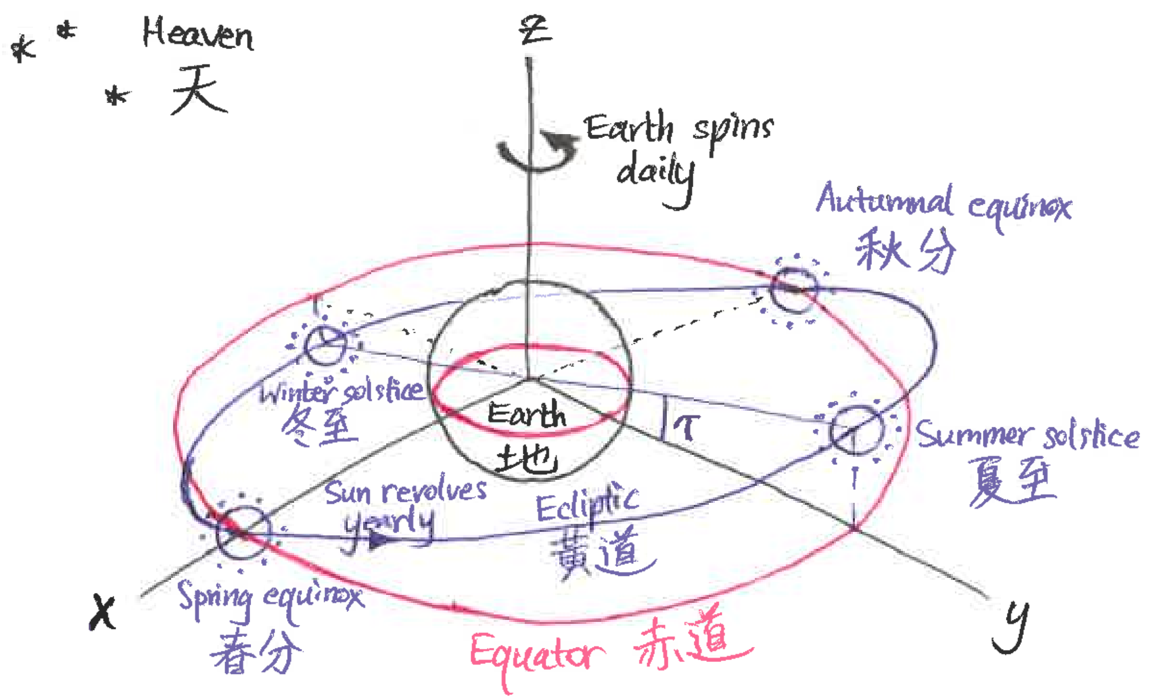
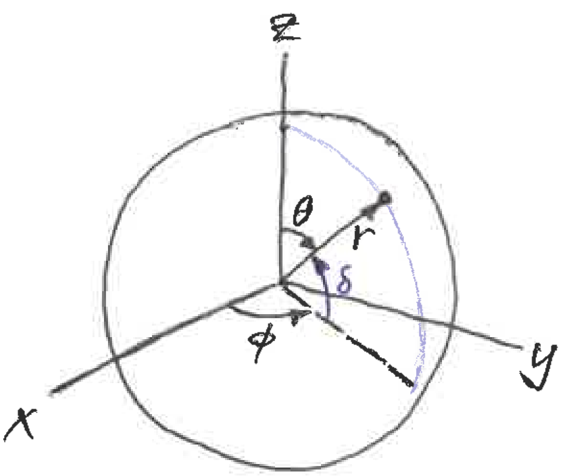
Conway daytime.pdf

