shuen-iii-26.pdf 20191117

A74 B162(157) C5222(3.26) D86

Vol. III 卷26

Now there be objects, and we know not their number. Counting them three by three, two be left over; counting them five by five, three be left over; and counting them seven by seven, two be left over.
We ask: how many objects be there?

答曰：閒賸數有二物三之，物十幾？七賸二，知數五之，五賸三，數三。

數：first 去聲 (number), all subsequent 上聲 (count)

賸二：two be left over; or there remain two
賸同剩

Answer saith: twenty-three.

With x being the number of objects, this is an archetypal Chinese Remainder Theorem problem

$$x \equiv 2 \pmod 3$$
$$x \equiv 3 \pmod 5$$
$$x \equiv 2 \pmod 7.$$

From modern number theory, since 3, 5 and 7 are pairwise coprime, the solution in x is unique modulo $N = 3 \times 5 \times 7 = 105$.
Seeking $y_3 \cdot 5 \cdot 7 \equiv 1 \pmod 3$, $y_5 \cdot 3 \cdot 7 \equiv 1 \pmod 5$ and $y_7 \cdot 3 \cdot 5 \equiv 1 \pmod 7$, we get $y_3 \cdot 5 \cdot 7 = 70$, $y_5 \cdot 3 \cdot 7 = 21$ and $y_7 \cdot 3 \cdot 5 = 15$,
thus $x \equiv 2(70) + 3(21) + 2(15) = 233 \equiv 23 \pmod{105}$.

Here 孫子 gives the quantities 70, 21 and 15 without explanation or derivation.

20191117

Method saith: for the counting them three by three, and two being left over, put down one hundred and forty; for the counting them five by five, and three being left over, put down sixty-three; and for the counting them seven by seven, and two being left over, put down thirty. Combining them, resulteth in two hundred two hundred and thirty-three. Diminishing it by two hundred and ten, we are done.

以三六百術
二十十四曰:
百并三十三
一之七五三
十得七五數
減二數數之,
之百之之,賸
即三賸賸一,
得。十二三置
三。置置一

Whenever counting them three by three, and one be left over, put down seventy; whenever counting them five by five, and one be left over, put down twenty-one; and whenever counting them seven by seven, and one be left over, put down fifteen. For one hundred and six or above, diminish it by one hundred and five, and we are done.

五十十十;凡
減五一;五三
之,一七五三
即百七數數
得。六數之,之,
以之賸賸,
上,賸一,一,
以一則則
一則置置
百置二七

END 4126