}^4\{G\} \Big/ \left|\frac{dG}{dz}\right| \le A \tag{r4.52}$$

or

$$\psi = \frac{\mathrm{Re}^4\{\gamma - \log\zeta\}\cdot|\zeta|}{\left|H_n(1)(1-\zeta^n)^{2/n}\right|} \le A. \tag{r4.53}$$

After some algebra,

$$\psi = \frac{4^{1/n}\sqrt{\pi}}{\Gamma(\frac{1}{2}-\frac{1}{n})\Gamma(1+\frac{1}{n})}\cdot \rho\,(\gamma - \log\rho)^4 \cdot \left[1+\rho^{2n} - 2\rho^n\cos(n\varphi)\right]^{-1/n} \tag{r4.54}$$

(compare with (r4.29)). The introduction of $\gamma > 0$ has interesting effects. Observe that along $\varphi = 2\pi k/n$, we now have

$$\psi = \mathrm{const}\cdot\frac{\rho\,(\gamma-\log\rho)^4}{[1-\rho^n]^{2/n}}. \tag{r4.55}$$

Putting $\rho = 1+\xi$ for a perturbation, this becomes

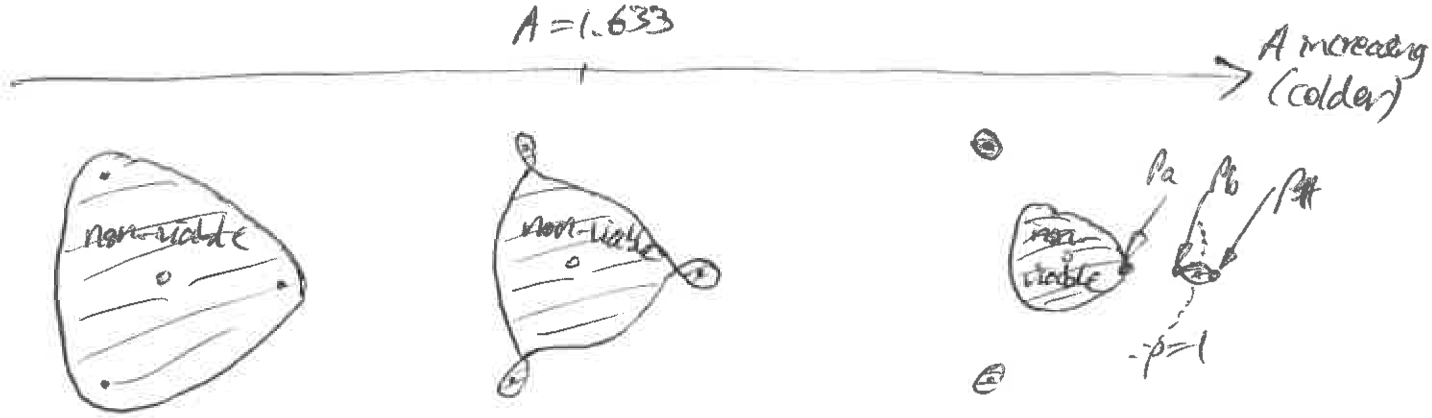
$$\psi = \text{const} \cdot \frac{(1+\xi)(\gamma - \xi + O(\xi^2))^4}{(1-[1+n\xi+O(\xi^2)])^{2/n}} \qquad (r4.56)$$

$$\sim \text{const} \cdot \frac{(\gamma-\xi)^4}{|\xi|^{2/n}} \qquad (r4.57)$$

and we see that whenever $\gamma > 0$, ψ will have a pole at $\xi = 0$, i.e. $\rho = 1$. (Only if $\gamma = 0$ does ψ remain finite.)

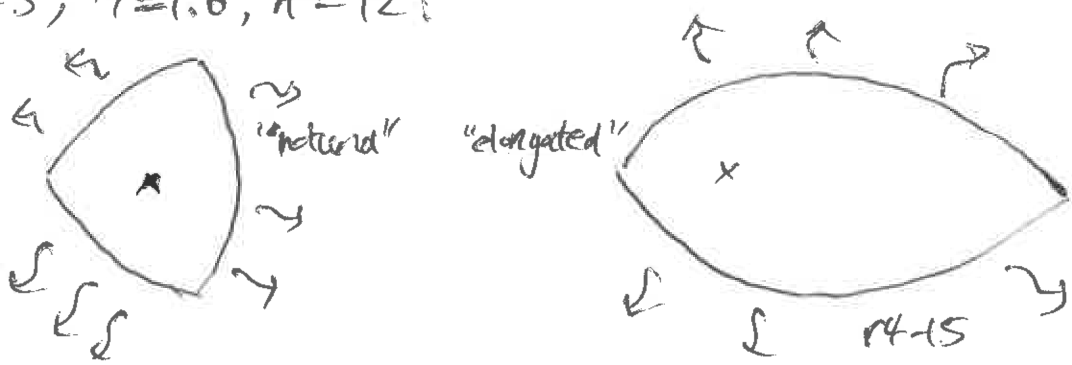
Thus in the present $\gamma > 0$ case, the points $\rho = 1$, $\varphi = 2\pi k/n$, which are the vertices of the polygon in ζ-space, are <u>always</u> viable. Note that the physical domain is $\rho = |\zeta| \leq e^{\gamma}$ (previously it was only the unit disk).

For example, if $n=3$ and $\gamma = 1$, the non-viable moat pincers off into three non-viable lakes containing $\rho = 1$, $\varphi = 2\pi k/n$:



In cases where the lakes have pincered off, we label ρ_a, ρ_b and $\rho_\#$ the ζ-space radii for the ~~outer~~ moat's outer tip, the lake's near tip and the lake's far tip, along $\varphi = 0$. The points at $\rho = \rho_a, \rho_b, \rho_\#$ along $\varphi = 2\pi k/n$ are [redacted] hyperbolic critical terminal, but the traced boundaries from ρ_b and $\rho_\#$ either terminate or never join up. Using the traced boundaries from ρ_a, and carefully choosing γ and A, we may construct convex domains.

E.g. $n=3$, $\gamma = 1.6$, $A = 12$:



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