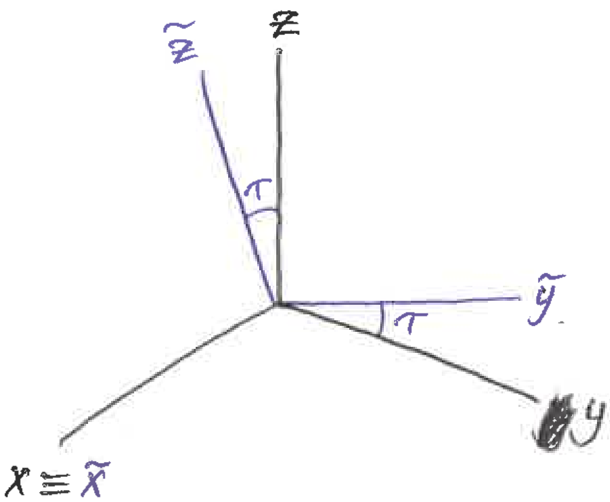
Manuscript for $\boxed{\text{Daytime: dependence on latitude and season}}$

Geometry

Equatorial coordinates: (x, y, z), (r, θ, ϕ)







Ecliptic coordinates: $(\tilde{x}, \tilde{y}, \tilde{z})$

Equatorial

Cartesian (x, y, z)

Spherical (r, θ, ϕ)

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

Note

$\phi = \alpha$, right ascension α

$\theta = \frac{\pi}{2} - \delta$, declination δ.

Equator (赤道): $r = \infty$, $\theta = \frac{\pi}{2}$ in the xy-plane.

Earth spins daily about z-axis.

Ecliptic

Cartesian $(\tilde{x}, \tilde{y}, \tilde{z})$

$$x = \tilde{x}$$
$$y = \tilde{y} \cos\tau - \tilde{z} \sin\tau$$
$$z = \tilde{y} \sin\tau + \tilde{z} \cos\tau$$

Ecliptic (黄道): path of the sun lies in the $\tilde{x}\tilde{y}$-plane, tilted at tilt τ from the xy-plane.

Sun orbits yearly about $\tilde{z}$-axis. $\tau = 23°26'$.

Let t be the time since northern spring (or March) equinox, 春分; in a solar year

$$yr = 365.242 \text{ day}$$

the sun goes once around the ecliptic (relative to the fixed stars), so the sun has position

$$(\tilde{x}, \tilde{y}, \tilde{z})_{sun} = \left(R_{es} \cos\frac{2\pi t}{yr},\ R_{es} \sin\frac{2\pi t}{yr},\ 0 \right).$$

In a day, the earth spins once on its own axis (relative to the fixed stars), so an observer at latitude δ north of the equator has position

$$(r, \theta, \phi)_{obs} = \left(R_e, \frac{\pi}{2} - \delta, \frac{2\pi t}{day}\right).$$

Actually, it should technically be $\phi_{obs} = \frac{2\pi t}{sd}$, where sd is a sidereal day, but since day $\ll$ yr, I shall be treating the sun as stationary over the course of one day (which is a zeroth-order approximation), and relative to the sun, the earth spins once on its own axis every solar day. Thus I have put

$$\phi_{obs} = \frac{2\pi t}{day},$$

where day $= 24$ hr $= 24 \times 60^2$ s. Trust me, I know what I'm doing.

Define day angle and year angle

$$D = \frac{2\pi t}{day}$$

$$Y = \frac{2\pi t}{yr}$$

(both modulo 2π). D runs through one turn (2π) each day, and Y each year. Note: $Y = 0, 90°, 180°, 270°$ correspond to 春分、夏至、秋分、冬至.

Thus

$$(\tilde{x}, \tilde{y}, \tilde{z})_{sun} = (R_{es} \cos Y, R_{es} \sin Y, 0)$$

$$(r, \theta, \phi)_{obs} = \left(R_e, \frac{\pi}{2} - \delta, D\right).$$

Now

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix}_{sun} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos T & -\sin T \\ 0 & \sin T & \cos T \end{pmatrix} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix}_{sun}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos T & -\sin T \\ 0 & \sin T & \cos T \end{pmatrix} R_{es} \begin{pmatrix} \cos Y \\ \sin Y \\ 0 \end{pmatrix}$$

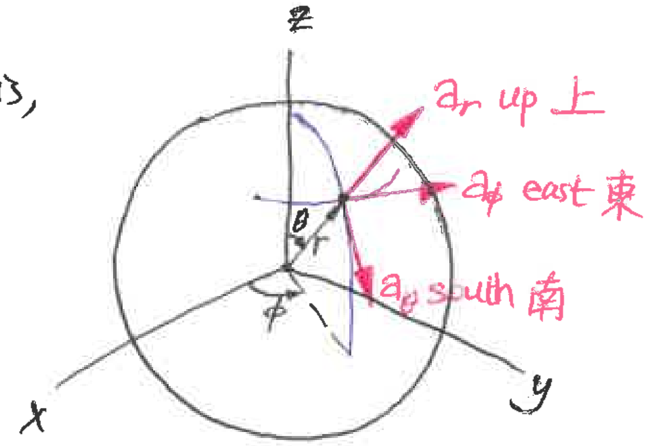
$$= R_{es} \begin{pmatrix} \cos Y \\ \cos T \sin Y \\ \sin T \sin Y \end{pmatrix}$$

Let $u = u_x a_x + u_y a_y + u_z a_z$ be the unit vector (direction) from the observer towards the sun. The observer lies at radius R_e from the origin; the sun at radius R_{es}. Since $R_e \ll R_{es}$, we simply have

$$u_x = \cos Y$$
$$u_y = \cos T \sin Y$$
$$u_z = \sin T \sin Y .$$

Transform to the local spherical basis, i.e. put

$$u = u_r a_r + u_\theta a_\theta + u_\phi a_\phi .$$



then

$$\begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & 0 \end{pmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$$

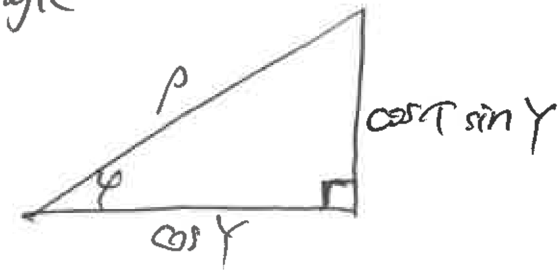
$$= \begin{pmatrix} \cos\delta\cos D & \cos\delta\sin D & \sin\delta \\ \sin\delta\cos D & \sin\delta\sin D & -\cos\delta \\ -\sin D & \cos D & 0 \end{pmatrix} \begin{pmatrix} \cos Y \\ \cos T \sin Y \\ \sin T \sin Y \end{pmatrix}$$

since $\theta_{obs} = \frac{\pi}{2} - \delta$ and $\phi_{obs} = D$.

Reduced quantities

Tilt-reduced unit radius and year angle

$$\rho = \sqrt{\cos^2 Y + \cos^2\tau \sin^2 Y}$$
$$\varphi = \tan^{-1}(\cos\tau \tan Y)$$



so that

$$\cos Y = \rho\cos\varphi$$
$$\cos\tau\sin Y = \rho\sin\varphi$$
$$\sin\tau\sin Y = \tan\tau\cdot\cos\tau\sin Y = \tan\tau\cdot\rho\sin\varphi \;.$$

Then

$$\begin{pmatrix} u_r \\ u_\theta \\ u_\phi \end{pmatrix} = \begin{pmatrix} \cos\delta\cos D & \cos\delta\sin D & \sin\delta \\ \sin\delta\cos D & \sin\delta\sin D & -\cos\delta \\ -\sin D & \cos D & 0 \end{pmatrix} \rho \begin{pmatrix} \cos\varphi \\ \sin\varphi \\ \tan\tau\sin\varphi \end{pmatrix}$$

$$= \rho\begin{pmatrix} \cos\delta[\cos D\cos\varphi + \sin D\sin\varphi] + \tan\tau\sin\delta\sin\varphi \\ \sin\delta[\cos D\cos\varphi + \sin D\sin\varphi] - \tan\tau\cos\delta\sin\varphi \\ -\sin D\cos\varphi + \cos D\sin\varphi \end{pmatrix}$$

$$= \rho\begin{pmatrix} \cos\delta\cos(D-\varphi) + \tan\tau\sin\delta\sin\varphi \\ \sin\delta\cos(D-\varphi) - \tan\tau\cos\delta\sin\varphi \\ -\sin(D-\varphi) \end{pmatrix} . \quad \begin{matrix} \text{(up)} \\ \text{(south)} \\ \text{(east)} \end{matrix}$$

Since day $\ll$ yr, ρ and φ are effectively constant over one day.
Thus the components are sinusoidal with period one day.
u_r maximized at $D=\varphi$. Probably $\tan^{-1}(u_r/\sqrt{u_\theta^2+u_\phi^2})$ too,
corresponding to sun being highest, or solar noon (or high noon).
Also $u_\phi = 0$ at $D=\varphi$. This makes sense.

Note: solar noon almost never at 12 pm.
(Tilt, orbit not circular, time zones)

Daytime

Sunrise & sunset occur when sun passes through horizon:

$$u_r = 0$$

or

$$\boxed{\cos\delta \cos(D-\varphi) + \tan\tau \sin\delta \sin\varphi = 0}.$$

$$\cos(D-\varphi) = -\tan\tau \tan\delta \sin\varphi$$

$$D-\varphi = \pm\cos^{-1}(-\tan\tau \tan\delta \sin\varphi)$$

$$= \pm\left\{\frac{\pi}{2} + \sin^{-1}(\tan\tau \tan\delta \sin\varphi)\right\}$$

$$D = D_\pm = \varphi \pm \left\{\frac{\pi}{2} + \sin^{-1}(\tan\tau \tan\delta \sin\varphi)\right\}.$$

$D = D_+$ corresponds to sunset; D_- sunrise.

$D = \varphi$ is noon, halfway between.

Thus the amount of day angle between sunrise & sunset is

$$D_+ - D_- = \pi + 2\sin^{-1}(\tan\tau \tan\delta \sin\varphi),$$

so the amount of day time is

$$T = t_+ - t_- = \frac{D_+ - D_-}{2\pi} \cdot \text{day}$$

$$= \left\{\frac{1}{2} + \frac{1}{\pi}\sin^{-1}(\tan\tau \tan\delta \sin\varphi)\right\} \text{day}$$

$$= 12\,\text{hr} + \frac{24\,\text{hr}}{\pi}\sin^{-1}(\tan\tau \tan\delta \sin\varphi).$$

Result

Daytime is

$$\mathrm{Re}\left\{\frac{1}{2} + \frac{1}{\pi}\sin^{-1}(\tan\tau \tan\delta \sin\varphi)\right\} \text{day}.$$

Note: argument to arcsine will exceed unity at some point if

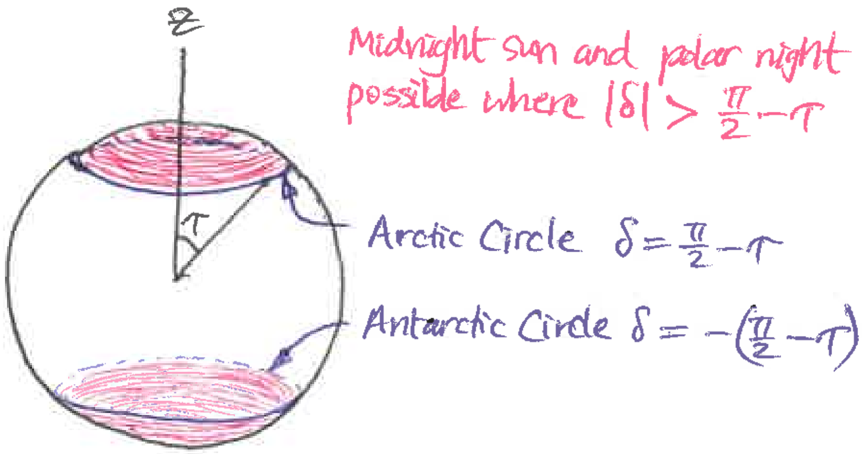
$$|\tan\tau \tan\delta| > 1$$

$$\tan|\delta| > \cot\tau$$

$$= \tan\left(\frac{\pi}{2} - \tau\right)$$

$$|\delta| > \frac{\pi}{2} - \tau \, .$$

In this case there is no real solution to the sunrise/sunset equation, corresponding to midnight sun and polar night, north of the Arctic Circle and south of the Antarctic Circle.



However, the real part of T still gives the correct daytime, since

$$\mathrm{Re}(\sin^{-1} w) = \begin{cases} +\pi/2, & w > 1 \\ -\pi/2, & w < -1 \end{cases}$$

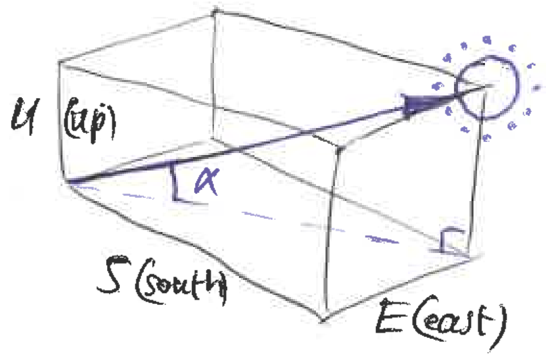
so we get

$$\mathrm{Re}(T) = 24 \text{ hr} \quad \text{for midnight sun},$$
$$\mathrm{Re}(T) = 0 \text{ hr} \quad \text{for polar night}.$$

Assorted quantities

Let

$$U = \cos\delta\cos(D-\varphi) + \tan\tau\sin\delta\sin\varphi$$
$$S = \sin\delta\cos(D-\varphi) - \tan\tau\cos\delta\sin\varphi$$
$$E = -\sin(D-\varphi).$$



Sun's elevation angle (or altitude)

$$\alpha = \tan^{-1}\frac{U}{\sqrt{S^2+E^2}}$$

Sun's bearing (clockwise from north)

$$\beta = \tan^{-1}\frac{E}{-S} \quad \text{(up to quadrant identification)}$$

Shadow length of a vertical pole of height h

$$\ell/h = \frac{\sqrt{S^2+E^2}}{U}$$

$$\ell = \frac{\sqrt{S^2+E^2}}{U}\cdot h.$$

Noon is at $D=\varphi$

Sunrise/sunset are at

$$D = D_{\pm} = \varphi \pm \left\{\frac{\pi}{2} + \sin^{-1}(\tan\tau\tan\delta\sin\varphi)\right\}.$$

Noon elevation & shadow length

$$\boxed{D=\varphi}$$

$$U = \cos\delta + \tan\tau \sin\delta \sin\varphi$$

$$S = \sin\delta - \tan\tau \cos\delta \sin\varphi$$

$$E = 0$$

$$\alpha = \tan^{-1}\frac{U}{|S|} = \tan^{-1}\frac{\cos\delta + \tan\tau\sin\delta\sin\varphi}{|\sin\delta - \tan\tau\cos\delta\sin\varphi|}$$

$$l = \frac{|S|}{U}\cdot h = \frac{|\sin\delta - \tan\tau\cos\delta\sin\varphi|}{\cos\delta + \tan\tau\sin\delta\sin\varphi}\cdot h .$$

Sunrise & sunset bearing

$$\boxed{D-\varphi = \pm\cos^{-1}(-\tan\tau\tan\delta\sin\varphi)}$$

$$U = 0$$

$$\begin{aligned} S &= \sin\delta[-\tan\tau\tan\delta\sin\varphi] - \tan\tau\cos\delta\sin\varphi \\ &= -[\sin\delta\tan\delta + \cos\delta]\tan\tau\sin\varphi \\ &= -\left(\frac{\sin^2\delta}{\cos\delta} + \frac{\cos^2\delta}{\cos\delta}\right)\tan\tau\sin\varphi \\ &= -\tan\tau\sec\delta\sin\varphi \end{aligned}$$

$$\begin{aligned} E &= -\sin\left\{\pm\cos^{-1}(-\tan\tau\tan\delta\sin\varphi)\right\} \\ &= \mp\sin\cos^{-1}(-\tan\tau\tan\delta\sin\varphi) \\ &= \mp\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi} \end{aligned}$$

$$\beta = \tan^{-1}\frac{E}{-S} = \tan^{-1}\frac{\mp\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi}}{\tan\tau\sec\delta\sin\varphi}$$

This goes imaginary for midnight sun and polar night, i.e. when $|\tan\tau\tan\delta\sin\varphi| > 1$.

Verification

At the time of writing it is Sunday 7 Jul 2019.
Spring equinox fell on Thursday 21 Mar 2019, 05:58 (UTC+8:00).
(Really I should use 20 Mar for places 6 hours or more west of Perth/HK in time zones, but I can't be bothered. Year angle error will not exceed (1/365.242 = 0.3%.) We have

(or even better, account for the time of the equinox)

$$t = [7\text{ Jul}] - [21\text{ Mar}]$$
$$= ([7\text{ Jul}] - [7\text{ Jun}]) + ([7\text{ Jun}] - [7\text{ May}]) + ([7\text{ May}] - [7\text{ Apr}]) + ([7\text{ Apr}] - [7\text{ Mar}]) + ([7\text{ Mar}] - [21\text{ Mar}])$$
$$= (30\text{ day}) + (31\text{ day}) + (30\text{ day}) + (31\text{ day}) + (7-21)\text{ day}$$
$$= 108\text{ day}$$

$$\Upsilon = \frac{2\pi t}{\text{yr}} = \frac{2\pi \cdot 108}{365.242} = 106.45°$$

$$\varphi = \tan^{-1}(\cos\tau \tan\Upsilon)$$
$$= \tan^{-1}(\cos 23.44° \tan 106.45°)$$
$$= 107.84°.$$

We compare the results above with data from timeanddate.com:

(Note $\tau = 23°26'$.)

Place	δ	Re{T}	noon α	rise β	set β
Station Nord	+81°36'	24 hr 24 hr nil	30.8° 31.0° −0.2°	N/A	
Reykjavik	+64°9'	19 hr 47' 20 hr 26'51" −40' −3.2%	48.3° 48.4° −0.1°	29.0° 24° +5°	331.0° 335° −4°
London	+51°30'	16 hr 10' 16 hr 25'57" −16' −1.6%	60.9° 61.1° −0.2°	52.2° 51° +1°	307.8° 309° −1°
Hong Kong	+22°19'	13 hr 18' 13 hr 26'41" −9' −1.1%	89.9° 89.7° +0.2°	65.7° 65° +1°	294.3° 295° −1°
Singapore	+1°17'	12 hr 4' 12 hr 11'31" −8' −1.0%	68.9° 68.7° +0.2°	67.6° 67° +1°	292.4° 293° −1°
Perth	−31°57'	10 hr 1' 10 hr 8'18" −7' −1.2%	35.6° 35.5° +0.1°	63.3° 64° −1°	296.7° 296° +1°
@663063 Enderby Land	−67°30'	41' 2 hr 3'28" −1 hr 22' −67%	0.1° 0.4° −0.3°	4.7° 14° −9°	355.3° 346° +9°
Concordia Station	−75°6'	0 hr 0 hr nil	−7.5° −7.7° +0.2°	N/A	

Entries are Predicted
Actual (timeanddate.com)
Error

Redone on Page 15

Not bad for something derived ~~ex~~ with just pen & paper.

~~[illegible]~~

Bulk of error caused by refraction, some by the sun not being a point. See

https://www.hko.gov.hk/m/article_e.htm?title=ele_00493
/gtt/astron2017/2017simp-paths-sun.pdf.

At actual sunrise/sunset, the sun's altitude is $\sim 50'$ below the horizon; roughly $16'$ for its nonzero apparent radius and $34'$ for refraction (seems to have been pulled off wikipedia).

We can do a crude correction for ~~ex~~ refraction by seeking the time at which the sun's altitude dips to $\alpha = -50'$.

In general recall

$$\alpha = \tan^{-1} \frac{U}{\sqrt{S^2 + E^2}}$$

where

$$U = \cos\delta \cos(D - \varphi) + \tan\tau \sin\delta \sin\varphi$$
$$S = \sin\delta \cos(D - \varphi) - \tan\tau \cos\delta \sin\varphi$$
$$E = \quad -\sin(D - \varphi),$$

and sunrise/sunset occur at

$$D = D_{\pm} = \varphi \pm \cos^{-1}(-\tan\tau \tan\delta \sin\varphi),$$

for which $U = 0$.

Now let $\alpha_r = 0°50'$ (altitude discrepancy caused by refraction).

We seek $D = D_\pm + \varepsilon_\pm$ for which

$$\frac{U}{\sqrt{S^2+E^2}} = -\tan\alpha_r \,, \quad \text{(which is close enough to } \alpha_r\text{)}$$

Assuming $\varepsilon_\pm \ll 2\pi$,

$$-\tan\alpha_r = \left.\frac{U}{\sqrt{S^2+E^2}}\right|_{D=D_\pm} + \varepsilon_\pm \cdot \frac{d}{dD} \left.\frac{U}{\sqrt{S^2+E^2}}\right|_{D=D_\pm}$$

$$= 0 + \varepsilon_\pm \left\{ U \cdot \frac{d}{dD}\frac{1}{\sqrt{S^2+E^2}} + \frac{dU/dD}{\sqrt{S^2+E^2}} \right\}\Bigg|_{D=D_\pm}$$

$$= \varepsilon_\pm \left\{ 0 + \left.\frac{-\cos\delta \sin(D-\varphi)}{\sqrt{S^2+E^2}}\right|_{D=D_\pm} \right\}$$

$$\tan\alpha_r = \varepsilon_\pm \cdot \left.\frac{\cos\delta \sin(D-\varphi)}{\sqrt{S^2+E^2}}\right|_{D=D_\pm}$$

$$= \varepsilon_\pm \cdot \frac{\cos\delta\left(\pm\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi}\right)}{\sqrt{\tan^2\tau\sec^2\delta\sin^2\varphi + 1 - \tan^2\tau\tan^2\delta\sin^2\varphi}}$$

(see bottom half of Page 9)

$$= \pm\varepsilon_\pm \cdot \frac{\cos\delta\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi}}{\sqrt{1+\tan^2\tau\left[\sec^2\delta - \tan^2\delta\right]\sin^2\varphi}}$$

$$= \pm\varepsilon_\pm \cdot \frac{\cos\delta\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi}}{\sqrt{1+\tan^2\tau\sin^2\varphi}}$$

$$\sec^2\delta - \tan^2\delta = \frac{1-\sin^2\delta}{\cos^2\delta} = 1$$

Thus

$$\varepsilon_\pm = \pm\frac{\tan\alpha_r\sqrt{1+\tan^2\tau\sin^2\varphi}}{\cos\delta\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi}}$$

and the amount of daytime is to be corrected by

$$\frac{\varepsilon_+ - \varepsilon_-}{2\pi}\,\text{day} = \frac{\tan\alpha_r\sqrt{1+\tan^2\tau\sin^2\varphi}}{\pi\cos\delta\sqrt{1-\tan^2\tau\tan^2\delta\sin^2\varphi}}\,\text{day} \,.$$

~~Check the Real parts for it to work~~

With effort, an expression for the correction to the sunrise/sunset bearings can be obtained, but it is much easier to evaluate the bearing at the corrected times, i.e.

$$\beta\Big|_{D=D_\pm \pm \varepsilon_\pm} = \tan^{-1}\frac{E}{-S}\Big|_{D_\pm+\varepsilon_\pm}.$$

(daytimes within 1%)

We see (Page 15) that the correction improves things quite well, with the exception of Enderby Land, probably because the sun is near horizon level for ~~an extended~~ a large proportion of the day.

16 Jun to 26 Jun   near 7 Jul   1 Dec to 12 Jan (2020)

Note that refraction correction does not correct for the earlier ending of polar night or the late ending of midnight sun.
Enderby Land has 11 days of polar night & 43 days of midnight sun.

Eg. Station Nord midnight sun duration:

$$\tan\tau \tan\delta \sin\varphi > 1$$

$$\sin\varphi > \cot\tau \cot\delta = \cot 23°26' \cot 81°36' = 0.3407$$

$$19.92° < \varphi < 160.08°$$

$$21.55° < Y < 158.45°$$

$\varphi = \tan^{-1}(\cos\tau \tan Y)$

$Y = \tan^{-1}(\sec\tau \tan\varphi)$

so Y spans 136.9°,
t spans 139 days.

$Y = \frac{2\pi t}{yr}$

$t = \frac{Y}{2\pi} yr = \frac{Y}{360°} \cdot 365.242$ day.

Actual: 10 Apr to 3 Sep incl, (2019)
147 days.

$\frac{139}{147} - 1 = -5\%.$

Entries are Actual
Predicted, no correction (error)
Predicted, with correction (error)

Place (δ)	Re{T}	noon α	rise β	set β
Station Nord (+81°36′)	24 hr 24 hr 24 hr	31.0° 30.8° (−0.2°)	N/A	
Reykjavik (+64° 9′)	20hr 26′ 51″ 19 hr 47′, (−3.2%) 20hr 19′ (−0.6%)	48.4° 48.3° (−0.1°)	24° 29.0° (+5°) 25.4° (+1°)	335° 331.0° (−4°) 334.6° (−0.°)
London (+51°30′)	~~[illegible]~~ 16hr 25′ 57″ 16hr 10′, (−1.6%) 16hr 24′ (−0.2%)	61.1° 60.9° (−0.2°)	51° 52.2° (+1°) 50.9° (−0.°)	309° 307.8° (−1°) 309.1° (+0.°)
Hong Kong (+22°19′)	13hr 26′ 41″ 13hr 18′, (−1.1%) 13hr 26′ (−0.1%)	89.7° 89.9° (+0.2°)	65° 65.7° (+1°) 65.3° (+0.°)	295° 294.3° (−1°) 294.7° (−0.°)
Singapore (+1°17′)	12hr 11′ 31″ 12hr 4′ (−1.0%) 12hr 11′ (−0.1%)	68.7° 68.9° (+0.2°)	67° 67.6° (+1°) 67.6° (+1°)	293° 292.4° (−1°) 292.4° (−1°)
Perth (−31°57′)	10hr 8′ 18″ 10 hr 1′ (−1.2%) 10hr 10′ (+0.3%)	35.5° 35.6° (+0.1°)	64° 63.3° (−1°) 63.9° (−0.°)	296° 296.7° (+1°) 296.1° (+0.°)
@ 6630103 Enderby Land ~~[illegible]~~ (−67°30′)	2 hr 3′ 28″ 41′ (−67%) 4hr 14′ (+106%)	0.4° 0.1° (−0.3°)	14° 4.7° (−9°) 29.1° (+15°)	346° 355.3° (+9°) 330.9° (−15°)
Concordia Station (−75°6′)	0 hr 0 hr 0 hr	−7.7° −7.5° (+0.2°)	N/A	

Eg. Enderby land midnight sun duration:

$$\tan\tau \tan\delta \sin\varphi > 1$$

$$\sin\varphi < \cot\tau \cot\delta$$

$$= \cot 23°26' \cot(-67°30')$$

$$= -0.9557$$

$$-107.12° < \varphi < -72.88°$$

$$-105.70° < \Upsilon < -74.30°$$

so Υ spans $31.4°$

t spans 32 days.

Actual: 43 days. (1 Dec, 2019 to 12 Jan, 2020) incl.

$\frac{32}{43} - 1 = -26\%$.

Concordia Station midnight sun duration

$$\sin\varphi < \cot\tau \cot\delta$$

$$= \cot 23°26' \cot(-75°6')$$

$$= -0.6139$$

$$-142.13° < \varphi < -37.87°$$

$$-139.72° < \Upsilon < -40.28°$$

So Υ spans $99.44°$

t spans 101 days

Actual 1 Nov, 2019 to 11 Feb, 2020 incl.

103 days.

$\frac{101}{103} - 1 = -2\%$.